ASTRONOMICAL

TABLES AND FORMULÆ.

ASTRONOMICAL

TABLES AND FORMULÆ

TOGETHER WITH A VARIETY OF

PROBLEMS

EXPLANATORY OF THEIR USE AND APPLICATION.

TO WHICH ARE PREFIXED

THE

ELEMENTS OF THE SOLAR SYSTEM.

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PREFACE.

THE present collection of TABLES and FORMULÆ is a part only of a more numerous collection which I had formed and originally designed for my own use, with a view to save time and trouble in the various astronomical computations and researches in which I have occasionally indulged; but without any view to their publication.

I found that many valuable *Tables*, of almost daily use in an active observatory, and which have been published from time to time by various authors, were to be met with only in works printed on the Continent, of considerable expense and not easily accessible; and that others were so scattered about in different publications, as to make it very troublesome to refer to them at the moment they were wanted. The whole of the Tables which I have here selected from such works, have been either re-computed, or carefully examined by means of differences: the rest are entirely new.

In the selection of the *Formulæ,* I have been guided more by their fitness for computation, than by their elegance: and have always adopted those which, after repeated trials, I have found to lead to the *practical* solution of the problem with the least expense of time and labour. Most of these Formulæ will be found in the works of other writers: but, it is too well known that in referring to different authors for a Formula, the reader is frequently distracted with a confusion of symbols, the values of which can only be obtained by a reference to other parts of the work: and, when obtained, may be found to be denoted differently by the authors whose writings form the subject of comparison. In order to prevent any confusion of this kind, I have changed the characters, used in the original Formulæ, into such as will, in most cases, more readily denote the quantities used in the solution of the problem. And, to remove every possibility of mistake or confusion on this point, I have inserted, at the bottom of each page, the quantities which are denoted by every symbol made use of in the given Formula.

The *Elements of the System* are taken, for the most part, from the *Système du Monde* of M. Laplace (5th edition, 1824): but with additions from other modern authors who have made certain branches of the science their more particular study. I have frequently experienced the want of a synopsis of this kind; where all the different facts relative to astronomy are brought under their respective heads, without the necessity of turning to a variety of works for information. Much time is frequently employed, and oftentimes wholly lost, in a research of this kind, which it is the object of the present abstract to prevent.

At the end of the work I have inserted a set of *Problems* with a view to introduce a few Examples of the use and application of some of the Formulæ and Tables, which may appear to be more in need of a practical explanation. And here it may be proper to allude to a new method of employing the *signs*, when annexed to the logarithms of the quantities under computation: a method which will be found to be very convenient and useful in the arithmetical solution of algebraical formulæ. The sign therefore prefixed to a logarithm in this work, is intended to affect only the *natural number* of such logarithm: for, in all cases, the logarithms themselves are to be added together or subtracted from each other (according to the conditions of the problem) exactly the same as if no such signs were annexed. With respect to the signs themselves, it must be observed that the addition of two *like* signs produces a *positive* natural number, and the addition of two *unlike* signs produces a *negative* natural number. Thus, in page 227, the natural number of the first sum of logarithms is *plus*, because the two negative signs produce a positive result: on the contrary, the natural number of the second sum in that page is *minus*, because there is only one negative sign. In complex trigonometrical computations the method is still more convenient and concise, and renders the result much less ambiguous; as will appear from the arithmetical operations in page 263.

Upon the whole, it is hoped that the present work will not be without its utility. I am aware that it might have been rendered much more extensive: but it would then have lost many advantages which attach to a small and portable volume; such as may be convenient to the intelligent and scientific traveller, and not the less useful in the observatory. Those who are desirous of possessing a more comprehensive set of Astronomical Tables, and more numerous and valuable than any that have ever yet appeared in this country, will procure the first volume of *Practical Astronomy* recently published by the Rev. Dr. Pearson: where they will find many Tables that do not come within the design and intention of the present work. For, I have in all cases avoided such Tables as are necessarily formed on the principle of *double entry*: and likewise those which are merely *local*; such as Tables of *parallax* &c. When problems involving quantities of this kind come before the computer, he must either have recourse to the Formulæ, or refer to such larger collections of Tables. The present work is intended as a *Manual* only: and aspires to no further merit than accuracy, utility and convenience may fairly lay claim to.

I have remarked in a former work that, in all astronomical calculations, in which the sexagesimal notation is so much involved, the computer will find considerable assistance in the use of Callet's *Tables portatives de logarithmes*: since, by the help of two additional collateral columns given in that work, the computation of any quantities composed of *sexagesimals* becomes, without any reduction, as easy and familiar as those which are formed according to the usual *decimal* notation. I am now happy in being able to state that a new *Table of Logarithms* of the natural numbers is about to appear in *this* country, agreeably to the above arrangement; with the additional advantage and convenience of having a mark attached to the *last* figure when it exceeds 5: whereby the table will be rendered more extensive and correct. This valuable addition to our list of mathematical Tables was suggested by Lieutenant Colonel Colby, for the use of the *Trigonometrical Survey;* and was immediately adopted and put in execution by Mr. Babbage: who has, with great care and diligence, examined and compared all the preceding works on this subject: and under whose able and active superintendence the work is now printing, in stereotype and on *coloured* paper.

January 1, 1827.

*** The reader is requested to make the corrections, pointed out in the list of *Errata* at the end, previous to a perusal of the work. And the Author will be obliged by the communication of any other errors that may have escaped his detection.

TABLE OF CONTENTS.

ELEMENTS OF THE SYSTEM

Sun .... *page* 3
Planets 6
Mercury 9
Venus 13
Earth 16
Mars 28
Vesta 31
Juno 32
Ceres 33
Pallas 34
Jupiter 35
Saturn 38
Uranus 41
Satellites 43
Moon 44
Satellites of Jupiter 55
Satellites of Saturn 58
Ring of Saturn 59
Satellites of Uranus 61
Recapitulation 62

FORMULÆ

1 Equivalent expressions for sin x . . . . *page* 71
2 ———————— cos x 72
3 ———————— tan x 73
4 Relative to two arcs A and B 74
5 Differences of trigonometrical lines 76
6 Differentials of do. do. 76
7 General analytical expressions for the sides and angles of any spherical triangle 77
8 Solutions of the cases of *right*-angled spherical triangles 78
9 ———————— of *oblique*-angled do. 79
10 Trigonometrical series 84
11 Multiple arcs 85
12 For computing the Longitude, Right Ascension and Declination of the *sun*; and the Angle of Position . 86
13 For computing the Longitude and Latitude, Right Ascension and Declination of the *moon or a star*; and the Angle of Position 87
14 For computing the Altitude and Azimuth of a heavenly body and its Angle of Variation 88
15 For computing the time &c, from *single altitudes* 89
16 For computing the time &c, from observations near the *prime vertical* 90
17 For computing the effect of atmospheric refraction . . . 91
18 For computing the *equation of equal altitudes* 92
19 For computing the correction for the Reduction to the Meridian 93
20 For computing the correction for the Reduction to the Solstice 94
21 For computing the Angle of the Vertical 95
22 For computing the horizontal Parallax of the moon at any given latitude 95
23 For computing the Moon's parallax in Altitude 96
24 For computing the longitude and altitude of the Nonagesimal 97

FORMULÆ
25 For computing the Moon's parallax in Longitude and Latitude .... *page* 98
26 For computing the Moon's parallax in Right Ascension 99
27 For computing the Moon's parallax in Declination .. 100
28 For computing the augmentation of the Moon's semidiameter 101
29 For computing the apparent distance between the centre of the moon and the sun or a star 102
30 For computing the place of the moon by means of differences 103
31 For computing the annual precession of the Equinoxes and the mean Obliquity of the ecliptic 104
32 For computing the Nutation of the obliquity of the Ecliptic, and of the Longitude 105
33 For computing the lunar and solar *nutation* of a star 106
34 For computing the *aberration* of a star 107
35 For computing the corrections to observations with a transit instrument 108
36 For computing the co-efficients used in the preceding formula 109
37 For computing the latitude of a place, from observations of the Pole star, at any time of the day 110
38 For computing the height of mountains with the barometer 111
39 For computing the increase of gravity, and the compression of the earth from the lengths of two *isochronous* pendulums 112
40 For computing the increase of gravity, and the compression of the earth from the vibrations of two *invariable* pendulums 113
41 For computing the corrections to be applied to an invariable pendulum, from various causes 114
42 For computing the principal geodetical quantities depending on the spheroidical figure of the earth .. 115

Formulæ

43 For computing the length of a degree of longitude and latitude, on the earth considered as a spheroid *page* 116
44 For computing the compression of the earth, from two measured arcs of the meridian 117
45 For computing the equations depending on the theory of the elliptical motion of the planets 118
46 For computing the greatest equation of the centre, the eccentricity of the orbit, and the eccentric anomaly 119
47 For computing the angular distance between the moon, and the sun or a star 120
48 For determining the *true* lunar distance 119*
49 The rule of the signs, in algebraic expressions of the circular functions 120*

Tables

1 Latitude and Longitude of various Observatories .. 123
2 Mean Æ of the sun, for every day of the year..... 125
3 —— Corrections for every year from 1800 to 1900 129
4 —— Correction for the Lunar Nutation 130
5 —— Correction for the Solar Nutation 131
6 For converting sidereal into mean solar time...... 132
7 For converting mean solar into sidereal time 133
8 For converting degrees into time; and *vice versa* .. 134
9 Mr. Ivory's Refractions.......................... 138
10 —— Corrections for thermometer and barometer.. 141
11 —— Corrections for low altitudes 142
12 Dr. Brinkley's Refractions 143
13 —— Corrections for low altitudes 144
14 Parallax of the sun, for the 1st day of every month . 145
15 Logarithms of $\sin^2 \frac{1}{2} P$........................ 146
16 Logarithms for the equation of equal altitudes 147
17 Altitude of a star, when on the prime vertical 153
18 For the reduction to the meridian 154

TABLES

19 For the reduction to the meridian, second part.. *page* 164
20 Mean obliquity of the ecliptic for every year from 1800 to 1900 165
21 Lunar Nutation of Longitude, and of Obliquity of the ecliptic 166
22 Solar Nutation of Longitude, and of Obliquity of the ecliptic 168
23 Selenographic positions of the principal lunar spots. 169
24 Angle of the Vertical, and Logarithm of the earth's radius 170
25 Augmentation of the moon's semidiameter 171
26 Logarithms to be used with the 1st, 2nd and 3rd differences of the moon's place 172
27 Annual precession of the equinoxes in Longitude; and the constants for Æ and D. 173
28 For aberration of a star in Æ, and *first* part of do. in Dec. 174
29 For aberration of a star in Dec. *second* part 175
30 For Lunar Nutation of a star in Æ and D. 176
31 For Solar Nutation of a star in Æ and D. 177
32 Circular arcs 178
33 Semidiurnal arcs 180
34 Length of a degree of Longitude and Latitude; and of the Pendulum 181
35 Expansion of various substances 182
36 For measuring heights by the barometer 183
37 For converting hours, minutes and seconds, into decimals of a day 184
38 For converting minutes and seconds into decimals of a degree 185
39 For converting any day into the decimal part of a year 186
40 For converting the centesimal division of the quadrant into the sexagesimal 188
41 For converting Fahrenheit's scale of the thermometer into Reaumur's and the Centesimal 191

Tables

42 Comparison of French and English measures .. *page* 192
43 Various logarithms used in astronomical calculations 193
44 For comparing French and English barometers 194
45 Depression of mercury in glass tubes............ 194*
Explanation of the Tables........................ 195

Problems

1 To convert sidereal time into mean solar time: and *vice versa* 217
2 To compute the Refraction of a heavenly body 220
3 To determine the time, from *single altitudes* of the sun, or a star 222
4 To determine the time, from *equal altitudes* of the sun 226
5 For the reduction to the meridian............... 228
6 To determine the latitude of a place............. 231
7 To determine the longitude of a place 236
8 To compute the obliquity of the ecliptic, from observations of the sun made near the solstice........ 247
9 To determine the place of the equinox, from observations of the sun made near that point 249
10 To compute the true place of the moon, by means of differences 250
11 To compute the parallax of the moon............ 252
12 To compute the aberration and nutation of a star .. 256
13 To determine the corrections for observations made with a transit instrument..................... 258
14 To compute a table of altitudes and azimuths 260
15 To compute the longitude and latitude of a heavenly body, from its right ascension and declination: and *vice versa*................................. 262
16 To determine the height of mountains, by means of the barometer 263
Appendix 265
Errata 301

ELEMENTS OF THE SYSTEM.

THE SUN.

THE sun, which is the source of light and heat to our system, is the most considerable of all the heavenly bodies; and governs all the planetary motions.

Its *mean distance* from the earth is 23984 times the semidiameter of the earth; or nearly 95 millions of miles.

Its *mean longitude*, at the commencement of the present century*, was in 280°. 39′. 10″,2: after subtracting 20″ for the effect of aberration.

His *mean motion* in the ecliptic (as it appears to a spectator on the earth) in 100 Julian years, or 36525 mean solar days, is, according to M. Damoiseau, 36000°.76472, or $100^{rev} + 0°. 45'. 53'',0$ †: whence we deduce his motion in a mean solar day to be 0°. 98564722 or 0°. 59′. 8″,32999; and consequently his mean motion in 365 days to be 359°. 7612354, or 359°. 45′. 40″,45.

The longitude of his *perigee*, at the commencement of the present century, was in 279°. 30′. 5″,0: and the line

* The epoch assumed in the following pages, as the *commencement of the present century*, is the moment of *mean noon* at Greenwich, on January 1, 1801; reckoning from the *mean* equinox.

† M. Lalande, in his solar tables, assumed this, in round numbers, equal to 46′. 0″. M. Delambre at first determined it to be 45′. 54″; but he afterwards adopted 45′. 45″: at the same time stating that he thought this latter value was too small. Baron Zach has assumed it equal to 45′. 48″.

of the apsides is subject to the same variation as that of the earth.

The *annual motion* of the sun in the ecliptic, and its *diurnal motion* from east to west, are in fact optical deceptions, arising from the real motion of the earth in its orbit and on its axis. These will be explained when we come to treat on the motions of the earth.

The *greatest equation of the centre*, as adopted by M. Delambre, is 1°. 55′. 26″,8: but M. Laplace has since proposed 1°. 55′. 27″,3. It diminishes at the rate of 17″,18 in a century.

The sun is surrounded by an *atmosphere*; and it is oftentimes obscured with *spots*. Some of these spots have been observed so large as to exceed the earth 4 or 5 times in magnitude: and they are generally confined within 33° of the solar equator.

The observation of these spots shows that the sun moves on its axis: and the duration of an entire sidereal *rotation* of the sun is about 25½ days. Whence we conclude that the sun is *flattened* at the poles.

The solar *axis* is inclined in an angle of 7°. 30′ to the axis of the ecliptic.

The mean horizontal *parallax* of the sun adopted by M. Laplace is 8″,66: but, as recently deduced by M. Encke from the transits of Venus in 1761 and 1769, it is 8″,5776: and this angle is the apparent semidiameter of the earth, as seen from the sun.

The *apparent diameter* of the sun, as seen from the earth, undergoes a periodical variation. It is greatest when the earth is in its perihelion; at which time it is 32′. 35″,6: and it is least when the earth is in its aphelion; at which time it is 31′. 31″,0. Its mean apparent diameter, or its diameter at its mean distance, is equal to 32′. 2″,9.

The *true diameter* of the sun is 111·454 times the mean diameter of the earth; or upwards of 882 thousand miles.

Whence its *volume* is 1384472 times greater than that of the earth.

Its *mass* is only 354936 times greater than that of the earth.

Whence we conclude that its *density* is $\frac{1}{3 \cdot 9326}$, or ·2543: which is about one quarter that of the earth.

A body which *weighs* one pound at the equator of the earth would, if removed to the equator of the sun, weigh 27·9 pounds. And bodies would fall there with a velocity of 334·65 feet in the first second of time.

The sun, and all the planets, move round the common centre of gravity of the system: which centre is nearly in the centre of the sun. This motion changes into epicycloids the ellipses of the planets and comets, which revolve round the sun.

The sun is supposed to have a particular motion, which carries our system towards the constellation of Hercules. But, this is doubted by M. Bessel.

THE PLANETS.

The number of planets belonging to our system is eleven. Six of these have been known and recognised from time immemorial: namely *Mercury*, *Venus*, the *Earth*, *Mars*, *Jupiter*, and *Saturn*. But, the remaining five, which are not visible to the naked eye, have lately been discovered by the help of the telescope; and are therefore called *telescopic planets:* namely,

Uranus, discov. by	Sir W. Herschel,	March 13, 1781.
Ceres, ...	M. Piazzi,	January 1, 1801.
Pallas, ...	M. Olbers,	March 28, 1802.
Juno, ...	M. Harding,	Septem. 1, 1804.
Vesta, ...	M. Olbers,	March 29, 1807.

All these planets revolve round the sun, as the centre of motion: and in performing their revolutions they follow the fundamental laws of planetary motion so happily discovered by Kepler; and which have been fully confirmed by subsequent observations. These laws are,

1°. The orbit of each planet is an ellipse; of which the sun occupies one of the foci.

2°. The areas described about the sun, by the radius vector of the planet, are proportional to the times employed in describing them.

3°. The squares of the times of the sidereal revolutions of the planets are to each other as the cubes of their mean distances.

The extremity of the major axis of the ellipse, nearest the sun, is called the *perihelion:* the opposite extremity of the same axis is called the *aphelion*. The line, which joins these two points, is called the line of the *apsides*.

The *anomaly* of a planet is its distance in degrees from the place of the perihelion.

The *radius vector* is an imaginary line drawn from the centre of the sun, to the centre of the planet, in any part of its orbit.

The *motion* of a planet in its orbit is always most rapid when in its perihelion. This velocity diminishes as the radius vector increases; till the planet arrives at its aphelion, when its motion is the slowest. It then increases, in an inverse manner, till the planet arrives again at its perihelion.

The first two laws of Kepler are sufficient for determining the motion of the planets round the sun: but, it is necessary to know, for each of these planets, seven quantities; which are called *the elements of their elliptical motions*. These are,

1°. The mean distance of the planet from the sun: or half the major axis of the orbit.

2°. The duration of a mean sidereal revolution of the planet.

3°. The mean longitude of the planet, at a given epoch.

4°. The longitude of the perihelion, at a given epoch.

5°. The inclination of the orbit to the ecliptic, at a given epoch.

6°. The longitude of the nodes, at a given epoch.

7°. The eccentricity of the orbit, described by the planet.

The ellipses, however, which the planets describe, are not unalterable. Their major axes and their mean motions in their orbits, appear to be always the same. But the position of their apsides, the inclination of their orbits to the ecliptic, the position of their nodes, and the amount of their eccentricities, seem to vary in a course of years. These inequalities, being sensible only in a series of ages, are called *secular variations.* There is no doubt of their existence: but the modern observations not being sufficiently extensive, and the ancient ones not sufficiently exact, there still rests some degree of uncertainty as to their magnitude.

There are indeed some inequalities which affect the *elliptical motion* of the planets. That of the earth, for instance, is a little altered. But, they are most sensible in Jupiter and Saturn: for, it appears that the duration of their mean sidereal revolution round the sun is subject to a periodical variation.

MERCURY.

MERCURY is the nearest planet to the sun: its *mean distance* being 0·3870981; that of the earth being considered as unity. This makes his mean distance above 36 millions of miles.

He performs his mean *sidereal revolution* in 87·9692580 mean solar days, or in $87^{d}.\ 23^{h}.\ 15^{m}.\ 43^{s},9$: and his mean *synodical revolution* in 115·877 mean solar days.

His *mean longitude*, at the commencement of the present century, was in $166^{\circ}.\ 0'.\ 48'',6$.

His *mean motion* in his orbit, in a mean solar day, is $4^{\circ}.09238$, or $4^{\circ}.\ 5'.\ 32'',6$. His mean motion in 365 days is consequently $1493^{\circ}.7175$ or $4^{rev} + 53^{\circ}.\ 43'.\ 3'',0$.

The longitude of the *perihelion* was, at the commencement of the present century, in $74^{\circ}.\ 21'.\ 46'',9$. The line of the apsides has a motion, according to the order of the signs, equal to $5'',84$ in a year: but, when referred to the ecliptic, this motion will (on account of the precession of the equinoctial points) be equal to $55'',9$ in a year. And it is in this manner that the value is assumed in the planetary tables.

His orbit is *inclined* to the plane of the ecliptic in an angle which, at the commencement of the present century, was $7^{\circ}.\ 0'.\ 9'',1$: and which angle is subject to a small increase of $0'',1818$ in a year.

His ascending *node* was, at the commencement of the present century, in $45^{\circ}.\ 57'.\ 30'',9$: having a motion, to the westward, every year, of $7'',82$. But, when referred to the ecliptic, the place of the node will (on account of the precession of the equinoxes) fall more to the eastward by $42'',3$ in a year. And it is in this manner that the value is assumed in the planetary tables.

The *eccentricity* of his orbit is 0·20551494; half the major axis being assumed equal to unity. This eccentricity is supposed to increase about ·000003866 in a century.

The *greatest equation of the centre,* deduced from this eccentricity, is $23°. 39'. 51''$; and subject to an increase of $1'',6$ in a century.

The *rotation* on his axis is accomplished in $24^h. 5^m. 28^s,3$.

The *inclination of its axis* to that of the ecliptic is not known.

His mean *apparent diameter*, or his apparent diameter at a distance equal to the mean distance of the earth from the sun, is $6'',9$: but, at the time of his superior conjunction it is only $5'',0$; whilst, at his inferior conjunction, it sometimes amounts to $12''$.

Mercury changes his *phases* like the moon, according to his various positions with regard to the sun and the earth: but this cannot be discovered without the aid of a powerful telescope.

His *true diameter*, compared with that of the earth considered as unity, is 0·398; which makes it about 3140 miles.

His *volume* is only 0·063, that of the earth being considered as unity.

His *mass*, compared with that of the sun considered as unity, is $\frac{1}{2025810}$ = ·0000004936.

A body, which *weighs* one pound at the equator of the earth, would, if removed to the equator of Mercury, weigh 1·03 pounds.

The proportion of *light and heat*, which it receives from the sun, is about 6·68 times greater than that received on the earth.

As seen from the earth, Mercury never appears at any great distance from the sun; either in the morning or the evening. His *elongation*, or angular distance, varies from 16°. 12′ to 28°. 48′.

His course sometimes appears *retrograde*. The arc which he describes in such cases varies from 9°. 22′ to 15°. 44′: its duration in the former case is 23½ days, and in the latter case 21½ days. This retrogradation commences when Mercury is at a distance from the sun, which varies from 15°. 24′ to 18°. 39′: and terminates when Mercury is at a distance which varies from 14°. 49′ to 20°. 51′.

Mercury is sometimes seen to *pass over the sun's disk:* which can happen only when he is in his nodes, and when the earth is in the same longitude. Consequently this phænomenon, for many centuries to come, can take place only in the months of May or November. The first observation of this kind was made by Gassendi in November 1631: since which period they have been frequent. The following is a list of all those which have happened since the above date inclusive, and of those that will happen till the end of the present century.

1631 Nov. 6	1690 Nov. 9
1644 Nov. 8	1697 Nov. 2
1651 Nov. 2	1707 May 5
1661 May 3	1710 Nov. 6
1664 Nov. 4	1723 Nov. 9
1674 May 6	1736 Nov. 10
1677 Nov. 7	1740 Nov. 2

1743	Nov. 4		1822	Nov. 4
1753	May 5	*	1832	May 5
1756	Nov. 6		1835	Nov. 7
1769	Nov. 9	*	1845	May 8
1776	Nov. 2	*	1848	Nov. 9
1782	Nov. 12	*	1861	Nov. 11
1786	May 3	*	1868	Nov. 4
1789	Nov. 5	*	1878	May 6
1799	May 7		1881	Nov. 7
1802	Nov. 8		1891	May 9
1815	Nov. 11		1894	Nov. 10

Those marked with an asterisk are such future ones as will be visible in this country.

VENUS.

The *mean distance* of Venus from the sun is 0·7233316; that of the earth being considered as unity. This makes her mean distance nearly 68 millions of miles.

She performs her mean *sidereal revolution* in 224·7007869 mean solar days, or in $224^d. 16^h. 49^m. 8^s,0$: and her mean *synodical revolution* in 583·920 mean solar days.

Her *mean longitude*, at the commencement of the present century, was in 11°. 33′. 3″,0.

The *mean motion* in her orbit, in a mean solar day, is 1°. 60217 or 1°. 36′. 7″,8. Her mean motion in 365 days is consequently 584°. 7916 or $1^{rev} + 224°. 47′. 30″,07$.

The longitude of her *perihelion* was, at the commencement of the present century, in 128°. 43′. 53″,1. The line of her apsides has a motion to the westward, of 2″,68 in a year: but, when referred to the ecliptic, this line will (on account of the precession of the equinoxes) appear to have a motion to the eastward, of 47″,4 in a year.

Her orbit is *inclined* to the plane of the ecliptic in an angle, which, at the commencement of the present century, was 3°. 23′. 28″,5: and which angle is subject to a small decrease of about 0″,0455 in a year.

Her ascending *node* was, at the commencement of the present century, in 74°. 54′. 12″,9: having a motion to the westward, every year, of 17″,6. But, when referred to the ecliptic, the place of the node will (on account of the precession of the equinoxes) fall more to the eastward by 32″,5 in a year.

The *eccentricity* of her orbit is 0·00686074; half the major axis being assumed equal to unity. This eccentricity is supposed to decrease about ·000062711 in a century.

The *greatest equation of the centre* is 0°. 47′. 15″; which is subject to an annual decrease of 0″,25.

The *rotation* on her axis is accomplished in $23^h. 21^m. 7^s,2$.

The *inclination of her axis*, to that of the ecliptic, is not exactly known.

Her mean *apparent diameter*, or her apparent diameter at a distance equal to the mean distance of the earth from the sun, is 16″,9: but, at the time of her superior conjunction it is only 9″,6; whilst at her inferior conjunction it sometimes amounts to 61″,2.

Venus changes her *phases*, like the moon, according to her various positions with respect to the sun and the earth: which causes a very considerable difference in her brilliancy.

Her *true diameter*, compared with that of the earth considered as unity, is 0·975; which makes it about 7700 miles.

Her *volume* is 0·927, that of the earth being considered as unity.

Her *mass*, compared with that of the sun considered as unity, is $\frac{1}{405871}$ = ·0000024638.

A body which *weighs* one pound at the equator of the earth, would, if removed to the equator of Venus, weigh only 0·98 pound.

The proportion of *light and heat*, which she receives from the sun, is about 1·91 times greater than that received on the earth.

She is surrounded by an *atmosphere*, the refractive powers of which differ very little from those of the terrestrial atmosphere.

As viewed from the earth, Venus is the most brilliant of all the planets; and may sometimes be seen with the naked eye at noon day. She is known and recognised as the morning and evening star: and never recedes far from the sun. Her *elongation*, or angular distance, varies from 45° to 47°. 12′.

Her course sometimes appears *retrograde*. The arc, which she describes in such cases, varies from 14°. 35′ to 17°. 12′: its duration, in the former case, is $40^d. 21^h$, and in the latter case $43^d. 12^h$. This retrogradation commences or finishes when she is at a distance from the sun, which varies from 27°. 40′ to 29°. 41′.

Venus is sometimes seen to *pass over the sun's disc*; which can happen only when she is in her nodes, and when the earth is in the same longitude. Consequently this phænomenon, for many centuries to come, can take place only in the months of June or December. It is a phænomenon indeed of very rare occurrence, as may be readily seen by the following list, which contains all those transits of Venus which have occurred since that which took place in December 1639 inclusive (the first that was ever known to have been seen by any human being) to the end of the 21st century.

1639 Dec. 4	1874 Dec. 8
1761 June 5	* 1882 Dec. 6
1769 June 3	* 2004 June 7
	2012 June 5

THE EARTH.

The earth which we inhabit is also one of the planets that revolve about the sun. Its *mean distance* from the sun is 23984 times its own semidiameter: whence it is nearly 95 millions of miles distant from that luminary. If this mean distance be assumed equal to unity, we shall have its distance at the perihelion equal to ·9832; and its distance at the aphelion equal to 1·0168.

It performs its mean *sidereal revolution* in 365·2563612 mean solar days, or $365^d. 6^h. 9^m. 9^s,6$: but the time employed in going from one equinox to the same again, or from one tropic to the same again (whence called the *tropical revolution*), is only 365·2422414 mean solar days, or $365^d. 5^h. 48^m. 49^s,7$ *. The tropical year is about $4^s,21$ shorter than it was at the time of Hipparchus.

Its *mean longitude*, at the commencement of the present century, was in 100°. 39′. 10″,2: after subtracting 20″ for the effect of aberration.

Its motion varies in different parts of its orbit. Like all the other planets, it is most rapid in its perihelion, and slowest in its aphelion. In the former point it describes an arc of 1°. 1′. 9″,9 in a mean solar day: and in the

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* M. Lalande makes this equal to $48^s,0$; whilst M. Delambre makes it $51^s,6$. In fact, if we augment the duration of the year 1^s, we must diminish the secular motion of the sun 4″,1. See the note in page 3.

latter point it describes an arc of only 57′. 11″,5 in the same period. Its *mean motion* is 0°·98564722, or 0°. 59′. 8″,32999 in a mean solar day; and 0°·98295603, or 0°. 58′. 58″,64172 in a sidereal day.

The mean longitude of its *perihelion*, at the commencement of the present century, was 99°. 30′. 5″,0. But the line of the apsides has a motion, to the eastward, of 11″,8 in a year: which line, being referred to the ecliptic, will (on account of the precession of the equinoxes) appear to have a motion of 61″,9 in a year. M. Laplace prefers 61″,76. A revolution of the earth, from one end of the apsides to the same point again, is called an *anomalistic year*: and, on the assumption of the quantity stated by M. Laplace, is performed in 365·2595981 mean solar days, or in $365^{d}. 6^{h}. 13^{m}. 49^{s},3$. The perihelion coincided with the vernal equinox about the year 4089 before the Christian era: it coincided with the summer solstice about the year 1250 after Christ: and will coincide with the autumnal equinox about the year 6483. A complete tropical revolution of the apsides is performed in 20984 years.

The axis of the earth is inclined to the pole of the ecliptic in an angle which, at the commencement of the present century, was 23°. 27′. 56″,5*: which angle is called the *obliquity of the ecliptic*. It is observed to decrease at the rate of 0″,4755 in a year. But, this variation is confined within certain limits; and cannot exceed 2°. 42′.

This angle is also subject to a periodical change called the *nutation*; depending principally on the place of the moon's node: whereby the axis of the earth appears to describe a small ellipse in the heavens. The semi major

* M. Bessel makes this only 54″,32 with an annual diminution of 0″,46.

axis of this ellipse is found by M. Laplace from theory to be 9″,40: but Dr. Brinkley, from a comparison of numerous observations, makes it only 9″,25. If a denote the semi axis major of this ellipse, the semi axis minor (b) will be $b = \frac{\cos 2\omega}{\cos \omega} \times a$: ω being the obliquity of the ecliptic. The sun has likewise an effect on the variation of this angle; which amounts, at a maximum, according to M. Laplace from theory, to 0″,493: but, according to Dr. Brinkley from observation, to 0″,545. These variations in the obliquity of the ecliptic affect the right ascensions and declinations of the stars, according to their positions in the heavens.

The intersection of the equator with the ecliptic is not always in the same point; but is constantly retrograding, or receding contrary to the order of the signs. Consequently the equinoctial points appear to move forward on the ecliptic: and whence this phænomenon is called *the precession of the equinoxes.* The quantity of this annual change caused by the action of the sun and moon, and which is called the *luni-solar* precession, is 50″,41; from which we must deduct the direct motion caused by the planets, equal to 0″,31: and the difference, or 50″,10 is the *general* precession in longitude. It is subject to a small secular variation. A complete revolution of the equinoxes is performed in 25868 years.

This precession is also subject to a periodical change, caused by the *nutation* of the earth's axis; and which affects the right ascensions of all the stars, by quantities depending (like the nutation of the obliquity) on the mean place of the moon's node and on the true longitude of the sun. M. Laplace's theory makes the constant of lunar nutation in longitude equal to 17″,579; and of the solar nutation equal to 1″,137. But Dr. Brinkley makes the former 17″299; and the latter 1″,255.

The *eccentricity* of the orbit of the earth is 0·016783568; half the major axis being considered as unity *. The major axis therefore will be to the minor axis of the orbit, as 1 to ·99986. The eccentricity of the earth's orbit is subject to a decrease of 0·00004163 in a century.

The *sidereal day*, or the time employed by the earth in revolving on its axis from any given star to the same star again, is always the same: and has not varied 0^s,003 since the time of Hipparchus. It is divided into 24 sidereal hours; and these are again subdivided into sidereal minutes and seconds. This mode of reckoning time, during the day, is now universally adopted by astronomers in their observatories: although the commencement of the day is still determined by the apparent culmination of the sun.

A *mean solar day*, as adopted by the public in this country, is the time employed by the earth in revolving on its axis, as compared with the sun, supposed to move at a *mean* rate in its orbit, and to make 365·2425 revolutions in a mean Gregorian year. But the mean solar day, adopted by astronomers, is founded on the assumption that the sun makes only 365·2422414 revolutions in a mean Gregorian year. It is divided into 24 mean solar hours; and these are again subdivided into mean solar minutes and seconds.

If the sidereal day be taken equal to 24 sidereal hours,

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* M. Laplace makes this equal to ·01685318, which appears to be too great. The present value is deduced from the formula $\frac{E}{2} - \frac{11\,E^3}{48}$; where E denotes the greatest equation of the centre, and which I have assumed equal to 1°.55′.27″,3.

the mean solar day will be equal to $24^h. 3^m. 56^s,55$ of those sidereal hours. And, if the mean solar day be taken equal to 24 mean solar hours, the sidereal day will be equal to $23^h. 56^m. 4^s,09$ of those hours. Or, in all cases, if we wish to determine in sidereal time the value of any given interval expressed in mean solar time, and *vice versa*, we shall have

$$\text{sidereal time} = 1\cdot00273791 \times \text{mean solar time}$$
$$\text{mean solar time} = 0\cdot99726957 \times \text{sidereal time.}$$

The *apparent day* is the time employed by the earth in revolving on its axis, as compared with the apparent place of the sun. This day is also divided into 24 apparent hours; which are again subdivided into apparent minutes and seconds. This mode of reckoning is still used by the public in many parts on the continent: and is frequently referred to by the practical astronomer on various occasions. In fact, the apparent culmination of the sun is the commencement of the astronomical day to every practical astronomer: and in most ephemerides the computations are made in apparent time.

Apparent time is constantly changing. This variation arises principally from two causes: 1° the unequal motion of the earth in its orbit; 2° the obliquity of that orbit to the plane of the equator. The mean and apparent solar days are never equal, except when the sun's daily motion in right ascension is equal to $59'. 8'',33$. This is the case about April 16th, June 16th, Sept. 1st, and Dec. 25th: on these days the difference vanishes, or nearly so. It is at its greatest about Novem. 1st, when it amounts to $16^m. 16^s$. The correction which is applied to apparent time, in order to reduce it to mean solar time, and *vice versa*, is called the *equation of time*. It depends on a variety of arguments which are given in all the solar tables.

The *astronomical year* is divided into four parts, determined by the two equinoxes and the two solstices. The interval between the vernal and autumnal equinoxes is (on account of the eccentricity of the earth's orbit, and its unequal velocity therein) nearly eight days longer than the interval between the autumnal and vernal equinoxes. These intervals were, in 1801, nearly as follow:

	d. h. m.	d. h. m.
From the vernal equinox to the summer solstice . .	= 92. 21. 50	= 186. 11. 34
From the summer solstice to the autumnal equinox	= 93. 13. 44	
From the autumnal equinox to the winter solstice . .	= 89. 16. 44	= 178. 18. 17
From the winter solstice to the vernal equinox . .	= 89. 1. 33	
		7. 17. 17

The *mass* of the earth, compared with that of the sun considered as unity, is $\frac{1}{354936}$ = ·0000028173.

Its *density* is 3·9326 times greater than that of the sun; and is, to that of water, as 11 to 2.

The *figure of the earth* is that of an oblate spheroid; the axis of the poles being to the diameter of the equator as 304 to 305. Whence the *compression* of the earth is $\frac{1}{305}$: which I shall denote by $\frac{1}{c}$. There is a considerable difference however in the results obtained by different astronomers and mathematicians. The mean *diameter* of the earth is about 7916 miles: its equatorial diameter is 7924 miles, and its polar diameter 7898 miles.

As a necessary consequence from this circumstance, the *degrees of latitude* increase in length, as we recede from the equator to the poles. But, different meridians under the same latitude present different results: the general fact however is well ascertained. If the length of a degree, divided in the middle by the equator, be denoted by d, the length of a degree, divided in the middle by any other latitude ($= \lambda$) will be increased by $\frac{3\,d}{c} \times \sin^2 \lambda$ nearly: c denoting the reciprocal of the compression above mentioned. Whence we conclude that the increase is proportional to the square of the sine of the latitude nearly.

The *centrifugal force* at the equator is nearly $\frac{1}{289}$ ($= \cdot 00346$) of gravity. If the rotation of the earth were 17 times more rapid, the centrifugal force would be equal to that of gravity: and bodies at the equator would not have any weight.

By reason of the circumstances mentioned in the last two paragraphs, bodies lose part of their weight by being taken towards the equator. If the *gravity* of a body at the equator be denoted by unity, its gravity at any other latitude ($= \lambda$) will be increased by $\cdot 00539 \sin^2 \lambda$ nearly.

A *pendulum* therefore, which vibrates seconds at the equator, must be lengthened in the same proportion, as we proceed towards the poles, in order that the oscillations may be rendered isochronous. If p denote the length of a pendulum at the equator, its length at any other latitude ($= \lambda$) must be $p\,(1 + \cdot 00539 \sin^2 \lambda)$ nearly, in order to be isochronous.

Light is supposed to take $8^m. 13^s,3$ to come from the sun to the earth. But, in this interval, the earth has moved $20'',25$ in its orbit.

This motion of the earth produces an optical illusion in the light which comes from all the heavenly bodies; and which is called the *aberration* of light. From a mean of 3326 observations made by Dr. Brinkley and Dr. Struve, the constant of aberration is found to be 20″,36. This would make the velocity of light equal to $8^m. 15^s,8$.

A rare and elastic fluid surrounds the earth, which is called the *atmosphere*. Neither the temperature nor density of this fluid is uniform; but diminishes in proportion to the distance from the surface of the earth, and is also affected by various other circumstances.

On the parallel of 45° of latitude, the temperature being at the freezing point, and the barometer at the level of the sea at its mean height (= 29·922 inches) the weight of the air is to that of a similar volume of mercury as 1 to 10477·9: whence it follows that, if the density of the atmosphere were every where the same, its height would be 26151 feet, or 4·95 miles. But its true height is much more considerable: since M. Gay-Lussac actually ascended in a balloon to the astonishing height of 23010 feet, or 4·36 miles; being the greatest elevation to which any person has yet ascended.

Air is generally supposed to expand in bulk $\frac{1}{480}$ for every degree of Fahrenheit's thermometer: but M. Laplace prefers $\frac{1}{450}$.

The rays of light do not move in a straight line through the atmosphere; but are inflected continually towards the earth: so that the heavenly bodies appear more elevated from the horizon than they really are. This phænomenon is called *refraction*.

We find, from the most accurate observations, that the refraction which the atmosphere produces, is for the most part independent of its temperature, and proportional to its density. But, as the density varies according to the temperature, it is necessary to attend not only to the state of the barometer, but also to that of the thermometer.

A ray of light, passing from a vacuum into air at the temperature of freezing water and under a pressure indicated by the barometer at 29·922 inches, will be refracted so that the sine of refraction is to the sine of incidence as 1 to 1·0002943321. It would be sufficient therefore, in order to determine the direction of a ray of light through the atmosphere, to know the law of the density of its strata. But, this law, which depends on the temperature, is very complicated, and varies every moment in the day.

The temperature of the whole atmosphere being supposed at the freezing point, the density of these strata will diminish in a geometrical progression according to their distances from the surface of the earth: and we find by analysis that, the barometer being at 29·922, the refraction at the horizon is 39′. 54″,68. It would be only 30′. 24″,1 if the density diminished in an arithmetical progression. The *horizontal refraction*, which we observe, (about 35′. 6″,0) is a mean between these limits.

When the apparent height of a star above the horizon exceeds 10°, its sensible refraction depends wholly on the state of the thermometer and barometer at the place of observation: and it is nearly proportional to the tangent of the apparent distance of the star from the zenith, diminished by 3·25 times the corresponding refraction at that distance: the thermometer being at the freezing point, and the barometer at 29·922 inches. Whence it follows that, at that temperature and under that pressure, the constant of refraction is 60″,66: but, at any other temperature and under any other pressure, corrections must be applied which are usually given in all Tables of refraction. When the apparent altitude of a star does not exceed 10°, the formula for determining the refraction becomes more complex: and has been the subject of much controversy between the most eminent mathematicians and astronomers.

The *humidity* of the air produces no sensible effect on its refractive powers, and may therefore be safely neglected.

The atmosphere is a heterogeneous substance. Out of 100 parts, 79 are azotic gas, and the remaining 21 are oxygen gas: with the exception of 3 or 4 parts of carbonic acid gas ont of every 1000. This is found to be universally the case in whatever season or whatever climate, or in whatever part of the world the experiment has been tried. This proportion is also found to exist in the highest points of the atmosphere that have been reached by means of balloons.

A body projected horizontally from the surface of the earth, to the distance of about 4·35 miles, if there were no resistance in the atmosphere, would not fall again to the earth; but would revolve round it as a satellite: the centrifugal force being then equal to its gravity.

The action of the sun and moon has a considerable effect on the waters of the ocean, and produces the phænomena of the *tides*.

The height of the tide, at high water, is not always the same; but varies from day to day: and these variations have an evident relation to the phases of the moon. It is greatest at the syzigies: after which it diminishes, and becomes the least at the quadratures.

The tides are also affected by the declinations of the sun and moon: for they diminish the tides of the syzigies which occur at the equinoxes; and augment the tides of the quadratures, which occur at the solstices. The diminution of the tides of the syzigies at the solstices, is only $\frac{3}{5}$ of the diminution of the tides of the syzigies at the equinoxes. And the increase of the tides at the quadratures, is twice as great at the equinoxes as it is at the solstices.

The distance of the moon from the earth has also a sensible influence on the tides. In general they increase and diminish as the diameter and parallax of the moon increases and diminishes; but in a greater degree. The diminution of the tides of the syzigies at the perigee is nearly three times greater than at the apogee.

The action of the moon upon the tides is three times that of the sun.

The sea rises and falls twice in each interval of time comprised between the consecutive returns of the moon to the same meridian. The mean interval of these returns is $1^d. 0^h. 50^m. 28^s,3$: consequently the mean interval between two following periods of high water is $12^h. 25^m. 14^s,1$. So that the *retardation* in the time of high water, from one day to another, is $50^m. 28^s,3$ in its mean state: and it is affected by all those causes which influence the moon's motion.

This retardation varies with the phases of the moon. It is at its minimum towards the syzigies, when the tides are at their maximum; and it is then only $39^m. 12^s,7$. But, towards the quadratures, when the tides are at their minimum, this retardation is the greatest possible; and amounts to $1^h. 14^m. 58^s,8$.

The variation in the distance of the sun and moon from the earth (and particularly the moon) has an influence also on this retardation. Each minute in the increase or diminution of the apparent diameter of the moon, augments or diminishes this retardation $3^m. 42^s,9$ towards the syzigies but towards the quadratures the effect is three times less.

The daily retardation of the tides varies likewise with the declination of the sun and moon. In the syzigies at the time of the solstices, it is about $1^m. 26^s,4$ greater than in its mean state: and it is diminished in the same proportion at the equinoxes. On the contrary, in the quadratures at the time of the equinoxes, it exceeds its mean state by $5^m. 45^s,6$: and is in a similar manner diminished by this quantity, in the quadratures at the time of the solstices.

But, the state of the tides is so modified by the nature and position of the coasts, the depth of the channel, the operation of the winds, and by other causes, that the above laws will not always be found to correspond with the actual state of the tides, particularly near the coast, or in rivers.

The general result however, from a mean of a number of observations, is that the inequalities, in the heights and intervals of the tides, have various periods. Some are of half a day and a day; others are of half a month and a month; whilst others are of half a year and a year: and some are the same as the times of the revolutions of the lunar nodes and apsides.

MARS.

THE *mean distance* of Mars from the sun is 1·5236923; that of the earth being considered as unity. This makes his mean distance above 142 millions of miles.

He performs his mean *sidereal revolution* in 686·9796458 mean solar days; or in $686^d. 23^h. 30^m. 41^s,4$: and his mean *synodical revolution* in 779·936 mean solar days.

His *mean longitude*, at the commencement of the present century, was in 64°. 22′. 55″,5.

His *mean motion* in his orbit, in a mean solar day, is 0°·524072, or 31′. 26″,66. His mean motion in 365 days is consequently 191°. 286280 or 191°. 17′. 10″,6.

The longitude of the *perihelion* was, at the commencement of the present century, in 332°. 23′. 56″,6. But, the line of the apsides has a motion, to the eastward, of 15″,8 in a year: which, on account of the precession of the equinoxes, will appear to move 65″,9 in a year.

His orbit is *inclined* to the plane of the ecliptic in an angle which, at the commencement of the present century, was 1°. 51′. 6″,2: and which angle decreases about 0″,014 in a year.

His ascending *node* was, at the commencement of the present century, in 48°. 0′. 3″,5; having a motion to the westward every year, of 23″,3. But, when referred to the ecliptic, the place of the node will (on account of the precession of the equinoxes) fall more to the eastward by 26″,8 in a year.

The *eccentricity* of his orbit is 0·0933070; half the major axis being considered as unity. This eccentricity is supposed to increase about 0·000090176 in a century.

The *greatest equation of the centre* is $10°. 40'. 50''$; which is subject to an annual increase of $0'',37$.

The *rotation* on his axis is performed in $24^h. 39^m. 21^s,3$.

The *inclination of his axis* to that of the ecliptic is $30°. 18'. 10'',8$.

His *parallax* is nearly double that of the sun.

His *apparent diameter*, at his mean distance from the earth, is $6'',29$. At its conjunction, it is sometimes not more than $3'',6$; but it increases as the planet approaches its opposition, when it sometimes amounts to $18'',28$. Sir W. Herschel states that the polar diameter is about $\frac{1}{16}$ less than the equatorial diameter.

Mars changes his *phases* (somewhat in the same manner as the moon does from her first to her third quarter) according to his various positions with respect to the sun and the earth. But he never becomes cornicular, as Venus and the moon do when near their conjunctions.

His *true diameter*, compared with the earth considered as unity, is ·517 or about 4100 miles; which is rather more than half the diameter of the earth.

His *volume* is 0·1386; that of the earth being considered as unity.

His *mass* compared with the sun considered as unity is $\frac{1}{2546320}$ = ·0000003927.

A body which *weighs* one pound at the equator of the earth, would, if removed to the equator of Mars, weigh only $\frac{1}{3}$ of a pound.

The proportion of *light and heat* received by him from the sun, is about 0·43; that received by the earth being considered as unity.

He has a very dense but moderate *atmosphere*: and he is not accompanied by any satellite. As viewed from the earth, he is known by his red and fiery appearance.

His course sometimes appears *retrograde.* The arc which he describes in such cases, varies from 10°. 6′ to 19°. 35′: its duration in the former case is $60^{d}. 18^{h}$; and in the latter case $80^{d}. 15^{h}$. This retrogradation commences or finishes when the planet is at a distance from the sun, which varies from 128°. 44′ to 146°. 37′.

VESTA.

THIS planet was discovered by Dr. Olbers on March 29, 1807: its *mean distance* from the sun is 2·367870; that of the earth being considered as unity.

It performs its *sidereal revolution* in 1325·7431 mean solar days: and its mean *synodical revolution* in 503·41 days.

Its *mean longitude*, at mean noon, at Greenwich, on Jan. 1, 1820, was in 278°. 30′. 0″,4.

Its *mean motion* in its orbit, in a mean solar day, is 16′. 17″,9516: its mean motion in 365 days is consequently 99°. 9′. 15″,33.

The longitude of its *perihelion*, on Jan. 1, 1820, was in 249°. 33′. 24″,4. According to M. Santini, it has an apparent annual motion of + 1′. 34″,24.

Its orbit is *inclined* to the plane of the ecliptic, in an angle of 7°. 8′. 9″: which, according to M. Santini, has an annual decrease of 0″,12.

Its ascending *node* was, on January 1, 1820, in 103°. 13′. 18″,2: which, according to M. Santini, has an apparent annual motion of + 15″,63.

The *eccentricity* of its orbit is 0·089130; half the major axis being considered as unity: subject to an annual increase, according to M. Santini, of ·000004009.

The *greatest equation of the centre* is 10°. 13′. 22″.

The elements of this planet however are not yet sufficiently determined to be depended on: and require correction from future observations.

JUNO.

This planet was first discovered by M. Harding on Sept. 1, 1804: its *mean distance* from the sun is 2·669009; that of the sun being considered as unity.

It performs its *sidereal revolution* in 1592·6608 mean solar days: and its mean *synodical revolution* in 473·95 days.

Its *mean longitude*, at mean noon, at Greenwich, on Jan. 1, 1820, was in 200°. 16′. 19″,1.

Its *mean motion* in its orbit, in a mean solar day, is 13′. 32″,9304: its mean motion in 365 days is consequently 82°. 25′. 19″,60.

The longitude of its *perihelion*, on Jan. 1, 1820, was in 53°. 33′. 46″.

Its orbit is *inclined* to the plane of the ecliptic, in an angle of 13°. 4′. 9″,7.

Its ascending *node* was, on Jan. 1, 1820, in 171°. 7′. 40″,4.

The *eccentricity* of its orbit is 0·257848; half the major axis being considered as unity.

The *greatest equation of the centre* is 29°. 46′. 19″.

The elements of this planet however are not yet sufficiently determined to be depended on: and require correction from future observations.

CERES.

This planet was first discovered by M. Piazzi, on Jan. 1, 1801: its *mean distance* from the sun is 2·767245; that of the earth being considered as unity.

It performs its mean *sidereal revolution* in 1681·3931 mean solar days: and its mean *synodical revolution* in 466·62 days.

Its *mean longitude*, at mean noon, at Greenwich, on Jan. 1, 1820, was in 123°. 16′. 11″,9.

Its *mean motion* in its orbit, in a mean solar day, is 12′. 50″,9230: its mean motion in 365 days is consequently 78°. 9′. 46″,89.

The longitude of its *perihelion*, on Jan. 1, 1820, was in 147°. 7′. 31″,5: which, according to M. Gauss, is subject to an apparent annual motion of + 2′. 1″,3.

Its orbit is *inclined* to the plane of the ecliptic in an angle of 10°. 37′. 26″,2: which, according to M. Gauss, has an annual decrease of 0″,44.

Its ascending *node* was, on Jan. 1, 1820, in 80°. 41′. 24″. According to M. Gauss, it has an apparent annual motion of + 1″,48.

The *eccentricity* of its orbit is 0·078439; half the major axis being considered as unity: which, according to M. Gauss, is subject to an annual decrease of ·00000583.

The *greatest equation of the centre* is 8°. 59′. 42″.

The elements of this planet however are not yet sufficiently determined to be depended on: and require correction from future observations.

PALLAS.

This planet was discovered by Dr. Olbers, on March 28, 1802: its *mean distance* from the sun is 2·772886; that of the earth being considered as unity.

It performs its *sidereal revolution* in 1686·5388 mean solar days: and its mean *synodical revolution* in 466·22 days.

Its *mean longitude*, at mean noon, at Greenwich, on Jan. 1, 1820, was in 108°. 24′. 57″,9.

Its *mean motion* in its orbit, in a mean solar day, is 12′. 48″,3934: its mean motion in 365 days is consequently 77°. 54′. 25″,59.

The longitude of its *perihelion*, on Jan. 1, 1820, was in 121°. 7′. 4″,3.

Its orbit is *inclined* to the plane of the ecliptic, in an angle of 34°. 34′. 55″,0.

Its ascending *node* was, on Jan. 1, 1820, in 172°. 39′. 26″,8.

The *eccentricity* of its orbit is 0·241648; half the major axis being considered as unity.

The *greatest equation of the centre* is 27°. 49′. 19″.

The elements of this planet however are not yet sufficiently determined to be depended on: and require correction from future observations. It appears subject to very considerable perturbations.

JUPITER.

The *mean distance* of this planet from the sun is 5·202776; that of the earth being considered as unity. This makes his mean distance above 485 millions of miles.

He performs his mean *sidereal revolution* in 4332·5848212 mean solar days, or in $4332^d. 14^h. 2^m. 8^s,5$: which is nearly 12 years. But this period is subject to some inequalities. His mean *synodical revolution* is performed in 398·867 mean solar days.

His *mean longitude* at the commencement of the present century, was in $112°. 15'. 23'',0$.

His *mean motion* in his orbit, in a mean solar day, is $0°.08312938$, or $4'. 59'',26$. His mean motion in 365 days is consequently $30°.3422228$, or $30°. 20'. 32'',0$: so that he passes through somewhat more than a sign in the course of a year.

The longitude of his *perihelion* was, at the commencement of the present century, in $11°. 8'. 34'',6$. The line of the apsides has a motion, to the eastward, of $6'',96$ in a year: which when referred to the ecliptic will (on account of the precession of the equinoxes) appear to be equal to $57'',06$ in a year.

His orbit is *inclined* to the plane of the ecliptic in an angle which, at the commencement of the present century, was $1°. 18'. 51'',3$; and which angle is subject to a small decrease of about $0'',226$ in a year.

His ascending *node* was, at the commencement of the present century, in $98°. 26'. 18'',9$; having a motion to the westward every year of 15″,8. But, when referred to the ecliptic, the place of the node will (on account of the precession of the equinoxes) fall more to the eastward by 34″,3 in a year.

The *eccentricity* of his orbit is 0·0481621, half the major axis being assumed equal to unity. This eccentricity is supposed to increase about 0·000159350 in a century.

The *greatest equation of the centre* is 5°. 31′. 13″,8: which is subject to an annual increase of 0″,6344.

The *rotation* on his axis is performed in $9^h. 55^m. 49^s,7$.

The *inclination of his axis* to that of the ecliptic is 3°. 5′. 30″.

His *apparent diameter* (measured equatorially) at his mean distance from the earth, is 36″,74. At its conjunction it is sometimes only 30″,0; but it increases as the planet approaches its opposition, when it sometimes amounts to 45″,88.

His *true diameter*, compared with that of the earth considered as unity, is 10·860; which makes it near 90000 miles. The axis of the poles is to his equatorial diameter, as 167 to 177.

His *volume* is 1280·9; that of the earth being considered as unity.

His *mass*, compared with that of the sun considered as unity, is $\frac{1}{1070·5}$ = ·0009341431.

His *density*, compared with that of the sun considered as unity, is ·99239: and is about $\frac{1}{4}$ of the density of the earth.

A body which *weighs* one pound at the equator of the earth would, if removed to the equator of Jupiter, weigh 2·716 pounds. But this must be diminished about a ninth part, on account of the centrifugal force due to each planet.

The proportion of *light and heat* which he receives from the sun is ·037: that received by the earth being considered as unity.

He is surrounded by faint substances which appear like zones or belts: and which are supposed to be parts of his *atmosphere.* As viewed from the earth he appears, next to Venus, the most brilliant of all the planets; whom he sometimes however surpasses in brightness.

His course sometimes appears *retrograde.* The arc which he describes in such cases varies from 9°. 51′ to 9°. 59′: its duration in the former case is $116^d. 18^h$; and in the latter case $122^d. 12^h$. This retrogradation commences or finishes when the planet is at a distance from the sun which varies from 113°. 35′ to 116°. 42′.

Jupiter is accompanied by four *satellites.*

SATURN.

The *mean distance* of Saturn from the sun is 9·5387861; that of the earth being considered as unity. This makes his mean distance above 890 millions of miles.

He performs his mean *sidereal revolution* in 10759·2198174 mean solar days; or in 29·456 Julian years. But this period is subject to some inequalities: and his motion at the present day appears to be less rapid than formerly. His mean *synodical revolution* is performed in 378·090 mean solar days.

His *mean longitude*, at the commencement of the present century, was in 135°. 20′. 6″,5.

His *mean motion* in his orbit, in a mean solar day, is 0°. 03349777 or 2′. 0″,6. His mean motion in 365 days is consequently 12°. 22668787 or 12°. 13′. 36″,08.

The longitude of his *perihelion* was, at the commencement of the present century, in 89°. 9′. 29″,8. The line of the apsides has a motion, to the eastward, of 19″,4 in a year: which, when referred to the ecliptic, will (on account of the precession of the equinoxes) appear to be equal to 69″,5 in a year.

His orbit is *inclined* to the plane of the ecliptic in an angle which, at the commencement of the present century, was 2°. 29′. 35″,7; and which angle is subject to a small decrease of 0″,155 in a year.

His ascending *node* was, at the commencement of the present century, in 111°. 56′. 37″,4; having a motion to the westward, every year, of 19″,4. But, when referred to the ecliptic, the place of the node will (on account of the precession of the equinoxes) fall more to the eastward, by 30″,7 in a year.

The *eccentricity* of his orbit is 0·05615050; half the major axis being assumed equal to unity. This eccentricity is supposed to decrease about ·000312402 in a century.

The *greatest equation of the centre* is 6°. 26′. 12″; which is subject to an annual decrease of 1″,279.

The *rotation* on his axis is performed in $10^h. 29^m. 16^s$,8.

The *inclination of his axis* to that of the ecliptic is 31°. 19′.

His *apparent diameter*, at his mean distance from the earth, is about 16″,20.

His *true diameter*, compared with that of the earth considered as unity, is 9·982; which makes it about 76068 miles. The axis of the poles is to the equatorial diameter as 11 to 12.

His *volume* is 995·00; that of the earth being considered as unity.

His *mass*, compared with that of the sun considered as unity, is $\frac{1}{3512}$ = ·0002847380.

His *density*, compared with that of the sun considered as unity, is ·550; which is about $\frac{1}{8}$ of the density of the earth; but there is some uncertainty in this determination.

A body which *weighs* one pound at the equator of the earth, would, if removed to the equator of Saturn, weigh 1·01 pounds.

The proportion of *light and heat* which it receives from the sun is about ·0011; that received by the earth being considered as unity.

He is sometimes marked by zones or belts; which are probably obscurations in his *atmosphere*.

His course sometimes appears *retrograde*. The arc which he describes in such cases varies from 6°. 41′ to 6°. 55′: its duration in the former case is $138^d. 18^h$; and in the latter case $135^d. 9^h$. This retrogradation commences or finishes when the planet is at a distance from the sun which varies from 107°. 25′ to 110°. 46′.

Saturn is accompanied by seven *satellites:* and also surrounded with a double *ring*.

URANUS.

Uranus was discovered to be a planet by Sir William Herschel on March 13, 1781; who gave it the name of the *Georgium sidus**. Its *mean distance* from the sun is 19·182390; that of the earth being considered as unity. This makes his mean distance upwards of 1800 millions of miles.

It performs its mean *sidereal revolution* in 30686·8208296 mean solar days; or in 84·02 Julian years. Its mean *synodical revolution* is performed in 369·656 mean solar days.

Its *mean longitude*, at the commencement of the present century, was in 177°. 48′. 23″,0.

The *mean motion* in its orbit in a mean solar day is 0°. 0117695; or 42″,37. His mean motion in 365 days is 4°. 295876 or 4°. 17′. 45″,16.

The longitude of his *perihelion* was, at the commencement of the present century, in 167°. 31′. 16″,1. The line of the apsides has an apparent motion, to the eastward, of 52″,50 in a year.

His orbit is *inclined* to the plane of the ecliptic in an angle of 46′. 28″,44.

His ascending *node* was, at the commencement of the

* It is remarkable that this star was observed as far back as 1690. It was seen three times by Flamsteed, once by Bradley, once by Mayer, and eleven times by Lemonnier: not one of whom suspected it to be a planet. That brilliant discovery was reserved for Herschel.

present century, in 72°. 59′. 35″,3; having an apparent motion to the eastward, every year, of 14″,16.

The *eccentricity* of his orbit is 0·04667938; half the major axis being considered as unity.

The *greatest equation of the centre* deduced from this eccentricity is 5°. 20′. 57″.

His *apparent diameter*, even at the time of his opposition, is scarcely 4″,0.

His *mass*, compared with that of the sun considered as unity, is $\frac{1}{17918}$ = ·0000558098.

His *density*, compared with that of the sun considered as unity, is supposed to be about 1·100.

The proportion of *light and heat* which it receives from the sun is about ·003; that received by the earth being considered as unity.

As seen from the earth the motion of Uranus sometimes appears *retrograde*. The mean arc which he describes in this case is about 3°. 36′: and its mean duration is about 151 days. This retrogradation commences or finishes when the planet is distant about 103°. 30′ from the sun.

This planet is accompanied by six *satellites*.

THE SATELLITES.

The number of satellites in our system, at present known, is eighteen: namely, the *Moon* which revolves round the Earth, *four* that belong to Jupiter, *seven* to Saturn, and *six* to Uranus. The moon is the only one visible to the naked eye.

They all move round their respective primary planets, as their centre, by the same laws as those primary ones move round the sun: namely,

1°. The orbit of each satellite is an ellipse, of which the primary planet occupies one of the foci.

2°. The areas, described about the primary planet, by the radius vector of the satellite, are proportional to the times employed in describing them.

3°. The squares of the times of the revolutions of the satellites, round their respective primary planets, are to each other as the cubes of their mean distances from the primary.

THE MOON.

THE *mean distance* of the moon from the earth is 29·982175 times the diameter of the terrestrial equator; or above 237 thousand miles.

She performs her mean *sidereal revolution* in 27·321661423 mean solar days, or $27^d. 7^h. 43^m. 11^s,5$: but the time employed in making a *tropical revolution* is only $27^d.321582418$, or $27^d. 7^h. 43^m. 4^s,7$. Her mean *synodical revolution* is 29·5305887215 mean solar days, or $29^d. 12^h. 44^m. 2^s,87$. But these periods are variable; and a comparison of the modern observations with the ancient ones proves incontestably an acceleration in the mean motions of the moon, to which we shall presently allude.

Her *mean longitude*, at the commencement of the present century, was in 118°. 17′. 8″,3 *.

Her *mean motion*, in 100 Julian years, or 36525 mean solar days, is 481267°.878222 or 1336^{rev} + 307°. 52′. 41″,6 †: whence we deduce her mean motion in a mean solar day to be 13°. 17639639 or 13°. 10′. 35″,027: and consequently her mean motion in 365 days to be 4809°. 38468235 or

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* M. Burg has adopted 16′. 56″,1: whilst M. Burckhardt assumes it 17′. 3″,0.

† Mayer, in his first tables, adopted 52′. 20″; but in his second tables, he increased it to 53′. 35″. M. Burg proposed 52′. 43″,48: whilst M. Burckhardt assumed it at 52′. 53″,5. Lastly, M. Damoiseau has retained the determination of M. Laplace, which makes it only 52′. 41″,6 as stated in the text.

$13^{rev} + 129^{\circ}.\ 23'.\ 4'',85646$. It is however subject to a secular variation, as will be shown presently.

The mean longitude of her *perigee* was, at the commencement of the present century, in $266^{\circ}.\ 10'.\ 7'',5$. But the line of the apsides has a motion to the eastward, which in 36525 mean solar days is $4069^{\circ}.046278$, or $11^{rev} + 109^{\circ}.\ 2'.\ 46'',6$ *: whence we deduce the mean motion in a mean solar day to be $6'.\ 41'',0$; and consequently the mean motion in 365 mean solar days to be $40^{\circ}.\ 39'.\ 45'',36$. It makes a sidereal revolution in 3232·575343 mean solar days, or in nearly 9 years. The period of a tropical revolution of the apsides is but 3231·4751 mean solar days. These periods however are not uniform: for they have a secular variation depending on the acceleration of the moon; and are retarded whilst the motion of the moon itself is accelerated. The amount of this secular variation, we shall allude to in the sequel.

If the mean place of the moon's perigee be deducted from her mean longitude, it will show her *mean anomaly*. This mean anomaly was, at the commencement of the present century, in $212^{\circ}.\ 7'.\ 0'',8$ †: and its motion in a mean solar day is $13^{\circ}.064992$; consequently its motion in 365 days is $4768^{\circ}.722057$, or $13^{rev} + 88^{\circ}.\ 43'.\ 19'',4$. The mean period of an anomalistic revolution of the moon is $27^{d}.5545995$ or $27^{d}.\ 13^{h}.\ 18^{m}.\ 37^{s},4$. But these motions are variable, as will be shown hereafter.

Her orbit is *inclined* to the plane of the ecliptic in an angle of $5^{\circ}.\ 8'.\ 47'',9$. But this inclination is subject to a periodical variation which principally depends on the

* M. Burg makes this $3'.\ 25'',7$, whilst M. Burckhardt makes it $3'.\ 48'',2$. M. Damoiseau has adopted the determination of M. Laplace.

† M. Burg assumes $6'.\ 56'',6$; whilst M. Burckhardt adopts $6'.\ 39'',8$.

cosine of twice the distance of the moon from the sun; and amounts, at a maximum, to 8′. 47″,15. The mean inclination however is constant, notwithstanding the secular variation in the plane of the ecliptic: a fact which is confirmed by all the observations, ancient and modern.

Her ascending *node* was, at the commencement of the present century, in 13°. 53′. 17″,7 *. It has a motion to the westward, which in 36525 mean solar days amounts to 1934°. 1659722, or 5^{rev} + 134°. 9′. 57″,5 †: whence we deduce the motion of the node in a mean solar day to be 0°. 052955 or 3′. 10″,64: and the motion in 365 mean solar days to be 19°. 328421 or 19°. 19′. 42″,316. The nodes make a sidereal revolution in 6793·39108 mean solar days; or in 18·6 Julian years. The place of the node is subject to many inequalities; of which the greatest is proportional to the sine of double the distance of the moon from the sun; which, at a maximum, amounts to 1°. 37′. 45″. A synodical revolution of the nodes is performed in 346·619851 mean solar days; or in $346^d. 14^h. 52^m. 35^s,1$. The mean period of a revolution of the moon, from node to node, is 27^d. 2122222, or $27^d. 5^h. 5^m. 36^s$. These mean motions however are not uniform: for the motion of the nodes is subject to a secular variation depending on the acceleration of the moon; and is retarded, whilst the motion of the moon is accelerated. The amount of this secular variation we shall now allude to.

The *acceleration of the moon's mean motion* arises from

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* M. Burg, in the *Supplement* to his tables, makes this 40″,6: whilst M. Burckhardt adopts 22″,2.

† M. Burg has assumed 42″,0, in the *Supplement* to his tables: whilst M. Burckhardt has adopted 48″,0. M. Damoiseau has followed the determination of M. Laplace.

the action of the sun, together with the secular variation of the eccentricity of the earth's orbit. Whilst this eccentricity diminishes (which is the case at present) the acceleration will increase: but, when the eccentricity shall begin to increase, this acceleration will be changed into a retardation of the moon's motion. The cause of this variation affects not only the position of the moon's longitude, but also the place of her perigee and node. M. Damoiseau has given the following formulæ for the secular variations: where x denotes the number of centuries from 1800.

$$\begin{aligned} \text{Long:} &= + 10'',7232x^2 + 0'',019361x^3 \\ \text{Anom:} &= + 50'',4203x^2 + 0'',091035x^3 \\ \text{Node} &= + 6'',5632x^2 + 0'',011850x^3 \end{aligned}$$

These quantities are related to each other, as the numbers 1, + 4·702 and + 0·612.

The *eccentricity* of the moon's orbit is 0·0548442; half the major axis being assumed equal to unity. It does not appear to be subject to any variation.

The *greatest equation of the centre* is $6^\circ.17'.12'',7$: which also appears to be invariable.

The *rotation* on her axis is equal and uniform; and is performed in precisely the same time as the tropical revolution in her orbit: whence she always presents nearly the same face to the earth.

But, as the motion of the moon, in her orbit, is periodically variable, we sometimes see more of her eastern edge, and sometimes more of her western edge. This appearance is called her *libration in longitude.*

The *inclination of her axis* to that of the ecliptic is $1^\circ.30'.10'',8$.

In consequence of this position of the moon, her poles alternately become visible to, and obscured from us. This phænomenon is called her *libration in latitude.*

There is also another phænomenon connected with this subject, arising from the moon being seen by us from the surface of the earth, instead of the centre. This is called her *diurnal libration.*

There are other inequalities in the moon's motions, arising from the action and influence of the sun. The principal of these are the three following ones; which are added as *equations* to the moon's mean longitude.

1°. The *evection,* whose constant effect is to diminish the equation of the centre in the syzigies, and to augment it in the quadratures. If this diminution and increase were always the same, the evection would depend only on the angular distance of the moon from the sun: but its absolute value varies also with the distance of the moon from the perigee of its orbit. After a long series of observations, we are enabled to represent this inequality by supposing it to depend on the sine of double the distance of the moon from the sun, minus the distance of the moon from its perigee. At its maximum it amounts to 1°. 20′. 29″,90.

2°. The *variation,* which disappears in the syzigies and quadratures, and is greatest in the octants. It is then equal to 35′. 41″,96. It is proportional to the sine of twice the distance of the moon from the sun: and its duration is half a synodical revolution of the moon.

3°. The *annual equation,* which follows exactly the same law as the equation of the centre of the sun, but with a contrary sign. For, when the earth is in its perihelion, the orbit of the moon is enlarged by the action of the sun; and the moon therefore requires more time to perform her revolution. But, as the earth proceeds towards its aphelion, the moon's orbit contracts. Hence, the period of this inequality is an anomalistic year: and, at its maximum it amounts to 11′. 11″,97. It is subject to a small secular variation.

The mean *horizontal parallax* of the moon, at the equator, is $57'.\,0'',9$. It varies from about $53'.\,48''$ to about $61'.\,24''$ according to the distance of the moon from the earth. The horizontal parallax at any other latitude $(=\lambda)$ is always less than that at the equator, by a quantity which is equal to the equatorial horizontal parallax multiplied by $\frac{1}{c}.\sin^2\lambda$ nearly: $\frac{1}{c}$ being the compression of the earth.

The *parallax of altitude* may, for most ordinary purposes, be considered as equal to the horizontal parallax (at the place) multiplied by the cosine of the apparent altitude.

The *apparent diameter* of the moon varies also according to her distance from the earth. When nearest to us, it is $33'.\,31'',07$; but at her greatest distance it is only $29'.\,21'',91$: the apparent diameter at her mean distance is $31'.\,7'',0$. It is always $\frac{6}{11}$ of the horizontal parallax of the moon: or, more correctly, equal to ·545 of the lunar parallax.

Her mean *true diameter* is, in proportion to that of the earth, as 5823 to 21332; or as 1 to 3·665. Whence her mean diameter is about 2160 miles. Her figure is that of an oblate spheroid, like that of the earth.

Her *volume* is $\frac{1}{49}$ of the volume of the earth.

Her *mass* is $\frac{1}{79{\cdot}89}$ of the mass of the earth *.

Her *density* is $\frac{1}{1{\cdot}623} =$ ·615 of the density of the earth.

* M. Laplace made this, at first, equal to $\frac{1}{68{\cdot}50}$: but he has since reduced it to $\frac{1}{75{\cdot}77}$. The value in the text is deduced from Dr. Brinkley's constant of nutation.

A body, which *weighs* one pound at the equator of the earth, would, if removed to the equator of the moon, weigh only $\frac{1}{6}$ of a pound.

The *light* of the moon is 300 thousand times more weak than that of the sun. Its rays, collected by the aid of powerful glasses, do not produce any sensible effect on the thermometer.

The *atmosphere* of the moon (if it has any) must be exceedingly attenuated; and must be more rare than that which we can produce with our best air-pumps.

The *refraction* of the rays of light at the surface of the earth must be at least a thousand times greater than at the surface of the moon. The horizontal refraction at the moon cannot exceed $1'',6$.

Volcanoes and *mountains* are discovered on her surface, by the aid of powerful telescopes.

A *body projected* from the surface of the moon, with a momentum that would cause it to proceed at the rate of about 8200 feet in the first second of time, and whose direction should be in a line which, at that moment, passed through the centre of the earth and moon, would not fall again to the surface of the moon; but would become a satellite to the earth. Its primitive impulse might, indeed, be such as to cause it even, after many revolutions, to precipitate to the earth. The stones, which have fallen from the air, may be accounted for in this manner.

The *phases* of the moon are caused by the reflection of the sun's light from her surface; and depend on the relative positions of the sun, the earth and the moon.

Eclipses can happen only when the moon is in her syzigies: and then only when she is near the place of her nodes. If, at the time of her mean conjunction with the sun, she be less than $13^\circ. 33'$ from her node, there will certainly be an eclipse of the sun in some part of the world: but, if this distance be greater than $19^\circ. 44'$, there cannot be one. Between these limits it will be necessary to make a more minute calculation. If at the time of her mean opposition, she be $7^\circ. 47'$ distant from her node, there will certainly be an eclipse of the moon: but if $13^\circ. 21'$ distant therefrom, there cannot be one. Between these limits it will also be necessary to make a more minute calculation.

The *number of eclipses* in a year cannot be less than two, nor more than seven. And when there are only two, they will both be solar.

A *solar eclipse* cannot take place unless the moon be in conjunction with the sun: and then only to a spectator on the earth under particular circumstances. When the centres of the sun and moon are in the same straight line with the eye of the spectator, and the apparent diameter of the moon is greater than that of the sun, the eclipse will be *total*: but, if her apparent diameter be less, the eclipse will be *annular*. In other cases, however, which are by far the most numerous, the sun will suffer only a *partial* eclipse. The greatest possible duration of the annular appearance of a solar eclipse is, according to M. Du Sejour, $12^{m}. 24^{s}$: and the greatest possible time during which the sun can be totally obscured is $7^{m}. 58^{s}$. The magnitude and duration of every solar eclipse will in fact differ at every point of the earth's surface.

The following is a list of all the subsequent solar eclipses that will be visible in this country during the present century. The hour of the day at which the eclipse commences, and the number of digits eclipsed, are adapted to the middle of England.

LIST OF SOLAR ECLIPSES *.					
Year.	Day and hour.				Digits eclipsed.
		d	h		° ′
1826	Nov.	29.	10.	A.M.	6. 41
1832	July	27.	2.	P.M.	0. 30
1833	July	17.	5.	A.M.	0. 36
1836	May	15.	2.	P.M.	11. 18
1841	July	18.	3.	——	contact
1842	July	8.	5.	A.M.	8. 54
1845	May	6.	8.	——	6. 15
1846	April	25.	6.	P.M.	2. 21
1847	Oct.	9.	6.	A.M.	11. 0
1851	July	28.	2.	P.M.	9. 43
1858	March	15.	11.	A.M.	11. 30
1860	July	18.	2.	P.M.	9. 12
1861	Dec.	31.	2.	——	5. 0
1863	May	17.	6.	——	3. 46
1865	Oct.	19.	4.	——	7. 36
1866	Oct.	8.	5.	——	5. 3
1867	March	6.	8.	A.M.	8. 42
1868	Feb.	23.	3.	P.M.	contact
1870	Dec.	22.	11.	A.M.	9. 36
1873	May	26.	8.	——	3. 43
1874	Oct.	10.	9.	——	6. 18
1875	Sept.	29.		noon	0. 33
1879	July	19.	7.	A.M.	4. 0
1880	Dec.	30.	2.	P.M.	4. 24
1882	May	17.	6.	A.M.	2. 18
1887	August	19.	3.	——	11. 58
1890	June	17.	8.	——	4. 39
1891	June	6.	5.	P.M.	3. 0
1895	March	26.	9.	A.M.	1. 0
1896	August	9.		sunrise	contact
1899	June	8.	5.	A.M.	3. 13
1900	May	28.	3.	P.M.	8. 0

* See Hutton's *Mathem. Dictionary*. Vol. i. page 453. 2nd edition. I have however added a few others, from the list published by M. Du Vaucel in the *Memoires des Savans Etrangers*. Vol. v. page 575.

A *lunar eclipse* takes place only when the moon is in opposition to the sun; and it is of the same magnitude and duration to every spectator on the surface of the earth. It is caused by her passing through the shadow of the earth; which is $3\frac{1}{2}$ times longer than the distance between the earth and the moon. The breadth of this shadow, in the part where it is traversed by the moon, is about $2\frac{3}{4}$ times greater than the diameter of the moon; and is equal to the sum of the horizontal parallaxes of the sun and moon, *minus* the semidiameter of the sun. The magnitude and duration of every lunar eclipse will consequently vary according to the magnitude of these quantities at the given time, and the relative positions of the luminaries.

Visible eclipses of the moon are so frequent, that a list of them cannot be conveniently inserted in this work. And, on account of the indistinctness of the border of the *penumbra*, the correct observation of such eclipses is generally difficult and unsatisfactory.

Eclipses generally *return again* nearly in the same order and magnitude at the end of 223 lunations. For in 223 mean synodical revolutions there are 6585·32 days; and in 6585·78 days there are 19 mean synodical revolutions of the moon's node. Therefore at the end of this period the sun and moon will be found nearly in the same position with respect to the place of the moon's node. This period consists of 18 Julian years and 11 days, if there are four leap years in the interval: but if there are five leap years, it will consist of no more than 18 Julian years, and 10 days. And it will be found that there are generally about 70 eclipses in this interval: of which, 29 will be lunar, and 41 solar.

During a mean synodical revolution of the moon, the motion of the sun's mean anomaly, the moon's mean anomaly, and the mean distance of the moon from her node, will be respectively

29°.1053533 25°.8169054 30°.6705153

These quantities are useful in the computation of *tables* for determining the periods of eclipses.

To an inhabitant of the moon, the earth always appears nearly in the *same place* in the heavens; from which it varies only in consequence of the libration.

JUPITER'S SATELLITES.

By the aid of the telescope we may observe *four satellites* revolving round Jupiter: the positions of which, with respect to each other, are continually changing. We sometimes see them pass over the disc of Jupiter, and to project their shadow on the body of the planet.

The shadow which Jupiter himself projects behind him, relatively to the sun, gives rise to another phænomenon of considerable importance. For, the satellites frequently disappear, or are *eclipsed* in that shadow, although they appear, with respect to us, to be at a distance from the disc of the planet. These eclipses are similar in principle to lunar eclipses; and vary in duration according to the relative position of the bodies with respect to the sun.

In the following table are given the mean *sidereal revolution* of the satellites, in mean solar days; together with their *mean distances* from Jupiter, the semidiameter of that planet's equator being considered as unity; and likewise their *masses* compared with that of Jupiter considered also as unity.

Sat.	Sidereal revolution.		Mean distance.	Mass.
	d h m	d		
1	1. 18. 28	1. 769137788148	6. 04853	·0000173281
2	3. 13. 14	3. 551810117849	9. 62347	·0000232355
3	7. 3. 43	7. 154552783970	15. 35024	·0000884972
4	16. 16. 32	16. 688769707084	26. 99835	·0000426591

First satellite. The plane of the orbit of this satellite coincides nearly with the plane of the equator of Jupiter; the inclination of which to the orbit of the planet, is 3°. 5'. 30''. Its eccentricity is insensible.

Second satellite. The eccentricity of the orbit of this satellite is also insensible. The inclination of its orbit to that of its primary is variable; as well as the position of its nodes. These variations are represented nearly by supposing the orbit of the satellite inclined 27', 49'',2 to the equator of Jupiter; and by giving the nodes a retrograde motion, on this plane, so as to make a revolution in 30 Julian years.

Third satellite. This satellite has a little eccentricity, which is subject to a very sensible variation. Towards the end of the century before the last, the equation of the centre was at its maximum; and was then as much as 13'. 16'',4. It afterwards diminished, and was at its minimum about the year 1777; when it was only 5'. 7'',5. The line of the apsides has a direct but variable motion. The inclination of its orbit to that of Jupiter, and the position of its nodes, are also variable. These variations may be represented nearly by supposing the orbit inclined 12'. 20'' to the equator of Jupiter; and by giving the nodes a retrograde motion, on this plane, so as to make a revolution in 142 Julian years.

Fourth satellite. The eccentricity of this satellite is greater than that of the other three. The line of the apsides has an annual and direct motion of 42'. 58'',7. The place of the nodes has a direct annual motion on the orbit of the planet, of 4'. 15'',3. The inclination of the orbit to that of Jupiter is about 2°. 58'. 48''. It is in consequence of this great inclination that this satellite frequently passes behind the planet, with respect to the sun, without being eclipsed. Since the middle of the last century, the inclination of the orbit has increased, and the motion of the nodes has diminished, very perceptibly.

Independent of these variations, the satellites are subject to perturbations which affect their elliptical motions; and which render their theory very complicated.

The motions of the first three satellites are related to each other by a most singular analogy. For, the mean sidereal or synodical revolution of the first, added to twice that of the third, is generally equal to three times that of the second. And the mean sidereal or synodical longitude of the first, *minus* three times that of the second, *plus* twice that of the third, is generally equal to two right angles.

It follows therefore that, for a great number of years at least, the first three satellites cannot be eclipsed at the same time. For, in the simultaneous eclipses of the second and third, the first will always be in conjunction with Jupiter: and *vice versa.*

The eclipses of Jupiter's satellites are of great utility in enabling us to *determine the longitude* of places, by their observation: and they likewise exhibit some curious phænomena with respect to light.

SATELLITES OF SATURN.

By the aid of the telescope also, we may observe *seven satellites* to revolve round Saturn: the elements of which are but little known on account of their great distance from us. The following table will show their mean *sidereal revolutions* in mean solar days, and their *mean distances* from the planet, in semidiameters of Saturn's equator.

Sat.	Sidereal revolution.		Mean distance.
	d h m	d	
1	0. 22. 38	0·94271	3·351
2	1. 8. 53	1·37024	4·300
3	1. 21. 18	1·88780	5·284
4	2. 17. 45	2·73948	6·819
5	4. 12. 25	4·51749	9·524
6	15. 22. 41	15·94530	22·081
7	79. 7. 55	79·32960	64·359

The orbits of the first six satellites appear to be nearly circular; and in the plane of Saturn's ring: whilst the seventh varies from that plane, and approaches nearer to that of the ecliptic.

The great distance of these satellites, and the difficulty of observing them, prevent us from ascertaining the ellipticity of their orbits, and still less the inequalities of their motions. We know however that the ellipticity of the sixth is very perceptible.

RING OF SATURN.

The most singular phænomenon attending Saturn is the *double ring*, with which he is surrounded: the apparent form and magnitude of which is very variable. Sometimes it appears nearly to surround the planet, and at other times is scarcely visible even in the most powerful telescopes. When it is approaching the latter state, it has the appearance of two handles, or *ansæ*; one on each side of the planet.

This ring, which is very thin and broad, is *inclined* to the plane of the ecliptic in an angle of $28°.\ 39'.\ 54''$.

It *revolves* from west to east, in a period of $10^h.\ 29^m.\ 16^s,8$, about an axis which is perpendicular to its plane, and which passes through the centre of the planet. And it is remarkable that this is the period in which a satellite, assumed to be at a mean distance equal to the mean distance of the ring, would revolve round the primary, according to the third law of Kepler.

The *breadth* of this ring is nearly equal to its distance from the surface of Saturn: that is, about $\frac{1}{3}$ of the diameter of the planet.

Its surface is separated nearly in the middle, by a black concentric band, which divides it into *two* distinct rings: the breadth of the exterior of which is rather less than that of the interior.

The *apparent diameter* of the ring, at the mean distance of the planet, is $38'',42$; and of its breadth $5'',78$.

The edges of the ring being very thin, and being occasionally presented obliquely to the earth, sometimes *disappear*. And as this edge will present itself to the sun twice in each revolution of the planet, it is obvious that the disappearance of the ring will occur about once in 15 years: but, under circumstances oftentimes very different.

The intersection of the ring and of the ecliptic is in 170°, and 350°: consequently, when Saturn is near either of those points, his ring will be invisible to us. On the contrary, when he is in 80° or 260°, we may see it to the greatest advantage. Regard however must be had to the position of the earth; which will cause some variations in this respect. The following are the dates, during the ensuing revolution of the planet, when the mean *heliocentric* longitude of Saturn is such that the ring will (if the earth be favourably situated) either be invisible, or seen to the greatest advantage.

Date.	Mean Long.	Phase.
1825 Nov.	80°	South side illumined
1833 April	170	Invisible
1838 July	260	North side illumined
1847 Dec.	350	Invisible
1855 April	80	South side illumined

SATELLITES OF URANUS.

By the aid of a very powerful telescope we may discover *six satellites* revolving round Uranus.

The following table will show their mean *sidereal revolutions* in mean solar days, and their *mean distances* from the planet in semidiameters of his equator.

Sat.	Sidereal revolution.		Mean distance.
	d h m	d	
1	5. 21. 25	5·8926	13·120
2	8. 16. 58	8·7068	17·022
3	10. 23. 4	10·9611	19·845
4	13. 10. 56	13·4559	22·752
5	38. 1. 48	38·0750	45·507
6	107. 16. 40	107·6944	91·008

All these satellites are stated by Sir Wm. Herschel (to whom we are indebted for all we know on the subject) to move in a plane which is nearly perpendicular to the plane of the planet's orbit.

RECAPITULATION.

Mean distance from the sun.

Mercury	0·3870981
Venus	0·7233316
Earth	1·0000000
Mars	1·5236923
Vesta	2·3678700
Juno	2·6690090
Ceres	2·7672450
Pallas	2·7728860
Jupiter	5·2027760
Saturn	9·5387861
Uranus	19·1823900

Mean sidereal revolution.

	d
Mercury	87·9692580
Venus	224·7007869
Earth	365·2563612
Mars	686·9796458
Vesta	1325·7431000
Juno	1592·6608000
Ceres	1681·3931000
Pallas	1686·5388000
Jupiter	4332·5848212
Saturn	10759·2198174
Uranus	30686·8208296

Synodical revolution.

	d
Mercury	115·877
Venus	583·920
Earth	365·242
Mars	779·936
Vesta	503·410
Juno	473·950
Ceres	466·620
Pallas	466·220
Jupiter	398·867
Saturn	378·090
Uranus	369·656

Mean longitude, Jan. 1, 1801.

		°	′	″
Mercury		166.	0.	48,6
Venus		11.	33.	3,0
Earth		100.	39.	10,2
Mars		64.	22.	55,5
Vesta	1820.	278.	30.	0,4
Juno	1820.	200.	16.	19,1
Ceres	1820.	123.	16.	11,9
Pallas . . .	1820.	108.	24.	57,9
Jupiter		112.	15.	23,0
Saturn		135.	20.	6,5
Uranus		177.	48.	23,0

Mean daily motion in the orbit.

	°	′	″
Mercury	4.	5.	32,6
Venus	1.	36.	7,8
Earth	0.	59.	8,3
Mars	0.	31.	26,7
Vesta	0.	16.	17,9
Juno	0.	13.	32,9
Ceres	0.	12.	50,9
Pallas	0.	12.	48,4
Jupiter	0.	4.	59,3
Saturn	0.	2.	0,6
Uranus	0.	0.	42,4

Longitude of perihelion.

		Jan. 1, 1801.	Ann. inc.
		° ′ ″	″
Mercury .		74. 21. 46,9	+ 55,9
Venus . .		128. 43. 53,1	+ 47,4
Earth . .		99. 30. 5,0	+ 61,8
Mars . . .		332. 23. 56,6	+ 65,9
Vesta .	1820.	249. 33. 24,4	+ 94,2
Juno .	1820.	53. 33. 46,0	
Ceres .	1820.	147. 7. 31,5	+ 121,3
Pallas .	1820.	121. 7. 4,3	
Jupiter . .		11. 8. 34,6	+ 57,1
Saturn . .		89. 9. 29,8	+ 69,5
Uranus . .		167. 31. 16,1	+ 52,5

Inclination of the orbit.

		Jan. 1, 1801.	Ann. var.
Mercury		7° 0′ 9″,1	+ 0″,18
Venus		3. 23. 28,5	− 0,04
Earth			
Mars		1. 51. 6,2	− 0,01
Vesta	1820.	7. 8. 9,0	− 0,12
Juno	1820.	13. 4. 9,7	
Ceres	1820.	10. 37. 26,2	− 0,44
Pallas	1820.	34. 34. 55,0	
Jupiter		1. 18. 51,3	− 0,23
Saturn		2. 29. 35,7	− 0,15
Uranus		0. 46. 28,4	+ 0,03

Longitude of the node.

		Jan. 1, 1801.	Ann. inc.
Mercury		45° 57′ 30″,9	+ 42″,3
Venus		74. 54. 12,9	+ 32,5
Earth			
Mars		48. 0. 3,5	+ 26,8
Vesta	1820.	103. 13. 18,2	+ 15,6
Juno	1820.	171. 7. 40,4	
Ceres	1820.	80. 41. 24,0	+ 1,5
Pallas	1820.	172. 39. 26,8	
Jupiter		98. 26. 18,9	+ 34,3
Saturn		111. 56. 37,4	+ 30,7
Uranus		72. 59. 35,3	+ 14,2

Eccentricity.

	Jan. 1, 1801.	*Secular* variation.
Mercury . .	0·205514940	+ ·000003866
Venus . . .	0·006860740	− ·000062711
Earth . . .	0·016783568	− ·000041630
Mars . . .	0·093307000	+ ·000090176
Vesta . . .	0·089130000	+ ·000004009
Juno	0·257848000	·
Ceres . . .	0·078439000	− ·000005830
Pallas . . .	0·241648000	·
Jupiter . . .	0·048162100	+ ·000159350
Saturn . . .	0·056150500	− ·000312402
Uranus . . .	0·046679380	

Greatest equation of the centre.

		Jan. 1, 1801.	Ann. var.
Mercury		23°. 39′. 51″,0	+ 0″,0160
Venus		0. 47. 15,0	− 0,2500
Earth		1. 55. 27,3	− 0,1718
Mars		10. 40. 50,0	+ 0,3700
Vesta . . .	1820.	10. 13. 22,0	
Juno	1820.	29. 46. 19,0	
Ceres . . .	1820.	8. 59. 42,0	
Pallas . . .	1820.	27. 49. 19,0	
Jupiter		5. 31. 13,8	+ 0,6344
Saturn		6. 26. 12,0	− 1,2790
Uranus		5. 20. 57,0	

	Apparent Diameter.			True Diameter.	Volume.
	Least.	Mean.	Greatest.		
Mercury	5″,0	6″,9	12″,0	0·398	0·063
Venus	9,6	16,9	61,2	0·975	0·927
Earth				1·000	1·000
Mars	3,6	6,3	18,3	0·517	0·139
Jupiter	30,0	36,7	45,9	10·860	1280·900
Saturn		16,2		9·982	995·000
Uranus		4,0		4·332	80·490
Sun	31′. 31″,0	32′. 2″,9	32′. 35″,6	111·454	1384472·000
Moon	29. 21,9	31. 7,0	33. 31,1	0·275	0·020

	Mass.	Density.	Gravity.	Sidereal Rotation.	Light and heat.
				h m s	
Mercury	$\frac{1}{2025810}$		1·03	24. 5. 28	6·680
Venus	$\frac{1}{405871}$		0·98	23. 21. 7	1·911
Earth	$\frac{1}{354936}$	3·9326	1·00	24. 0. 0	1·000
Mars	$\frac{1}{2546320}$		0·33	24. 39. 21	·431
Jupiter	$\frac{1}{1070·5}$	·9924	2·72	9. 55. 50	·037
Saturn	$\frac{1}{3512}$	·5500	1·01	10. 29. 17	·011
Uranus	$\frac{1}{17918}$	1·1000			·003
Sun	1	1·0000	27·90	25. 12. 0	
Moon	$\frac{1}{26620200}$	2·4185	0·16	27. 7. 43	

Revolutions of the moon.

	d	h	m	s		d
Synodical . .	29.	12.	44.	2,9	=	29·53058872
Anomalistic .	27.	13.	18.	37,4	=	27·55459950
Sidereal . .	27.	7.	43.	11,5	=	27·32166142
Tropical . .	27.	7.	43.	4,7	=	27·32158242
Nodical . .	27.	5.	5.	36,0	=	27·21222222

FORMULÆ.

I. Equivalent expressions for $\sin x$.

1 $\cos x \cdot \tan x$

2 $\dfrac{\cos x}{\cot x}$

3 $\sqrt{(1 - \cos^2 x)}$

4 $\dfrac{1}{\sqrt{(1 + \cot^2 x)}}$

5 $\dfrac{\tan x}{\sqrt{(1 + \tan^2 x)}}$

6 $2 \sin \frac{1}{2} x \cdot \cos \frac{1}{2} x$

7 $\sqrt{\dfrac{1 - \cos 2x}{2}}$

8 $\dfrac{2 \tan \frac{1}{2} x}{1 + \tan^2 \frac{1}{2} x}$

9 $\dfrac{2}{\cot \frac{1}{2} x + \tan \frac{1}{2} x}$

10 $\dfrac{\sin(30° + x) - \sin(30° - x)}{\sqrt{3}}$

11 $2 \sin^2(45° + \frac{1}{2} x) - 1$

12 $1 - 2 \sin^2(45° - \frac{1}{2} x)$

13 $\dfrac{1 - \tan^2(45° - \frac{1}{2} x)}{1 + \tan^2(45° - \frac{1}{2} x)}$

14 $\dfrac{\tan(45° + \frac{1}{2} x) - \tan(45° - \frac{1}{2} x)}{\tan(45° + \frac{1}{2} x) + \tan(45° - \frac{1}{2} x)}$

15 $\sin(60° + x) - \sin(60° - x)$

16 $\dfrac{1}{\text{cosecant } x}$

II. Equivalent expressions for $\cos x$.

1 $\dfrac{\sin x}{\tan x}$

2 $\sin x \,.\, \cot x$

3 $\sqrt{(1 - \sin^2 x)}$

4 $\dfrac{1}{\sqrt{(1 + \tan^2 x)}}$

5 $\dfrac{\cot x}{\sqrt{(1 + \cot^2 x)}}$

6 $\cos^2 \frac{1}{2} x - \sin^2 \frac{1}{2} x$

7 $1 - 2 \sin^2 \frac{1}{2} x$

8 $2 \cos^2 \frac{1}{2} x - 1$

9 $\sqrt{\dfrac{1 + \cos 2 x}{2}}$

10 $\dfrac{1 - \tan^2 \frac{1}{2} x}{1 + \tan^2 \frac{1}{2} x}$

11 $\dfrac{\cot \frac{1}{2} x - \tan \frac{1}{2} x}{\cot \frac{1}{2} x + \tan \frac{1}{2} x}$

12 $\dfrac{1}{1 + \tan x \,.\, \tan \frac{1}{2} x}$

13 $\dfrac{2}{\tan (45° + \frac{1}{2} x) + \cot (45° + \frac{1}{2} x)}$

14 $2 \cos (45° + \frac{1}{2} x) \cos (45° - \frac{1}{2} x)$

15 $\cos (60° + x) + \cos (60° - x)$

16 $\dfrac{1}{\text{secant}\, x}$

III. Equivalent expressions for $\tan x$.

1 $\dfrac{\sin x}{\cos x}$

2 $\dfrac{1}{\cot x}$

3 $\sqrt{\left(\dfrac{1}{\cos^2 x} - 1\right)}$

4 $\dfrac{\sin x}{\sqrt{(1 - \sin^2 x)}}$

5 $\dfrac{\sqrt{(1 - \cos^2 x)}}{\cos x}$

6 $\dfrac{2 \tan \frac{1}{2} x}{1 - \tan^2 \frac{1}{2} x}$

7 $\dfrac{2 \cot \frac{1}{2} x}{\cot^2 \frac{1}{2} x - 1}$

8 $\dfrac{2}{\cot \frac{1}{2} x - \tan \frac{1}{2} x}$

9 $\cot x - 2 \cot 2 x$

10 $\dfrac{1 - \cos 2 x}{\sin 2 x}$

11 $\dfrac{\sin 2 x}{1 + \cos 2 x}$

12 $\sqrt{\dfrac{1 - \cos 2 x}{1 + \cos 2 x}}$

13 $\dfrac{\tan (45° + \frac{1}{2} x) - \tan (45° - \frac{1}{2} x)}{2}$

IV. Relative to two arcs A and B.

1 $\sin(A + B) = \sin A.\cos B + \cos A.\sin B$

2 $\sin(A - B) = \sin A.\cos B - \cos A.\sin B$

3 $\cos(A + B) = \cos A.\cos B - \sin A.\sin B$

4 $\cos(A - B) = \cos A.\cos B + \sin A.\sin B$

5 $\tan(A + B) = \dfrac{\tan A + \tan B}{1 - \tan A.\tan B}$

6 $\tan(A - B) = \dfrac{\tan A - \tan B}{1 + \tan A.\tan B}$

7 $\sin(45° \pm B)$
8 $\cos(45° \mp B)$ $\left.\right\} = \dfrac{\cos B \pm \sin B}{\sqrt{2}}$

9 $\tan(45° \pm B) = \dfrac{1 \pm \tan B}{1 \mp \tan B}$

10 $\tan^2(45° \pm \frac{1}{2}B) = \dfrac{1 \pm \sin B}{1 \pm \sin B}$

11 $\tan(45° \pm \frac{1}{2}B) = \dfrac{1 \pm \sin B}{\cos B} = \dfrac{\cos B}{1 \mp \sin B}$

12 $\dfrac{\sin(A + B)}{\sin(A - B)} = \dfrac{\tan A + \tan B}{\tan A - \tan B} = \dfrac{\cot B + \cot A}{\cot B - \cot A}$

13 $\dfrac{\cos(A + B)}{\cos(A - B)} = \dfrac{\cot B - \tan A}{\cot B + \tan A} = \dfrac{\cot A - \tan B}{\cot A + \tan B}$

14 $\dfrac{\sin A + \sin B}{\sin A - \sin B} = \dfrac{\tan \frac{1}{2}(A + B)}{\tan \frac{1}{2}(A - B)}$

15 $\dfrac{\cos B + \cos A}{\cos B - \cos A} = \dfrac{\cot \frac{1}{2}(A + B)}{\tan \frac{1}{2}(A - B)}$

[continued.

IV. continued. Relative to two arcs A and B.

16	$\sin A . \cos B$	$=$	$\frac{1}{2}\sin(A+B) + \frac{1}{2}\sin(A-B)$
17	$\cos A . \sin B$	$=$	$\frac{1}{2}\sin(A+B) - \frac{1}{2}\sin(A-B)$
18	$\sin A . \sin B$	$=$	$\frac{1}{2}\cos(A-B) - \frac{1}{2}\cos(A+B)$
19	$\cos A . \cos B$	$=$	$\frac{1}{2}\cos(A+B) + \frac{1}{2}\cos(A-B)$
20	$\sin A + \sin B$	$=$	$2\sin\frac{1}{2}(A+B).\cos\frac{1}{2}(A-B)$
21	$\cos A + \cos B$	$=$	$2\cos\frac{1}{2}(A+B).\cos\frac{1}{2}(A-B)$
22	$\tan A + \tan B$	$=$	$\frac{\sin(A+B)}{\cos A.\cos B}$
23	$\cot A + \cot B$	$=$	$\frac{\sin(A+B)}{\sin A.\sin B}$
24	$\sin A - \sin B$	$=$	$2\sin\frac{1}{2}(A-B).\cos\frac{1}{2}(A+B)$
25	$\cos B - \cos A$	$=$	$2\sin\frac{1}{2}(A-B).\sin\frac{1}{2}(A+B)$
26	$\tan A - \tan B$	$=$	$\frac{\sin(A-B)}{\cos A.\cos B}$
27	$\cot B - \cot A$	$=$	$\frac{\sin(A-B)}{\sin A.\sin B}$
28	$\sin^2 A - \sin^2 B$	}	$\sin(A-B).\sin(A+B)$
29	$\cos^2 B - \cos^2 A$	}	$\sin(A-B).\sin(A+B)$
30	$\cos^2 A - \sin^2 B$	$=$	$\cos(A-B).\cos(A+B)$
31	$\tan^2 A - \tan^2 B$	$=$	$\frac{\sin(A-B).\sin(A+B)}{\cos^2 A.\cos^2 B}$
32	$\cot^2 B - \cot^2 A$	$=$	$\frac{\sin(A-B).\sin(A+B)}{\sin^2 A.\sin^2 B}$

V. Differences of trigonometrical lines.

1 $\Delta \sin x = + 2 \sin \tfrac{1}{2} \Delta x . \cos (x + \tfrac{1}{2} \Delta x)$

2 $\Delta \cos x = - 2 \sin \tfrac{1}{2} \Delta x . \sin (x + \tfrac{1}{2} \Delta x)$

3 $\Delta \tan x = + \dfrac{\sin \Delta x}{\cos x . \cos (x + \Delta x)}$

4 $\Delta \cot x = - \dfrac{\sin \Delta x}{\sin x . \sin (x + \Delta x)}$

5 $\Delta \sin^2 x = + \sin \Delta x . \sin (2 x + \Delta x)$

6 $\Delta \cos^2 x = - \sin \Delta x . \sin (2 x + \Delta x)$

7 $\Delta \tan^2 x = + \dfrac{\sin \Delta x . \sin (2 x + \Delta x)}{\cos^2 x . \cos^2 (x + \Delta x)}$

8 $\Delta \cot^3 x = - \dfrac{\sin \Delta x . \sin (2 x + \Delta x)}{\sin^2 x . \sin^2 (x + \Delta x)}$

VI. Differentials of trigonometrical lines.

1 $d \sin x = + d x . \cos x$

2 $d \cos x = - d x . \sin x$

3 $d \tan x = + \dfrac{d x}{\cos^2 x}$

4 $d \cot x = - \dfrac{d x}{\sin^2 x}$

5 $d \sin^2 x = + 2 d x . \sin x . \cos x$

6 $d \cos^2 x = - 2 d x . \sin x . \cos x$

7 $d \tan^2 x = + \dfrac{2 d x . \tan x}{\cos^2 x}$

8 $d \cot^2 x = - \dfrac{2 d x . \cot x}{\sin^2 x}$

VII. General analytical expressions for the sides and angles of any spherical triangle.

1 $\cos S = \cos A . \sin S' . \sin S'' + \cos S' . \cos S''$

2 $\cos S' = \cos A' . \sin S'' . \sin S + \cos S'' . \cos S$

3 $\cos S'' = \cos A'' . \sin S . \sin S' + \cos S . \cos S'$

4 $\cos A = \cos S . \sin A' . \sin A'' - \cos A' . \cos A''$

5 $\cos A' = \cos S' . \sin A'' . \sin A - \cos A'' . \cos A$

6 $\cos A'' = \cos S'' . \sin A . \sin A' - \cos A . \cos A'$

7 $\cos S . \cos A' = \cot S'' . \sin S - \sin A' . \cot A''$

8 $\cos S' . \cos A'' = \cot S . \sin S' - \sin A'' . \cot A$

9 $\cos S'' . \cos A = \cot S' . \sin S'' - \sin A . \cot A'$

10 $\dfrac{\sin A}{\sin S} = \dfrac{\sin A'}{\sin S'} = \dfrac{\sin A''}{\sin S''}$

11 $\sin \frac{1}{2}(S' + S) : \sin \frac{1}{2}(S' - S) :: \cot \frac{1}{2} A'' : \tan \frac{1}{2}(A' - A)$

12 $\cos \frac{1}{2}(S' + S) : \cos \frac{1}{2}(S' - S) :: \cot \frac{1}{2} A'' : \tan \frac{1}{2}(A' + A)$

13 $\sin \frac{1}{2}(A' + A) : \sin \frac{1}{2}(A' - A) :: \tan \frac{1}{2} S'' : \tan \frac{1}{2}(S' - S)$

14 $\cos \frac{1}{2}(A' + A) : \cos \frac{1}{2}(A' - A) :: \tan \frac{1}{2} S'' : \tan \frac{1}{2}(S' + S)$

In these formulæ A, A′, A″, denote the several *angles* of the triangle; and S, S′, S″, the *sides* opposite those angles respectively. For the more convenient computation of the formulæ No. 1—9, certain auxiliary angles are introduced, which will be alluded to in the formulæ for the solution of the several cases of oblique-angled spherical triangles.

VIII. Solutions of the cases of *right*-angled spherical triangles.

Given.	*Required.*		*Solution.*
Hypothen. and an angle	side op. giv. ang.	1	$\sin x = \sin h . \sin a$
	side adj. giv. ang.	2	$\tan x = \tan h . \cos a$
	the other angle	3	$\cot x = \cos h . \tan a$
Hypothen. and a side	the other side	4	$\cos x = \dfrac{\cos h}{\cos s}$
	ang. adj. giv. side	5	$\cos x = \tan s . \cot h$
	ang. op. giv. side	6	$\sin x = \dfrac{\sin s}{\sin h}$
A side and the angle opposite	the hypothen.	7	$\sin x = \dfrac{\sin s}{\sin a}$ (the ambiguous cases.)
	the other side	8	$\sin x = \tan s . \cot a$ (the ambiguous cases.)
	the other angle	9	$\sin x = \dfrac{\cos a}{\cos s}$ (the ambiguous cases.)
A side and the angle adjacent	the hypothen.	10	$\cot x = \cos a . \cot s$
	the other side	11	$\tan x = \tan a . \sin s$
	the other angle	12	$\cos x = \sin a . \cos s$
The two sides	the hypothen.	13	$\cos x =$ rectang. cos. of the giv. sides
	an angle	14	$\cot x =$ sin. adj. side × cot. op. side
The two angles	the hypothen.	15	$\cos x =$ rectang. cot. of the given angles
	a side	16	$\cos x = \dfrac{\text{cos. opp. ang.}}{\text{sin. adj. ang.}}$

In these formulæ, x denotes the quantity sought.

a = the *given* angle
s = the *given* side
h = the hypothenuse.

IX. Solutions of the cases of *oblique*-angled spherical triangles.

GIVEN, *Two sides and an angle opposite one of them.*

Required, 1°. The angle opposite the other given side.

$$\sin x = \frac{\text{sin. side op. ang. sought} \times \text{sin. giv. ang.}}{\text{sin. side oppos. given angle}}$$

Required, 2°. The angle included between the given sides.

$$\cot a' = \text{tan. giv. ang.} \times \text{cos. adj. side}$$

$$\cos a'' = \frac{\cos a' \times \text{tan. side adj. giv. ang.}}{\text{tan. side op. given angle}}$$

$$x = (a' \pm a'')$$

Required, 3°. The third side.

$$\tan a' = \text{cos. giv. ang.} \times \text{tan. adj. side}$$

$$\cos a'' = \frac{\cos a' \times \text{cos. side op. giv. ang.}}{\text{cos. side adj. given angle}}$$

$$x = (a' \pm a'')$$

In these formulæ, x denotes the quantity sought: a' and a'' are auxiliary angles introduced for the purpose of facilitating the computations.

The angle sought in formula 1 is, in certain cases, ambiguous. In the formulæ 2 and 3, when the angles opposite to the given sides are of the *same species*, we must take the *upper* sign: on the contrary, the *lower* sign. The whole of these formulæ therefore are, in certain cases, ambiguous.

[continued.

IX. continued. Solutions of the cases of *oblique*-angled spherical triangles.

GIVEN, Two angles and a side opposite one of them.

Required, 4°. The side opposite the other given angle.

$$\sin x = \frac{\text{sin. ang. op. side sought} \times \text{sin. giv. side}}{\text{sin. ang. op. given side}}$$

Required, 5°. The side included between the given angles.

$$\tan a' = \text{tan. giv. side} \times \text{cos. ang. adj. giv. side}$$

$$\sin a'' = \frac{\sin a' \times \text{tan. ang. adj. giv. side}}{\text{tan. ang. op. given side}}$$

$$x = (a' \pm a'')$$

Required, 6°. The third angle.

$$\cot a' = \text{cos given side} \times \text{tan. adj. angle}$$

$$\sin a'' = \frac{\sin a' \times \text{cos. ang. op. giv. side}}{\text{cos. ang. adj. given side}}$$

$$x = (a' \pm a'')$$

In these formulæ, x denotes the quantity sought: a' and a'' are auxiliary angles introduced for the purpose of facilitating the computations.

The side sought in formula 4 is, in certain cases, ambiguous. In the formulæ 5 and 6, when the sides opposite the given angles are of the *same species*, we must take the *upper* sign: on the contrary, the *lower* sign. The whole of these formulæ therefore are, in certain cases, ambiguous.

[continued.

IX. continued. Solutions of the cases of *oblique*-angled spherical triangles.

GIVEN, *Two sides and the included angle.*

Required, 7°. One of the other angles.

$$\tan a' = \cos \text{given angle} \times \tan \textit{given} \text{ side}$$
$$a'' = \text{the } \textit{base} - a'$$
$$\tan x = \tan \text{given angle} \times \frac{\sin a'}{\sin a''}$$

In this formula, the *given* side is assumed to be the side opposite the angle sought: the other known side is called the *base*.

Required, 8°. The third side.

$$\tan a' = \cos \text{given angle} \times \tan \textit{given} \text{ side}$$
$$a'' = \text{the } \textit{base} - a'$$
$$\cos x = \cos \textit{given} \text{ side} \times \frac{\cos a''}{\cos a'}$$

In this formula, either of the given sides may be assumed as the *base*: and the other, as the *given* side.

In these formulæ, x denotes the quantity sought: a' and a'' are auxiliary angles introduced for the purpose of facilitating the computations.

If the side sought in formula 8 be small, the formula may not give the value to a sufficient degree of accuracy: and some other mode must be adopted for obtaining the correct value.

 [continued.

IX. continued. Solutions of the cases of *oblique*-angled spherical triangles.

Given, *A side and the two adjacent angles.*

Required, 9°. One of the other sides.

$$\cot a' = \tan \textit{given} \text{ angle} \times \cos \text{given side}$$

$$a'' = \text{the } \textit{vertical} \text{ angle} \sim a'$$

$$\tan x = \tan \text{given side} \times \frac{\cos a'}{\cos a''}$$

In this formula, the angle, opposite the side sought, is assumed as the *given* angle: the other known angle is called the *vertical* angle.

Required, 10°. The third angle.

$$\cot a' = \tan \textit{given} \text{ angle} \times \cos \text{given side}$$

$$a'' = \text{the } \textit{vertical} \text{ angle} - a'$$

$$\cos x = \cos \textit{given} \text{ angle} \times \frac{\sin a''}{\sin a'}$$

In this formula, either of the given angles may be assumed as the *vertical* angle: and the other as the *given* angle.

In these formulæ, x denotes the quantity sought: a' and a'' are auxiliary angles introduced for the purpose of facilitating the computations.

If the angle sought in formula 10 be small, the formula may not give the value to a sufficient degree of accuracy: and some other mode must be adopted for obtaining the correct value.

[continued.

IX. continued. Solutions of the cases of *oblique*-angled spherical triangles.

Given, *The three sides.*

Required, 11°. An angle.

$$\sin^2 \tfrac{1}{2} x = \frac{\sin\left(\frac{A+B+C}{2} - B\right) \times \sin\left(\frac{A+B+C}{2} - C\right)}{\sin B \, . \, \sin C}$$

$$\cos^2 \tfrac{1}{2} x = \frac{\sin\left(\frac{A+B+C}{2}\right) \times \sin\left(\frac{A+B+C}{2} - A\right)}{\sin B \, . \, \sin C}$$

In these formulæ, A, B, C are the three sides of the triangle: and A is assumed as the side opposite to the angle required.

Given, *The three angles.*

Required, 12°. A side.

$$\sin^2 \tfrac{1}{2} x = -\frac{\cos\left(\frac{a+b+c}{2}\right) \times \cos\left(\frac{a+b+c}{2} \backsim a\right)}{\sin b \, . \, \sin c}$$

$$\cos^2 \tfrac{1}{2} x = \frac{\cos\left(\frac{a+b+c}{2} \backsim b\right) \times \cos\left(\frac{a+b+c}{2} \backsim c\right)}{\sin b \, . \, \sin c}$$

In these formulæ, a, b, c are the three angles of the triangle: and a is assumed as the angle opposite to the side required.

In these formulæ, x denotes the quantity sought. The formulæ, which are resolved by the *cosine*, are used only when the angle or side x is small.

X. Trigonometrical series.

1 $$\sin x = x - \frac{x^3}{2\,.\,3} + \frac{x^5}{2\,.\,3\,.\,4\,.\,5} - \&c.$$

2 $$\cos x = 1 - \frac{x^2}{2} + \frac{x^4}{2\,.\,3\,.\,4} - \frac{x^6}{2\,.\,3\,.\,4\,.\,5\,.\,6} + \&c.$$

3 $$\tan x = x + \frac{x^3}{3} + \frac{2\,x^5}{3\,.\,5} + \frac{17\,x^7}{3^2\,.\,5\,.\,7} + \&c.$$

4 $$\cot x = \frac{1}{x} - \frac{x}{3} - \frac{x^3}{3^2\,.\,5} - \frac{2\,x^5}{3^3\,.\,5\,.\,7} - \&c.$$

5 $$\text{ver-sin}\, x = \frac{x^2}{2} - \frac{x^4}{2\,.\,3\,.\,4} + \frac{x^6}{2\,.\,3\,.\,4\,.\,5\,.\,6} - \&c.$$

6 $$x = \sin x + \frac{\sin^3 x}{2\,.\,3} + \frac{1\,.\,3 \sin^5 x}{2\,.\,4\,.\,5} + \&c.$$

7 $$x = \frac{\pi}{2} - \cos x - \frac{\cos^3 x}{2\,.\,3} - \frac{1\,.\,3 \cos^5 x}{2\,.\,4\,.\,5} - \&c.$$

8 $$x = \tan x - \tfrac{1}{3}\tan^3 x + \tfrac{1}{5}\tan^5 x - \&c.$$

In the series No. 7, π denotes the periphery of the circle, or 3·14159265.

XI. Multiple arcs.

$$\sin 0 = 0$$
$$\sin x = \sin x$$
$$\sin 2x = 2 \sin x . \cos x$$
$$\sin 3x = 2 \sin x . \cos 2x + \sin x$$
$$\sin 4x = 2 \sin x . \cos 3x + \sin 2x$$

&c. &c. &c.

$$\cos 0 = 1$$
$$\cos x = \cos x$$
$$\cos 2x = 2 \cos x . \cos x - 1$$
$$\cos 3x = 2 \cos x . \cos 2x - \cos x$$
$$\cos 4x = 2 \cos x . \cos 3x - \cos 2x$$

&c. &c. &c.

$$\tan x = \tan x$$
$$\tan 2x = \frac{2 \tan x}{1 - \tan^2 x}$$
$$\tan 3x = \frac{\tan x + \tan 2x}{1 - \tan x . \tan 2x}$$
$$\tan 4x = \frac{\tan x + \tan 3x}{1 - \tan x . \tan 3x}$$

&c. &c. &c.

XII. For computing the Longitude, Right Ascension and Declination of the *Sun*; any one of those quantities, together with the Obliquity of the ecliptic, being given. Also for computing the angle of Position.

1 $\sin \odot = \dfrac{\sin \mathrm{D}}{\sin \omega}$

2 $\cot \odot = \cos \omega . \cot Æ$

3 $\sin Æ = \cot \omega . \tan \mathrm{D}$

4 $\tan Æ = \cos \omega . \tan \odot$

5 $\sin \mathrm{D} = \sin \omega . \sin \odot$

6 $\tan \mathrm{D} = \tan \omega . \sin Æ$

7 $\cos Æ = \dfrac{\cos \odot}{\cos \mathrm{D}}$

8 $\sin p = \sin \omega . \cos Æ$

9 $\tan p = \tan \omega . \cos \odot$

$\odot$ = the Longitude
Æ = the Right Ascension
D = the Declination: (*minus* when South)
ω = the Obliquity of the ecliptic
p = the angle of Position

XIII. For computing the Longitude and Latitude of the *Moon* or a *Star* (the Obliquity of the ecliptic being given) from the Right Ascension and Declination: and *vice versa*. Also for computing the angle of Position.

$$\text{Make } \tan a = \sin Æ . \cot D$$

$$1 \quad \tan L = \frac{\sin (a + \omega)}{\sin a} \tan Æ$$

$$2 \quad \tan l = \cot (a + \omega) \sin L$$

$$3 \quad \sin l = \frac{\cos (a + \omega)}{\cos a} \sin D$$

$$4 \quad \sin L = \tan (a + \omega) \tan l$$

$$\text{Make } \cot a = \sin L . \cot l$$

$$5 \quad \tan Æ = \frac{\cos (a + \omega)}{\cos a} \tan L$$

$$6 \quad \tan D = \tan (a + \omega) \sin Æ$$

$$7 \quad \sin D = \frac{\sin (a + \omega)}{\sin a} \sin l$$

$$8 \quad \sin Æ = \cot (a + \omega) \tan D$$

$$9 \quad \sin p = \frac{\sin \omega . \cos Æ}{\cos l} = \frac{\sin \omega . \cos L}{\cos D}$$

L = the Longitude
l = the Latitude
Æ = the Right Ascension
D = the Declination: (*minus* when South)
ω = the Obliquity of the ecliptic
p = the angle of Position

XIV. For computing the Azimuth, angle of Variation and Zenith distance of a star; the Co-latitude of the place, the north-polar distance of the star and its hour angle at the Pole, being given.

$$\tan \tfrac{1}{2}(A + V) = \frac{\cos \tfrac{1}{2}(\psi \backsim \Delta)}{\cos \tfrac{1}{2}(\psi + \Delta)} \times \cot \tfrac{1}{2} P$$

$$\tan \tfrac{1}{2}(A - V) = \frac{\sin \tfrac{1}{2}(\psi \backsim \Delta)}{\sin \tfrac{1}{2}(\psi + \Delta)} \times \cot \tfrac{1}{2} P$$

$$\tfrac{1}{2}(A + V) - \tfrac{1}{2}(A - V) = V$$

$$\tfrac{1}{2}(A + V) + \tfrac{1}{2}(A - V) = A$$

$$\sin Z = \frac{\sin \Delta}{\sin A} \times \sin P$$

A = the Azimuth, reckoned from the *north:* which must be subtracted from 180°, if reckoned from the *south.*
V = the angle of Variation
Z = the Zenith distance
P = the hour angle of the star, at the Pole
ψ = the Co-latitude of the place
Δ = the North-polar distance of the star

XV. For computing the hour angle at the Pole: the Latitude of the place, and the Declination and Zenith distance of the sun, or star, being given.

$$\sin^2 \tfrac{1}{2} P = \frac{\sin\left[\frac{Z + (L - D)}{2}\right] \times \sin\left[\frac{Z - (L - D)}{2}\right]}{\cos L \,.\, \cos D}$$

$$Æ + P = \text{the sidereal time of observation}$$

If we assume $Z = 90°$, we shall have the expression for the *semi-diurnal arc*, equal to

$$\cos P = - \tan D \,.\, \tan L$$

exclusive of the effect caused by parallax and refraction.

L = the Latitude of the place
D = the apparent Declination: (*minus* when South)
Z = the observed Zenith distance, corrected for parallax and refraction
P = the hour angle at the Pole: *plus*, when the observation is made to the *west* of the meridian; *minus*, when *east*. In the expression for the semi-diurnal arc, P is negative (and consequently greater than 90°) when the declination and latitude are both on the *same* side of the equator
Æ = the apparent Right Ascension of the sun or star; increased by 24^h if necessary

XVI. For computing the horary angle at the Pole, and the Zenith distance of a star when on the *Prime vertical*, together with its Declination, and the Latitude of the place: any two of those quantities being given.

1 $\cos P = \cot L \cdot \tan D$

2 $\sin P = \dfrac{\sin Z}{\cos D}$

3 $\cot P = \cos L \cdot \cot Z$

4 $\sin Z = \sin P \cdot \cos D$

5 $\cos Z = \dfrac{\sin D}{\sin L}$

6 $\tan Z = \cos L \cdot \tan P$

7 $\cos L = \cot P \cdot \tan Z$

8 $\sin L = \dfrac{\sin D}{\cos Z}$

9 $\cot L = \cos P \cdot \cot D$

In these cases, the star must be on the same side of the equator as the observer: and its declination must not exceed the latitude of the place.

When the declination of the star exceeds the latitude of the place, we shall have, at the moment when the *vertical becomes a tangent* to the circle of declination,

10 $\cos P = \tan L \cdot \cot D$

11 $\cos Z = \dfrac{\sin L}{\sin D}$

Z = the Zenith distance of the star
P = the horary angle of the star at the Pole
D = the Declination of the star
L = the Latitude of the place

XVII. For computing the effect of atmospheric Refraction.

1 $r = a.\tan(Z - br)$

2 $r = \cdot 99918827 \times c.\tan Z - \cdot 001105603 \times c.\tan^3 Z$

In these formulæ, Z denotes the apparent Zenith distance; and *r* is the Refraction required: *a*, *b*, *c* are constants, to be determined from observations.

No. 1 is Bradley's formula, who assumed $a = 57''$ and $b = 3$. Dr. Brinkley has proposed $a = 56'',9$ and $b = 3\cdot 2$: but Mr. Groombridge prefers $a = 58'',133$ and $b = 3\cdot 634$.

No. 2 is Laplace's formula, reduced to its most simple terms. In the formation of the French Tables of refraction, *c* is assumed equal to 60″,616; but M. Laplace has since proposed 60″,66. This latter formula will not give the correct values for greater zenith distances than 74°: at lower altitudes those Tables, as well as most others, are computed from more complex formulæ.

In the computation of Tables of refraction, a mean temperature and a mean pressure of the atmosphere are assumed. Let β denote the height of the barometer, τ the height of the thermometer (Fahr.) attached thereto, and *t* the height of the thermometer in the air, which are assumed in the formation of the tables. Then, for any other height (β') of the barometer, and for any other height (τ') and (t') of the thermometers, we have the following expression, by which the mean refraction must be multiplied, in order to obtain the true refraction: viz.

$$\frac{\beta'}{\beta} \times \frac{1}{1 + \cdot 0020833\,(t' - t)} \times \frac{1}{1 + \cdot 0001\,(\tau' - \tau)}$$

XVIII. For computing the correction in time, to be applied as an Equation to the mean of the times of observed Equal Altitudes of the sun: in order to obtain the time of its meridional passage.

$$x = \frac{\delta}{48^{h}} \times \frac{T}{30}\left(\frac{\tan D}{\tan 7\frac{1}{2}\,T} - \frac{\tan L}{\sin 7\frac{1}{2}\,T}\right)$$

$$= \delta.\tan D \, \frac{T}{1440 \tan 7\frac{1}{2}\,T} - \delta.\tan L \, \frac{T}{1440 \sin 7\frac{1}{2}\,T}$$

$$\text{Make} \quad \frac{T}{1440 \sin 7\frac{1}{2}\,T} = A$$

$$\frac{T}{1440 \tan 7\frac{1}{2}\,T} = B$$

$$x = \mp A.\delta.\tan L + B.\delta.\tan D$$

T = the interval of Time between the observations, expressed in hours

L = the Latitude of the place of observation: (*minus*, when South)

D = the Declination, at the time of noon, on the given day: (*minus*, when South)

δ = the double daily variation in the declination, deduced from the noon of the preceding day to the noon of the following day: (*minus*, when the sun is proceeding towards the South)

x = the required correction, in *seconds:* where A is to be *minus* when the time of noon is required; and *plus* when the time of midnight is required.

XIX. For computing the correction for the Reduction to the Meridian; or the correction to be applied to the Zenith distances, observed near the meridian, in order to obtain the true meridional Zenith distance.

$$x = -\frac{2\sin^2\frac{1}{2}P}{\sin 1''}\cdot\frac{\cos L.\cos D}{\sin(L-D)} + \frac{2\sin^4\frac{1}{2}P}{\sin 1''}\left(\frac{\cos L.\cos D}{\sin(L-D)}\right)^2\cot(L-D)$$

$$\text{Make } \frac{2\sin^2\frac{1}{2}P}{\sin 1''} = A \qquad \frac{2\sin^4\frac{1}{2}P}{\sin 1''} = B$$

$$x = -A\times\frac{\cos L.\cos D}{\sin(L-D)} + B\times\left(\frac{\cos L.\cos D}{\sin(L-D)}\right)^2\times\cot(L-D)$$

When the *sun* is the object observed, we must apply the following *additional* correction,

$$-\delta\times\frac{E-W}{n}$$

P = the correct horary angle of the star at the Pole, as shown by a well regulated clock; which angle will change its sign after the meridional passage of the star

L = the Latitude of the place

D = the Declination of the star: (*minus* when South)

δ = the change of declination in one minute of time: (*minus*, when decreasing)

E = the sum of the horary angles observed to the East; &

W = the sum of the horary angles observed to the West of the meridian

n = the Number of those observations

x = the required correction, in *seconds*

XX. For computing the correction for the Reduction to the Solstice; or the correction to be applied to the declination observed on days near the solstice, in order to determine the obliquity of the ecliptic from observation.

1 $$x = 2 \sin^2 \tfrac{1}{2} d\,\text{Æ} . \cos \text{D} \times \frac{\sin \omega}{\sin 1''}$$

2 $$x = \frac{2 \tan^2 \frac{1}{2} \delta}{\sin 1''} \times \tan \left(\text{D} + \frac{x}{2}\right)$$

3 $$x = \frac{(3600)^2 \sin 1'' \tan \omega}{2} \delta^2$$
$$- \frac{(3600)^4 \sin^3 1'' \tan \omega}{24} (1 + 3 \tan^2 \omega) \delta^4$$
$$+ \frac{(3600)^6 \sin^5 1'' \tan \omega}{720} (1 + 30 \tan^2 \omega + 45 \tan^4 \omega) \delta^6$$

In which latter equation, if we make $\omega = 23^\circ . 27' . 40''$ we have

$$x = 13'',63469 \delta^2 - 0'',000541699 \delta^4 + 0'',0000000289847 \delta^6$$

and the variation, on account of the diminution of every 1″ in the obliquity of the ecliptic, will be

$$- \frac{\sin 2''}{\sin 2 \omega} x = - \cdot 0000132748\, x$$

$d\,\text{Æ}$ = the distance of the sun's true Right Ascension, from the solstice, at the time of observation, converted into degrees

δ = the distance of the sun's true longitude, at the same time, converted also into *degrees*, and decimal parts of a degree

D = the observed Declination corrected for refraction, &c.

ω = the Obliquity of the ecliptic

x = the required correction, in *seconds*

XXI. For computing the Angle of the Vertical; or the angle which should be deducted from the latitude of the place, in order to obtain the *reduced* latitude on a spheroid.

$$\text{Make } a = \frac{2c - 1}{2c^2 - (2c - 1)} = \frac{1}{c} \quad \text{nearly}$$

$$x = \frac{a}{\sin 1''} \times \sin 2\,\mathrm{L} + \frac{a^2}{\sin 2''} \times \sin 4\,\mathrm{L}$$

$$= \frac{a}{\sin 1''} \times \sin 2\,\mathrm{L} \quad \text{very nearly}$$

XXII. For computing the horizontal Parallax of the Moon at any given latitude of the earth, considered as a spheroid: the horizontal parallax at the equator being given.

$$p = \mathrm{P}\left(1 - a.\sin^2\mathrm{L} + \frac{5}{2} \times a^2.\sin^2\mathrm{L}.\cos^2\mathrm{L}\right)$$

$$= \mathrm{P}\,(1 - a.\sin^2\mathrm{L}) \quad \text{very nearly.}$$

The quantity, within the parenthesis, denotes the radius of the earth at the given latitude; or the distance from the centre of the earth to that point on the earth's surface: the radius at the equator being considered as unity.

$\frac{1}{c}$ = the Compression of the earth; or the quantity by which the equatorial diameter (considered as unity) exceeds the polar

L = the Latitude of the place

x = the required Angle

P = the horizontal Parallax at the equator

p = the required horizontal Parallax at the place

XXIII. For computing the Moon's parallax in Altitude.

1 $\sin\pi = \sin p . \sin(Z + \pi)$ or

2 $\pi = p . \sin(Z + \pi)$ very nearly

3 $\tan\pi = \dfrac{\sin p . \sin Z}{1 - \sin p . \cos Z}$ or

4 $\pi = \dfrac{\sin p . \sin Z}{\sin 1''} + \dfrac{\sin^2 p . \sin 2Z}{\sin 2''} + \dfrac{\sin^3 p . \sin 3Z}{\sin 3''} + \&c.$

very nearly.

When the *apparent* zenith distance (as affected by parallax) is known, we must make use of the formula 1 or 2. But, when we know only the *true* zenith distance, we must adopt the formula 3 or 4.

The zenith distance on the meridian is $= (L - D)$.

p = the horizontal Parallax at the place
Z = the true Zenith distance of the moon
$(Z + \pi)$ = the apparent Zenith distance of the moon, as affected by parallax
π = the required Parallax

XXIV. For computing the longitude and co-latitude of the Zenith: or (as it is frequently termed) the longitude and altitude of the Nonagesimal.

$$\text{Make } \tan a = \sin S . \cot L$$

$$1 \quad \cos A = \frac{\cos(a + \omega)}{\cos a} \sin L$$

$$2 \quad \sin N = \tan(a + \omega) \cot A$$

$$3 \quad \tan N = \frac{\sin(a + \omega)}{\sin a} \tan S$$

$$4 \quad \cot A = \cot(a + \omega) \sin N$$

When S is between 80° and 100°, or between 260° and 280°, the equations 3 and 4 will be the most proper for use.

S = the Sidereal Time of observation: or the Right Ascension of the meridian, converted into degrees

L = the *reduced* Latitude of the place of observation: deduced from Formula XXI.

ω = the Obliquity of the ecliptic

N = the required longitude of the zenith, or of the Nonagesimal

A = the required co-latitude of the zenith, or Altitude of the nonagesimal

XXV. For computing the Moon's parallax in Longitude and Latitude.

1°. In Longitude.

$$\text{Make } a = \frac{\sin p . \sin A}{\cos \lambda}$$

1 $$\tan \Pi = \frac{a . \sin(☽ - N)}{1 - a . \cos(☽ - N)}$$

2 $$\Pi = \frac{a . \sin(☽ - N)}{\sin 1''} + \frac{a^2 . \sin 2(☽ - N)}{\sin 2''} + \frac{a^3 . \sin 3(☽ - N)}{\sin 3''} + \&c.$$

2°. In Latitude.

$$\text{Make } \cot b = \frac{\cos(☽ - N + \frac{1}{2}\Pi) \tan A}{\cos \frac{1}{2}\Pi}$$

$$c = \frac{\sin p . \cos A}{\sin b}$$

3 $$\tan \varpi = \frac{c . \sin(b - \lambda)}{1 - c . \cos(b - \lambda)}$$

4 $$\varpi = \frac{c . \sin(b - \lambda)}{\sin 1''} + \frac{c^2 . \sin 2(b - \lambda)}{\sin 2''} + \frac{c^3 . \sin 3(b - \lambda)}{\sin 3''} + \&c$$

☽ = the true longitude of the Moon
N = the longitude of the zenith, or the Nonagesimal
A = the co-lat. of the zenith, or the Altitude of the nonag.
λ = the Latitude of the moon: (*minus*, when South)
p = the horizontal Parallax *at the place*, by Form. XXII.
Π = the required Parallax in longitude
ϖ = the required Parallax in latitude

XXVI. For computing the Moon's parallax in Right Ascension.

$$\text{Make } a = \frac{\sin p \,.\cos \mathrm{L}}{\cos \mathrm{D}}$$

1 $\quad \sin \Pi = a.\sin(\mathrm{P} + \Pi)$

2 $\quad \Pi = \frac{p.\cos \mathrm{L}}{\cos \mathrm{D}} \times \sin(\mathrm{P} + \Pi)$ very nearly

3 $\quad \tan \Pi = \frac{a.\sin \mathrm{P}}{1 - a.\cos \mathrm{P}}$

4 $\quad \Pi = \frac{a.\sin \mathrm{P}}{\sin 1''} + \frac{a^2.\sin 2\,\mathrm{P}}{\sin 2''} + \frac{a^3.\sin 3\,\mathrm{P}}{\sin 3''} + \&c.$

When the *apparent* hour angle (as affected by parallax) is known, we must make use of the equation 1 or 2. But, when we know only P, we must adopt the equation 3 or 4.

P = the true horary angle at the Pole; or the true horary distance of the moon from the meridian

L = the *reduced* Latitude of the place of observation, by Formula XXI.

D = the true Declination of the moon: (*minus*, when South)

p = the horizontal Parallax *at the place*, by Form. XXII.

Π = the required Parallax in right ascension

XXVII. For computing the Moon's parallax in Declination.

$$\text{Make } \cot b = \frac{\cos(\mathrm{P} + \frac{1}{2}\Pi)\cot \mathrm{L}}{\cos\frac{1}{2}\Pi}$$

$$c = \frac{\sin p . \sin \mathrm{L}}{\sin b}$$

1 $\sin \varpi = c . \cos(\mathrm{D} + b - \varpi)$

2 $\varpi = \frac{p . \sin \mathrm{L}}{\sin b} \times \cos(\mathrm{D} + b - \varpi)$ very nearly

3 $\tan \varpi = \frac{c . \sin(b - \mathrm{D})}{1 - c . \cos(b - \mathrm{D})}$

4 $\varpi = \frac{c . \sin(b-\mathrm{D})}{\sin 1''} + \frac{c^2 . \sin 2(b-\mathrm{D})}{\sin 2''} + \frac{c^3 . \sin 3(b-\mathrm{D})}{\sin 3''} + \&c.$

When the apparent declination (as affected by parallax) is known, we must make use of the equation 1 or 2. But, when D only is known, we must adopt the equation 3 or 4.

P = the true horary angle at the Pole

L = the *reduced* Latitude of the place of observation, by Formula XXI.

D = the true Declination of the moon: (*minus*, when South)

p = the horizontal Parallax *at the place*, by Form. XXII.

Π = the Parallax in right ascension, by Form. XXVI.

ϖ = the required Parallax in declination

XXVIII. For computing the Augmentation of the Moon's semidiameter, on account of her altitude above the horizon: or the correction to be applied to her true semidiameter, in order to obtain her apparent semidiameter.

$$1 \quad x = s^2\,(\cdot 000017767 \sin A) + \frac{s^3}{2}\,(\cdot 000017767 \sin A)^2$$

$$2 \quad x = s \,.\, \sin \varpi \,.\, \cot\,(b - \lambda) - \frac{s}{2} \sin^2 \varpi$$

$$3 \quad x = s \,.\, \sin \varpi \,.\, \cot\,(b - D) - \frac{s}{2} \sin^2 \varpi$$

In neither of these equations will the second term ever exceed 0″,15. And in the first equation, in lieu of the second term, we may always assume it equal to the first term, multiplied by ·008379 sin A, without the risk of an error amounting to the $\frac{1}{100}$th part of a second.

s = the true Semidiameter of the moon
A = the apparent Altitude of the moon
$\varpi = \begin{cases} \text{in equation 2, the Parallax in Latitude} \\ \text{in equation 3, the Parallax in Declination} \end{cases}$
x = the correction required
$(b - \lambda)$ is determined by Formula XXV.
$(b - D)$ is determined by Formula XXVII.

XXIX. For computing the apparent distance between the centre of the Moon and of the sun, or a star, when near her.

$$\text{Make } \tan a = (M \backsim S) \times \frac{\cos \frac{1}{2}(m + s)}{m \backsim s}$$

$$d = \frac{m \backsim s}{\cos a}$$

M = the apparent longitude of the Moon

S = the apparent longitude of the Sun or Star

m = the apparent latitude of the Moon

s = the apparent latitude of the Sun or Star

d = the required apparent Distance of the centres

If we substitute the apparent Right Ascension and Declination, for the apparent Longitude and Latitude respectively, the formula will still give the correct value of d.

XXX. For computing the true place of the Moon by means of the equation of Second and Third Differences.

Series.	*1st Diff.*	*2d Diff.*	*3d Diff.*
M'			
	Δ'		
M''		d'	
	Δ''		δ'
M'''		d''	
	Δ'''		
M^{iv}			

$$M = M'' + \frac{h}{12}\Delta'' + \frac{h(h-12)}{2\,(12)^2} \times \frac{d' + d''}{2} + \frac{h(h-12)\,(h-24)}{2.3\,(12)^3}\,\delta'$$

M', M'', M''', M^{iv} = the place of the Moon (in Longitude or Latitude, Right Ascension or Declination) at four successive equal intervals, of 12 hours each, as shown by the ephemeris: and taken out so that the required place, M, may fall between M'' and M'''

Δ', Δ'', Δ''' = the successive Differences between those values

d', d'' = the successive Differences between those first differences

δ' = the Difference between those second differences

h = the number of Hours from the time, for which M'' is computed

M = the required place of the Moon

XXXI. For computing the Annual Precession of the Equinoxes in Longitude, Right Ascension and Declination. Also for computing the mean Obliquity of the Ecliptic, for any given year.

$$P = 50'',340499 - 0'',0002435890 \times y$$
$$p = 50'',176068 + 0'',0002442966 \times y$$
$$m = 45'',99592 \ + 0'',0003086450 \times y$$
$$n = 20'',05039 \ - 0'',0000970204 \times y$$
$$\mu = \ 0'',17926 \ - 0'',0005320798 \times y$$
$$\omega = 23^\circ.28'.18'',0$$
$$\omega + d\omega = 23^\circ.28'.18'',0 + 0'',0000098423 \times y^2$$
$$\omega + d\omega' = 23^\circ.28'.18'',0 - 0'',48368 \times y - 0'',00000272295 \times y^2$$

Annual Prec. in Æ $= m + n.\sin$ Æ$.\tan D$
Annual Prec. in D $= n.\cos$ Æ

P = the annual *lunisolar* Precession in longitude
p = the annual *general* Precession in longitude
ω = the mean Obliquity of the ecliptic in 1750
$d\omega$ = the variation of the angle of the *fixed* ecliptic
$d\omega'$ = the variation of the angle of the *moveable* ecliptic
m = $(P.\cos\omega - \mu)$ = the constant of the ann. prec. in Æ
n = $(P.\sin\omega)$ = the constant of the ann. prec. in D
Æ = the Right Ascension of the star
D = the Declination of the star
y = the Years from 1750: *plus* after, *negative* before
μ = the annual Motion of the equinoctial points along the equator

XXXII. For computing the Nutation of the Obliquity of the Ecliptic, and of the Equinoxes in Longitude. Also for computing the Mass of the Moon.

$$\Delta\omega = [+9'',648 \cos ☊ - 0'',09423 \cos 2 ☊ + 0'',0939 \cos 2 ☽]\,(1+z) + (0'',49333 - 1'',2452 \times z) \sin 2 \odot$$

$$\Delta L = [-18'',0377 \sin ☊ + 0'',21707 \sin 2 ☊ - 0'',21632 \sin 2 ☽]\,(1+z) - (1'',13645 - 2'',8686 \times z) \sin 2 \odot$$

$$\text{Mass of the Moon} = \frac{1+z}{69{\cdot}2376 - 178{\cdot}2918 \times z}$$

If we make $z = 0$, we shall have the values as determined by M. Laplace in the *Méchanique Céleste.* But the recent determinations of Dr. Brinkley warrant the assumption of $z = -\,{\cdot}04125$. Whence

$$\Delta\omega = +9'',250 \cos ☊ - 0'',090 \cos 2 ☊ + 0'',545 \cos 2 \odot + 0'',090 \cos 2 ☽$$

$$\Delta L = -17'',298 \sin ☊ + 0'',208 \sin 2 ☊ - 1'',255 \sin 2 \odot - 0'',207 \sin 2 ☽$$

$$\text{Mass of the Moon} = \frac{1}{79{\cdot}888}$$

☊ = the mean longitude of the moon's Node
☽ = the true longitude of the Moon
$\odot$ = the true longitude of the Sun
z = a quantity to be determined from observation
$\Delta\omega$ = the required Nutation of the Obliquity
ΔL = the required Nutation of Longitude: which is found by multiplying the first term of $\Delta\omega$ by $2 \cot 2\,\omega$, and the remaining terms by $\cot \omega$, then changing the signs of the quantities, and converting the cosines into sines.

XXXIII. For computing the Lunar and Solar Nutation of a star, in Right Ascension and Declination.

$$\text{Nut. Æ} = \Delta L(\cos\omega + \sin\omega . \sin \text{Æ} . \tan D) - \Delta\omega . \cos \text{Æ} . \tan D$$

$$\text{Nut. } D = \Delta L . \sin\omega . \cos \text{Æ} + \Delta\omega . \sin \text{Æ}$$

By assuming $\omega = 23°. 27'. 40''$, and the values of $\Delta\omega$ and ΔL as determined in Formula XXXII, we have

$$\begin{aligned} \text{Nut. Æ} = & - (15'',868 + 6'',887 \sin \text{Æ} . \tan D) \sin ☊ \\ & - 9'',250 \cos \text{Æ} . \tan D . \cos ☊ \\ & + (0'',191 + 0'',083 \sin \text{Æ} . \tan D) \sin 2 ☊ \\ & + 0'',090 \cos \text{Æ} . \tan D . \cos 2 ☊ \\ & - (1'',151 + 0'',500 \sin \text{Æ} . \tan D) \sin 2 ☉ \\ & - 0'',545 \cos \text{Æ} . \tan D . \cos 2 ☉ \\ & - (0'',190 + 0'',082 \sin \text{Æ} . \tan D) \sin 2 ☽ \\ & - 0'',090 \cos \text{Æ} . \tan D . \cos 2 ☽ \end{aligned}$$

$$\begin{aligned} \text{Nut. D} = & + 9'',250 \sin \text{Æ} . \cos ☊ - 6'',887 \cos \text{Æ} . \sin ☊ \\ & - 0'',090 \sin \text{Æ} . \cos 2 ☊ + 0'',083 \cos \text{Æ} . \sin 2 ☊ \\ & + 0'',545 \sin \text{Æ} . \cos 2 ☉ - 0'',500 \cos \text{Æ} . \sin 2 ☉ \\ & + 0'',090 \sin \text{Æ} . \cos 2 ☽ - 0'',082 \cos \text{Æ} . \sin 2 ☽ \end{aligned}$$

ω = the Obliquity of the ecliptic
$\Delta\omega$ = the nutation of the obliq. of Ecliptic } by Form.
ΔL = the nutation in Longitude } XXXII.
Æ = the Right Ascension of the star
D = the Declination of the star: (*minus* when South)
☊ = the mean longitude of the moon's Node
☉ = the true longitude of the Sun
☽ = the true longitude of the Moon

XXXIV. For computing the Aberration of a star in Longitude and Latitude; and in Right Ascension and Declination.

$$\text{Aber. in Lon.} = -A \cdot \cos(\odot - L) \cdot \sec l$$

$$\text{Aber. in Lat.} = -A \cdot \sin(\odot - L) \cdot \sin l$$

$$\text{Aber. in Æ} = -A(\sin \text{Æ} \cdot \sin\odot + \cos\omega \cdot \cos\text{Æ} \cdot \cos\odot)\sec D$$

$$\text{Aber. in D} = -A(\cos\text{Æ} \cdot \sin\odot - \cos\omega \cdot \sin\text{Æ} \cdot \cos\odot)\sin D - A \cdot \sin\omega \cdot \cos\odot \cdot \cos D$$

If we assume $A = 20'',36$, and $\omega = 23^\circ.\ 27'.\ 40''$, we have

$$\text{Aber. Æ} = -(20'',36 \sin\text{Æ} \cdot \sin\odot + 18'',677 \cos\text{Æ} \cdot \cos\odot)\sec D$$

$$\text{Aber. D} = -(20'',36 \cos\text{Æ} \cdot \sin\odot - 18'',677 \sin\text{Æ} \cdot \cos\odot)\sin D - 8'',106 \cos\odot \cdot \cos D$$

$$\text{Diurnal Aber. in} \begin{cases} \text{Æ} = 0'',309 \cos\phi \cdot \cos P \cdot \sec D \\ D = 0'',309 \cos\phi \cdot \sin P \cdot \sin D \end{cases}$$

$\odot$ = the true longitude of the Sun
L = the Longitude of the star
l = the Latitude of the star: (*minus* when South)
Æ = the Right Ascension of the star
D = the Declination of the star: (*minus* when South)
ω = the Obliquity of the ecliptic
A = the constant of Aberration: which has been usually assumed equal to $20'',255$; but more recently $= 20'',36$
ϕ = the Latitude of the place
P = the hour angle at the Pole

XXXV. For computing the corrections to be applied to the observed transit of a star on account of the error of the Clock, and on account of the three principal errors of the transit instrument, in Azimuth, in the Inclination of the axis, and in Collimation; in order to obtain the correct apparent Right Ascension.

$$Æ = (T + dt) + a.\frac{\sin(\phi - \delta)}{\cos\delta} + b.\frac{\cos(\phi - \delta)}{\cos\delta} + \frac{c}{\cos\delta}$$

T = the observed Time of transit, as shown by the clock

dt = the correction for the error of the Clock: *plus* when the clock is too *slow*

ϕ = the Latitude of the place

δ = the Declination of the star: *plus* when North, and *minus* when South, for the upper culminations; and *vice versa* for the lower culminations

a = the deviation of the telescope in Azimuth: *plus*, when (pointing to the south) the vertical which it describes falls to the east; and *minus*, when it falls to the west: and *vice versa* when pointing to the north

b = the Bias or inclination of the axis of the telescope: *plus*, when the west end of the axis is too high

c = the error in Collimation: *plus*, when the vertical, described by the optical axis of the telescope (pointing to the south), falls to the east; and *minus*, when it falls to the west: and *vice versa* when pointing to the north

Æ = the apparent Right Ascension required

XXXVI. For computing the value (in *time*) of the coefficients a, b, c, in the preceding Formula.

$$b = \frac{d}{60}\left[(w+w') - (e+e')\right]$$

where w' and e' denote respectively the values of w and e, after *reversing* the *level.*

$$c = \tfrac{1}{2}(t'-t)\cos D + \tfrac{1}{2}(b'-b)\cos(\phi - D)$$

where t' and b' denote respectively the values of t and b, after *reversing* the *instrument.*

By observations of a circumpolar star,

$$a = \frac{12^{h} - (T'-T)}{2\cos\phi \,.\, \tan D} + \frac{b\,.\cos(\phi - D) - b'\,.\cos(\phi + D) + 2c}{2\cos\phi \,.\, \sin D}$$

where T′ and b' denote respectively the values of T and b, at the *lower* culmination.

By observations of a high and low star,

$$a = \left[(T'-T) - (Æ R' - Æ R)\right] \times \frac{\cos\delta' \,.\, \cos\delta}{\cos\phi \,.\, \sin(\delta' - \delta)}$$

where T′, ÆR′ and δ' denote respectively the values of T, ÆR, and δ of the *second* star observed.

d = the value of each Division of level, in seconds of space
w = the inclination of the level to the West
e = the inclination of the level to the East
D = the Declination of the circumpolar star
t = the time of the Transit of the circumpolar star, deduced from an obs. at a given *side* wire of the instrument.

The other quantities are the same as in Formula XXXV.

The values of a and c, when once determined, may be afterwards re-examined and corrected by means of a well divided meridian mark. If the level of the axis be well adjusted, the quantities depending on b and b' may be safely neglected.

XXXVII. For computing the Latitude of a place, from observations of the altitude of a star near the pole, at any time of the day.

$$\text{Make } \tan a = \tan \Delta . \cos P$$

$$\sin(\varphi + a) = \frac{\cos a}{\cos \Delta} \times \sin A$$

$$\varphi = (\varphi + a) - a$$

For a *fixed* observatory, we have

$$\varphi = A - \Delta . \cos P + \frac{\Delta^2}{2} . \sin^2 P . \tan \varphi$$

$$+ \frac{\Delta^3}{2} . \sin^2 P . \cos P . \tan^2 \varphi + \frac{\Delta^3}{6} . \sin^2 P . \cos P$$

$$\begin{aligned} \text{Make } \alpha &= - \Delta . \cos P \\ \beta &= + \Delta^2 . \sin^2 P . \tan \varphi . \tfrac{1}{2} \sin 1'' \\ \gamma &= + (\tfrac{1}{3} \cot \varphi + \tan \varphi) \sin 1'' \end{aligned}$$

$$\varphi = A + \alpha + \beta + \alpha\beta\gamma$$

As the latitude is always supposed to be very nearly known in a fixed observatory, this series will be found very convenient; in as much as not only γ, but also the multiplier $(\tan \varphi . \frac{1}{2} \sin 1'')$, may be considered as *constant* quantities.

Δ = the apparent north polar Distance of the star
P = the correct hour angle at the Pole
A = the observed Altitude, corrected for refraction
φ = the Latitude required

XXXVIII. For computing the Difference in the Heights of two places, by means of the Barometer.

$$x = c\left[1+\tfrac{1}{2}(t+t'-64^\circ)a\right]\times\left[1+\tfrac{1}{2}g.\cos 2\varphi\right]\times\left[1+\frac{2h}{r}\right]$$
$$\times\left[\left(1+\frac{x}{r+h}\right).\log\frac{\beta}{\beta'\left[1+(\tau-\tau')m\right]}+\frac{x}{r+h}\times 2\,\mathrm{M}\right]$$

By expanding the last term, agreeably to the method adopted by M. Biot, and assuming the numerical values for the several quantities as below, we have

$$x = 60345{\cdot}51\ \times\left[1+{\cdot}00111111\,(t+t'-64^\circ)\right]$$
$$\times \log.\text{ of}\left[\frac{\beta}{\beta'}\times\frac{1}{1+{\cdot}0001\,(\tau-\tau')}\right]\times\left[1\ +\ {\cdot}002695\cos 2\varphi\right]$$

c = a Constant determined by observation; and which is here assumed equal to 60158·53 English feet
g = the incr. of Grav. from equator to the poles, = ·00539
φ = the latitude of the Place
β = the height of the Barometer } at the lower station.
τ = the Temperature (Fahr.) of the mercury } at the lower station.
t = the Temperature (Fahr.) of the air } at the lower station.
β' = the height of the Barometer } at the upper station.
τ' = the Temperature (Fahr.) of the mercury } at the upper station.
t' = the Temperature (Fahr.) of the air } at the upper station.
a = the expansion of *moist* Air for 1° Fahr. = ·0022222
m = the expansion of Mercury for 1° Fahr. = ·0001001
r = Radius of the earth at φ: assumed = 20898240 feet
h = the Height of the lower station above the level of the sea: assumed = 4000 feet
M = the Modulus of the common logarithms = ·434294
x = the Difference required, in English feet

XXXIX. For computing the increase of Gravity from the equator to the poles, and thence the Compression of the earth, from the difference in the lengths of two isochronous Pendulums, at different latitudes.

$$P = \varpi + (\Pi - \varpi)\sin^2 L = \varpi(1 + g.\sin^2 L)$$

$$p = \varpi + (\Pi - \varpi)\sin^2 l = \varpi(1 + g.\sin^2 l)$$

$$(\Pi - \varpi) = \frac{P - p}{\sin(L - l).\sin(L + l)}$$

$$g = \frac{\Pi - \varpi}{\varpi} = \frac{P - p}{p.\sin^2 L - P.\sin^2 l}$$

$$\frac{1}{c} = \frac{5f}{2} - g$$

If we assume $f = \frac{1}{289}$, we have

$$\frac{1}{c} = \cdot 00865052 - g \qquad \text{and } c = \frac{578}{5 - 578 \times g}$$

f = the ratio of the centrifugal Force at the equator, to the force of gravity there
L = the Latitude of the place farthest from the equator
l = the Latitude of the place nearest to the equator
P = the length of the Pendulum at the latitude L
p = the length of the Pendulum at the latitude l
Π = the length of the Pendulum at the pole
ϖ = the length of the Pendulum at the equator
g = the required increase of Gravity
$\frac{1}{c}$ = the required Compression of the earth

XL. For computing the increase of Gravity from the equator to the poles, and thence the Compression of the earth, from the difference in the number of Vibrations made in equal times, by an invariable pendulum, at different latitudes.

$$g = \frac{2n}{N} \times \frac{1}{\sin(L - l) . \sin(L + l)} + g^2 . \sin^2 l$$

$$n = \frac{N}{2} \times g . \sin(L - l) . \sin(L + l) \times (1 - g . \sin^2 l)$$

N = the Number of vibrations in a day, made by the invariable pendulum, at a given latitude l

n = the *additional* Number of vibrations in a day, made by the same pendulum, and in the same time, at any other latitude L, *greater* than l

g = the required increase of Gravity. The last term in this equation will never exceed ·00003

The other quantities are the same as in Formula XXXIX. and the Compression is found in a similar manner.

In all these cases it is presumed that the vibrations of the pendulum are made at a given temperature, and at a given height of the barometer; that the arc described by the pendulum is infinitely small: and that the experiments are made *in vacuo* and at the level of the sea. Or that they are reduced to these several standards, by certain known methods of reduction, which will be explained in the next Formula.

XLI. For computing the corrections to be applied to the number of vibrations of an invariable pendulum, on account of the amplitude of the Arc, the Rate of the clock, the Expansion of the pendulum, the pressure of the Atmosphere, and the Height of the place above the level of the sea.

$$\text{for Arc} = + \text{N} . \frac{\sin(\text{A}+a).\sin(\text{A}-a)}{32\,\text{M}.(\log\sin\text{A} - \log\sin a)}$$

$$\text{for Rate} = + \text{N} . \frac{r}{86400'' + r}$$

$$\text{for Expan.} = + \text{N} . \tfrac{1}{2} e (t' - \theta)$$

$$\text{for Atm.} = + \text{N} . \frac{1}{2\left(\frac{\text{G}}{g} - 1\right)} \times \frac{\beta'}{\beta} \times \frac{1}{1 + \cdot 002083\,(t' - t)} \times \frac{1}{1 + \cdot 0001\,(\tau' - t)}$$

$$\text{for Height} = + \text{N} . \frac{h}{\text{R}} \times x$$

N = the Numb. of vibrations in 24^{h}, as shown by the clock
A = the semi-Arc of vibration at the commencement
a = the semi-Arc of vibration at the end
M = the logarithmic Modulus = 2·302585093
r = the Rate of the clock: *minus*, if losing
β = the height of the Barometer } assumed as standards
t = the Temp. (Fahr.) of the air } for the specific gravity
θ = the Temp. (Fahr.) assumed as a standard for the pend.
β' = the height of the Barometer }
t' = the Temperature of the air } during the vibrations
τ' = the Temp. of the mercury }
G = the spec. Grav. of the pend. } compared with water
g = the specific Grav. of the air } considered as unity
h = the Height of the place, above the level of the sea
R = the Radius of the earth, at the latitude of the place
x = a quantity determined from theory: assumed by Dr. Young, from ·50 to ·75

XLII. For computing the principal geodetical quantities, depending on the spheroidical figure of the earth, at any given latitude.

Ellipticity of the earth . . .	$e=\left(\frac{a^2-b^2}{a^2}\right)^{\frac{1}{2}}=\left(\frac{2}{c}-\frac{1}{c^2}\right)^{\frac{1}{2}}$
Normal, ending at minor axis	$n=\frac{a}{(1-e^2.\sin^2 L)^{\frac{1}{2}}}$
Normal, ending at major axis	$N=n(1-e^2)$
Tangent, ending at minor axis	$t=n.\cot L$
Tangent, ending at major axis	$T=n.\tan L.(1-e^2)$
Radius of the parallel . . .	$\varrho=n.\cos L$
Radius of curvature of merid.	$R=\frac{n^3}{a^2}(1-e^2)$
Radius of the earth . . .	$r=n(1-e^2).(1-e^2.\sin^2 L)$
Length of arc of meridian .	$dM=n.(1-e^2)dL=N.dL$
Do. perpendicular to do. . .	$dP=n.dL$

a = the semi-axis major of the earth

b = the semi-axis minor of the earth

$\frac{1}{c}$ = the Compression of the earth $=1-\frac{b}{a}$

L = the given Latitude

XLIII. For computing the length of a degree of Longitude and Latitude at any point on the surface of the Earth, considered as an ellipsoid; the length of a degree at the equator being considered as unity, and the Compression of the earth being given.

$$\text{Assume } e^2 = \frac{2}{c}\left(1 - \frac{1}{2c}\right) \qquad = \frac{2}{c} \text{ nearly}$$

$$\frac{1}{c} = \tfrac{1}{2}e^2 + \tfrac{1}{8}e^4 + \&c. \qquad = \frac{e^2}{2} \text{ nearly}$$

$$\text{Deg. of Long.} = \cos L\,(1 + \tfrac{1}{2}e^2 \sin^2 L + \tfrac{3}{8}e^4 \sin^4 L + \&c.)$$

$$\text{Deg. of Lat.} = 1 + \tfrac{3}{2}e^2 \sin^2 L + \tfrac{15}{4}e^4 \sin^4 L + \&c.$$

e = the Eccentricity of the ellipse forming the ellipsoid

$\frac{1}{c}$ = the Compression of the earth

L = the Latitude of the given point, which is assumed to be equidistant from the ends of the degree of Latitude required

XLIV. For computing the Eccentricity and Compression of the Earth, from the lengths of two measured arcs of the meridian, differing from each other in latitude.

$$e^2 = \frac{d^{\frac{2}{3}} - D^{\frac{2}{3}}}{d^{\frac{2}{3}} \sin^2 l - D^{\frac{2}{3}} \sin^2 L} \qquad \text{nearly}$$

$$c = 2 \times \frac{d^{\frac{2}{3}} \sin^2 l - D^{\frac{2}{3}} \sin^2 L}{d^{\frac{2}{3}} - D^{\frac{2}{3}}} \qquad \text{nearly}$$

l = the degree of Latitude nearest to the equator
L = the degree of Latitude farthest from the equator
d = the measure of a Degree, of which l is the middle point
D = the measure of a Degree, of which L is the middle point
e = the required Eccentricity of the ellipse, forming the ellipsoid
$\frac{1}{c}$ = the required Compression of the earth

In these formulæ, the values of L and l denote respectively the latitude of the *middle* points of the degrees in question.

XLV. For computing the Equations depending on the theory of the elliptical motion of the planets.

$$a^2 = b^2 + e^2$$

$$m = x + \frac{e}{a}.\sin x$$

$$\tan^2 \tfrac{1}{2}\, t = \frac{a - e}{a + e} \tan^2 \tfrac{1}{2}\, x$$

$$r = \frac{a^2 - e^2}{a - e.\cos t} = a + e.\cos x = b.\frac{\sin x}{\sin t}$$

a = the semi-axis major of the orbit; generally assumed equal to unity: in which case, b, e and r must be taken proportional thereto

b = the semi-axis minor of the orbit

e = the Eccentricity of the orbit

r = the Radius vector of the planet

m = the Mean anomaly } reckoned from the perihelion

t = the True anomaly } reckoned from the perihelion

x = the Eccentric anomaly = $(m - t)$. The determination of this quantity, from which the others are deduced, is one of the most difficult in the science of astronomy: and can only be obtained by approximation. It goes by the name of *Kepler's Problem*

XLVI. For computing the greatest Equation of the centre, the Eccentricity being given: and *vice versa*. Also for computing, *at that time*, the eccentric anomaly; and thence, the true and the mean anomaly.

$$E = 2e + \frac{11}{2^4 . 3} e^3 + \frac{599}{2^{10} . 5} e^5 + \frac{17219}{2^{16} . 7} e^7 + \&c.$$

$$e = \tfrac{1}{2} E - \frac{11}{2^4 . 3} E^3 - \frac{587}{2^{16} . 3 . 5} E^5 - \frac{40583}{2^{23} . 5 . 7 . 9} E^7 - \&c.$$

$$\cos x = -\tfrac{1}{4} e - \tfrac{1}{4} \cdot \tfrac{3}{8} e^3 - \tfrac{1}{4} \cdot \tfrac{3}{8} \cdot \tfrac{7}{12} e^5 - \tfrac{1}{4} \cdot \tfrac{3}{8} \cdot \tfrac{7}{12} \cdot \tfrac{11}{16} e^7 - \&c.$$

$$\tan^2 \tfrac{1}{2} x = \frac{1 - \cos x}{1 + \cos x}$$

$$\tan^2 \tfrac{1}{2} t = \frac{1 - e}{1 + e} \tan^2 \tfrac{1}{2} x$$

$$m = x + e . \sin x$$

E = the greatest Equation of the centre, $= m - t$

e = the Eccentricity of the orbit, the semi-axis major being considered as unity

x = the Eccentric anomaly

m = the Mean anomaly

t = the True anomaly

XLVII. For computing the angular distance between the centre of the Moon and of the Sun (or a Star) their Right Ascensions and Declinations, or their Longitudes and Latitudes being given.

$$\cot a' = \cos (S \backsim M) . \cot D$$

$$a'' = d - a'$$

$$\cos x = \sin D . \frac{\cos a''}{\cancel{\cos a''}\ \sin a'}$$

S = the Right Ascension of the sun
M = the Right Ascension of the moon
D = the Declination of the moon
d = the Declination of the sun, or star
x = the angular distance required

If we substitute the Longitude and Latitude for the Right Ascension and Declination of the Moon and Sun (or Star) respectively, the formula will still give the correct value of x.

XLVIII. To determine the *true* distance of the Moon from the Sun (or a Star), the *apparent* distance, together with the *apparent* altitudes of the Moon and the Sun (or Star) being given.

$$\text{Make } \beta = \tfrac{1}{2}(\Delta + \text{H} + h)$$

$$\sin a = \frac{\left(\cos\beta.\cos(\beta - \Delta)\,\dfrac{\cos \text{H}'.\cos h'}{\cos \text{H}.\cos h}\right)^{\frac{1}{2}}}{\cos\tfrac{1}{2}(\text{H}' + h')}$$

$$\sin\tfrac{1}{2}x = \cos\tfrac{1}{2}(\text{H}' + h')\cos a$$

H = the apparent } altitude of the Moon
H' = the true } altitude of the Moon
h = the apparent } altitude of the Sun, or Star
h' = the true } altitude of the Sun, or Star
Δ = the apparent Distance
x = the true distance required

N.B. The *true* altitude is easily deduced from the *apparent* altitude, by subtracting the refraction, and (in the case of the Moon and Sun) adding the parallax in altitude.

XLIX. The rule of the *signs* in the algebraic expressions of the circular functions.

	0°	For an angle a between 0° and 90°	90°	For an angle a between 90° and 180°	180°	For an angle a between 180° and 270°	270°	For an angle a between 270° and 360°	360°
Sine a	0	$+\sin a$	+1	$+\sin(180^\circ-a)$ $+\cos(a-90^\circ)$	0	$-\sin(a-180^\circ)$ $-\cos(270^\circ-a)$	−1	$-\sin(360^\circ-a)$ $-\cos(a-270^\circ)$	0
Cos a	+1	$+\cos a$	0	$-\cos(180^\circ-a)$ $-\sin(a-90^\circ)$	−1	$-\cos(a-180^\circ)$ $-\sin(270^\circ-a)$	0	$+\cos(360^\circ-a)$ $+\sin(a-270^\circ)$	+1
Tan a	0	$+\tan a$	∞	$-\tan(180^\circ-a)$ $-\cot(a-90^\circ)$	0	$+\tan(a-180^\circ)$ $+\cot(270^\circ-a)$	∞	$-\tan(360^\circ-a)$ $-\cot(a-270^\circ)$	0
Cot a	∞	$+\cot a$	0	$-\cot(180^\circ-a)$ $-\tan(a-90^\circ)$	∞	$+\cot(a-180^\circ)$ $+\tan(270^\circ-a)$	0	$-\cot(360^\circ-a)$ $-\tan(a-270^\circ)$	∞
Sec a	+1	$+\sec a$	∞	$-\sec(180^\circ-a)$ $-\text{cosec}(a-90^\circ)$	−1	$-\sec(a-180^\circ)$ $-\text{cosec}(270^\circ-a)$	∞	$+\sec(360^\circ-a)$ $+\text{cosec}(a-270^\circ)$	+1
CoSec a	∞	$+\text{cosec}\ a$	+1	$+\text{cosec}(180^\circ-a)$ $+\sec(a-90^\circ)$	∞	$-\text{cosec}(a-180^\circ)$ $-\sec(270^\circ-a)$	−1	$-\text{cosec}(360^\circ-a)$ $-\sec(a-270^\circ)$	∞
Ver-sin a	0	$+\text{versin}\ a$	+1	$2-\text{versin}(180^\circ-a)$ $2-\text{coversin}(a-90^\circ)$	+2	$2-\text{versin}(a-180^\circ)$ $2-\text{coversin}(270^\circ-a)$	+1	$+\text{versin}(360^\circ-a)$ $+\text{coversin}(a-270^\circ)$	0
Co-Versin a	+1	$+\text{coversin}\ a$	0	$+\text{coversin}(180^\circ-a)$ $+\text{versin}(a-90^\circ)$	+1	$2-\text{versin}(270^\circ-a)$ $2-\text{coversin}(a-180^\circ)$	+2	$2-\text{coversin}(a-270^\circ)$ $2-\text{versin}(360^\circ-a)$	+1

TABLES.

atitude and Longitude of various places, where astronomical observations have been made.

Places.		Latitude.	Longitude.
		° ′ ″	h m s
Abo	Obs.	+ 60 27 0	− 1 29 10
Alexandria		+ 31 13 5	− 1 59 41
Altona	Obs.	+ 53 32 51	− 0 39 50
Amsterdam		+ 52 22 17	− 0 19 33
Archangel		+ 64 34 0	− 2 42 52
Bagdad		+ 33 19 40	− 2 57 39
Barcelona		+ 41 21 44	+ 0 8 40
Berlin	Obs.	+ 52 31 45	− 0 53 29
Bordeaux		+ 44 50 14	+ 0 2 16
Breslau		+ 51 6 30	− 1 8 9
Brussels		+ 50 50 59	− 0 17 29
Buda		+ 47 29 44	− 1 16 10
Bushey Heath	Obs.	+ 51 37 44	+ 0 1 21
Cadiz		+ 36 32 0	+ 0 25 9
Cairo		+ 30 3 21	− 2 5 13
Calcutta		+ 22 34 15	− 5 53 44
Cambridge	Obs.	+ 52 12 43	− 0 0 30
Cape of Good Hope	Obs.	− 33 55 42	− 1 13 32
Coimbra		+ 40 12 30	+ 0 33 38
Constantinople		+ 41 1 27	− 1 55 41
Copenhagen		+ 55 41 4	− 0 50 20
Dantzig		+ 54 20 48	− 1 14 31
Dijon		+ 47 19 25	− 0 20 8
Dorpat	Obs.	+ 58 22 47	− 1 46 48
Dublin	Obs.	+ 53 23 13	+ 0 25 22
Edinburgh	Obs.	+ 55 56 42	+ 0 12 49
Florence		+ 43 46 41	− 0 45 3
Geneva	Obs.	+ 46 12 0	− 0 24 38
Genoa		+ 44 25 0	− 0 35 52
Gotha	Obs.	+ 50 56 8	− 0 42 56
Gottingen	Obs.	+ 51 31 50	− 0 39 46
Greenwich	Obs.	+ 51 28 40	0 0 0
Kew	Obs.	+ 51 28 37	+ 0 1 3
Königsberg	Obs.	+ 54 42 12	− 1 21 57

Places.		Latitude.	Longitude.
		° ′ ″	h m s
Lilienthal		+ 53 8 30	− 0 35 37
Leghorn		+ 43 33 5	− 0 41 7
Lisbon	Obs.	+ 38 42 24	+ 0 36 16
London (St. Paul's)		+ 51 30 49	+ 0 0 23
Madras (Flagstaff)		+ 13 5 0	− 5 21 28
Madrid		+ 40 24 57	+ 0 15 9
Manheim	Obs.	+ 49 29 18	− 0 33 52
Marseilles	Obs.	+ 43 17 49	− 0 21 29
Mexico		+ 19 25 45	+ 6 36 21
Milan	Obs.	+ 45 28 2	− 0 36 46
Montauban	Obs.	+ 44 0 55	− 0 5 23
Naples		+ 40 50 15	− 0 57 3
Oxford	Obs.	+ 51 45 39	+ 0 5 1
Palermo	Obs.	+ 38 6 44	− 0 53 28
Paramatta	Obs.	− 33 48 45	−10 4 5
Paris	Obs.	+ 48 50 14	− 0 9 21
Pekin	Obs.	+ 39 54 13	− 7 45 51
Petersburg		+ 59 56 23	− 2 1 15
Philadelphia		+ 39 56 55	+ 5 0 46
Prague		+ 50 5 19	− 0 57 41
Quebec		+ 46 47 30	+ 4 44 39
Quito		− 0 13 17	+ 5 15 0
Rome		+ 41 53 54	− 0 49 59
Slough	Obs.	+ 51 30 20	+ 0 2 24
Stockholm		+ 59 20 31	− 1 12 14
Toulouse		+ 43 35 46	− 0 5 46
Tubingen	Obs.	+ 48 31 10	− 0 36 14
Turin		+ 45 4 0	− 0 30 41
Uraniburg	Obs.	+ 55 54 38	− 0 50 52
Verona	Obs.	+ 45 26 7	− 0 44 5
Vienna	Obs.	+ 48 12 40	− 1 5 31
Viviers	Obs.	+ 44 29 14	− 0 18 44
Wilna		+ 54 41 2	− 1 41 12

In the column of Latitudes the sign *plus* denotes North; and the sign *minus*, South. In the column of Longitudes the sign *plus* denotes West; and the sign *minus*, East, from Greenwich.

Mean Right Ascension of the Sun.

	January.	☊	February.	☊	March.	☊
	h m s		h m s		h m s	
1	18 40 0,000	0	20 42 13,215	5	22 32 36,765	9
2	43 56,555	0	46 9,771	5	36 33,320	9
3	47 53,111	0	50 6,326	5	40 29,875	9
4	51 49,666	0	54 2,881	5	44 26,431	9
5	55 46,221	1	57 59,437	5	48 22,986	9
6	59 42,777	1	21 1 55,992	5	52 19,541	9
7	19 3 39,332	1	5 52,547	5	56 16,097	10
8	7 35,887	1	9 49,103	6	23 0 12,652	10
9	11 32,443	1	13 45,658	6	4 9,207	10
10	15 28,998	1	17 42,213	6	8 5,763	10
11	19 25,553	1	21 38,769	6	12 2,318	10
12	23 22,109	2	25 35,324	6	15 58,873	10
13	27 18,664	2	29 31,879	6	19 55,429	10
14	31 15,219	2	33 28,435	6	23 51,984	11
15	35 11,775	2	37 24,990	7	27 48,539	11
16	39 8,330	2	41 21,545	7	31 45,095	11
17	43 4,885	2	45 18,101	7	35 41,650	11
18	47 1,441	2	49 14,656	7	39 38,205	11
19	50 57,996	3	53 11,211	7	43 34,761	11
20	54 54,551	3	57 7,767	7	47 31,316	11
21	58 51,107	3	22 1 4,322	7	51 27,871	12
22	20 2 47,662	3	5 0,877	8	55 24,427	12
23	6 44,217	3	8 57,433	8	59 20,982	12
24	10 40,773	3	12 53,988	8	0 3 17,537	12
25	14 37,328	4	16 50,543	8	7 14,093	12
26	18 33,883	4	20 47,099	8	11 10,648	12
27	22 30,439	4	24 43,654	8	15 7,203	12
28	26 26,994	4	28 40,209	9	19 3,759	13
29	30 23,549	4			23 0,314	13
30	34 20,105	4			26 56,869	13
31	38 16,660	4			30 53,425	13

In Leap years, enter the table, in January and February, with the given date, *minus* unity.

	April.	☊	May.	☊	June.	☊
	h m s		h m s		h m s	
1	0 34 49,980	13	2 33 6,640	18	4 35 19,855	21
2	38 46,535	13	37 3,195	18	39 16,410	21
3	42 43,091	14	40 59,750	18	43 12,966	21
4	46 39,646	14	44 56,306	18	47 9,521	22
5	50 36,201	14	48 52,861	18	51 6,076	22
6	54 32,757	14	52 49,416	18	55 2,632	22
7	58 29,312	14	56 45,972	18	58 59,187	22
8	1 2 25,867	14	3 0 42,527	19	5 2 55,742	22
9	6 22,422	14	4 39,082	19	6 52,298	22
10	10 18,978	15	8 35,638	19	10 48,853	22
11	14 15,533	15	12 32,193	19	14 45,408	23
12	18 12,088	15	16 28,748	19	18 41,964	23
13	22 8,644	15	20 25,304	19	22 38,519	23
14	26 5,199	15	24 21,859	19	26 35,074	23
15	30 1,754	15	28 18,414	20	30 31,630	23
16	33 58,310	15	32 14,970	20	34 28,185	23
17	37 54,865	16	36 11,525	20	38 24,740	23
18	41 51,420	16	40 8,080	20	42 21,296	24
19	45 47,976	16	44 4,636	20	46 17,851	24
20	49 44,531	16	48 1,191	20	50 14,406	24
21	53 41,086	16	51 57,746	20	54 10,962	24
22	57 37,642	16	55 54,302	21	58 7,517	24
23	2 1 34,197	16	59 50,857	21	6 2 4,072	24
24	5 30,752	17	4 3 47,412	21	6 0,628	24
25	9 27,308	17	7 43,968	21	9 57,183	25
26	13 23,863	17	11 40,523	21	13 53,738	25
27	17 20,418	17	15 37,078	21	17 50,294	25
28	21 16,974	17	19 33,634	21	21 46,849	25
29	25 13,529	17	23 30,189	21	25 43,404	25
30	29 10,084	17	27 26,744	21	29 39,960	25
31			31 23,300	21		

	July.	☊	August.	☊	September.	☊
	h m s		h m s		h m s	
1	6 33 36,515	25	8 35 49,731	30	10 38 2,946	35
2	37 33,070	26	39 46,286	30	41 59,501	35
3	41 29,626	26	43 42,841	30	45 56,057	35
4	45 26,181	26	47 39,397	30	49 52,612	35
5	49 22,736	26	51 35,952	31	53 49,167	35
6	53 19,292	26	55 32,507	31	57 45,723	35
7	57 15,847	26	59 29,063	31	11 1 42,278	35
8	7 1 12,402	27	9 3 25,618	31	5 38,833	36
9	5 8,958	27	7 22,173	31	9 35,389	37
10	9 5,513	27	11 18,729	31	13 31,944	37
11	13 2,068	27	15 15,284	32	17 28,499	37
12	16 58,624	27	19 11,839	32	21 25,055	37
13	20 55,179	27	23 8,395	32	25 21,610	37
14	24 51,734	27	27 4,950	32	29 18,165	38
15	28 48,290	28	31 1,505	32	33 14,721	38
16	32 44,845	28	34 58,061	32	37 11,276	38
17	36 41,400	28	38 54,616	32	41 7,831	38
18	40 37,956	28	42 51,171	33	45 4,387	38
19	44 34,511	28	46 47,727	33	49 0,942	38
20	48 31,067	28	50 44,282	33	52 57,497	38
21	52 27,622	28	54 40,837	33	56 54,053	39
22	56 24,177	29	58 37,393	33	12 0 50,608	39
23	8 0 20,733	29	10 2 33,948	33	4 47,163	39
24	4 17,288	29	6 30,503	33	8 43,719	39
25	8 13,843	29	10 27,059	34	12 40,274	39
26	12 10,399	29	14 23,614	34	16 36,829	39
27	16 6,954	29	18 20,169	34	20 33,385	39
28	20 3,509	29	22 16,725	34	24 29,940	40
29	24 0,065	30	26 13,280	34	28 26,495	40
30	27 56,620	30	30 9,835	34	32 23,051	40
31	31 53,175	30	34 6,391	34		

	October.	☊	November.	☊	December.	☊
	h m s		h m s		h m s	
1	12 36 19,606	40	14 38 32,821	45	16 36 49,481	49
2	40 16,161	40	42 29,377	45	40 46,037	49
3	44 12,717	40	46 25,932	45	44 42,592	49
4	48 9,272	40	50 22,487	45	48 39,147	49
5	52 5,827	41	54 19,043	45	52 35,703	50
6	56 2,383	41	58 15,598	45	56 32,258	50
7	59 58,938	41	15 2 12,153	45	17 0 28,813	50
8	13 3 55,493	41	6 8,709	46	4 25,369	50
9	7 52,049	41	10 5,264	46	8 21,924	50
10	11 48,604	41	14 1,819	46	12 18,479	50
11	15 45,159	41	17 58,375	46	16 15,035	50
12	19 41,715	42	21 54,930	46	20 11,590	51
13	23 38,270	42	25 51,485	46	24 8,145	51
14	27 34,825	42	29 48,041	46	28 4,701	51
15	31 31,381	42	33 44,596	47	32 1,256	51
16	35 27,936	42	37 41,151	47	35 57,811	51
17	39 24,491	42	41 37,707	47	39 54,367	51
18	43 21,047	43	45 34,262	47	43 50,922	51
19	47 17,602	43	49 30,817	47	47 47,477	52
20	51 14,157	43	53 27,373	47	51 44,033	52
21	55 10,713	43	57 23,928	48	55 40,588	52
22	59 7,268	43	16 1 20,483	48	59 37,143	52
23	14 3 3,823	43	5 17,039	48	18 3 33,699	52
24	7 0,379	43	9 13,594	48	7 30,254	52
25	10 56,934	44	13 10,149	48	11 26,809	53
26	14 53,489	44	17 6,705	48	15 23,365	53
27	18 50,045	44	21 3,260	48	19 19,920	53
28	22 46,600	44	24 59,815	49	23 16,475	53
29	26 43,155	44	28 56,371	49	27 13,030	53
30	30 39,711	44	32 52,926	49	31 9,586	53
31	34 36,266	44			35 6,141	53

Correction to be added to the values in Table II.

Year	Correction	☊	Year	Correction	☊	Year	Correction	☊
	m s			m s			m s	
C 1800	3 33,993	92	1834	2 38,004	265	B 1868	5 38,569	438
1801	2 36,686	39	1835	1 40,697	212	1869	4 41,262	385
1802	1 39,380	985	B 1836	4 39,945	158	1870	3 43,956	331
1803	0 42,073	931	1837	3 42,638	104	1871	2 46,649	277
B 1804	3 41,321	877	1838	2 45,332	50	B 1872	5 45,897	224
1805	2 44,014	824	1839	1 48,025	997	1873	4 48,590	170
1806	1 46,708	770	B 1840	4 47,273	943	1874	3 51,284	116
1807	0 49,401	716	1841	3 49,966	890	1875	2 53,977	63
B 1808	3 48,649	662	1842	2 52,660	836	B 1876	5 53,225	9
1809	2 51,342	609	1843	1 55,353	782	1877	4 55,918	955
1810	1 54,036	555	B 1844	4 54,601	728	1878	3 58,612	901
1811	0 56,729	501	1845	3 57,294	675	1879	3 1,305	848
B 1812	3 55,977	448	1846	2 59,988	621	B 1880	6 0,553	794
1813	2 58,670	394	1847	2 2,681	567	1881	5 3,246	740
1814	2 1,364	340	B 1848	5 1,929	513	1882	4 5,940	687
1815	1 4,057	287	1849	4 4,622	460	1883	3 8,633	633
B 1816	4 3,305	233	1850	3 7,316	406	B 1884	6 7,881	579
1817	3 5,998	179	1851	2 10,009	352	1885	5 10,574	526
1818	2 8,692	125	B 1852	5 9,257	299	1886	4 13,268	472
1819	1 11,385	72	1853	4 11,950	245	1887	3 15,961	418
B 1820	4 10,633	18	1854	3 14,644	191	B 1888	6 15,209	364
1821	3 13,326	963	1855	2 17,337	138	1889	5 17,902	311
1822	2 16,020	910	B 1856	5 16,585	84	1890	4 20,596	257
1823	1 18,713	856	1857	4 19,278	30	1891	3 23,289	202
B 1824	4 17,961	802	1858	3 21,972	976	B 1892	6 22,537	149
1825	3 20,654	749	1859	2 24,665	923	1893	5 25,230	95
1826	2 23,348	695	B 1860	5 23,913	869	1894	4 27,924	41
1827	1 26,041	641	1861	4 26,606	814	1895	3 30,617	988
B 1828	4 25,289	587	1862	3 29,300	761	B 1896	6 29,865	934
1829	3 27,982	534	1863	2 31,993	707	1897	5 32,558	880
1830	2 30,676	480	B 1864	5 31,241	653	1898	4 35,252	826
1831	1 33,369	426	1865	4 33,934	600	1899	3 37,945	773
B 1832	4 32,617	373	1866	3 36,628	546	C 1900	2 40,638	719
1833	3 35,310	319	1867	2 39,321	492			

TABLE IV.

Correction for the Lunar Nutation.

Argument = The mean place of the Moon's node.

☊		Equation.	☊		Equation.
500	0	− 0,000	500	1000	+ 0,000
490	10	0,065	510	990	0,068
480	20	0,129	520	980	0,136
470	30	0,193	530	970	0,203
460	40	0,257	540	960	0,269
450	50	0,319	550	950	0,334
440	60	0,381	560	940	0,398
430	70	0,441	570	930	0,460
420	80	0,499	580	920	0,520
410	90	0,555	590	910	0,578
400	100	0,610	600	900	0,634
390	110	0,662	610	890	0,687
380	120	0,711	620	880	0,737
370	130	0,758	630	870	0,784
360	140	0,803	640	860	0,828
350	150	0,844	650	850	0,868
340	160	0,882	660	840	0,905
330	170	0,916	670	830	0,938
320	180	0,947	680	820	0,967
310	190	0,975	690	810	0,992
300	200	0,999	700	800	1,014
290	210	1,019	710	790	1,031
280	220	1,034	720	780	1,044
270	230	1,046	730	770	1,053
260	240	1,054	740	760	1,057
250	250	− 1,058	750	750	+ 1,058

Correction for the Solar Nutation.

Argument = Sun's true longitude.

☉				Equation.	☉			
270°	180°	90°	0°	− 0,000 + (s)	90°	180°	270°	360°
267	183	87	3	0,008	93	177	273	357
264	186	84	6	0,016	96	174	276	354
261	189	81	9	0,024	99	171	279	351
258	192	78	12	0,031	102	168	282	348
255	195	75	15	0,038	105	165	285	345
252	198	72	18	0,045	108	162	288	342
249	201	69	21	0,051	111	159	291	339
246	204	66	24	0,057	114	156	294	336
243	207	63	27	0,062	117	153	297	333
240	210	60	30	0,066	120	150	300	330
237	213	57	33	0,070	123	147	303	327
234	216	54	36	0,073	126	144	306	324
231	219	51	39	0,075	129	141	309	321
228	222	48	42	0,076	132	138	312	318
225	225	45	45	− 0,077 +	135	135	315	315

TABLE VI.

For converting *sidereal* into *mean solar* time.

Hours.		Minutes.				Seconds.			
	m s		s		s		s		s
1	0 9,830	1	0,164	31	5,079	1	0,003	31	0,085
2	0 19,659	2	0,328	32	5,242	2	0,005	32	0,087
3	0 29,489	3	0,491	33	5,406	3	0,008	33	0,090
4	0 39,318	4	0,655	34	5,570	4	0,011	34	0,093
5	0 49,148	5	0,819	35	5,734	5	0,014	35	0,096
6	0 58,977	6	0.983	36	5,898	6	0,016	36	0,098
7	1 8,807	7	1,147	37	6,062	7	0,019	37	0,101
8	1 18,636	8	1,311	38	6,225	8	0,022	38	0,104
9	1 28,466	9	1,474	39	6,389	9	0,025	39	0,106
10	1 38,296	10	1,638	40	6,553	10	0,027	40	0,109
11	1 48,125	11	1,802	41	6,717	11	0,030	41	0,112
12	1 57,955	12	1,966	42	6,881	12	0,033	42	0,115
13	2 7,784	13	2.130	43	7,044	13	0,036	43	0,118
14	2 17,614	14	2,294	44	7,208	14	0,038	44	0,120
15	2 27,443	15	2,457	45	7,372	15	0,041	45	0,123
16	2 37,273	16	2,621	46	7,536	16	0,044	46	0,126
17	2 47,103	17	2,785	47	7,700	17	0,047	47	0,128
18	2 56,932	18	2,949	48	7,864	18	0,049	48	0,131
19	3 6,762	19	3,113	49	8,027	19	0,052	49	0,134
20	3 16,591	20	3,277	50	8,191	20	0,055	50	0,137
21	3 26,421	21	3,440	51	8,355	21	0,057	51	0,140
22	3 36,250	22	3,604	52	8,519	22	0,060	52	0,142
23	3 46,080	23	3,768	53	8,683	23	0,063	53	0,145
24	3 55,909	24	3,932	54	8,847	24	0,066	54	0,148
		25	4,096	55	9,010	25	0,068	55	0,150
		26	4,259	56	9,174	26	0,071	56	0,153
		27	4,423	57	9,338	27	0,074	57	0,156
		28	4,587	58	9,502	28	0,076	58	0,159
		29	4,751	59	9,666	29	0,079	59	0,161
		30	4,915	60	9,830	30	0,082	60	0,164

For converting *mean solar* into *sidereal* time.

Hours.		Minutes.				Seconds.			
	m s		s		s		s		s
1	0 9,856	1	0,164	31	5,092	1	0,003	31	0,085
2	0 19,713	2	0,329	32	5,257	2	0,005	32	0,087
3	0 29,569	3	0,493	33	5,421	3	0,008	33	0,090
4	0 39,426	4	0,657	34	5,585	4	0,011	34	0,093
5	0 49,282	5	0,821	35	5,750	5	0,014	35	0,096
6	0 59,139	6	0,986	36	5,914	6	0,016	36	0,098
7	1 8,995	7	1,150	37	6,078	7	0,019	37	0,101
8	1 18,852	8	1,314	38	6,242	8	0,022	38	0,104
9	1 28,708	9	1,478	39	6,407	9	0,025	39	0,106
10	1 38,565	10	1,643	40	6,571	10	0,027	40	0,109
11	1 48,421	11	1,807	41	6,735	11	0,030	41	0,112
12	1 58,278	12	1,971	42	6,900	12	0,033	42	0,115
13	2 8,134	13	2,136	43	7,064	13	0,036	43	0,118
14	2 17,991	14	2,300	44	7,228	14	0,038	44	0,120
15	2 27,847	15	2,464	45	7,392	15	0,041	45	0,123
16	2 37,704	16	2,628	46	7,557	16	0,044	46	0,126
17	2 47,560	17	2,793	47	7,721	17	0,047	47	0,128
18	2 57,416	18	2,957	48	7,885	18	0,049	48	0,131
19	3 7,273	19	3,121	49	8,050	19	0,052	49	0,134
20	3 17,129	20	3,285	50	8,214	20	0,055	50	0,137
21	3 26,986	21	3,450	51	8,378	21	0,057	51	0,140
22	3 36,842	22	3,614	52	8,542	22	0,060	52	0,142
23	3 46,699	23	3,778	53	8,707	23	0,063	53	0,145
24	3 56,555	24	3,943	54	8,871	24	0,066	54	0,148
		25	4,107	55	9,035	25	0,068	55	0,150
		26	4,271	56	9,199	26	0,071	56	0,153
		27	4,436	57	9,364	27	0,074	57	0,156
		28	4,600	58	9,528	28	0,076	58	0,159
		29	4,764	59	9,692	29	0,079	59	0,161
		30	4,928	60	9,856	30	0,082	60	0,164

TABLE VIII.

For converting *degrees* into *time*: and *vice versa*.

Degrees.											
Space.	Time.		Space.	Time.		Space.	Time.		Space.	Time.	
°	h	m	°	h	m	°	h	m	°	h	m
1	0	4	31	2	4	61	4	4	91	6	4
2	0	8	32	2	8	62	4	8	92	6	8
3	0	12	33	2	12	63	4	12	93	6	12
4	0	16	34	2	16	64	4	16	94	6	16
5	0	20	35	2	20	65	4	20	95	6	20
6	0	24	36	2	24	66	4	24	96	6	24
7	0	28	37	2	28	67	4	28	97	6	28
8	0	32	38	2	32	68	4	32	98	6	32
9	0	36	39	2	36	69	4	36	99	6	36
10	0	40	40	2	40	70	4	40	100	6	40
11	0	44	41	2	44	71	4	44	101	6	44
12	0	48	42	2	48	72	4	48	102	6	48
13	0	52	43	2	52	73	4	52	103	6	52
14	0	56	44	2	56	74	4	56	104	6	56
15	1	0	45	3	0	75	5	0	105	7	0
16	1	4	46	3	4	76	5	4	106	7	4
17	1	8	47	3	8	77	5	8	107	7	8
18	1	12	48	3	12	78	5	12	108	7	12
19	1	16	49	3	16	79	5	16	109	7	16
20	1	20	50	3	20	80	5	20	110	7	20
21	1	24	51	3	24	81	5	24	111	7	24
22	1	28	52	3	28	82	5	28	112	7	28
23	1	32	53	3	32	83	5	32	113	7	32
24	1	36	54	3	36	84	5	36	114	7	36
25	1	40	55	3	40	85	5	40	115	7	40
26	1	44	56	3	44	86	5	44	116	7	44
27	1	48	57	3	48	87	5	48	117	7	48
28	1	52	58	3	52	88	5	52	118	7	52
29	1	56	59	3	56	89	5	56	119	7	56
30	2	0	60	4	0	90	6	0	120	8	0

Degrees.							
Space.	Time.	Space.	Time.	Space.	Time.	Space.	Time.
°	h m	°	h m	°	h m	°	h m
121	8 4	151	10 4	181	12 4	211	14 4
122	8 8	152	10 8	182	12 8	212	14 8
123	8 12	153	10 12	183	12 12	213	14 12
124	8 16	154	10 16	184	12 16	214	14 16
125	8 20	155	10 20	185	12 20	215	14 20
126	8 24	156	10 24	186	12 24	216	14 24
127	8 28	157	10 28	187	12 28	217	14 28
128	8 32	158	10 32	188	12 32	218	14 32
129	8 36	159	10 36	189	12 36	219	14 36
130	8 40	160	10 40	190	12 40	220	14 40
131	8 44	161	10 44	191	12 44	221	14 44
132	8 48	162	10 48	192	12 48	222	14 48
133	8 52	163	10 52	193	12 52	223	14 52
134	8 56	164	10 56	194	12 56	224	14 56
135	9 0	165	11 0	195	13 0	225	15 0
136	9 4	166	11 4	196	13 4	226	15 4
137	9 8	167	11 8	197	13 8	227	15 8
138	9 12	168	11 12	198	13 12	228	15 12
139	9 16	169	11 16	199	13 16	229	15 16
140	9 20	170	11 20	200	13 20	230	15 20
141	9 24	171	11 24	201	13 24	231	15 24
142	9 28	172	11 28	202	13 28	232	15 28
143	9 32	173	11 32	203	13 32	233	15 32
144	9 36	174	11 36	204	13 36	234	15 36
145	9 40	175	11 40	205	13 40	235	15 40
146	9 44	176	11 44	206	13 44	236	15 44
147	9 48	177	11 48	207	13 48	237	15 48
148	9 52	178	11 52	208	13 52	238	15 52
149	9 56	179	11 56	209	13 56	239	15 56
150	10 0	180	12 0	210	14 0	240	16 0

Degrees.							
Space.	Time.	Space.	Time.	Space.	Time.	Space.	Time.
°	h m	°	h m	°	h m	°	h m
241	16 4	271	18 4	301	20 4	331	22 4
242	16 8	272	18 8	302	20 8	332	22 8
243	16 12	273	18 12	303	20 12	333	22 12
244	16 16	274	18 16	304	20 16	334	22 16
245	16 20	275	18 20	305	20 20	335	22 20
246	16 24	276	18 24	306	20 24	336	22 24
247	16 28	277	18 28	307	20 28	337	22 28
248	16 32	278	18 32	308	20 32	338	22 32
249	16 36	279	18 36	309	20 36	339	22 36
250	16 40	280	18 40	310	20 40	340	22 40
251	16 44	281	18 44	311	20 44	341	22 44
252	16 48	282	18 48	312	20 48	342	22 48
253	16 52	283	18 52	313	20 52	343	22 52
254	16 56	284	18 56	314	20 56	344	22 56
255	17 0	285	19 0	315	21 0	345	23 0
256	17 4	286	19 4	316	21 4	346	23 4
257	17 8	287	19 8	317	21 8	347	23 8
258	17 12	288	19 12	318	21 12	348	23 12
259	17 16	289	19 16	319	21 16	349	23 16
260	17 20	290	19 20	320	21 20	350	23 20
261	17 24	291	19 24	321	21 24	351	23 24
262	17 28	292	19 28	322	21 28	352	23 28
263	17 32	293	19 32	323	21 32	353	23 32
264	17 36	294	19 36	324	21 36	354	23 36
265	17 40	295	19 40	325	21 40	355	23 40
266	17 44	296	19 44	326	21 44	356	23 44
267	17 48	297	19 48	327	21 48	357	23 48
268	17 52	298	19 52	328	21 52	358	23 52
269	17 56	299	19 56	329	21 56	359	23 56
270	18 0	300	20 0	330	22 0	360	24 0

Minutes.				Seconds.			
Space.	Time.	Space.	Time.	Space.	Time.	Space.	Time.
′	m s	′	m s	″	s	″	s
1	0 4	31	2 4	1	0,067	31	2,067
2	0 8	32	2 8	2	0,133	32	2,133
3	0 12	33	2 12	3	0,200	33	2,200
4	0 16	34	2 16	4	0,267	34	2,267
5	0 20	35	2 20	5	0,333	35	2,333
6	0 24	36	2 24	6	0,400	36	2,400
7	0 28	37	2 28	7	0,467	37	2,467
8	0 32	38	2 32	8	0,533	38	2,533
9	0 36	39	2 36	9	0,600	39	2,600
10	0 40	40	2 40	10	0,667	40	2,667
11	0 44	41	2 44	11	0,733	41	2,733
12	0 48	42	2 48	12	0,800	42	2,800
13	0 52	43	2 52	13	0,867	43	2,867
14	0 56	44	2 56	14	0,933	44	2,993
15	1 0	45	3 0	15	1,000	45	3,000
16	1 4	46	3 4	16	1,067	46	3,067
17	1 8	47	3 8	17	1,133	47	3,133
18	1 12	48	3 12	18	1,200	48	3,200
19	1 16	49	3 16	19	1,267	49	3,267
20	1 20	50	3 20	20	1,333	50	3,333
21	1 24	51	3 24	21	1,400	51	3,400
22	1 28	52	3 28	22	1,467	52	3,467
23	1 32	53	3 32	23	1,533	53	3,533
24	1 36	54	3 36	24	1,600	54	3,600
25	1 40	55	3 40	25	1,667	55	3,667
26	1 44	56	3 44	26	1,733	56	3,733
27	1 48	57	3 48	27	1,800	57	3,800
28	1 52	58	3 52	28	1,867	58	3,867
29	1 56	59	3 56	29	1,933	59	3,933
30	2 0	60	4 0	30	2,000	60	4,000

Mr. Ivory's Mean Refractions; with the logarithms and their differences annexed.

Zenith Dist.	Mean Refrac.	Log.	Diff.	Zenith Dist.	Mean Refrac.	Log.	Diff.
1°	0′ 1″,02	0.0085		31°	0′ 35″,09	1.5452	173
2	2,04	0.3097	3012	32	36,49	1.5622	170
3	3,06	0.4860	1763	33	37,93	1.5790	168
4	4,08	0.6112	1252	34	39,39	1.5954	164
5	5,11	0.7086	974	35	40,89	1.6116	162
6	6,14	0.7882	796	36	42,42	1.6276	160
7	7,17	0.8557	675	37	44,00	1.6435	159
8	8,21	0.9144	587	38	45,61	1.6591	156
9	9,25	0.9663	519	39	47,27	1.6746	155
10	10,30	1.0129	466	40	48,99	1.6901	155
11	11,35	1.0553	424	41	50,75	1.7055	154
12	12,42	1.0941	388	42	52,57	1.7207	152
13	13,49	1.1300	359	43	54,43	1.7358	151
14	14,56	1.1634	334	44	56,35	1.7510	152
15	15,66	1.1947	313	45	58,36	1.76611	151
16	16,75	1.2241	294	46	1 0,43	1.78123	1512
17	17,86	1.2519	278	47	2,57	1.79637	1514
18	18,98	1.2784	265	48	4,80	1.81155	1518
19	20,11	1.3036	252	49	7,11	1.82678	1523
20	21,26	1.3277	241	50	9,52	1.84208	1530
21	22,42	1.3507	230	51	12,02	1.85747	1539
22	23,60	1.3729	222	52	14,64	1.87298	1551
23	24,80	1.3944	215	53	17,38	1.88863	1565
24	26,01	1.4151	207	54	20,24	1.90440	1577
25	27,24	1.4352	201	55	23,25	1.92036	1596
26	28,49	1.4547	195	56	26,41	1.93653	1617
27	29,76	1.4736	189	57	29,73	1.95291	1638
28	31,05	1.4921	185	58	33,23	1.96955	1664
29	32,38	1.5102	181	59	36,93	1.98646	1691
30	0 33,72	1.5279	177	60	1 40,85	2.00368	1722

Zenith Dist.	Mean Refrac.	Log.	Diff.	Zenith Dist.	Mean Refrac.	Log.	Diff.
61° 0′	1′ 45″,01	2.02124	1756	74° 00′	3′ 21″,01	2.30322	462
62 0	49,44	2.03918	1794	10	23,18	2.30789	467
63 0	54,17	2.05754	1836	20	25,39	2.31259	470
64 0	59,22	2.07635	1881	30	27,66	2.31734	475
65 0	2 4,65	2.09567	1932	40	29,95	2.32213	479
66 0	10,48	2.11555	1988	50	32,30	2.32696	483
67 0	16,78	2.13603	2048	75 00	34,70	2.33184	488
68 0	23,61	2.15719	2116	10	37,16	2.33677	493
69 0	31,04	2.17910	2191	20	39,65	2.34174	497
70 00	39,16	2.20185	2275	30	42,21	2.34676	502
10	40,59	2.20573	388	40	44,82	2.35183	507
20	42,04	2.20963	390	50	47,48	2.35695	512
30	43,52	2.21356	393	76 00	50,21	2.36212	517
40	45,02	2.21752	396	10	53,00	2.36735	523
50	46,53	2.22150	398	20	55,85	2.37263	528
71 00	48,08	2.22552	402	30	58,76	2.37796	533
10	49,65	2.22956	404	40	4 1,74	2.38334	538
20	51,25	2.23363	407	50	4,79	2.38879	545
30	52,87	2.23773	410	77 00	7,91	2.39430	551
40	54,53	2.24186	413	10	11,11	2.39987	557
50	56,21	2.24603	417	20	14,39	2.40550	563
72 00	57,92	2.25022	419	30	17,74	2.41119	569
10	59,66	2.25445	423	40	21,19	2.41695	576
20	3 1,43	2.25870	425	50	24,72	2.42278	583
30	3,23	2.26299	429	78 00	28,33	2.42867	589
40	5,06	2.26732	433	10	32,04	2.43463	596
50	6,93	2.27168	436	20	35,84	2.44066	603
73 00	8,83	2.27608	440	30	39,75	2.44677	611
10	10,77	2.28051	443	40	43,76	2.45295	618
20	12,74	2.28498	447	50	47,88	2.45921	626
30	14,75	2.28948	450	79 00	52,12	2.46556	635
40	16,80	2.29402	454	10	56,47	2.47198	642
50	3 18,88	2.29860	458	20	5 0,94	2.47848	650

Zenith Dist.	Mean Refrac.	Log.	Diff.	Zenith Dist.	Mean Refrac.	Log.	Diff.
79° 30′	5′ 5″,54	2.48507	659	84° 50′	9′ 38″,12	2.76202	1139
40	10,28	2.49176	669	85 00	9 53,84	2.77367	1165
50	15,16	2.49853	677	10	10 10,35	2.78558	1191
80 00	20,19	2.50541	688	20	27,73	2.79777	1219
10	25,36	2.51237	696	30	46,03	2.81025	1248
20	30,70	2.51944	707	40	11 5,30	2.82302	1277
30	36,20	2.52660	716	50	25,66	2.83611	1309
40	41,88	2.53387	727	86 00	47,15	2.84951	1340
50	47,74	2.54125	738	10	12 9,88	2.86325	1374
81 00	53,79	2.54874	749	20	33,97	2.87735	1410
10	6 0,04	2.55635	761	30	59,51	2.89182	1447
20	6,50	2.56407	772	40	13 26,61	2.90666	1484
30	13,18	2.57192	785	50	13 55,40	2.92189	1523
40	20,09	2.57989	797	87 00	14 26,04	2.93754	1565
50	27,26	2.58800	811	10	14 58,71	2.95362	1608
82 00	34,68	2.59624	824	20	15 33,60	2.97016	1654
10	42,37	2.60462	838	30	16 10,89	2.98717	1701
20	50,33	2.61313	851	40	16 50,8	3.00466	1749
30	58,59	2.62179	866	50	17 33,6	3.02267	1801
40	7 7,19	2.63062	883	88 00	18 19,6	3.04122	1855
50	16,13	2.63961	899	10	19 9,0	3.06031	1909
83 00	25,40	2.64875	914	20	20 2,2	3.07998	1967
10	35,05	2.65806	931	30	20 59,6	3.10024	2026
20	45,10	2.66755	949	40	22 1,7	3.12113	2089
30	55,58	2.67722	967	50	23 8,9	3.14268	2155
40	8 6,50	2.68708	986	89 00	24 21,8	3.16489	2221
50	17,90	2.69714	1006	10	25 40,9	3.18779	2290
84 00	29,80	2.70740	1026	20	27 7,1	3.21140	2361
10	42,24	2.71787	1047	30	28 40,8	3.23574	2434
20	55,25	2.72856	1069	40	30 23,2	3.26083	2509
30	9 8,88	2.73948	1092	50	32 15,0	3.28667	2584
40	9 23,16	2.75063	1115	90 00	34 17,5	3.31334	2667

Mr. Ivory's Refractions continued: showing the logarithms of the corrections, on account of the state of the Thermometer and Barometer.

Thermometer				Barometer	
	Logarithm		Logarithm		Logarithm
80°	9.97237	50°	0.00000	in. 31.0	0.01424
79	9.97326	49	0.00094	30.9	0.01248
78	9.97416	48	0.00190	8	0.01143
77	9.97506	47	0.00285	7	0.01002
76	9.97596	46	0.00380	6	0.00860
75	9.97686	45	0.00476	5	0.00718
74	9.97777	44	0.00572	4	0.00575
73	9.97867	43	0.00668	3	0.00432
72	9.97958	42	0.00764	2	0.00289
71	9.98049	41	0.00861	1	0.00145
70	9.98140	40	0.00957	30.0	0.00000
69	9.98231	39	0.01053	29.9	9.99855
68	9.98323	38	0.01151	8	9.99709
67	9.98414	37	0.01248	7	9.99563
66	9.98506	36	0.01346	6	9.99417
65	9.98598	35	0.01444	5	9.99270
64	9.98690	34	0.01541	4	9.99123
63	9.98783	33	0.01640	3	9.98975
62	9.98875	32	0.01738	2	9.98826
61	9.98969	31	0.01837	1	9.98677
60	9.99061	30	0.01935	29.0	9.98528
59	9.99154	29	0.02033	28.9	9.98378
58	9.99248	28	0.02133	8	9.98227
57	9.99341	27	0.02232	7	9.98076
56	9.99434	26	0.02331	6	9.97924
55	9.99529	25	0.02432	5	9.97772
54	9.92623	24	0.02531	4	9.97620
53	9.99717	23	0.02630	3	9.97466
52	9.99811	22	0.02730	2	9.97313
51	9.99906	21	0.02832	1	9.97158
50	0.00000	20	0.02933	28.0	9.97004

Mr. Ivory's Refractions continued: showing the *further* quantities by which the refraction at *low* altitudes is to be corrected, on account of the state of the Thermometer and Barometer.

Zenith Distance	T	B	Zenith Distance	T	B
75° 0′	− 0″,009		86° 30′	− 0″,317	+ 0″,51
76 0	0,012		86 40	0,345	0,56
77 0	0,015		86 50	0,376	0,62
78 0	0,018		87 0	0,410	0,68
79 0	0,023		87 10	0,448	0,75
80 0	0,030	+ 0″,04	87 20	0,490	0,83
81 0	0,040	0,05	87 30	0,538	0,91
81 30	0,046	0,07	87 40	0,593	1,01
82 0	0,053	0,08	87 50	0,654	1,13
82 30	0,063	0,10	88 0	0,722	1,26
83 0	0,074	0,11	88 10	0,799	1,41
83 30	0,089	0,13	88 20	0,887	1,59
84 0	0,107	0,16	88 30	0,987	1,79
84 30	0,130	0,20	88 40	1,101	2,02
85 0	0,159	0,25	88 50	1,231	2,29
85 10	0,171	0,26	89 0	1,380	2,61
85 20	0,184	0,28	89 10	1,551	2,98
85 30	0,198	0,31	89 20	1,749	3,41
85 40	0,213	0,33	89 30	1,977	3,93
85 50	0,229	0,36	89 40	2,241	4,54
86 0	0,248	0,39	89 50	2,549	5,26
86 10	0,269	0,43	90 0	− 2,909	+ 6,12
86 20	− 0,292	+ 0,47			

The column marked T is to be multiplied by $(t-50°)$: and the column marked B is to be multiplied by $(\beta-30^{in}.00)$. The results are to be applied to the approximate refraction obtained by the preceding tables.

Dr. Brinkley's Refractions: containing the logarithms of the quantities depending on the state of the Thermometer.

Far. Therm.	Log. T	Far. Therm.	Log. T	Far. Therm.	Log. T
10°	0.3283	34°	0.3048	58°	0.2827
11	0.3273	35	0.3039	59	0.2818
12	0.3263	36	0.3030	60	0.2809
13	0.3253	37	0.3020	61	0.2800
14	0.3243	38	0.3011	62	0.2791
15	0.3233	39	0.3001	63	0.2782
16	0.3223	40	0.2992	64	0.2773
17	0.3213	41	0.2983	65	0.2764
18	0.3203	42	0.2974	66	0.2755
19	0.3193	43	0.2965	67	0.2746
20	0.3183	44	0.2956	68	0.2737
21	0.3173	45	0.2946	69	0.2728
22	0.3163	46	0.2937	70	0.2720
23	0.3154	47	0.2928	71	0.2711
24	0.3144	48	0.2919	72	0.2703
25	0.3134	49	0.2910	73	0.2694
26	0.3124	50	0.2900	74	0.2685
27	0.3114	51	0.2891	75	0.2677
28	0.3105	52	0.2881	76	0.2668
29	0.3095	53	0.2872	77	0.2660
30	0.3086	54	0.2863	78	0.2652
31	0.3076	55	0.2854	79	0.2644
32	0.3067	56	0.2845	80	0.2636
33	0.3058	57	0.2836	81	0.2627

Approximate Refraction $= \mathrm{T} . \beta . \tan \mathrm{Z}$

Correct Refraction $= \mathrm{T} . \beta . \tan \mathrm{Z} - c$

Dr. Brinkley's Refractions cont[d]: containing the quantity *c*, depending on the state of the Barometer and Zenith distance, to be deducted from the approximate refraction.

Zenith Dist.	Barometer				
	28.50	29.00	29.50	30.00	30.50
0°	0″,0				0″,0
30	0,0				0,0
40	0,1				0,1
45	0,2				0,2
50	0,2				0,2
52	0,2				0,2
54	0,3				0,3
56	0,3				0,3
58	0,4				0,4
60	0,5				0,5
61	0,5				0,5
62	0,6				0,6
63	0,6				0,6
64	0,7				0,7
65	0,8				0,8
66	0,9				0,9
67	1,0				1,0
68	1,2	1″,2	1″,2	1″,2	1,2
69	1,3	1,3	1,3	1,4	1,4
70	1,5	1,5	1,5	1,6	1,6
71	1,8	1,8	1,9	1,9	1,9
72	2,1	2,1	2,2	2,2	2,2
73	2,5	2,5	2,6	2,6	2,6
74	3,0	3,0	3,1	3,1	3,2
75	3,4	3,4	3,5	3,6	3,7
76	4,1	4,2	4,3	4,4	4,5
77	5,1	5,2	5,3	5,5	5,6
78	6,3	6,4	6,6	6,7	6,9
79	8,1	8,3	8,5	8,5	8,9
80	10,5	10,7	10,9	11,1	11,4

Parallax of the Sun, on the first day of each month: the mean horizontal parallax being assumed = 8″,60.

Zenith Dist.	Jan.	Feb. Dec.	March Nov.	April Oct.	May Sept.	June Aug.	July
0°	0″,00	0″,00	0″,00	0″,00	0″,00	0″,00	0″,00
5	0,76	0,76	0,76	0,75	0,74	0,74	0,74
10	1,52	1,52	1,51	1,49	1,48	1,47	1,47
15	2,26	2,26	2,25	2,23	2,21	2,19	2,19
20	2,99	2,98	2,97	2,94	2,92	2,90	2,89
25	3,70	3,69	3,67	3,63	3,60	3,58	3,57
30	4,37	4,36	4,34	4,30	4,26	4,24	4,23
35	5,02	5,01	4,98	4,93	4,89	4,86	4,85
40	5,62	5,61	5,58	5,53	5,48	5,45	5,44
45	6,19	6,17	6,13	6,08	6,03	5,99	5,98
50	6,70	6,68	6,64	6,59	6,53	6,49	6,48
55	7,17	7,15	7,11	7,04	6,99	6,94	6,93
60	7,58	7,56	7,51	7,45	7,39	7,34	7,33
65	7,93	7,91	7,86	7,79	7,73	7,68	7,67
70	8,22	8,20	8,15	8,08	8,01	7,97	7,95
75	8,45	8,43	8,38	8,30	8,24	8,19	8,17
80	8,62	8,59	8,54	8,47	8,40	8,35	8,33
85	8,73	8,69	8,64	8,56	8,50	8,44	8,42
90	8,75	8,73	8,67	8,60	8,53	8,48	8,46

Logarithms of $\sin^2 \frac{1}{2}$ P, in time.

Minutes	3 hours	4 hours	5 hours	6 hours	7 hours
0	9·165679	9·397940	9·568894	9·698970	9·798933
1	·170240	·401214	·571358	·700865	·800384
2	·174773	·404471	·573811	·702743	·801828
3	·179278	·407713	·576253	·704618	·803266
4	·183756	·410938	·578684	·706484	·804697
5	·188207	·414147	·581104	·708342	·806122
6	·192631	·417340	·583513	·710192	·807540
7	·197028	·420517	·585911	·712034	·808952
8	·201399	·423679	·588299	·713868	·810357
9	·205745	·426825	·590676	·715694	·811756
10	·210064	·429955	·593042	·717512	·813149
11	·214358	·433070	·595398	·719322	·814535
12	·218627	·436170	·597744	·721124	·815915
13	·222870	·439255	·600078	·722919	·817289
14	·227089	·442325	·602403	·724705	·818656
15	·231284	·445379	·604717	·726484	·820017
16	·235454	·448419	·607021	·728255	·821372
17	·239600	·451445	·609315	·730018	·822721
18	·243722	·454455	·611598	·731774	·824063
19	·247821	·457451	·613872	·733522	·825399
20	·251897	·460433	·616135	·735262	·826729
21	·255949	·463400	·618388	·736994	·828053
22	·259978	·466354	·620632	·738719	·829370
23	·263985	·469293	·622865	·740437	·830682
24	·267969	·472218	·625089	·742147	·831987
25	·271930	·475129	·627303	·743849	·833287
26	·275870	·478026	·629507	·745544	·834580
27	·279788	·480909	·631701	·747232	·835867
28	·283684	·483779	·633886	·748912	·837148
29	9·287558	9·486635	9·636061	9·750585	9·838424

Logarithms of $\sin^2 \frac{1}{2} P$, in time.

Minutes	3 hours	4 hours	5 hours	6 hours	7 hours
30	9·291412	9·489478	9·638227	9·752251	9·839693
31	·295244	·492307	·640383	·753909	·840956
32	·299055	·495123	·642529	·755560	·842213
33	·302845	·497926	·644666	·757203	·843464
34	·306615	·500716	·646794	·758840	·844710
35	·310364	·503492	·648913	·760469	·845949
36	·314094	·506256	·651022	·762091	·847183
37	·317803	·509007	·653122	·763706	·848410
38	·321492	·511745	·655213	·765314	·849632
39	·325161	·514470	·657294	·766914	·850848
40	·328811	·517183	·659367	·768508	·852058
41	·332442	·519883	·661430	·770094	·853263
42	·336053	·522570	·663485	·771674	·854461
43	·339645	·525245	·665530	·773247	·855654
44	·343219	·527908	·667567	·774812	·856841
45	·346773	·530559	·669594	·776371	·858022
46	·350309	·533197	·671613	·777922	·859198
47	·353827	·535823	·673623	·779467	·860367
48	·357326	·538437	·675624	·781005	·861532
49	·360807	·541040	·677617	·782536	·862690
50	·364270	·543630	·679601	·784061	·863843
51	·367715	·546208	·681576	·785578	·864990
52	·371142	·548775	·683543	·787089	·866131
53	·374552	·551330	·685501	·788593	·867267
54	·377945	·553874	·687450	·790090	·868397
55	·381320	·556406	·689391	·791580	·869522
56	·384678	·558926	·691324	·793064	·870641
57	·388018	·561435	·693248	·794541	·871754
58	·391342	·563933	·695163	·796012	·872862
59	9·394650	9·566419	9·697071	9·797476	9·873964

For the equation of Equal Altitudes of the Sun.

Interval	Log. A	Log. B	Interval	Log. A	Log. B
h m			h m		
2 0	7·7297	7·7146	3 0	7·7359	7·7015
2	·7298	·7143	2	·7362	·7010
4	·7300	·7139	4	·7364	·7005
6	·7302	·7136	6	·7367	·6999
8	·7304	·7132	8	·7369	·6993
10	·7305	·7128	10	·7372	·6988
12	·7307	·7125	12	·7374	·6982
14	·7309	·7121	14	·7377	·6976
16	·7311	·7117	16	·7380	·6970
18	·7313	·7113	18	·7383	·6964
20	·7315	·7109	20	·7386	·6958
22	·7317	·7105	22	·7388	·6952
24	·7319	·7101	24	·7391	·6946
26	·7321	·7097	26	·7394	·6940
28	·7323	·7092	28	·7397	·6934
30	·7325	·7088	30	·7400	·6927
32	·7327	·7083	32	·7403	·6921
34	·7329	·7079	34	·7406	·6914
36	·7331	·7075	36	·7409	·6908
38	·7333	·7070	38	·7412	·6901
40	·7336	·7065	40	·7415	·6894
42	·7338	·7061	42	·7418	·6888
44	·7340	·7056	44	·7421	·6881
46	·7342	·7051	46	·7424	·6874
48	·7345	·7046	48	·7428	·6867
50	·7347	·7041	50	·7431	·6859
52	·7349	·7036	52	·7434	·6852
54	·7352	·7031	54	·7437	·6845
56	·7354	·7026	56	·7441	·6838
58	7·7357	7·7021	58	7·7444	7·6830

For the equation of Equal Altitudes of the Sun.

Interval h m	Log. A	Log. B	Interval h m	Log. A	Log. B
4 0	7·7447	7·6823	5 0	7·7562	7·6556
2	·7451	·6815	2	·7566	·6546
4	·7454	·6807	4	·7570	·6536
6	·7458	·6800	6	·7575	·6525
8	·7461	·6792	8	·7579	·6514
10	·7464	·6784	10	·7583	·6504
12	·7468	·6776	12	·7588	·6493
14	·7472	·6768	14	·7592	·6482
16	·7475	·6759	16	·7597	·6471
18	·7479	·6751	18	·7601	·6460
20	·7482	·6743	20	·7606	·6448
22	·7486	·6734	22	·7610	·6437
24	·7490	·6726	24	·7615	·6425
26	·7494	·6717	26	·7620	·6414
28	·7497	·6708	28	·7624	·6402
30	·7501	·6700	30	·7629	·6390
32	·7505	·6691	32	·7634	·6378
34	·7509	·6682	34	·7638	·6366
36	·7513	·6673	36	·7643	·6354
38	·7517	·6663	38	·7648	·6342
40	·7521	·6654	40	·7653	·6329
42	·7525	·6645	42	·7658	·6317
44	·7529	·6635	44	·7663	·6304
46	·7533	·6626	46	·7668	·6291
48	·7537	·6616	48	·7673	·6278
50	·7541	·6606	50	·7678	·6265
52	·7545	·6597	52	·7683	·6252
54	·7549	·6587	54	·7688	·6239
56	·7553	·6577	56	·7693	·6225
58	7·7557	7·6567	58	7·7698	7·6212

For the equation of Equal Altitudes of the Sun.

Interval	Log. A	Log. B	Interval	Log. A	Log. B
h m 6 0	7·7703	7·6198	h m 7 0	7·7873	7·5717
2	·7708	·6184	2	·7879	·5699
4	·7713	·6170	4	·7885	·5680
6	·7719	·6156	6	·7891	·5661
8	·7724	·6142	8	·7898	·5641
10	·7729	·6127	10	·7904	·5622
12	·7735	·6113	12	·7910	·5602
14	·7740	·6098	14	·7916	·5582
16	·7745	·6083	16	·7923	·5562
18	·7751	·6068	18	·7929	·5542
20	·7756	·6053	20	·7936	·5522
22	·7762	·6038	22	·7942	·5501
24	·7767	·6023	24	·7949	·5480
26	·7773	·6007	26	·7955	·5459
28	·7779	·5991	28	·7962	·5437
30	·7784	·5975	30	·7969	·5416
32	·7790	·5959	32	·7975	·5394
34	·7796	·5943	34	·7982	·5372
36	·7801	·5927	36	·7989	·5350
38	·7807	·5910	38	·7995	·5327
40	·7813	·5894	40	·8002	·5304
42	·7819	·5877	42	·8009	·5281
44	·7825	·5860	44	·8016	·5258
46	·7831	·5843	46	·8023	·5234
48	·7836	·5825	48	·8030	·5211
50	·7842	·5808	50	·8037	·5186
52	·7848	·5790	52	·8044	·5162
54	·7854	·5772	54	·8051	·5137
56	·7860	·5754	56	·8058	·5112
58	7·7867	7·5736	58	7·8065	7·5087

For the equation of Equal Altitudes of the Sun.

Interval	Log. A	Log. B	Interval	Log. A	Log. B
h m 8 0	7·8072	7·5062	h m 9 0	7·8302	7·4131
2	·8079	·5036	2	·8311	·4093
4	·8086	·5010	4	·8319	·4055
6	·8094	·4983	6	·8328	·4016
8	·8101	·4957	8	·8336	·3977
10	·8108	·4930	10	·8344	·3937
12	·8116	·4902	12	·8353	·3896
14	·8123	·4874	14	·8361	·3855
16	·8130	·4846	16	·8370	·3813
18	·8138	·4818	18	·8378	·3771
20	·8145	·4789	20	·8387	·3728
22	·8153	·4760	22	·8396	·3684
24	·8160	·4731	24	·8404	·3639
26	·8168	·4701	26	·8413	·3594
28	·8176	·4671	28	·8422	·3548
30	·8183	·4640	30	·8430	·3501
32	·8191	·4609	32	·8439	·3454
34	·8199	·4578	34	·8448	·3406
36	·8206	·4546	36	·8457	·3357
38	·8214	·4514	38	·8466	·3307
40	·8222	·4482	40	·8475	·3256
42	·8230	·4449	42	·8484	·3205
44	·8238	·4415	44	·8493	·3152
46	·8246	·4381	46	·8502	·3099
48	·8254	·4347	48	·8511	·3045
50	·8262	·4312	50	·8520	·2989
52	·8270	·4277	52	·8530	·2933
54	·8278	·4241	54	·8539	·2876
56	·8286	·4205	56	·8548	·2817
58	7·8294	7·4168	58	7·8558	7·2758

For the equation of Equal Altitudes of the Sun.

Interval	Log. A	Log. B	Interval	Log. A	Log. B
h m 10 0	7·8567	7·2697	h m 11 0	7·8868	7·0025
2	·8576	·2635	2	·8878	6·9889
4	·8586	·2572	4	·8889	·9748
6	·8595	·2507	6	·8900	·9602
8	·8605	·2442	8	·8911	·9449
10	·8614	·2374	10	·8922	·9290
12	·8624	·2306	12	·8932	·9125
14	·8634	·2236	14	·8943	·8953
16	·8643	·2164	16	·8954	·8770
18	·8653	·2091	18	·8965	·8580
20	·8663	·2016	20	·8977	·8379
22	·8673	·1940	22	·8988	·8168
24	·8683	·1861	24	·8999	·7945
26	·8693	·1781	26	·9010	·7709
28	·8703	·1699	28	·9021	·7457
30	·8713	·1615	30	·9033	·7189
32	·8723	·1529	32	·9044	·6901
34	·8733	·1440	34	·9055	·6591
36	·8743	·1349	36	·9067	·6255
38	·8753	·1256	38	·9078	·5889
40	·8763	·1160	40	·9090	·5487
42	·8773	·1061	42	·9102	·5041
44	·8784	·0960	44	·9113	·4541
46	·8794	·0855	46	·9125	·3973
48	·8804	·0748	48	·9137	·3316
50	·8815	·0637	50	·9148	·2536
52	·8825	·0522	52	·9160	·1579
54	·8836	·0404	54	·9172	6·0341
56	·8846	·0282	56	·9184	5·8593
58	7·8857	7·0156	58	7·9196	5·5594

For the equation of Equal Altitudes of the Sun.

Interval	Log. A	Log. B	Interval	Log. A	Log. B
h m 12 0	7·9208	B = 0	h m 13 0	7·9593	— 7·0750
2	·9220	— 5·5549	2	·9607	·0905
4	·9232	5·8641	4	·9620	·1056
6	·9245	6·0414	6	·9634	·1203
8	·9257	·1675	8	·9648	·1345
10	·9269	·2657	10	·9662	·1484
12	·9281	·3461	12	·9676	·1619
14	·9294	·4142	14	·9690	·1751
16	·9306	·4734	16	·9704	·1880
18	·9319	·5258	18	·9718	·2006
20	·9331	·5728	20	·9732	·2129
22	·9344	·6154	22	·9746	·2249
24	·9357	·6545	24	·9761	·2367
26	·9369	·6905	26	·9775	·2482
28	·9382	·7239	28	·9789	·2595
30	·9395	·7551	30	·9804	·2706
32	·9408	·7843	32	·9818	·2815
34	·9421	·8119	34	·9833	·2922
36	·9433	·8380	36	·9848	·3026
38	·9446	·8627	38	·9862	·3129
40	·9460	·8863	40	·9877	·3231
42	·9473	·9087	42	·9892	·3330
44	·9486	·9302	44	·9907	·3428
46	·9499	·9507	46	·9922	·3524
48	·9512	·9705	48	·9937	·3619
50	·9526	6·9895	50	·9952	·3712
52	·9539	7·0078	52	·9967	·3804
54	·9552	·0254	54	·9982	·3894
56	·9566	·0425	56	7·9998	·3984
58	7·9580	— 7·0590	58	8·0013	— 7·4071

TABLE XVI. continued.

For the equation of Equal Altitudes of the Sun.

Interval	Log. A	Log. B	Interval	Log. A	Log. B
h m 14 0	8·0028	− 7·4158	h m 15 0	8·0521	− 7·6350
2	·0044	·4244	2	·0539	·6413
4	·0059	·4328	4	·0556	·6475
6	·0075	·4412	6	·0574	·6537
8	·0090	·4494	8	·0592	·6599
10	·0106	·4575	10	·0610	·6660
12	·0122	·4655	12	·0628	·6721
14	·0138	·4735	14	·0646	·6781
16	·0154	·4813	16	·0664	·6841
18	·0170	·4890	18	·0682	·6900
20	·0186	·4967	20	·0700	·6959
22	·0202	·5043	22	·0718	·7018
24	·0218	·5118	24	·0737	·7077
26	·0234	·5192	26	·0755	·7135
28	·0250	·5265	28	·0774	·7192
30	·0267	·5338	30	·0792	·7249
32	·0283	·5410	32	·0811	·7306
34	·0300	·5481	34	·0830	·7363
36	·0316	·5551	36	·0849	·7419
38	·0333	·5621	38	·0868	·7475
40	·0350	·5690	40	·0887	·7531
42	·0367	·5759	42	·0906	·7586
44	·0384	·5827	44	·0925	·7641
46	·0400	·5894	46	·0945	·7696
48	·0417	·5961	48	·0964	·7751
50	·0435	·6027	50	·0983	·7805
52	·0452	·6092	52	·1003	·7859
54	·0469	·6158	54	·1023	·7912
56	·0486	·6222	56	·1042	·7966
58	8·0504	− 7·6286	58	8·1062	− 7·8019

For the equation of Equal Altitudes of the Sun.

Interval	Log. A	Log. B	Interval	Log. A	Log. B
h m 16 0	8·1082	— 7·8072	h m 17 0	8·1726	— 7·9571
2	·1102	·8125	2	·1749	·9618
4	·1122	·8177	4	·1773	·9666
6	·1143	·8229	6	·1796	·9713
8	·1163	·8281	8	·1819	·9761
10	·1183	·8333	10	·1843	·9808
12	·1204	·8385	12	·1867	·9855
14	·1224	·8436	14	·1890	·9902
16	·1245	·8487	16	·1914	·9949
18	·1266	·8538	18	·1938	7·9996
20	·1287	·8589	20	·1963	8·0043
22	·1308	·8640	22	·1987	·0090
24	·1329	·8690	24	·2011	·0137
26	·1350	·8740	26	·2036	·0184
28	·1371	·8790	28	·2061	·0230
30	·1393	·8840	30	·2086	·0277
32	·1414	·8890	32	·2111	·0323
34	·1436	·8939	34	·2136	·0370
36	·1458	·8989	36	·2161	·0416
38	·1479	·9038	38	·2186	·0462
40	·1501	·9087	40	·2212	·0508
42	·1523	·9136	42	·2237	·0555
44	·1545	·9185	44	·2263	·0601
46	·1568	·9234	46	·2289	·0647
48	·1590	·9282	48	·2315	·0693
50	·1612	·9330	50	·2341	·0739
52	·1635	·9379	52	·2367	·0785
54	·1658	·9427	54	·2394	·0831
56	·1680	·9475	56	·2420	·0877
58	8·1703	— 7·9523	58	8·2447	— 8·0923

For the equation of Equal Altitudes of the Sun.

Interval	Log. A	Log. B	Interval	Log. A	Log. B
h m 18 0	8·2474	— 8·0969	h m 19 0	8·3359	— 8·2354
2	·2501	·1015	2	·3392	·2401
4	·2529	·1061	4	·3424	·2448
6	·2556	·1107	6	·3457	·2495
8	·2583	·1153	8	·3490	·2542
10	·2611	·1199	10	·3524	·2589
12	·2639	·1245	12	·3557	·2637
14	·2667	·1291	14	·3591	·2684
16	·2695	·1336	16	·3625	·2732
18	·2723	·1382	18	·3659	·2779
20	·2752	·1428	20	·3694	·2827
22	·2781	·1474	22	·3728	·2875
24	·2809	·1520	24	·3763	·2923
26	·2838	·1566	26	·3798	·2971
28	·2868	·1612	28	·3834	·3019
30	·2897	·1658	30	·3869	·3068
32	·2926	·1704	32	·3905	·3116
34	·2956	·1750	34	·3941	·3165
36	·2986	·1797	36	·3978	·3214
38	·3016	·1842	38	·4015	·3263
40	·3046	·1889	40	·4052	·3312
42	·3077	·1935	42	·4089	·3361
44	·3107	·1981	44	·4126	·3410
46	·3138	·2028	46	·4164	·3460
48	·3169	·2074	48	·4202	·3510
50	·3200	·2121	50	·4241	·3560
52	·3232	·2167	52	·4279	·3610
54	·3263	·2214	54	·4318	·3660
56	·3295	·2261	56	·4357	·3711
58	8·3327	— 8·2307	58	8·4397	— 8·3761

For the equation of Equal Altitudes of the Sun.

Interval	Log. A	Log. B	Interval	Log. A	Log. B
h m 20 0	8·4437	— 8·3812	h m 21 0	8·5810	— 8·5466
2	·4477	·3863	2	·5863	·5527
4	·4518	·3915	4	.5917	·5588
6	·4559	·3966	6	·5971	·5650
8	·4600	·4018	8	·6025	·5712
10	·4641	·4070	10	·6081	·5775
12	·4683	·4122	12	·6136	·5838
14	·4726	·4175	14	·6193	·5902
16	·4768	·4227	16	·6250	·5966
18	·4811	·4280	18	·6308	·6031
20	·4854	·4334	20	·6366	·6096
22	·4898	·4387	22	·6426	·6162
24	·4942	·4441	24	·6486	·6229
26	·4987	·4495	26	·6546	·6296
28	·5032	·4549	28	·6608	·6364
30	·5077	·4604	30	·6670	·6433
32	·5123	·4659	32	·6733	·6502
34	·5169	·4714	34	·6796	·6572
36	·5215	·4770	36	·6861	·6643
38	·5262	·4826	38	·6927	·6715
40	·5310	·4882	40	·6993	·6788
42	·5357	·4939	42	·7060	·6860
44	·5406	·4996	44	·7128	·6934
46	·5455	·5053	46	·7197	·7009
48	·5504	·5111	48	·7268	·7085
50	·5554	·5169	50	·7339	·7162
52	·5604	·5228	52	·7411	·7239
54	·5655	·5287	54	·7484	·7318
56	·5706	·5346	56	·7558	·7398
58	8·5758	— 8·5406	58	8·7634	— 8·7478

For the equation of Equal Altitudes of the Sun.

Interval	Log. A	Log. B	Interval	Log. A	Log. B
h m 22 0	8·7711	— 8·7560	h m 23 0	9·0877	— 9·0839
2	·7789	·7643	2	·1029	·0995
4	·7868	·7727	4	·1187	·1155
6	·7948	·7813	6	·1351	·1321
8	·8030	·7899	8	·1520	·1492
10	·8113	·7987	10	·1696	·1670
12	·8198	·8076	12	·1879	·1855
14	·8284	·8167	14	·2069	·2047
16	·8372	·8259	16	·2268	·2248
18	·8461	·8353	18	·2476	·2456
20	·8553	·8448	20	·2693	·2677
22	·8645	·8545	22	·2922	·2907
24	·8740	·8 644	24	·3162	·3149
26	·8837	·8745	26	·3416	·3404
28	·8935	·8847	28	·3685	·3674
30	·9036	·8952	30	·3971	·3962
32	·9139	·9058	32	·4276	·4268
34	·9244	·9167	34	·4604	·4597
36	·9351	·9278	36	4957	·4952
38	·9461	·9391	38	·5341	·5336
40	·9574	·9507	40	·5761	·5757
42	·9689	·9626	42	·6224	·6221
44	·9807	·9747	44	·6742	·6739
46	8·9928	·9871	46	·7328	·7326
48	9·0052	8·9999	48	·8003	·8001
50	·0180	9·0129	50	·8801	·8800
52	·0311	·0263	52	9·9776	9·9775
54	·0446	·0401	54	0·1031	0·1031
56	·0585	·0543	56	0·2798	0·2798
58	9·0729	— 9·0689	58	0·5814	— 0·5814

Showing the Altitude of a star, whose Declination is *less* than the Latitude of the place, at the moment of its passing the Prime Vertical: also of a star, whose Declination is *greater* than the Latitude of the place, at the time of its greatest Azimuth, or at the moment when the vertical becomes a tangent to the circle of declination.

N.B. The Declination must be on the same side of the equator as the Latitude of the place.

Lat.	Declination of the star						
	5°	10°	15°	20°	25°	30°	35°
5°	90° 0′	30° 8′	19° 41′	14° 46′	11° 54′	10° 2′	8° 44′
10	30 8	90 0	42 8	30 31	24 16	20 19	17 37
15	19 41	42 8	90 0	49 11	37 46	31 10	26 49
20	14 46	30 31	49 11	90 0	54 40	43 10	36 36
25	11 54	24 16	37 46	54 40	90 0	57 45	47 28
30	10 2	20 19	31 10	43 10	57 45	90 0	60 40
35	8 44	17 37	26 49	36 36	47 28	60 40	90 0
40	7 48	15 40	23 45	32 9	41 6	51 4	63 10
45	7 5	14 13	21 28	28 56	36 42	45 0	54 12
50	6 32	13 6	19 45	26 31	33 29	40 45	48 29
55	6 6	12 14	18 25	24 41	31 4	37 37	44 27
60	5 47	11 34	17 23	23 16	29 40	35 16	41 29
65	5 31	11 3	16 36	22 10	27 48	33 29	39 16
70	5 19	10 39	15 59	21 21	26 44	32 9	37 37

The change of altitude, on the *Prime Vertical*, in one second of time is $= 15'' \times \sin \text{Lat.}$

For the Reduction to the Meridian: showing the value of

$$A = \frac{2 \sin^2 \frac{1}{2} P}{\sin 1''}$$

Sec.	0m	1m	2m	3m	4m	5m	6m	7m
0	0,0	2,0	7,8	17,7	31,4	49,1	70,7	96,2
1	0,0	2,0	8,0	17,9	31,7	49,4	71,1	96,7
2	0,0	2,1	8,1	18,1	31,9	49,7	71,5	97,1
3	0,0	2,2	8,2	18,3	32,2	50,1	71,9	97,6
4	0,0	2,2	8,4	18,5	32,5	50,4	72,3	98,0
5	0,0	2,3	8,5	18,7	32,7	50,7	72,7	98,5
6	0,0	2,4	8,7	18,9	33,0	51,1	73,1	99,0
7	0,0	2,4	8,8	19,1	33,3	51,4	73,5	99,4
8	0,0	2,5	8,9	19,3	33,5	51,7	73,9	99,9
9	0,0	2,6	9,1	19,5	33,8	52,1	74,3	100,4
10	0,1	2,7	9,2	19,7	34,1	52,4	74,7	100,8
11	0,1	2,7	9,4	19,9	34,4	52,7	75,1	101,3
12	0,1	2,8	9,5	20,1	34,6	53,1	75,5	101,8
13	0,1	2,9	9,6	20,3	34,9	53,4	75,9	102,3
14	0,1	3,0	9,8	20,5	35,2	53,8	76,3	102,7
15	0,1	3,1	9,9	20,7	35,5	54,1	76,7	103,2
16	0,1	3,1	10,1	20,9	35,7	54,5	77,1	103,7
17	0,2	3,2	10,2	21,2	36,0	54,8	77,5	104,2
18	0,2	3,3	10,4	21,4	36,3	55,1	77,9	104,6
19	0,2	3,4	10,5	21,6	36,6	55,5	78,3	105,1
20	0,2	3,5	10,7	21,8	36,9	55,8	78,8	105,6
21	0,2	3,6	10,8	22,0	37,2	56,2	79,2	106,1
22	0,3	3,7	11,0	22,3	37,4	56,5	79,6	106,6
23	0,3	3,8	11,2	22,5	37,7	56,9	80,0	107,0
24	0,3	3,8	11,3	22,7	38,0	57,3	80,4	107,5
25	0,3	3,9	11,5	22,9	38,3	57,6	80,8	108,0
26	0,4	4,0	11,6	23,1	38,6	58,0	81,3	108,5
27	0,4	4,1	11,8	23,4	38,9	58,3	81,7	109,0
28	0,4	4,2	11,9	23,6	39,2	58,7	82,1	109,5
29	0,5	4,3	12,1	23,8	39,5	59,0	82,5	110,0

For the Reduction to the Meridian: showing the value of

$$A = \frac{2 \sin^2 \frac{1}{2} P}{\sin 1''}$$

Sec.	0m	1m	2m	3m	4m	5m	6m	7m
30	0,5	4,4	12,3	24,0	39,8	59,4	83,0	110,4
31	0,5	4,5	12,4	24,3	40,1	59,8	83,4	110,9
32	0,6	4,6	12,6	24,5	40,3	60,1	83,8	111,4
33	0,6	4,7	12,8	24,7	40,6	60,5	84,2	111,9
34	0,6	4,8	12,9	25,0	40,9	60,8	84,7	112,4
35	0,7	4,9	13,1	25,2	41,2	61,2	85,1	112,9
36	0,7	5,0	13,3	25,4	41,5	61,6	85,5	113,4
37	0,7	5,1	13,4	25,7	41,8	61,9	86,0	113,9
38	0,8	5,2	13,6	25,9	42,1	62,3	86,4	114,4
39	0,8	5,3	13,8	26,2	42,5	62,7	86,8	114,9
40	0,9	5,4	14,0	26,4	42,8	63,0	87,3	115,4
41	0,9	5,6	14,1	26,6	43,1	63,4	87,7	115,9
42	1,0	5,7	14,3	26,9	43,4	63,8	88,1	116,4
43	1,0	5,8	14,5	27,1	43,7	64,2	88,6	116,9
44	1,1	5,9	14,7	27,4	44,0	64,5	89,0	117,4
45	1,1	6,0	14,8	27,6	44,3	64,9	89,5	117,9
46	1,2	6,1	15,0	27,9	44,6	65,3	89,9	118,4
47	1,2	6,2	15,2	28,1	44,9	65,7	90,3	118,9
48	1,3	6,4	15,4	28,3	45,2	66,0	90,8	119,5
49	1,3	6,5	15,6	28,6	45,5	66,4	91,2	120,0
50	1,4	6,6	15,8	28,8	45,9	66,8	91,7	120,5
51	1,4	6,7	15,9	29,1	46,2	67,2	92,1	121,0
52	1,5	6,8	16,1	29,4	46,5	67,6	92,6	121,5
53	1,5	7,0	16,3	29,6	46,8	68,0	93,0	122,0
54	1,6	7,1	16,5	29,9	47,1	68,3	93,5	122,5
55	1,6	7,2	16,7	30,1	47,5	68,7	93,9	123,1
56	1,7	7,3	16,9	30,4	47,8	69,1	94,4	123,6
57	1,8	7,5	17,1	30,6	48,1	69,5	94,8	124,1
58	1,8	7,6	17,3	30,9	48,4	69,9	95,3	124,6
59	1,9	7,7	17,5	31,1	48,8	70,3	95,7	125,1

For the Reduction to the Meridian: showing the value of

$$A = \frac{2 \sin^2 \frac{1}{2} P}{\sin 1''}$$

Sec.	8m	9m	10m	11m	12m	13m	14m
0	125,7	159,0	196,3	237,5	282,7	331,8	384,7
1	126,2	159,6	197,0	238,3	283,5	332,6	385,6
2	126,7	160,2	197,6	239,0	284,2	333,4	386,6
3	127,2	160,8	198,3	239,7	285,0	334,3	387,5
4	127,8	161,4	198,9	240,4	285,8	335,2	388,4
5	128,3	162,0	199,6	241,2	286,6	336,0	389,3
6	128,8	162,6	200,3	241,9	287,4	336,9	390,2
7	129,3	163,2	200,9	242,6	288,2	337,7	391,1
8	129,9	163,8	201,6	243,3	289,0	338,6	392,1
9	130,4	164,4	202,2	244,1	289,8	339,4	393,0
10	131,0	165,0	202,9	244,8	290,6	340,3	393,9
11	131,5	165,6	203,6	245,5	291,4	341,2	394,8
12	132,0	166,2	204,2	246,3	292,2	342,0	395,8
13	132,6	166,8	204,9	247,0	293,0	342,9	396,7
14	133,1	167,4	205,6	247,7	293,8	343,7	397,6
15	133,6	168,0	206,3	248,5	294,6	344,6	398,6
16	134,2	168,6	206,9	249,2	295,4	345,5	399,5
17	134,7	169,2	207,6	249,9	296,2	346,4	400,5
18	135,3	169,8	208,3	250,7	297,0	347,2	401,4
19	135,8	170,4	208,9	251,4	297,8	348,1	402,3
20	136,3	171,0	209,6	252,2	298,6	349,0	403,3
21	136,9	171,6	210,3	253,0	299,4	349,8	404,2
22	137,4	172,2	211,0	253,6	300,2	350,7	405,1
23	138,0	172,9	211,7	254,4	301,0	351,6	406,0
24	138,5	173,5	212,3	255,1	301,8	352,5	407,0
25	139,1	174,1	213,0	255,9	302,6	353,3	408,0
26	139,6	174,7	213,7	256,6	303,5	354,2	408,9
27	140,2	175,3	214,4	257,4	304,3	355,1	409,9
28	140,7	175,9	215,1	258,1	305,1	356,0	410,8
29	141,3	176,6	215,8	258,9	305,9	356,9	411,7

For the Reduction to the Meridian: showing the value of

$$A = \frac{2 \sin^2 \frac{1}{2} P}{\sin 1''}$$

Sec.	8m	9m	10m	11m	12m	13m	14m
30	141,8	177,2	216,4	259,6	306,7	357,7	412,7
31	142,4	177,8	217,1	260,4	307,5	358,6	413,6
32	143,0	178,4	217,8	261,1	308,4	359,5	414,6
33	143,5	179,0	218,5	261,9	309,2	360,4	415,5
34	144,1	179,7	219,2	262,6	310,0	361,3	416,5
35	144,6	180,3	219,9	263,4	310,8	362,2	417,5
36	145,2	180,9	220,6	264,1	311,6	363,1	418,4
37	145,8	181,6	221,3	264,9	312,5	364,0	419,4
38	146,3	182,2	222,0	265,7	313,3	364,8	420,3
39	146,9	182,8	222,7	266,4	314,1	365,7	421,3
40	147,5	183,5	223,4	267,2	315,0	366,6	422,2
41	148,0	184,1	224,1	267,9	315,8	367,5	423,2
42	148,6	184,7	224,8	268,7	316,6	368,4	424,2
43	149,2	185,4	225,5	269,5	317,4	369,3	425,1
44	149,7	186,0	226,2	270,3	318,3	370,2	426,1
45	150,3	186,6	226,9	271,0	319,1	371,1	427,0
46	150,9	187,3	227,6	271,8	319,9	372,0	428,0
47	151,5	187,9	228,3	272,6	320,8	372,9	429,0
48	152,0	188,5	229,0	273,3	321,6	373,8	429,9
49	152,6	189,2	229,7	274,1	322,4	374,7	430,9
50	153,2	189,8	230,4	274,9	323,3	375,6	431,9
51	153,8	190,5	231,1	275,6	324,1	376,5	432,8
52	154,4	191,1	231,8	276,4	325,0	377,4	433,8
53	154,9	191,8	232,5	277,2	325,8	378,3	434,8
54	155,5	192,4	233,2	278,0	326,7	379,3	435,8
55	156,1	193,1	234,0	278,8	327,5	380,2	436,7
56	156,7	193,7	234,7	279,5	328,4	381,1	437,7
57	157,3	194,4	235,4	280,3	329,2	382,0	438,7
58	157,8	195,0	236,1	281,1	330,0	382,9	439,7
59	158,4	195,7	236,8	281,9	330,9	383,8	440,6

For the Reduction to the Meridian: showing the value of

$$A = \frac{2 \sin^2 \frac{1}{2} P}{\sin 1''}$$

Sec.	15m	16m	17m	18m	19m	20m	21m
0	441,6	502,5	567,2	635,9	708,4	784,9	865,3
1	442,6	503,5	568,3	637,0	709,7	786,2	866,6
2	443,6	504,6	569,4	638,2	710,9	787,5	868,0
3	444,6	505,6	570,5	639,4	712,1	788,8	869,4
4	445,6	506,7	571,6	640,6	713,4	790,1	870,8
5	446,5	507,7	572,8	641,7	714,6	791,4	872,1
6	447,5	508,8	573,9	642,9	715,9	792,7	873,5
7	448,5	509,8	575,0	644,1	717,1	794,0	874,9
8	449,5	510,9	576,1	645,3	718,4	795,4	876,3
9	450,5	511,9	577,2	646,5	719,6	796,7	877,6
10	451,5	513,0	578,4	647,7	720,9	798,0	879,0
11	452,5	514,0	579,5	648,9	722,1	799,3	880,4
12	453,5	515,1	580,6	650,0	723,4	800,7	881,8
13	454,5	516,1	581,7	651,2	724,6	802,0	883,2
14	455,5	517,2	582,9	652,4	725,9	803,3	884,6
15	456,5	518,3	584,0	653,6	727,2	804,6	886,0
16	457,5	519,3	585,1	654,8	728,4	806,0	887,4
17	458,5	520,4	586,2	656,0	729,7	807,3	888,8
18	459,5	521,5	587,4	657,2	730,9	808,6	890,2
19	460,5	522,5	588,5	658,4	732,2	809,9	891,6
20	461,5	523,6	589,6	659,6	733,5	811,3	893,0
21	462,5	524,6	590,8	660,8	734,7	812,6	894,4
22	463,5	525,7	591,9	662,0	736,0	813,9	895,8
23	464,5	526,8	593,0	663,2	737,3	815,2	897,2
24	465,5	527,9	594,2	664,4	738,5	816,6	898,6
25	466,5	528,9	595,3	665,6	739,8	817,9	900,0
26	467,5	530,0	596,5	666,8	741,1	819,2	901,4
27	468,5	531,1	597,6	668,0	742,3	820,5	902,8
28	469,5	532,2	598,7	669,2	743,6	821,9	904,2
29	470,5	533,2	599,9	670,4	744,9	823,2	905,6

For the Reduction to the Meridian: showing the value of

$$A = \frac{2\sin^2\frac{1}{2}P}{\sin 1''}$$

Sec.	15^m	16^m	17^m	18^m	19^m	20^m	21^m
30	471,5″	534,3″	601,0″	671,6″	746,2″	824,6″	907,0″
31	472,6	535,4	602,2	672,8	747,4	825,9	908,4
32	473,6	536,5	603,3	674,1	748,7	827,3	909,8
33	474,6	537,6	604,5	675,3	750,0	828,6	911,2
34	475,6	538,7	605,6	676,5	751,3	829,9	912,6
35	476,6	539,7	606,8	677,7	752,6	831,2	914,0
36	477,6	540,8	607,9	678,9	753,8	832,6	915,5
37	478,7	541,9	609,1	680,1	755,1	833,9	916,9
38	479,7	543,0	610,2	681,3	756,4	835,3	918,3
39	480,7	544,1	611,4	682,6	757,7	836,6	919,7
40	481,7	545,2	612,5	683,8	759,0	838,0	921,1
41	482,8	546,3	613,7	685,0	760,2	839,3	922,5
42	483,8	547,4	614,8	686,2	761,5	840,7	923,9
43	484,8	548,4	616,0	687,4	762,8	842,0	925,3
44	485,8	549,5	617,2	688,7	764,1	843,4	926,8
45	486,9	550,6	618,3	689,9	765,4	844,7	928,2
46	487,9	551,7	619,5	691,1	766,7	846,1	929,6
47	488,9	552,8	620,6	692,4	768,0	847,5	931,0
48	490,0	553,9	621,8	693,6	769,3	848,9	932,4
49	491,0	555,0	623,0	694,8	770,6	850,2	933,8
50	492,0	556,1	624,1	696,0	771,9	851,6	935,2
51	493,1	557,2	625,3	697,3	773,1	852,9	936,6
52	494,1	558,3	626,5	698,5	774,5	854,3	938,1
53	495,2	559,4	627,6	699,7	775,8	855,7	939,5
54	496,2	560,5	628,8	701,0	777,1	857,1	940,9
55	497,2	561,6	630,0	702,2	778,4	858,4	942,3
56	498,3	562,7	631,2	703,5	779,7	859,8	943,8
57	499,3	563,9	632,3	704,7	781,0	861,1	945,2
58	500,3	565,0	633,5	705,9	782,3	862,5	946,6
59	501,4	566,1	634,7	707,1	783,6	863,9	948,1

For the Reduction to the Meridian: showing the value of

$$A = \frac{2 \sin^2 \frac{1}{2} P}{\sin 1''}$$

Sec.	22m	23m	24m	25m	26m	27m	28m
0	949,6″	1037,8″	1129,9″	1225,9″	1325,9″	1429,7″	1537,5″
1	951,0	1039,3	1131,4	1227,5	1327,6	1431,4	1539,3
2	952,4	1040,8	1133,0	1229,2	1329,3	1433,2	1541,1
3	953,8	1042,3	1134,6	1230,8	1331,0	1434,9	1542,9
4	955,3	1043,8	1136,2	1232,5	1332,7	1436,7	1544,8
5	956,7	1045,3	1137,8	1234,1	1334,4	1438,5	1546,6
6	958,2	1046,8	1139,3	1235,7	1336,1	1440,3	1548,4
7	959,6	1048,3	1140,9	1237,3	1337,8	1442,1	1550,2
8	961,1	1049,8	1142,5	1239,0	1339,5	1443,9	1552,1
9	962,5	1051,3	1144,0	1240,6	1341,2	1445,6	1553,9
10	963,9	1052,8	1145,6	1242,3	1342,9	1447,4	1555,8
11	965,4	1054,3	1147,2	1243,9	1344,6	1449,2	1557,6
12	966,9	1055,9	1148,8	1245,6	1346,3	1451,0	1559,5
13	968,3	1057,4	1150,4	1247,2	1348,0	1452,8	1561,3
14	969,8	1058,9	1152,0	1248,9	1349,7	1454,5	1563,2
15	971,2	1060,4	1153,6	1250,5	1351,4	1456,3	1565,0
16	972,7	1062,0	1155,2	1252,2	1353,2	1458,1	1566,9
17	974,1	1063,5	1156,8	1253,8	1354,9	1459,9	1568,7
18	975,5	1065,0	1158,3	1255,5	1356,6	1461,6	1570,5
19	977,0	1066,5	1159,9	1257,1	1358,3	1463,4	1572,4
20	978,5	1068,1	1161,5	1258,8	1360,1	1465,2	1574,3
21	979,9	1069,6	1163,1	1260,4	1361,8	1466,9	1576,1
22	981,4	1071,1	1164,7	1262,1	1363,5	1468,7	1578,0
23	982,9	1072,6	1166,3	1263,7	1365,2	1470,5	1579,8
24	984,4	1074,2	1167,9	1265,4	1367,0	1472,3	1581,7
25	985,8	1075,7	1169,5	1267,0	1368,7	1474,0	1583,5
26	987,3	1077,2	1171,1	1268,7	1370,4	1475,9	1585,3
27	988,8	1078,7	1172,7	1270,3	1372,1	1477,7	1587,2
28	990,3	1080,3	1174,3	1272,1	1373,9	1479,5	1589,1
29	991,8	1081,8	1175,9	1273,7	1375,6	1481,3	1590,9

For the Reduction to the Meridian: showing the value of

$$A = \frac{2 \sin^2 \frac{1}{2} P}{\sin 1''}$$

Sec.	22m	23m	24m	25m	26m	27m	28m
	"	"	"	"	"	"	"
30	993,2	1083,3	1177,5	1275,4	1377,4	1483,1	1592,7
31	994,7	1084,8	1179,1	1277,1	1379,0	1484,9	1594,6
32	996,2	1086,4	1180,7	1278,8	1380,8	1486,7	1596,5
33	997,6	1087,9	1182,3	1280,4	1382,5	1488,5	1598,3
34	999,1	1089,5	1183,9	1282,1	1384,2	1490,3	1600,2
35	1000,6	1091,0	1185,5	1283,8	1385,9	1492,1	1602,1
36	1002,1	1092,6	1187,1	1285,5	1387,7	1493,9	1604,0
37	1003,5	1094,1	1188,7	1287,1	1389,4	1495,7	1605,9
38	1005,0	1095,7	1190,3	1288,8	1391,2	1497,5	1607,7
39	1006,5	1097,2	1191,9	1290,5	1392,9	1499,3	1609,6
40	1008,0	1098,8	1193,5	1292,2	1394,7	1501,1	1611,5
41	1009,4	1100,3	1195,1	1293,8	1396,4	1502,9	1613,3
42	1010,9	1101,9	1196,7	1295,5	1398,2	1504,7	1615,2
43	1012,4	1103,4	1198,3	1297,2	1399,9	1506,5	1617,1
44	1013,9	1105,0	1199,9	1298,9	1401,7	1508,4	1619,0
45	1015,4	1106,5	1201,5	1300,5	1403,4	1510,2	1620,8
46	1016,9	1108,1	1203,1	1302,2	1405,2	1512,0	1622,7
47	1018,4	1109,6	1204,7	1303,9	1406,9	1513,8	1624,6
48	1019,9	1111,2	1206,4	1305,6	1408,7	1515,6	1626,5
49	1021,4	1112,7	1208,0	1307,3	1410,4	1517,4	1628,3
50	1022,8	1114,3	1209,6	1309,0	1412,2	1519,2	1630,2
51	1024,3	1115,8	1211,2	1310,7	1413,9	1521,0	1632,1
52	1025,8	1117,4	1212,9	1312,4	1415,7	1522,9	1634,0
53	1027,3	1118,9	1214,5	1314,1	1417,4	1524,7	1635,9
54	1028,8	1120,5	1216,1	1315,7	1419,2	1526,5	1637,7
55	1030,3	1122,0	1217,7	1317,4	1420,9	1528,3	1639,6
56	1031,8	1123,6	1219,4	1319,1	1422,7	1530,2	1641,5
57	1033,3	1125,1	1221,0	1320,8	1424,4	1532,0	1643,3
58	1034,8	1126,7	1222,6	1322,5	1426,2	1533,8	1645,2
59	1036,3	1128,3	1224,2	1324,2	1427,9	1535,6	1647,1

For the Reduction to the Meridian: showing the value of

$$A = \frac{2 \sin^2 \frac{1}{2} P}{\sin 1''}$$

Sec.	29m	30m	31m	32m	33m	34m	35m
0	1649,0	1764,6	1884,0	2007,4	2134,6	2265,6	2400,6
1	1650,9	1766,6	1886,0	2009,4	2136,8	2267,8	2402,9
2	1652,8	1768,5	1888,0	2011,5	2138,9	2270,0	2405,2
3	1654,7	1770,5	1890,0	2013,6	2141,1	2272,2	2407,5
4	1656,6	1772,4	1892,1	2015,7	2143,2	2274,5	2409,8
5	1658,5	1774,4	1894,1	2017,8	2145,3	2276,7	2412,0
6	1660,4	1776,3	1896,1	2019,9	2147,5	2278,9	2414,3
7	1662,3	1778,3	1898,1	2022,0	2149,7	2281,2	2416,6
8	1664,2	1780,3	1900,2	2024,1	2151,8	2283,4	2418,9
9	1666,1	1782,3	1902,2	2026,2	2153,9	2285,6	2421,2
10	1668,0	1784,2	1904,3	2028,3	2156,1	2287,8	2423,5
11	1669,9	1786,2	1906,3	2030,5	2158,3	2290,0	2425,8
12	1671,9	1788,2	1908,4	2032,5	2160,5	2292,3	2428,1
13	1673,8	1790,1	1910,4	2034,6	2162,6	2294,5	2430,4
14	1675,7	1792,1	1912,4	2036,7	2164,8	2296,8	2432,7
15	1677,6	1794,1	1914,4	2038,8	2166,9	2299,0	2435,0
16	1679,5	1796,1	1916,5	2040,9	2169,1	2301,3	2437,3
17	1681,4	1798,1	1918,5	2043,0	2171,2	2303,6	2439,6
18	1683,3	1800,0	1920,6	2045,1	2173,4	2305,8	2441,9
19	1685,2	1802,0	1922,6	2047,2	2175,6	2308,0	2444,2
20	1687,2	1804,0	1924,7	2049,3	2177,8	2310,2	2446,5
21	1689,1	1805,9	1926,7	2051,4	2179,9	2312,4	2448,8
22	1691,0	1807,9	1928,8	2053,5	2182,1	2314,7	2451,1
23	1692,9	1809,9	1930,8	2055,7	2184,3	2316,9	2453,4
24	1694,8	1811,9	1932,9	2057,8	2186,5	2319,2	2455,7
25	1696,7	1813,9	1935,0	2059,9	2188,6	2321,5	2458,0
26	1698,6	1815,8	1937,0	2062,0	2190,8	2323,7	2460,3
27	1700,5	1817,8	1939,0	2064,1	2193,0	2325,9	2462,6
28	1702,5	1819,8	1941,1	2066,2	2195,2	2328,2	2464,9
29	1704,4	1821,8	1943,1	2068,3	2197,3	2330,4	2467,2

For the Reduction to the Meridian: showing the value of

$$A = \frac{2 \sin^2 \frac{1}{2} P}{\sin 1''}$$

Sec.	29m	30m	31m	32m	33m	34m	35m
30	1706,3	1823,8	1945,2	2070,4	2199,5	2332,7	2469,5
31	1708,2	1825,8	1947,2	2072,6	2201,7	2334,9	2471,8
32	1710,2	1827,8	1949,3	2074,7	2203,9	2337,2	2474,2
33	1712,1	1829,8	1951,3	2076,8	2206,1	2339,4	2476,5
34	1714,0	1831,8	1953,4	2078,9	2208,3	2341,7	2478,8
35	1715,9	1833,8	1955,5	2081,0	2210,5	2343,9	2481,1
36	1717,9	1835,8	1957,6	2083,2	2212,7	2346,2	2483,5
37	1719,8	1837,8	1959,6	2085,3	2214,9	2348,5	2485,8
38	1721,7	1839,8	1961,7	2087,4	2217,1	2350,7	2488,1
39	1723,6	1841,8	1963,7	2089,6	2219,3	2353,0	2490,4
40	1725,6	1843,8	1965,8	2091,7	2221,5	2355,2	2492,8
41	1727,5	1845,8	1967,8	2093,8	2223,7	2357,5	2495,1
42	1729,5	1847,8	1969,9	2095,9	2225,9	2359,7	2497,4
43	1731,5	1849,8	1972,0	2098,0	2228,1	2361,9	2499,7
44	1733,4	1851,8	1974,1	2100,2	2230,3	2364,2	2502,1
45	1735,3	1853,8	1976,1	2102,3	2232,5	2366,4	2504,4
46	1737,2	1855,8	1978,2	2104,5	2234,7	2368,7	2506,7
47	1739,2	1857,8	1980,3	2106,6	2236,9	2371,0	2509,0
48	1741,2	1859,8	1982,4	2108,8	2239,1	2373,3	2511,4
49	1743,1	1861,8	1984,4	2110,9	2241,3	2375,5	2513,7
50	1745,1	1863,8	1986,5	2113,1	2243,5	2377,8	2516,1
51	1747,0	1865,8	1988,6	2115,2	2245,7	2380,1	2518,4
52	1749,0	1867,8	1990,7	2117,4	2247,9	2382,4	2520,8
53	1750,9	1869,8	1992,7	2119,6	2250,1	2384,6	2523,1
54	1752,9	1871,8	1994,8	2121,7	2252,3	2386,9	2525,4
55	1754,8	1873,8	1996,9	2123,8	2254,5	2389,2	2527,7
56	1756,8	1875,9	1999,0	2126,0	2256,7	2391,5	2530,1
57	1758,7	1877,9	2001,0	2128,1	2258,9	2393,7	2532,4
58	1760,7	1879,9	2003,1	2130,3	2261,1	2396,0	2534,8
59	1762,6	1882,0	2005,3	2132,4	2263,4	2398,3	2537,1

TABLE XIX.

For the second part of the Reduction to the Meridian: showing the value of $B = \frac{2 \sin^4 \frac{1}{2} P}{\sin 1''}$

Minutes	0ˢ	10ˢ	20ˢ	30ˢ	40ˢ	50ˢ
5	0,01″	0,01″	0,01″	0,01″	0,01″	0,01″
6	0,01	0,01	0,01	0,02	0,02	0,02
7	0,02	0,02	0,03	0,03	0,03	0,04
8	0,04	0,04	0,05	0,05	0,05	0,06
9	0,06	0,07	0,08	0,08	0,08	0,09
10	0,09	0,10	0,11	0,11	0,12	0,13
11	0,14	0,15	0,15	0,16	0,17	0,18
12	0,19	0,20	0,22	0,23	0,24	0,25
13	0,27	0,28	0,30	0,31	0,33	0,34
14	0,36	0,38	0,39	0,41	0,43	0,45
15	0,47	0,49	0,52	0,54	0,56	0,59
16	0,61	0,64	0,67	0,69	0,72	0,75
17	0,78	0,81	0,84	0,88	0,91	0,95
18	0,98	1,02	1,06	1,09	1,13	1,18
19	1,22	1,26	1,30	1,35	1,40	1,44
20	1,49	1,54	1,60	1,65	1,70	1,76
21	1,82	1,87	1,93	1,99	2,06	2,12
22	2,19	2,25	2,32	2,39	2,46	2,54
23	2,61	2,69	2,77	2,85	2,93	3,01
24	3,10	3,18	3,27	3,36	3,45	3,55
25	3,64	3,74	3,84	3,94	4,05	4,15
26	4,26	4,37	4,48	4,60	4,72	4,83
27	4,96	5,08	5,20	5,33	5,46	5,60
28	5,73	5,87	6,01	6,15	6,30	6,44
29	6,59	6,75	6,90	7,06	7,22	7,38
30	7,55	7,72	7,89	8,06	8,24	8,42
31	8,61	8,79	8,98	9,17	9,37	9,57
32	9,77	9,97	10,18	10,39	10,61	10,82
33	11,04	11,27	11,50	11,73	11,96	12,20
34	12,44	12,69	12,94	13,19	13,45	13,71
35	13,97	14,24	14,51	14,78	15,06	15,35

Mean Obliquity of the Ecliptic, on Jan. 1. in every year from 1800 to 1900.

Year	23° 27′	Year	23° 27′	Year	23° 27′
C 1800	54″,78	1834	39″,24	B 1868	23″,70
1801	54,32	1835	38,78	1869	23,24
1802	53,86	B 1836	38,32	1870	22,79
1803	53,41	1837	37,87	1871	22,33
B 1804	52,95	1838	37,41	B 1872	21,87
1805	52,49	1839	36,95	1873	21,42
1806	52,03	B 1840	36,50	1874	20,96
1807	51,58	1841	36,04	1875	20,50
B 1808	51,12	1842	35,58	B 1876	20,04
1809	50,66	1843	35,13	1877	19,59
1810	50,21	B 1844	34,67	1878	19,13
1811	49,75	1845	34,21	1879	18,67
B 1812	49,29	1846	33,75	B 1880	18,22
1813	48,84	1847	33,30	1881	17,76
1814	48,38	B 1848	32,84	1882	17,30
1815	47,92	1849	32,38	1883	16,85
B 1816	47,46	1850	31,93	B 1884	16,39
1817	47,01	1851	31,47	1885	15,93
1818	46,55	B 1852	31,01	1886	15,47
1819	46,09	1853	30,56	1887	15,02
B 1820	45,64	1854	30,10	B 1888	14,56
1821	45,18	1855	29,64	1889	14,10
1822	44,72	B 1856	29,18	1890	13,65
1823	44,27	1857	28,73	1891	13,19
B 1824	43,81	1858	28,27	B 1892	12,73
1825	43,35	1859	27,81	1893	12,28
1826	42,89	B 1860	27,36	1894	11,82
1827	42,44	1861	26,90	1895	11,36
B 1828	41,98	1862	26,44	B 1896	10,90
1829	41,52	1863	25,99	1897	10,45
1830	41,07	B 1864	25,53	1898	9,99
1831	40,61	1865	25,07	1899	9,53
B 1832	40,15	1866	24,61	C 1900	9,08
1833	39,70	1867	24,16		

Lunar Nutation in Longitude, and in the Obliquity of the Ecliptic.

Argument = The mean place of the Moon's node.

☊	Long.	Obliq.	☊	Long.	Obliq.
0	− 0,00″	+ 9,16″	250	− 17,30″	+ 0,09″
10	1,04	9,14	260	17,29	− 0,49
20	2,12	9,09	270	17,21	1,07
30	3,16	9,00	280	17,07	1,65
40	4,20	8,88	290	16,85	2,22
50	5,22	8,72	300	16,57	2,79
60	6,23	8,53	310	16,23	3,34
70	7,20	8,31	320	15,81	3,88
80	8,16	8,06	330	15,33	4,41
90	9,08	7,77	340	14,79	4,92
100	9,97	7,46	350	14,19	5,41
110	10,82	7,11	360	13,53	5,88
120	11,63	6,74	370	12,82	6,33
130	12,40	6,34	380	12,05	6,75
140	13,12	5,91	390	11,23	7,14
150	13,80	5,46	400	10,37	7,51
160	14,42	4,99	410	9,46	7,85
170	14,98	4,50	420	8,51	8,15
180	15,49	4,00	430	7,52	8,43
190	15,94	3,47	440	6,51	8,67
200	16,33	2,93	450	5,47	8,87
210	16,65	2,38	460	4,40	9,04
220	16,91	1,82	470	3,32	9,17
230	17,11	1,25	480	2,22	9,26
240	17,24	0,67	490	1,09	9,32
250	− 17,30	+ 0,09	500	− 0,00	− 9,34

Lunar Nutation in Longitude, and in the Obliquity of the Ecliptic.

Argument = The mean place of the Moon's node.

☊	Long.	Obliq.	☊	Long.	Obliq.
500	+ 0,00″	− 9,34″	750	+ 17,30″	+ 0,09″
510	1,09	9,32	760	17,24	0,67
520	2,22	9,26	770	17,11	1,25
530	3,32	9,17	780	16,91	1,82
540	4,40	9,04	790	16,65	2,38
550	5,47	8,87	800	16,33	2,93
560	6,51	8,67	810	15,94	3,47
570	7,52	8,43	820	15,49	4,00
580	8,51	8,15	830	14,98	4,50
590	9,46	7,85	840	14,42	4,99
600	10,37	7,51	850	13,80	5,46
610	11,23	7,14	860	13,12	5,91
620	12,05	6,75	870	12,40	6,34
630	12,82	6,33	880	11,63	6,74
640	13,53	5,88	890	10,82	7,11
650	14,19	5,41	900	9,97	7,46
660	14,79	4,92	910	9,08	7,77
670	15,33	4,41	920	8,16	8,06
680	15,81	3,88	930	7,20	8,31
690	16,23	3,34	940	6,23	8,53
700	16,57	2,79	950	5,22	8,72
710	16,85	2,22	960	4,20	8,88
720	17,07	1,65	970	3,16	9,00
730	17,21	1,07	980	2,12	9,09
740	17,29	− 0,49	990	1,04	9,14
750	+ 17,30	+ 0,09	1000	+ 0,00	+ 9,16

Solar Nutation in Longitude; and the solar nutation, added to the mean diminution, of the Obliquity of the Ecliptic.

Argument = The day of the year.

Day	Long.	Obliq.	Day	Long.	Obliq.
Jan. 1	+ 0″,47	− 0″,50	July 10	+ 0″,74	− 0″,68
11	0,85	0,41	20	1,03	0,56
21	1,12	0,27	30	1,21	0,41
31	1,25	− 0,10	August 9	1,25	0,24
Feb. 10	1,22	+ 0,08	19	1,16	− 0,08
20	1,05	0,24	29	0,93	+ 0,06
March 2 *	0,74	0,36	Sept. 8	0,60	0,16
12	+ 0,35	0,43	18	+ 0,20	0,21
22	− 0,08	0,44	28	− 0,23	0,20
April 1	0,50	0,39	Oct. 8	0,63	+ 0,12
11	0,85	0,27	18	0,96	− 0,01
21	1,11	+ 0,11	28	1,18	0,19
May 1	1,24	− 0,07	Nov. 7	1,25	0,39
11	1,23	0,27	17	1,18	0,59
21	1,08	0,45	27	0,96	0,77
31	0,81	0,60	Dec. 7	0,60	0,90
June 10	0,45	0,71	17	− 0,20	0,98
20	− 0,05	0,76	27	+ 0,25	0,98
30	+ 0,37	− 0,75	37	+ 0,66	− 0,93

N.B. The Longitude ought to be further corrected by $-0'',207 \sin 2☽$: and the Obliquity by $+0'',090 \cos 2☽$.

* In *Leap years*, we must deduct unity from all these tabular dates after February, in order to obtain the corresponding civil date.

Selenographic positions of the principal Lunar Spots.

No.	Riccioli's Names	Long^{e}.	Lat^{e}.
1	Zoroaster	West 72°	+ 58°
2	Mercurius	67	+ 40
3	Petavius	64	− 24
4	Langrenus	62	− 8
5	Endymion	60	+ 53
6	Cleomedes	55	+ 26
7	Atlas	48	+ 47
8	Hercules	42	+ 48
9	CENSORINUS	32	0
10	Fracastorius	32	− 22
11	Possidonius	32	+ 31
12	Theophilus	27	− 12
13	Cyrillus	25	− 13
14	St. Catharina	24	− 18
15	Menelaus	15	+ 16
16	Aristoteles	West 15	+ 50
17	Ptolomæus	East 2	− 10
18	Arzachel	3	− 20
19	Archimedes	5	+ 28
20	Tycho	10	− 43
21	Plato	10	+ 52
22	Pitatus	12	− 29
23	Eratosthenes	12	+ 14
24	Clavius	16	− 60
25	Copernicus	19	+ 9
26	Bullialdus	21	− 21
27	Blancanus	25	− 65
28	Heraclides	38	+ 41
29	Keplerus	38	+ 7
30	Gassendus	39	− 19
31	Aristarchus	48	+ 24
32	Hevelius	67	− 1
33	Schickardus	68	− 49
34	Grimaldus	East 68	− 5

The *enlightened part* of the Moon's disc is denoted by the versed sine of the angular distance of the moon from the sun.

Showing the Angle of the Vertical, and the Logarithm of the earth's Radius, at different Latitudes: the Compression of the earth being assumed equal to $\frac{1}{300}$.

Lat.	Angle of the Vertical	Logarithm of Earth's Radius
0°	0′ 0″,0	0·0000000
5	1 59,2	9·9999890
10	3 54,8	9·9999566
15	5 43,4	9·9999037
20	7 21,6	9·9998318
25	8 46,5	9·9997431
30	9 55,4	9·9996402
35	10 46,4	9·9995261
40	11 17,9	9·9994044
45	11 28,7	9·9992786
50	11 18,6	9·9991525
55	10 47,9	9·9990302
60	9 57,4	9·9989151
65	8 48,7	9·9988111
70	7 23,8	9·9987210
75	5 45,4	9·9986479
80	3 56,3	9·9985940
85	2 0,0	9·9985610
90	0 0,0	9·9985499

Greenwich	11′ 11″,6	9·9991159

Showing the Augmentation of the Moon's Semidiameter, on account of her apparent altitude.

Apparent Altitude	Horizontal Semidiameter					
	14′ 30″	15′ 0″	15′ 30″	16′ 0″	16′ 30″	17′ 0″
0°	0″,00	0″,00	0″,00	0″,00	0″,00	0″,00
3	0,71	0,75	0,80	0,86	0,92	0,97
6	1,41	1,50	1,60	1,71	1,83	1,94
9	2,11	2,25	2,40	2,56	2,73	2,90
12	2,81	3,00	3,20	3,41	3,63	3,86
15	3,50	3,74	3,99	4,25	4,52	4,80
18	4,17	4,46	4,76	5,07	5,39	5,73
21	4,84	5,18	5,52	5,89	6,26	6,65
24	5,49	5,88	6,27	6,68	7,11	7,54
27	6,13	6,56	7,00	7,46	7,93	8,42
30	6,75	7,23	7,71	8,22	8,74	9,28
33	7,35	7,88	8,40	8,96	9,52	10,12
36	7,93	8,50	9,07	9,67	10,28	10,92
39	8,49	9,10	9,72	10,36	11,02	11,66
42	9,03	9,68	10,34	11,02	11,72	12,44
45	9,55	10,23	10,93	11,65	12,39	13,15
48	10,05	10,76	11,49	12,25	13,03	13,83
51	10,52	11,26	12,02	12,81	13,63	14,46
54	10,95	11,72	12,52	13,34	14,19	15,06
57	11,35	12,15	12,98	13,83	14,72	15,62
60	11,72	12,55	13,40	14,29	15,20	16,13
63	12,06	12,91	13,79	14,70	15,64	16,60
66	12,37	13,24	14,14	15,08	16,04	17,03
69	12,64	13,53	14,46	15,41	16,39	17,40
72	12,88	13,79	14,73	15,70	16,70	17,73
75	13,08	14,01	14,96	15,95	16,96	18,01
78	13,24	14,18	15,15	16,15	17,18	18,24
81	13,37	14,32	15,30	16,31	17,35	18,42
84	13,46	14,42	15,41	16,42	17,47	18,55
87	13,52	14,48	15,47	16,49	17,54	18,62
90	13,54	14,50	15,49	16,51	17,57	18,65

Logarithms for the equations of the first, second, and third differences of the Moon's place.

Hour from Noon or Midnight		Logarithms of the factors of the First differences		2nd diff.	3rd diff.	Hour from Noon or Midnight	
h	m	+	+	−		h	m
0	10	8·1427	9·9939	7·8356	+ 7·0452 −	11	50
	20	8·4437	9·9878	8·1304	7·3275		40
	30	8·6198	9·9815	8·3003	7·4843		30
	40	8·7447	9·9752	8·4189	7·5896		20
	50	8·8416	9·9687	8·5094	7·6663		10
1	0	8·9208	9·9622	8·5820	7·7247	11	0
	10	8·9878	9·9559	8·6423	7·7703		50
	20	9·0458	9·9488	8·6936	7·8063		40
	30	9·0969	9·9420	8·7379	7·8348		30
	40	9·1427	9·9351	8·7767	7·8572		20
	50	9·1841	9·9280	8·8110	7·8745		10
2	0	9·2218	9·9208	8·8416	7·8874	10	0
	10	9·2566	9·9135	8·8691	7·8964		50
	20	9·2888	9·9061	8·8939	7·9018		40
	30	9·3188	9·8985	8·9163	7·9040		30
	40	9·3468	9·8909	8·9366	7·9032		20
	50	9·3731	9·8830	8·9551	7·8994		10
3	0	9·3979	9·8751	8·9720	7·8928	9	0
	10	9·4214	9·8669	8·9873	7·8833		50
	20	9·4437	9·8587	9·0013	7·8710		40
	30	9·4649	9·8502	9·0141	7·8557		30
	40	9·4851	9·8416	9·0257	7·8374		20
	50	9·5044	9·8329	9·0362	7·8157		10
4	0	9·5229	9·8239	9·0458	7·7905	8	0
	10	9·5406	9·8148	9·0543	7·7613		50
	20	9·5577	9·8054	9·0620	7·7276		40
	30	9·5740	9·7959	9·0689	7·6887		30
	40	9·5898	9·7861	9·0749	7·6436		20
	50	9·6051	9·7761	9·0802	7·5908		10
5	0	9·6198	9·7659	9·0847	7·5284	7	0
	10	9·6340	9·7555	9·0884	7·4230		50
	20	9·6478	9·7447	9·0915	7·3591		40
	30	9·6612	9·7337	9·0939	7·2366		30
	40	9·6741	9·7225	9·0956	7·0621		20
	50	9·6867	9·7109	9·0966	6·7621		10
6	0	9·6990	9·6990	9·0969	+ ∞ −	6	0

TABLE XXVII.

Showing the Annual Precession of the Equinoxes in Longitude: and the Constants for computing the annual precession in Right Ascension and Declination.

Year	Precession in Longitude		Prec. in Æ const. $= m$	Prec. in Æ and D const. $= n$
	General	Luni-solar		
1800	50″,22350	50″,36354	46″,04367	20″,05690
1805	50,22472	50,36232	46,04521	20,05641
1810	50,22594	50,36110	46,04676	20,05593
1815	50,22716	50,35988	46,04830	20,05544
1820	50,22839	50,35866	46,04984	20,05496
1825	50,22961	50,35745	46,05138	20,05447
1830	50,23083	50,35623	46,05293	20,05399
1835	50,23205	50,35502	46,05447	20,05350
1840	50,23328	50,35380	46,05601	20,05302
1845	50,23450	50,35258	46,05755	20,05253
1850	50,23572	50,35136	46,05910	20,05205
1855	50,23694	50,35014	46,06065	20,05156
1860	50,23816	50,34892	46,06219	20,05108
1865	50,23938	50,34771	46,06374	20,05059
1870	50,24060	50,34649	46,06528	20,05011
1875	50,24182	50,34527	46,06682	20,04962
1880	50,24305	50,34405	46,06836	20,04914
1885	50,24427	50,34283	46,06991	20,04865
1890	50,24549	50,34162	46,07115	20,04817
1895	50,24671	50,34040	46,07299	20,04768
1900	50,24793	50,33918	46,07454	20,94720

Annual Prec. in Æ $= m + n \cdot \sin$ Æ $\cdot \tan D$

Annual Prec. in $D = n \cdot \cos$ Æ

For determining the Aberration of a star in Æ, and the *first* part of the Aberration of a star in Declination.

Argument, ☉ = the true longitude of the sun.

	0ˢ	VIˢ	Iˢ	VIIˢ	IIˢ	VIIIˢ	
	Log. *a*	A +	Log. *a*	A +	Log. *a*	A +	
0°	1·2690	0° 0′	1·2790	2° 11′	1·2977	2° 6′	30°
1	1·2690	0 5	1·2796	2 14	1·2983	2 3	29
2	1·2691	0 11	1·2802	2 16	1·2988	2 0	28
3	1·2692	0 16	1·2808	2 18	1·2993	1 57	27
4	1·2692	0 22	1·2815	2 20	1·2998	1 54	26
5	1·2693	0 27	1·2821	2 21	1·3003	1 51	25
6	1·2695	0 32	1·2827	2 23	1·3008	1 47	24
7	1·2696	0 37	1·2834	2 24	1·3012	1 44	23
8	1·2698	0 43	1·2840	2 25	1·3017	1 40	22
9	1·2700	0 48	1·2847	2 26	1·3021	1 36	21
10	1·2703	0 53	1·2853	2 27	1·3025	1 32	20
11	1·2705	0 58	1·2860	2 28	1·3028	1 28	19
12	1·2708	1 3	1·2866	2 28	1·3032	1 24	18
13	1·2711	1 8	1·2873	2 28	1·3036	1 20	17
14	1·2714	1 12	1·2879	2 28	1·3039	1 16	16
15	1·2718	1 17	1·2886	2 28	1·3042	1 11	15
16	1·2721	1 22	1·2892	2 28	1·3045	1 7	14
17	1·2725	1 26	1·2899	2 27	1·3048	1 3	13
18	1·2729	1 30	1·2905	2 27	1·3050	0 58	12
19	1·2733	1 34	1·2912	2 26	1·3053	0 53	11
20	1·2738	1 39	1·2918	2 25	1·3055	0 49	10
21	1·2742	1 42	1·2924	2 24	1·3057	0 44	9
22	1·2747	1 46	1·2931	2 22	1·3059	0 39	8
23	1·2752	1 50	1·2938	2 21	1·3060	0 34	7
24	1·2757	1 53	1·2944	2 19	1·3061	0 30	6
25	1·2762	1 57	1·2949	2 17	1·3063	0 25	5
26	1·2768	2 0	1·2956	2 15	1·3064	0 20	4
27	1·2773	2 3	1·2961	2 13	1·3064	0 15	3
28	1·2779	2 6	1·2966	2 11	1·3065	0 10	2
29	1·2785	2 9	1·2972	2 8	1·3065	0 5	1
30	1·2790	2 11	1·2977	2 6	1·3065	0 0	0
	Log. *a*	A −	Log. *a*	A −	Log. *a*	A −	
	Vˢ	XIˢ	IVˢ	Xˢ	IIIˢ	IXˢ	

For the *second* part of the Aberration of a star in Dec.

Arguments, (☉ + D) and (☉ − D).

	0^s VI^s	I^s VII^s	II^s $VIII^s$	
	− +	− +	− +	
0°	4,03″	3,49″	2,02″	30°
1	4,03	3,46	1,96	29
2	4,03	3,42	1,89	28
3	4,03	3,38	1,83	27
4	4,02	3,34	1,77	26
5	4,02	3,30	1,70	25
6	4,01	3,26	1,64	24
7	4,00	3,22	1,58	23
8	3,99	3,18	1,51	22
9	3,98	3,13	1,45	21
10	3,97	3,09	1,38	20
11	3,96	3,04	1,31	19
12	3,95	3,00	1,25	18
13	3,93	2,95	1,18	17
14	3,91	2,90	1,11	16
15	3,90	2,85	1,04	15
16	3,88	2,80	0,98	14
17	3,86	2,75	0,91	13
18	3,84	2,70	0,84	12
19	3,81	2,65	0,77	11
20	3,79	2,59	0,70	10
21	3,77	2,54	0,63	9
22	3,74	2,48	0,56	8
23	3,71	2,43	0,49	7
24	3,68	2,37	0,42	6
25	3,66	2,31	0,35	5
26	3,63	2,26	0,28	4
27	3,59	2,20	0,21	3
28	3,56	2,14	0,14	2
29	3,53	2,08	0,07	1
30	3,49	2,02	0,00	0
	+ −	+ −	+ −	
	V^s XI^s	IV^s X^s	III^s IX^s	

TABLE XXX.

For the Lunar Nutation of a star in Æ and Dec.

Argument, ☊ = the mean longitude of Moon's node.

	O^s	VI^s		I^s	VII^s		II^s	VIII^s		
	Log. b	B −	c − +	Log. b	B −	c − +	Log. b	B −	c − +	
0°	0·9844	0° 0′	0,00″	0·9588	6° 45′	8,27″	0·8960	7° 48′	14,33″	30°
1	·9844	0 15	0,29	·9571	6 54	8,52	·8939	7 40	14,47	29
2	·9843	0 31	0,58	·9554	7 3	8,77	·8917	7 32	14,61	28
3	·9842	0 46	0,87	·9536	7 12	9,01	·8896	7 23	14,74	27
4	·9840	1 1	1,15	·9518	7 20	9,25	·8875	7 14	14,87	26
5	·9837	1 16	1,44	·9500	7 28	9,49	·8854	7 4	14,99	25
6	·9834	1 32	1,73	·9481	7 36	9,72	·8834	6 53	15,11	24
7	·9830	1 47	2,02	·9462	7 43	9,96	·8814	6 42	15,23	23
8	·9825	2 2	2,30	·9442	7 49	10,19	·8795	6 29	15,34	22
9	·9821	2 17	2,59	·9422	7 55	10,41	·8776	6 17	15,45	21
10	·9815	2 31	2,87	·9402	8 1	10,63	·8758	6 3	15,55	20
11	·9809	2 46	3,16	·9382	8 6	10,85	·8740	5 49	15,64	19
12	·9802	3 1	3,44	·9361	8 10	11,07	·8723	5 35	15,73	18
13	·9795	3 15	3,72	·9340	8 14	11,28	·8707	5 20	15,82	17
14	·9787	3 29	4,00	·9318	8 17	11,49	·8691	5 4	15,90	16
15	·9779	3 43	4,28	·9297	8 20	11,70	·8677	4 48	15,98	15
16	·9770	3 57	4,56	·9275	8 23	11,90	·8663	4 31	16,05	14
17	·9760	4 11	4,84	·9253	8 24	12,10	·8649	4 14	16,12	13
18	·9750	4 24	5,11	·9231	8 25	12,30	·8637	3 56	16,18	12
19	·9739	4 37	5,39	·9208	8 25	12,49	·8625	3 38	16,24	11
20	·9728	4 50	5,66	·9186	8 25	12,67	·8615	3 20	16,29	10
21	·9716	5 3	5,93	·9163	8 24	12,86	·8605	3 1	16,34	9
22	·9704	5 16	6,20	·9140	8 23	13,04	·8596	2 41	16,38	8
23	·9691	5 28	6,46	·9118	8 21	13,21	·8588	2 22	16,42	7
24	·9678	5 40	6,73	·9095	8 18	13,38	·8582	2 2	16,45	6
25	·9664	5 51	6,99	·9072	8 15	13,55	·8576	1 42	16,48	5
26	·9650	6 3	7,25	·9050	8 11	13,72	·8571	1 22	16,50	4
27	·9635	6 14	7,51	·9027	8 6	13,88	·8568	1 2	16,52	3
28	·9620	6 24	7,77	·9005	8 1	14,03	·8565	0 41	16,53	2
29	·9604	6 35	8,02	·8983	7 55	14,18	·8563	0 21	16,54	1
30	0·9588	6 45	8,27	0·8960	7 48	14,33	0·8563	0 0	16,54	0
	Log. b	+ B	− + c	Log. b	+ B	− + c	Log. b	+ B	− + c	
	V^s	XI^s		IV^s	X^s		III^s	IX^s		

For the Solar Nutation of a star in Right Ascension and Declination.

Degrees	In Right Ascension		In Dec.	Degrees
	1st part Argument = 2 ☉	2nd part Argument = 2 ☉ − Æ	Argument = 2 ☉ − Æ	
0°	− 0″,00 +	− 0″,47 −	− 0″,00 +	360°
10	0,18	0,46	0,08	350
20	0,35	0,44	0,16	340
30	0,51	0,41	0,24	330
40	0,66	0,36	0,30	320
50	0,79	0,30	0,36	310
60	0,89	0,24	0,41	300
70	0,96	0,16	0,44	290
80	1,01	− 0,08 −	0,46	280
90	1,03	0,00	0,47	270
100	1,01	+ 0,08 +	0,46	260
110	0,96	0,16	0,44	250
120	0,89	0,24	0,41	240
130	0,79	0,30	0,36	230
140	0,66	0,36	0,30	220
150	0,51	0,41	0,24	210
160	0,35	0,44	0,16	200
170	0,18	0,46	0,08	190
180	− 0,00 +	+ 0,47 +	− 0,00 +	180

The *second* part of the solar nutation in Æ must be multiplied by the tangent of Declination.

TABLE XXXII.

Circular Arcs.

Degrees							
1°	0·0174533	31°	0·5410521	61°	1·0646508	95°	1·6580628
2	0·0349066	32	0·5585054	62	1·0821041	100	1·7453293
3	0·0523599	33	0·5759587	63	1·0995574	105	1·8325957
4	0·0698132	34	0·5934119	64	1·1170107	110	1·9198622
5	0·0872665	35	0·6108652	65	1·1344640	115	2·0071286
6	0·1047198	36	0·6283185	66	1·1519173	120	2·0943951
7	0·1221730	37	0·6457718	67	1·1693706	130	2·2689280
8	0·1396263	38	0·6632251	68	1·1868239	140	2·4434610
9	0·1570796	39	0·6806784	69	1·2042772	150	2·6179939
10	0·1745329	40	0·6981317	70	1·2217305	160	2·7925268
11	0·1919862	41	0·7155850	71	1·2391838	170	2·9670597
12	0·2094395	42	0·7330383	72	1·2566371	180	3·1415927
13	0·2268928	43	0·7504916	73	1·2740904	190	3·3161256
14	0·2443461	44	0·7679449	74	1·2915436	200	3·4906585
15	0·2617994	45	0·7853982	75	1·3089969	210	3·6651914
16	0·2792527	46	0·8028515	76	1·3264502	220	3·8397243
17	0·2967060	47	0·8203047	77	1·3439035	230	4·0142573
18	0·3141593	48	0·8377580	78	1·3613568	240	4·1887902
19	0·3316126	49	0·8552113	79	1·3788101	250	4·3633231
20	0·3490659	50	0·8726646	80	1·3962634	260	4·5378561
21	0·3665191	51	0·8901179	81	1·4137167	270	4·7123890
22	0·3839724	52	0·9075712	82	1·4311700	280	4·8869219
23	0·4014257	53	0·9250245	83	1·4486233	290	5·0614548
24	0·4188790	54	0·9424778	84	1·4660766	300	5·2359878
25	0·4363323	55	0·9599311	85	1·4835299	310	5·4105207
26	0·4537856	56	0·9773844	86	1·5009832	320	5·5850536
27	0·4712389	57	0·9948377	87	1·5184364	330	5·7595865
28	0·4886922	58	1·0122910	88	1·5358897	340	5·9341195
29	0·5061455	59	1·0297443	89	1·5533430	350	6·1086524
30	0·5235988	60	1·0471976	90	1·5707963	360	6·2831853

Circular Arcs.

Minutes				Seconds			
1′	0·0002909	31′	0·0090175	1″	0·0000048	31″	0·0001503
2	·0005818	32	·0093084	2	·0000097	32	·0001551
3	·0008727	33	·0095993	3	·0000145	33	·0001600
4	·0011636	34	·0098902	4	·0000194	34	·0001648
5	·0014544	35	·0101811	5	·0000242	35	·0001697
6	·0017453	36	·0104720	6	·0000291	36	·0001745
7	·0020362	37	·0107629	7	·0000339	37	·0001794
8	·0023271	38	·0110538	8	·0000388	38	·0001842
9	·0026180	39	·0113446	9	·0000436	39	·0001891
10	·0029089	40	·0116355	10	·0000485	40	·0001939
11	·0031998	41	·0119264	11	·0000533	41	·0001988
12	·0034907	42	·0122173	12	·0000582	42	·0002036
13	·0037815	43	·0125082	13	·0000630	43	·0002085
14	·0040724	44	·0127991	14	·0000679	44	·0002133
15	·0043633	45	·0130900	15	·0000727	45	·0002182
16	·0046542	46	·0133809	16	·0000776	46	·0002230
17	·0049451	47	·0136717	17	·0000824	47	·0002279
18	·0052360	48	·0139626	18	·0000873	48	·0002327
19	·0055269	49	·0142535	19	·0000921	49	·0002376
20	·0058178	50	·0145444	20	·0000970	50	·0002424
21	·0061087	51	·0148353	21	·0001018	51	·0002473
22	·0063995	52	·0151262	22	·0001067	52	·0002521
23	·0066904	53	·0154171	23	·0001115	53	·0002570
24	·0069813	54	·0157080	24	·0001164	54	·0002618
25	·0072722	55	·0159989	25	·0001212	55	·0002666
26	·0075631	56	·0162897	26	·0001261	56	·0002715
27	·0078540	57	·0165806	27	·0001309	57	·0002763
28	·0081449	58	·0168715	28	·0001357	58	·0002812
29	·0084358	59	·0171624	29	·0001406	59	·0002860
30	0·0087266	60	0·0174533	30	0·0001454	60	0·0002909

TABLE XXXIII.

Semidiurnal Arcs.

Lat.	Declination 1°	5°	10°	15°	20°	25°	30°
	h m	h m	h m	h m	h m	h m	h m
5°	6 0	6 2	6 4	6 5	6 7	6 9	6 12
10	6 1	6 4	6 7	6 11	6 15	6 19	6 24
15	6 1	6 5	6 11	6 16	6 22	6 29	6 36
20	6 1	6 7	6 15	6 22	6 30	6 39	6 49
25	6 2	6 9	6 19	6 29	6 39	6 50	7 2
30	6 2	6 12	6 23	6 36	6 49	7 2	7 18
35	6 3	6 14	6 28	6 43	6 59	7 16	7 35
40	6 3	6 17	6 34	6 52	7 11	7 32	7 56
45	6 4	6 20	6 41	7 2	7 25	7 51	8 21
50	6 5	6 24	6 49	7 14	7 43	8 15	8 54
55	6 6	6 29	6 58	7 30	8 5	8 47	9 42
60	6 7	6 35	7 11	7 51	8 36	9 35	12 0
65	6 9	6 43	7 29	8 20	9 25	12 0	

Showing the length of a degree of Longitude and Latitude on the Earth's surface (compression = $\frac{1}{300}$); together with the length of the Pendulum beating seconds there (compression = $\frac{1}{307}$): the measures at the equator being considered as unity. Also the increase in the number of vibrations, of an invariable pendulum beating seconds at the equator, on proceeding towards the pole.

Lat^{e}.	Degree of Longitude	Degree of Latitude	Length of the Pendulum	Increase of Vibrations
0°	1·00000	1·000000	1·00000	0″,00
5	0·99622	1·000076	1·00004	1,77
10	·98490	1·000301	1·00016	7,02
15	·96614	1·000669	1·00036	15,60
20	·94006	1·001168	1·00063	27,24
25	·90685	1·001783	1·00096	41,59
30	·86675	1·002496	1·00135	58,21
35	·82005	1·003284	1·00177	76,60
40	·76710	1·004125	1·00223	96,21
45	·70828	1·004992	1·00269	116,42
50	·64404	1·005858	1·00316	136,64
55	·57485	1·006699	1·00362	156,25
60	·50126	1·007487	1·00404	174,63
65	·42377	1·008200	1·00443	191,26
70	·34302	1·008815	1·00476	205,61
75	·25960	1·009315	1·00503	217,25
80	·17421	1·009682	1·00523	225,82
85	·08764	1·009907	1·00535	231,08
90	0·00000	1·009983	1·00539	232,85

If the equatorial diameter of the earth be assumed equal to 7924 miles, a degree of longitude at the equator will be equal to 69·15 miles = 365110 feet: and consequently 1 second in time at the equator will be equal to 1521 feet.

TABLE XXXV.

Showing the expansion of various substances for 1° Fahren.

White deal	·0000022685	Kater	
Glass, barom. tubes . .	43119	Roy	
——, English flint . .	45092	Lavoisier	
——, with lead . . .	48444	do.	
——, without lead . .	50973	do.	
Platina	47583	Borda	
——	49121	Dulong and Petit	
Iron, cast	61632	Roy	
——	63333	Borda	
——	65668	Dulong and Petit	
—— bar	69844	Hasslar	
——	69907	Smeaton	
Steel	59305	Lavoisier	
—— rod	63596	Roy	
—— blistered . . .	63900	Smeaton	
——	66100	Troughton	
—— hard	68056	Smeaton	
—— tempered . . .	68866	Lavoisier	
Gold	86197	do.	
Copper	94444	Smeaton	
Brass	95456	Dulong and Petit	
——	98888	Borda	
——	99590	Kater	
——, scale	·0000103077	Roy	
——, cast	104166	Smeaton	
——, bar	104850	Hasslar	
——, rod	105155	Roy	
——	106666	Troughton	
——, wire	107407	Smeaton	
Silver	106038	Lavoisier	
Pewter	126852	Smeaton	
Tin, grain	137963	do.	
Lead	159259	do.	
Zinc	163426	do.	
——, hammered . . .	172685	do.	
Mercury (*Apparent*) .	·0000857339	cubical expan.	$\frac{1}{6480}$
—— (*Absolute*) . .	·0001001001		$\frac{1}{5520}$
Air, dry	·0020833333		
— moist	·0022222222		

For determining Altitudes with the Barometer.

Thermometers in open air				Thermometers to the Baromˢ		Latitude of the place	
$t+t'$	A	$t+t'$	A	$\tau-\tau'$	B	φ	C
40°	4·76891	102°	4·79860	0°	0·00000	0°	0·00117
42	4·76989	104	4·79953	1	0·00004	5	0·00115
44	4·77089	106	4.80045	2	0·00009	10	0·00110
46	4·77187	108	4·80137	3	0·00013	15	0·00100
48	4·77286	110	4·80229	4	0·00017	20	0·00090
50	4·77383	112	4·80321	5	0·00022	25	0·00075
52	4·77482	114	4·80412	6	0·00026	30	0·00058
54	4·77579	116	4·80504	7	0·00030	35	0·00040
56	4·77677	118	4·80595	8	0·00035	40	0·00020
58	4·77774	120	4·80687	9	0·00039	45	0·00000
60	4·77871	122	4·80777	10	0·00043	50	9·99980
62	4·77968	124	4·80869	11	0·00048	55	9·99960
64	4·78065	126	4·80958	12	0·00052	60	9·99942
66	4·78161	128	4·81048	13	0·00056	65	9·99925
68	4·78257	130	4·81138	14	0·00061	70	9·99910
70	4·78353	132	4·81228	15	0·00065	75	9·99900
72	4·78449	134	4·81317	16	0·00069	80	9·99890
74	4·78544	136	4·81407	17	0·00074	85	9·99885
76	4·78640	138	4·81496	18	0·00078	90	9·99883
78	4·78735	140	4·81585	19	0·00083		
80	4·78830	142	4·81675	20	0·00087		
82	4·78925	144	4·81763	21	0·00091		
84	4·79019	146	4·81851	22	0·00096		
86	4·79113	148	4·81940	23	0·00100		
88	4·79207	150	4·82027	24	0·00104		
90	4·79301	152	4·82116	25	0·00109		
92	4·79395	154	4·82204	26	0·00113		
94	4·79488	156	4·82291	27	0·00117		
96	4·79582	158	4·82379	28	0·00122		
98	4·79675	160	4·82466	29	0·00126		
100	4·79768	162	4·82553	30	0·00130		

Make D equal to $\log\beta-(\log\beta'+\mathrm{B})$: then will the logarithm of the difference of altitudes in English feet be equal to

$$\mathrm{A}+\mathrm{C}+\log\mathrm{D}$$

TABLE XXXVII.

For converting time into decimal parts of a day.

Hours		Minutes				Seconds			
h		′		′		″		″	
1	·04167	1	·00069	31	·02153	1	·00001	31	·00036
2	·08333	2	·00139	32	·02222	2	·00002	32	·00037
3	·12500	3	·00208	33	·02292	3	·00003	33	·00038
4	·16667	4	·00278	34	·02361	4	·00005	34	·00039
5	·20833	5	·00347	35	·02430	5	·00006	35	·00040
6	·25000	6	·00417	36	·02500	6	·00007	36	·00042
7	·29167	7	·00486	37	·02569	7	·00008	37	·00043
8	·33333	8	·00556	38	·02639	8	·00009	38	·00044
9	·37500	9	·00625	39	·02708	9	·00010	39	·00045
10	·41667	10	·00694	40	·02778	10	·00012	40	·00046
11	·45833	11	·00764	41	·02847	11	·00013	41	·00047
12	·50000	12	·00833	42	·02917	12	·00014	42	·00049
13	·54167	13	·00903	43	·02986	13	·00015	43	·00050
14	·58333	14	·00972	44	·03056	14	·00016	44	·00051
15	·62500	15	·01042	45	·03125	15	·00017	45	·00052
16	·66667	16	·01111	46	·03194	16	·00018	46	·00053
17	·70833	17	·01180	47	·03264	17	·00020	47	·00054
18	·75000	18	·01250	48	·03333	18	·00021	48	·00056
19	·79167	19	·01319	49	·03403	19	·00022	49	·00057
20	·83333	20	·01389	50	·03472	20	·00023	50	·00058
21	·87500	21	·01458	51	·03542	21	·00024	51	·00059
22	·91667	22	·01528	52	·03611	22	·00025	52	·00060
23	·95833	23	·01597	53	·03680	23	·00027	53	·00061
24	1·00000	24	·01667	54	·03750	24	·00028	54	·00062
		25	·01736	55	·03819	25	·00029	55	·00064
		26	·01805	56	·03889	26	·00030	56	·00065
		27	·01875	57	·03958	27	·00031	57	·00066
		28	·01944	58	·04028	28	·00032	58	·00067
		29	·02014	59	·04097	29	·00034	59	·00068
		30	·02083	60	·04167	30	·00035	60	·00069

For converting Minutes and Seconds of a *degree*, into the decimal division of the same.

Minutes				Seconds			
′		′		″		″	
1	·01667	31	·51667	1	·00028	31	·00861
2	·03333	32	·53333	2	·00056	32	·00889
3	·05000	33	·55000	3	·00083	33	·00917
4	·06667	34	·56667	4	·00111	34	·00944
5	·08333	35	·58333	5	·00139	35	·00972
6	·10000	36	·60000	6	·00167	36	·01000
7	·11667	37	·61667	7	·00194	37	·01028
8	·13333	38	·63333	8	·00222	38	·01056
9	·15000	39	·65000	9	·00250	39	·01083
10	·16667	40	·66667	10	·00278	40	·01111
11	·18333	41	·68333	11	·00306	41	·01139
12	·20000	42	·70000	12	·00333	42	·01167
13	·21667	43	·71667	13	·00361	43	·01194
14	·23333	44	·73333	14	·00389	44	·01222
15	·25000	45	·75000	15	·00417	45	·01250
16	·26667	46	·76667	16	·00444	46	·01278
17	·28333	47	·78333	17	·00472	47	·01306
18	·30000	48	·80000	18	·00500	48	·01333
19	·31667	49	·81667	19	·00528	49	·01361
20	·33333	50	·83333	20	·00556	50	·01389
21	·35000	51	·85000	21	·00583	51	·01417
22	·36667	52	·86667	22	·00611	52	·01444
23	·38333	53	·88333	23	·00639	53	·01472
24	·40000	54	·90000	24	·00667	54	·01500
25	·41667	55	·91667	25	·00694	55	·01528
26	·43333	56	·93333	26	·00722	56	·01556
27	·45000	57	·95000	27	·00750	57	·01583
28	·46667	58	·96667	28	·00778	58	·01611
29	·48333	59	·98333	29	·00806	59	·01639
30	·50000	60	1.00000	30	·00833	60	·01667

TABLE XXXIX.

For converting any given day into the decimal part of a year of 365 days.

Day	Jan.	Feb.	March	April	May	June
1	·000	·085	·162	·247	·329	·414
2	·003	·088	·164	·249	·331	·416
3	·006	·090	·167	·252	·334	·419
4	·008	·093	·170	·255	·337	·422
5	·011	·096	·173	·258	·340	·425
6	·014	·099	·175	·260	·342	·427
7	·016	·101	·178	·263	·345	·430
8	·019	·104	·181	·266	·348	·433
9	·022	·107	·184	·268	·351	·436
10	·025	·110	·186	·271	·353	·438
11	·027	·112	·189	·274	·356	·441
12	·030	·115	·192	·277	·359	·444
13	·033	·118	·195	·279	·362	·446
14	·036	·121	·197	·282	·364	·449
15	·038	·123	·200	·285	·367	·452
16	·041	·126	·203	·288	·370	·455
17	·044	·129	·205	·290	·373	·458
18	·046	·132	·208	·293	·375	·460
19	·049	·134	·211	·296	·378	·463
20	·052	·137	·214	·299	·381	·466
21	·055	·140	·216	·301	·384	·468
22	·058	·142	·219	·304	·386	·471
23	·060	·145	·222	·307	·389	·474
24	·063	·148	·225	·310	·392	·477
25	·066	·151	·227	·312	·395	·479
26	·068	·153	·230	·315	·397	·482
27	·071	·156	·233	·318	·400	·485
28	·074	·159	·236	·321	·403	·488
29	·077		·238	·323	·405	·490
30	·079		·241	·326	·408	·493
31	·082		·244		·411	

For converting any given day into the decimal part of a year of 365 days.

Day	July	August	Sept.	Oct.	Nov.	Dec.
1	·496	·581	·666	·748	·833	·915
2	·499	·584	·668	·751	·836	·918
3	·501	·586	·671	·753	·838	·921
4	·504	·589	·674	·756	·841	·923
5	·507	·592	·677	·759	·844	·926
6	·510	·595	·679	·762	·846	·929
7	·512	·597	·682	·764	·849	·931
8	·515	·600	·685	·767	·852	·934
9	·518	·603	·688	·770	·855	·937
10	·521	·605	·690	·773	·858	·940
11	·523	·608	·693	·775	·860	·942
12	·526	·611	·696	·778	·863	·945
13	·529	·614	·699	·781	·866	·948
14	·532	·616	·701	·784	·868	·951
15	·534	·619	·704	·786	·871	·953
16	·537	·622	·707	·789	·874	·956
17	·540	·625	·710	·792	·877	·959
18	·542	·627	·712	·795	·879	·962
19	·545	·630	·715	·797	·882	·964
20	·548	·633	·718	·800	·885	·967
21	·551	·636	·721	·803	·888	·970
22	·553	·638	·723	·805	·890	·973
23	·556	·641	·726	·808	·893	·975
24	·559	·644	·729	·811	·896	·978
25	·562	·647	·731	·814	·899	·981
26	·564	·649	·734	·816	·901	·984
27	·567	·652	·737	·819	·904	·986
28	·570	·655	·740	·822	·907	·989
29	·573	·658	·742	·825	·910	·992
30	·575	·660	·745	·827	·912	·995
31	·578	·663		·830		·997

TABLE XL.

For converting the centesimal division of the *quadrant* into the sexagesimal division of the same.

Centesimal *degrees.*

Centes.	Sexages.		Centes.	Sexages.		Centes.	Sexages.	
°	°	′	°	°	′	°	°	′
1	0	54	34	30	36	67	60	18
2	1	48	35	31	30	68	61	12
3	2	42	36	32	24	69	62	6
4	3	36	37	33	18	70	63	0
5	4	30	38	34	12	71	63	54
6	5	24	39	35	6	72	64	48
7	6	18	40	36	0	73	65	42
8	7	12	41	36	54	74	66	36
9	8	6	42	37	48	75	67	30
10	9	0	43	38	42	76	68	24
11	9	54	44	39	36	77	69	18
12	10	48	45	40	30	78	70	12
13	11	42	46	41	24	79	71	6
14	12	36	47	42	18	80	72	0
15	13	30	48	43	12	81	72	54
16	14	24	49	44	6	82	73	48
17	15	18	50	45	0	83	74	42
18	16	12	51	45	54	84	75	36
19	17	6	52	46	48	85	76	30
20	18	0	53	47	42	86	77	24
21	18	54	54	48	36	87	78	18
22	19	48	55	49	30	88	79	12
23	20	42	56	50	24	89	80	6
24	21	36	57	51	18	90	81	0
25	22	30	58	52	12	91	81	54
26	23	24	59	53	6	92	82	48
27	24	18	60	54	0	93	83	42
28	25	12	61	54	54	94	84	36
29	6	6	62	55	48	95	85	30
30	27	0	63	56	42	96	86	24
31	27	54	64	57	36	97	87	18
32	28	48	65	58	30	98	88	12
33	29	42	66	59	24	99	89	6
34	30	36	67	60	18	100	90	0

For converting the centesimal division of the *quadrant* into the sexagesimal division of the same.

Centesimal *minutes*.

Centes.	Sexages.		Centes.	Sexages.		Centes.	Sexages.	
·01	0′	32″,4	·34	18′	21″,6	·67	36′	10″,8
·02	1	4,8	·35	18	54,0	·68	36	43,2
·03	1	37,2	·36	19	26,4	·69	37	15,6
·04	2	9,6	·37	19	58,8	·70	37	48,0
·05	2	42,0	·38	20	31,2	·71	38	20,4
·06	3	14,4	·39	21	3,6	·72	38	52,8
·07	3	46,8	·40	21	36,0	·73	39	25,2
·08	4	19,2	·41	22	8,4	·74	39	57,6
·09	4	51,6	·42	22	40,8	·75	40	30,0
·10	5	24,0	·43	23	13,2	·76	41	2,4
·11	5	56,4	·44	23	45,6	·77	41	34,8
·12	6	28,8	·45	24	18,0	·78	42	7,2
·13	7	1,2	·46	24	50,4	·79	42	39,6
·14	7	33,6	·47	25	22,8	·80	43	12,0
·15	8	6,0	·48	25	55,2	·81	43	44,4
·16	8	38,4	·49	26	27,6	·82	44	16,8
·17	9	10,8	·50	27	0,0	·83	44	49,2
·18	9	43,2	·51	27	32,4	·84	45	21,6
·19	10	15,6	·52	28	4,8	·85	45	54,0
·20	10	48,0	·53	28	37,2	·86	46	26,4
·21	11	20,4	·54	29	9,6	·87	46	58,8
·22	11	52,8	·55	29	42,0	·88	47	31,2
·23	12	25,2	·56	30	14,4	·89	48	3,6
·24	12	57,6	·57	30	46,8	·90	48	36,0
·25	13	30,0	·58	31	19,2	·91	49	8,4
·26	14	2,4	·59	31	51,6	·92	49	40,8
·27	14	34,8	·60	32	24,0	·93	50	13,2
·28	15	7,2	·61	32	56,4	·94	50	45,6
·29	15	39,6	·62	33	28,8	·95	51	18,0
·30	16	12,0	·63	34	1,2	·96	51	50,4
·31	16	44,4	·64	34	33,6	·97	52	22,8
·32	17	16,8	·65	35	6,0	·98	52	55,2
·33	17	49,2	·66	35	38,4	·99	53	27,6
·34	18	21,6	·67	36	10,8	1·00	54	0,0

For converting the centesimal division of the *quadrant* into the sexagesimal division of the same.

Centesimal *seconds*.

Centes.	Sexages.	Centes.	Sexages.	Centes.	Sexages.
·0001	″ 0,324	·0034	″ 11,016	·0067	″ 21,708
·0002	0,648	·0035	11,340	·0068	22,032
·0003	0,972	·0036	11,664	·0069	22,356
·0004	1,296	·0037	11,988	·0070	22,680
·0005	1,620	·0038	12,312	·0071	23,004
·0006	1,944	·0039	12,636	·0072	23,328
·0007	2,268	·0040	12,960	·0073	23,652
·0008	2,592	·0041	13,284	·0074	23,976
·0009	2,916	·0042	13,608	·0075	24,300
·0010	3,240	·0043	13,932	·0076	24,624
·0011	3,564	·0044	14,256	·0077	24,948
·0012	3,888	·0045	14,580	·0078	25,272
·0013	4,212	·0046	14,904	·0079	25,596
·0014	4,536	·0047	15,228	·0080	25,920
·0015	4,860	·0048	15,552	·0081	26,244
·0016	5,184	·0049	15,876	·0082	26,568
·0017	5,508	·0050	16,200	·0083	26,892
·0018	5,832	·0051	16,524	·0084	27,216
·0019	6,156	·0052	16,848	·0085	27,540
·0020	6,480	·0053	17,172	·0086	27,864
·0021	6,804	·0054	17,496	·0087	28,188
·0022	7,128	·0055	17,820	·0088	28,512
·0023	7,452	·0056	18,144	·0089	28,836
·0024	7,776	·0057	18,468	·0090	29,160
·0025	8,100	·0058	18,792	·0091	29,484
·0026	8,424	·0059	19,116	·0092	29,808
·0027	8,748	·0060	19,440	·0093	30,132
·0028	9,072	·0061	19,764	·0094	30,456
·0029	9,396	·0062	20,088	·0095	30,780
·0030	9,720	·0063	20,412	·0096	31,104
·0031	10,044	·0064	20,736	·0097	31,428
·0032	10,368	·0065	21,060	·0098	31,752
·0033	10,692	·0066	21,384	·0099	32,076
·0034	11,016	·0067	21,708	·0100	32,400

Comparison of Fahrenheit's thermometer, with Reaumur's and the Centesimal.

Fahr.	Reaum.	Centes.	Fahr.	Reaum.	Centes.	Fahr.	Reaum.	Centes.
			33°	+ 0°,4	+ 0°,6	67°	+ 15°,6	+ 19°,4
0°	−14°,2	−17°,8	34	0,9	1,1	68	16,0	20,0
1	13,8	17,2	35	1,3	1,7	69	16,4	20,6
2	13,3	16,7	36	1,8	2,2	70	16,9	21,1
3	12,9	16,1	37	2,2	2,8	71	17,3	21,7
4	12,4	15,6	38	2,7	3,3	72	17,8	22,2
5	12,0	15,0	39	3,1	3,9	73	18,2	22,8
6	11,6	14,4	40	3,6	4,4	74	18,7	23,3
7	11,1	13,9	41	4,0	5,0	75	19,1	23,9
8	10,7	13,3	42	4,4	5,6	76	19,6	24,4
9	10,2	12,8	43	4,9	6,1	77	20,0	25,0
10	9,8	12,2	44	5,3	6,7	78	20,4	25,6
11	9,3	11,7	45	5,8	7,2	79	20,9	26,1
12	8,9	11,1	46	6,2	7,8	80	21,3	26,7
13	8,4	10,6	47	6,7	8,3	81	21,8	27,2
14	8,0	10,0	48	7,1	8,9	82	22,2	27,8
15	7,6	9,4	49	7,6	9,4	83	22,7	28,3
16	7,1	8,9	50	8,0	10,0	84	23,1	28,9
17	6,7	8,3	51	8,4	10,6	85	23,6	29,4
18	6,2	7,8	52	8,9	11,1	86	24,0	30,0
19	5,8	7,2	53	9,3	11,7	87	24,4	30,6
20	5,3	6,7	54	9,8	12,2	88	24,9	31,1
21	4,9	6,1	55	10,2	12,8	89	25,3	31,7
22	4,4	5,6	56	10,7	13,3	90	25,8	32,2
23	4,0	5,0	57	11,1	13,9	91	26,2	32,8
24	3,6	4,4	58	11,6	14,4	92	26,7	33,3
25	3,1	3,9	59	12,0	15,0	93	27,1	33,9
26	2,7	3,3	60	12,4	15,6	94	27,6	34,4
27	2,2	2,8	61	12,9	16,1	95	28,0	35,0
28	1,8	2,2	62	13,3	16,7	96	28,4	35,6
29	1,3	1,7	63	13,8	17,2	97	28,9	36,1
30	0,9	1,1	64	14,2	17,8	98	29,3	36,7
31	− 0,4	− 0,6	65	14,7	18,3	99	29,8	37,2
32	0,0	0,0	66	+ 15,1	+ 18,9	100	+ 30,2	+ 37,8

Comparison of French and English Measures, &c.

French Metre	at 32° Fahr.	=	39·37079	Eng. Inches of Sir G. Shuckburgh's scale, at 62° Fahr.
—— Decimetre		=	3·93708	
—— Centimetre		=	0·39371	
—— Millimetre		=	0.03937	
French Toise	at 56°,3 Fahr.	=	76·739400	English Inches of the same, at 56°,3 Fahr.
—— Foot		=	12·789900	
—— Inch		=	1·065825	
—— Line		=	0·088819	
French Toise	at 61°,3 F.	=	1·949036	French Metre at 61°,3 Fahr.
—— Foot		=	0·324839	
—— Inch		=	0·027070	
English Foot	at 62° F.	=	0·304794	French Metre at 32° Fahr.
—— Inch		=	0·025399	

Centes. Deg. of the Quadrant	or 1□·0000	=	0° 54′ 0″000	Sexagesimal
—— Minute	or 0□·0100	=	32,400	
—— Seconds	or 0□·0001	=	0,324	

Comparison of different *thermometers*

$x°$ Reaumur $= (32° + \frac{9}{4}\,x°)$ Fahr. $= \frac{5}{4}\,x°$ Centes.

$x°$ Centes. $= (32° + \frac{9}{5}\,x°)$ Fahr. $= \frac{4}{5}\,x°$ Reaum.

$x°$ Fahren. $= (x° - 32°)\,\frac{4}{9}$ Reaum. $= (x° - 32°)\,\frac{5}{9}$ Centes.

The length of the *pendulum* vibrating seconds of mean solar time, in London, in vacuo, and at the level of the sea, is 39·1393 English Inches of Sir G. Shuckburgh's scale, at 62° Fahr.

The *velocity of sound*, in one second of time at 32° Fahr. in dry air, is about 1090 English feet. For any higher temperature, add 1 foot for every degree of the thermometer above 32°.

Logarithms of various quantities.

			Logarithms
Radius, reduced to seconds of the arc	=	206264″,8	5·3144251
Circumference of the circle, diameter=1	=	3·1415926	0·4971499
Length of 1° in parts of do.	=	·01745329	8·2418773
Length of 1′ in parts of do.	=	·00029089	6·4637262
sin 1″	=	·00000485	4·6855749
sin 2″	=	·00000970	4·9866049
sin 3″	=	·00001454	5·1626961
Number whose Hyper. log. is = 1 . .	=	2·7182818	0·4342945
Modulus of the common logarithms . .	=	·43429448	9·6377843
Complement to the same . . .	=	2·3025851	0·3622157
12 hours, expressed in seconds . . .	=	43200	4·6354837
Complement to the same . . .	=	·00002315	5·3645163
24 hours, expressed in seconds . . .	=	86400	4·9365137
Complement to the same . . .	=	·00001157	5·0634863
360 degrees, expressed in seconds . .	=	1296000	6·1126050
Compression of the earth = $\frac{1}{300}$ . .	=	·003333	7·5228787
Sidereal revolution of the Earth, in days	=	365·25636	2·5625978
Tropical revolution of do.	=	365·24224	2·5625810

Constant logarithms (always *additive*) for converting			
	Sidereal time into Mean Solar time		9·9988126
	French Toises into Metres . . .	at 61°·3	0·2898200
	——— Feet into Metres . . .	at 61°·3	9·5116687
	——— Toises into English Feet .	at 56°·3	0·8058372
	——— Feet into English Feet .	at 56°·3	0·0276860
	——— Metre into English Feet .	at their standards	0·5159929
	——— Millimetres into Eng. Inch.	at their standards	8·5951741
	Centes. Degrees of Quadrant into Sexages. Degrees		9·9542425
	——— Minutes of do. into ——— Minutes		1·7323938
	——— Seconds of do. into ——— Seconds		3·5105450

The arithmetical complements of these last 10 logarithms will serve to convert Mean Solar time into Sidereal time, Metres into Toises &c, English Feet into Toises &c, and the Sexagesimal divisions of the Quadrant into the Centesimal divisions.

TABLE XLIV.

For the comparison of French and English Barometers.

Milli-metres	English inches	French inches & lines		Milli-metres	English inches	French inches & lines	
711	27·9927	26	3,183	741	29·1738	27	4,482
712	28·0320		3,627	742	·2131		4,926
713	·0714		4,070	743	·2525		5,369
714	·1108		4,513	744	·2919		5,812
715	·1501		4,956	745	·3312		6,256
716	·1895		5,400	746	·3706		6,699
717	·2289		5,843	747	·4100		7,142
718	·2683		6,286	748	·4494		7,585
719	·3076		6,730	749	·4887		8,029
720	·3470		7,173	750	·5281		8,472
721	·3864		7,616	751	·5675		8,915
722	·4257		8,060	752	·6068		9,359
723	·4651		8,503	753	·6462		9,802
724	·5045		8,946	754	·6856		10,245
725	·5438		9.389	755	·7249		10,688
726	·5832		9,833	756	·7643		11,132
727	·6226		10,276	757	·8037	27	11,575
728	·6620		10,719	758	·8431	28	0,018
729	·7013		11,163	759	·8824		0,462
730	·7407	26	11,606	760	·9218		0,905
731	·7800	27	0,049	761	29·9612		1,348
732	·8194		0,493	762	30·0005		1,792
733	·8588		0,936	763	·0399		2,235
734	·8982		1,379	764	·0793		2,678
735	·9375		1,823	765	·1187		3,121
736	28·9769		2,266	766	·1580		3,565
737	29·0163		2,709	767	·1974		4,008
738	·0556		3,152	768	·2368		4,451
739	·0950		3,596	769	·2761		4,895
740	29·1344	27	4,039	770	30·3155	28	5,338

For the comparison of French and English Barometers.

Millimetres	English inches	French inches & lines	
771	30·3549	28	5,781
772	·3942		6,224
773	·4336		6,668
774	·4730		7,111
775	·5124		7,554
776	·5517		7,998
777	·5911		8,441
778	·6305		8,884
779	·6698		9,328
780	·7092		9,771
781	·7486		10,214
782	·7879		10,658
783	·8273		11,101
784	·8667		11,544
785	·9060	28	11,978
786	·9454	29	0,431
787	30·9848		0,874
788	31·0242		1,317
789	·0635		1,761
790	31·1029	29	2,204

Proportional parts

Millimetres	English inches	French lines
0·1	0·0039	0,044
0·2	0·0079	0,089
0·3	0·0118	0,133
0·4	0·0157	0,177
0·5	0·0197	0,222
0·6	0·0236	0,266
0·7	0·0276	0,310
0·8	0·0315	0,355
0·9	0·0354	0,399
1·0	0·0394	0,443

1 Metre . . { =39·3707 Eng. inch. / =443,296 Paris lines

1 Eng. foot . { =0·304794 Metre / =135,114 Paris lines

1 French foot { =1·0658 Eng. foot / =0·32484 Metre

For the comparison of French and English Barometers.

English inches	Milli-metres	French inches & lines	
28·0	711·19	26	3.266
·1	713·73		4,392
·2	716·27		5,518
·3	718·81		6,644
·4	721·35		7,770
·5	723·89		8,896
·6	726·43		10,022
·7	728·97	26	11,148
·8	731·51	27	0,274
28·9	734·05		1,400
29·0	736·59		2,526
·1	739·13		3,652
·2	741·67		4,778
·3	744·21		5,904
·4	746·75		7,030
·5	749·29		8,156
·6	751·83		9,282
·7	754·37		10,408
·8	756·91	27	11,534
29·9	759·45	28	0,659
30·0	761·99		1,785
·1	764·53		2,911
·2	767·07		4,037
·3	769·61		5,163
·4	772·15		6,289
·5	774·69		7,415
·6	777·23		8,541
·7	779·77		9,667
·8	782·31		10,793
30·9	784·85	28	11,919

Proportional parts

English inches	Milli-metres	French lines
0·01	·000254	0,1126
0·02	·000508	0,2252
0·03	·000762	0,3378
0·04	·001016	0,4504
0·05	·001270	0,5630
0·06	·001524	0,6756
0·07	·001778	0,7882
0·08	·002032	0,9008
0·09	·002286	1,0134
0·10	·002540	1,1260

TABLE XLV.

Depression of Mercury in glass tubes

Diameter	Ivory	Young	Laplace
in. 0·05	in. 0·2949	in. 0·2964	in. 0· . . .
·10	·1404	·1424	·1394
·15	·0865	·0880	·0854
·20	·0583	·0589	·0580
·25	·0409	·0404	·0412
·30	·0293	·0280	·0296
·35	·0212	·0196	·0216
·40	·0154	·0139	·0159
·45	·0112	·0100	·0117
·50	·0082	·0074	·0087
·60	·0043	·0045	·0046
·70	·0023	....	·0024
0·80	0·0012	....	0·0013

EXPLANATION OF THE TABLES.

Table I does not appear to require any explanation. It contains the Longitudes (from Greenwich) and the Latitudes of various places where astronomical observations have been made; and has been taken, for the most part, from the *Connaissance des tems.* Some of the places, however, have been corrected from more recent observations.

Tables II and III serve to convert sidereal time into mean solar time; and *vice versâ.* They show the sidereal time of mean noon on any given day in any given year.

The values in the first of these tables have been formed on the assumption that the sun's mean right ascension at mean noon at Greenwich on January 1st in any year is exactly equal to $18^h. 40^m. 0^s.$ (which, though not strictly correct, is very nearly so): and that its increase in right ascension in a mean solar day is $3^m. 56^s,555.$ To this table is annexed, in a separate column, the amount of the motion of the moon's node from January 1st to any other day in the year; for a purpose to which I shall presently allude.

The second of these tables serves to correct the values given in the preceding table. For, since the mean right ascension of the sun will never be the same on the 1st of January in any two years, it becomes necessary to correct the values in Table II by the addition of the quantities here set against the given year. The sum of these two values gives the correct right ascension of the sun at mean noon at Greenwich, as reckoned from the *mean* equinox. These values have been computed from the formula

$$\tfrac{1}{15}\,[53'.\ 29'',8 - (y - 4\,\beta)\ 14'.\ 47'',08 + 27'',48\,y]$$
$$= 3^{m}.\ 33^{s},993 - (y - 4\,\beta)\ 59^{s},1387 + 1^{s},8320\,y$$

where y denotes the number of years from 1800 (negative *before*, and positive *after* that period): and β the number of bissextile *days* between 1800 and the month of March in the given year, which also changes its sign with y.

As these tables are formed for the meridian of Greenwich, it is evident that a correction must be applied when they are intended for any other place situated east or west of that observatory. The amount of this correction is

$$\lambda \times 0^{s},0027379$$

where λ denotes the longitude in time (expressed in seconds) from Greenwich; + when west, and − when east.* In the third column is given the mean place of the moon's node on the 1st of January; the whole circle being supposed to be divided into 1000 parts.

For an example of the use and application of these tables, see Problem I.

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* These corrections will be, for the following observatories, as under: viz.

Altona	− 6·544
Dorpat	− 17,544
Dublin	+ 4,167
Konigsberg . . .	− 13,462
Milan	− 6.040
Palermo	− 8,783
Paramatta	− 99,235
Paris	− 1,536
Vienna	− 10,763

Tables IV and V show the correction for the Lunar and Solar Nutation; or the values which ought to be applied to those given in Tables II and III, in order to obtain the sun's mean right ascension as reckoned from the *apparent* equinox. The true correction, as deduced from Formula XXXIII in page 106, is

$$-15'',868 \sin ☊ + 0'',191 \sin 2 ☊ - 1'',151 \sin 2 ☉ - 0'',190 \sin 2 ☽$$
$$= -1^s,058 \sin ☊ + 0^s,013 \sin 2 ☊ - 0^s,077 \sin 2 ☉ - 0^s,013 \sin 2 ☽$$

Table IV contains the correction depending on the moon's node (the position of which may be taken from the two preceding tables), the whole circle being divided into 1000 parts. Table V contains the correction depending on the sun's true longitude, the place of which must be taken from an ephemeris. The last quantity, depending on the moon's true longitude, has been omitted as too small to affect the results in any material degree.

For an example of the use and application of these tables, see also Problem I.

Table VI expresses the motion of the sun's mean right ascension in a *sidereal day*, for hours, minutes and seconds of *mean solar* time; and will be found very useful and convenient for converting sidereal time into mean solar time.

For an example of the use and application of this table, see also Problem I.

Table VII is a similar table for the motion of the sun's mean right ascension in a *mean solar day*, expressed in hours, minutes and seconds of *sidereal* time; and will be found occasionally useful and convenient for converting mean solar time into sidereal time.

For an example of the use and application of this table, see also Problem I.

Table VIII serves to convert degrees, minutes and seconds of *space* into similar denominations of *time*: and *vice versâ*. It is founded on the ratio of 15° to 1^h. and does not require any further explanation.

Tables IX, X and XI contain Mr. Ivory's refractions, as given in the *Phil. Trans.* for 1823. The first of these denotes the mean refraction, for the several degrees of zenith distance therein stated: to which are annexed, in a contiguous column, for the convenience of computation, the logarithms of those values, with their differences. The mean values are computed on the assumption that the temperature of the air is 50° of Fahrenheit's thermometer, and that the barometer stands at 30 inches. Table X serves to correct these values when the thermometer or barometer is higher or lower than the mean state above mentioned *. The value, thus corrected, shows the *approximate* refraction: which, in most instances, will be sufficiently correct. But, in the case of low altitudes and where great accuracy is required, this value must be further corrected by Table XI, agreeably to the rule there given.

For an example of the use and application of these tables, see Problem II.

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* In this table I have united Mr. Ivory's tables II and IV; which is the only alteration that I have made in his arrangement. No error will arise from this disposition of the tables, if the *interior* thermometer be used; nor if the temperature out of doors differs but little from that within.

Tables XII and XIII are Dr. Brinkley's very convenient tables for the computation of refraction: but they are not adapted to altitudes lower than 10 degrees. The argument in the first of these tables is the height of Fahrenheit's thermometer: corresponding to which is the logarithm of a quantity which I shall denote by T, and which must be multiplied not only by the height of the barometer (expressed in inches) but also by the tangent of the zenith distance of the star, which must not be greater than 80°. This will give the *approximate* refraction: which, in the case of low altitudes, and where great accuracy is required, must be further corrected by subtracting the value (which I shall call *c*) in Table XIII, set against the given zenith distance, and under the given height of the barometer.

For an example of the use and application of these tables, see also Problem II.

Table XIV has been computed from the assumed horizontal parallax of the sun, at its mean distance, being equal to 8″,60: and gives the required parallax of altitude on the first day of every month. The proportional parts are easily found for any intermediate day.

Since parallax always tends to lower the true altitude of a body, we must *add* the parallax to the *observed* altitude, in order to obtain the *true* altitude. Or, which is the same thing, we must *deduct* it from the *observed* zenith distance, in order to obtain the *true* zenith distance.

The use and application of this table are sufficiently evident without an example.

Table XV will be found very convenient for determining the time as deduced from observations of *single* altitudes of the sun or a star. The formula, for the reduction of such observations, is given in page 89: and it will be there seen that the hour angle P is always deduced from a logarithm which expresses the value of $\sin^2 \frac{1}{2}$ P. In order to save the time and labour of computing the values of such logarithms and to prevent the occurrence of error, I have given the corresponding hour angles, for every minute of time, from 3 hours to 8 hours; which will be sufficient for all ordinary purposes. The time corresponding to any intermediate logarithm may be readily determined by direct proportion.

For an example of the use and application of this table, see Problem III.

Table XVI is for determining the equation of *equal* altitudes of the sun, which will be found more convenient and correct than the table in general use for that purpose. It is computed from the Formula XVIII in page 92. The value of A (which is always negative) must be multiplied by the double daily variation of the sun's declination (considered as *negative* when *decreasing*) and also by the tangent of the latitude of the place: to which must be added the value of B, multiplied also by the double daily variation of the sun's declination, and likewise by the tangent of the sun's declination at the time of apparent noon on the given day. The sum of these two quantities is the correction required.

For an example of the use and application of this table, see Problem IV.

Table XVII is computed from the Formula XVI in page 90 (No. 5 and 11). The *hour angle*, at which the star passes the prime vertical, may be deduced from No. 1, and the hour angle at which the vertical becomes a tangent to the circle of declination may be deduced from No. 10.

The hour angle, thus deduced, being *added* to the right ascension of the star will give the sidereal time when the star is on the prime vertical, or becomes a tangent to the circle of declination, to the *westward*; or being *subtracted* from the right ascension of the star (increased by 24^h if necessary) will give the sidereal time of the same position to the *eastward*. To persons in the practice of making observations on or near the prime vertical, it would be useful to have a table calculated to show the *altitude* of those stars which they usually observe in such case, and the *time* at which they pass the prime vertical. See Problem XIV.

Tables XVIII and XIX are the well known tables for the *reduction to the meridian*, computed agreeably to the Formula XIX in page 93. The first of these tables shows the value of $A = \frac{2 \sin^2 \frac{1}{2} P}{\sin 1''}$*: and the argument of the table is the distance (in time) of the sun or star from the meridian. This value (or the sum of those values divided by the number of observations, if more than one observa-

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* Since $2 \sin^2 \frac{1}{2} P$ denotes the *versed sine* of the arc P, it is evident that the expression in the text is equal to $\frac{1}{\sin 1''} \times \text{ver. sine} P$. It is in this manner that the tables for the reduction to the meridian have been published by the Board of Longitude. They have printed a table of the versed sines of arcs from 0^s to 30^m of time, and direct that the arithmetical complement of the logarithm of sin $1''$ be added to the constant logarithm (with its index increased by 4) of the number by which the result is multiplied: thus increasing the trouble and the liability to error. We are also directed to use the logarithm 5·3168000, instead of the arithmetical complement of the logarithm of sin $1''$, for observations on the *stars*, when *solar* time is employed: but the true logarithm is 5·3167966: a slight difference, which however is of no great moment.

tion has been made) must be multiplied by $\frac{\cos L \cdot \cos D}{\sin Z}$ *; and the product subtracted from the zenith distance (corrected for refraction &c) of the sun or star observed near the meridian. The *difference* † thus obtained will give the true meridional zenith distance of the sun or star, as correctly as if it had been observed precisely on the meridian. It must however be remarked that, when the distance from the meridian is considerable, and when great accuracy is required, this value must be further corrected by the addition of the value of B in Table XIX ‡, multiplied by $\left(\frac{\cos L \cdot \cos D}{\sin Z}\right)^2 \times \cot Z$.

The distance (in time) of the sun or star from the meridian is determined by a well regulated clock; and, for greater simplicity, the motion of the clock should correspond with the object observed: that is, if the sun be the object, the clock should be regulated to mean solar time; and if a star be the object, the clock should be regulated to sidereal time. This however is not absolutely necessary, since we may readily correct the errors arising from the rate of the clock. For if the sun be the object, and the clock be regulated to sidereal time, we must multiply A by ·99455418 (log = 9·9976285): and if a star be the object and the clock be regulated to mean solar time, we must multiply A by 1·00547562 (log = 0·0023715). There is however, in all cases, a correction necessary when the

* See the list of *Errata*, in this work.

† If the star has been observed *under* the pole, we must take the *sum* of these two quantities.

‡ Or the sum of the values of B, divided by their number, if more than one observation has been made.

clock does not go accurately during the observations: and whenever this occurs, we must multiply the value of A still further by $1 + \cdot00002315\,r$ ($\log = 0\cdot000010053 \times r$): where r denotes the daily rate of the clock, expressed in seconds, which must be assumed *minus* when *gaining*, and *plus* when *losing*.

For an example of the use and application of these Tables, see Problem V.

Table XX shows the *mean obliquity* of the ecliptic, on the 1st of January in every year during the present century. It is deduced from the assumption that the mean obliquity in 1750 was 23°. 28′. 17″,63; and that the annual diminution is 0″,457. This corresponds with the determination of M. Bessel, as given in Professor Schumacher's *Astronomische Nachrichten*, No. 34.

Tables XXI and XXII serve to determine the corrections for lunar and solar nutation, which should be applied to the values in the preceding table, in order to obtain the *apparent obliquity* of the ecliptic: and also the value of the lunar and solar *nutation in longitude*. The correction for the obliquity is deduced from the Formula XXXII in page 105, and is equal to

$$+9'',250 \cos ☊ - 0'',090 \cos 2\,☊ + 0'',545 \cos 2\odot + 0'',090 \cos 2\,☽$$

to which must be added the diminution of the mean obliquity from the 1st of January to the given day, which is equal to

$$-\frac{0'',457}{365} \times d$$

where d denotes the number of days elapsed from the commencement of the year.

The correction for the longitude is (as in page 105) equal to

$$-17'',298 \sin ☊ + 0'',208 \sin 2 ☊ - 1'',255 \sin 2 ☉ - 0'',207 \sin 2 ☽$$

Table XXI contains the corrections for the Longitude and Obliquity depending on the place of the moon's node (which may be taken from Tables II and III), the whole circle being divided into 1000 parts. Table XXII contains the corrections for the same quantities depending on the sun's true longitude: the day of the month being made the argument. In the column marked *Obliq.* the value of $-\frac{0'',457}{365} \times d$ has been included in the computation. The last quantities, depending on the moon's true longitude, are omitted, as being too small to affect the results in any material degree: but they may be taken into the computation (if required) by actual calculation.

For an example of the use and application of these Tables, see Problem VIII.

Table XXIII is introduced with a view to preserve some uniformity in referring to the principal *lunar spots.* Much confusion has arisen from the various names given to the same spots on the moon, by different authors. The present list is taken from the plate in Russel's *Description of the Selenographia,* who has proposed these spots as a basis from which the situation of other less remarkable spots may be determined. The latitudes are computed from a line passing through *Censorinus;* the longitude of which is assumed equal to 32° west: and the positions are taken from the table in the *Recueil de Tables Astronomiques,* Berlin 1776.

Table XXIV shows the *angle of the vertical,* deduced from the Formula XXI, in page 95; where c is assumed equal to 300. The logarithms of the *earth's radius*, in the last column, are deduced from the Formula XXII, on the same assumption. These quantities are necessary in computing the parallax of the moon: for, in such calculations, the latitude of the place and the equatorial parallax of the moon must be diminished in consequence of the compression of the earth. The values for the latitude of Greenwich are given separately at the bottom of the table.

For an example of the use and application of this Table, see the note to Problem XI.

Table XXV shows the *augmentation of the moon's semidiameter* on account of her altitude, deduced from the Formula XXVIII No. 1, in page 101; according to the several values of s, from 14′. 30″ to 17′. 0″.

Its use and application are too obvious to require an example.

Table XXVI contains the logarithms for the *equations of the first, second and third differences* of the moon's place; and will be found very convenient for obtaining the correct values required. They are computed from the Formula XXX in page 103, for every ten minutes of time from noon or midnight. The two columns, entitled *first* differences, contain the logarithms of $\frac{h}{12^h}$; to which must be added the logarithm of Δ'': the natural number corresponding to the sum of these two logarithms will be the equation of the first difference *. The column,

* The logarithms in the Table are carried to *four* places of decimals only; which is sufficient for the equations of the second and third differences: but probably may not be considered in many instances

entitled *second* differences, contains the logarithm of $\frac{h(h-12)}{2(12)^2}$; to which must be added the logarithm of $\frac{d'+d''}{2}$: the natural number corresponding to the sum of these two logarithms will be the equation of the second difference. The column, entitled *third* differences, contains the logarithm of $\frac{h(h-12)(h-6)}{6(12)^3}$; to which must be added the logarithm of δ'. This last quantity is however generally so small, that it may in most ordinary cases be omitted. All these equations are to be applied to the moon's place, according to the *signs*: and it should be remembered that the moon's latitude, and declination, when *south*, is considered as a *negative* quantity.

For an example of the use and application of this Table, see Problem X.

Table XXVII was originally computed from the values given in Formula XXXI in page 104, and which are the same as those given by M. Bessel in his *Fundamenta Astronomiæ*, page 297. But that distinguished astronomer has (since that formula was printed) re-investigated the subject of the *precession of the equinoxes*, and the result of his inquiry is the table here given*.

The use and application of this Table are too well known to require an example. The formulæ, for determining the *annual precession* of a star in right ascension and declination, are given at the bottom of the Table. In

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accurate enough for the first differences. In such case, we must compute the logarithm from the formula $\frac{h}{12^h}$.

* The alterations in the values in the Formula are noticed in the list of *Errata*.

these formulæ, and indeed in all the formulæ in the present work, the declination when *south* is considered as a negative quantity, and treated as such in the algebraic and arithmetical computation. Therefore when in such case the amount of precession, aberration &c in declination is a *positive* quantity, it must be *subtracted* from the mean declination: and, on the contrary, when it is a *negative* quantity, it must be *added* thereto, in order to obtain the apparent declination. Some astronomers still consider the declination as always *positive;* and *change* the sign of the precession &c when the star has south declination. But the former plan is the most convenient; and is now more generally adopted.

Tables XXVIII—XXXI are M. Gauss's very convenient and general tables of *aberration and nutation.* They are deduced from a transformation of the Formulæ XXXIII and XXXIV in pages 106 and 107; and will be found very useful when we have not a ready access to more comprehensive tables. The constant of aberration is assumed equal to 20″,255; the constant of the lunar nutation of the obliquity equal to 9″,65; and the constant of the solar nutation of the same equal to 0″,493. These values differ in a slight degree from those given in the formulæ above mentioned: but this difference will in most cases be insensible in practice. The use and application of these Tables may be understood from the following expressions.

For the correction in Right Ascension

$$\text{Aber} = -a.\cos(\odot + \text{A} - \text{Æ}) \sec \text{D}$$

$$\text{Lunar Nut} = -b.\cos(\odot + \text{B} - \text{Æ}) \tan \text{D}$$

Solar Nut = the two equations from Table XXXI

For the correction in Declination

$$\text{Aber} = -a.\sin(\odot + \text{A} - \text{Æ})\sin\text{D} + \text{the 2 eq. of Tab. XXIX}$$
$$\text{Lunar Nut} = -b.\sin(\odot + \text{B} - \text{Æ})$$
$$\text{Solar Nut} = \text{the equation from Table XXXI}$$

The values of A and of the logarithm of a are to be taken from Table XXVIII: and the values of B and c, and the logarithm of b, are to be taken from Table XXX. The *solar* nutation is taken from Table XXXI: and it should be remarked that the second part of the solar nutation in right ascension must be multiplied by the tangent of declination. The computer should bear in mind that, in the whole of these operations, the declination when *south* is always considered as a *negative* quantity, and the results must be applied accordingly with the proper signs.

For an example of the use and application of these Tables, see Problem XII.

Table XXXII is the well known table of the length of *circular arcs* (radius = 1) deduced from the expression $l = a.\sin 1''$. Whence, the length being known, we have the arc $a = \frac{l}{\sin 1''}$: where l denotes the length of the arc, and a the arc itself expressed in seconds. Its use and application are too well known to require an example.

Table XXXIII shows the *semidiurnal arc,* or the interval of time employed by the sun or a star in passing from the horizon to its point of culmination, and *vice versâ;* according to its declination, and the latitude of the place. These values have been deduced from the equation in Formula XV in page 89, without considering the effect of refraction; which would increase the duration according to the circumstances of the case. The use and application of this Table are sufficiently evident without an example.

Table XXXIV has been introduced merely for the purpose of showing the relative values of the quantities therein stated, according to the latitude of the place of observation. The lengths of the *degrees of longitude and latitude* are computed from the Formula XLIII in page 116. The *length of the pendulum* is computed from the formula in page 22: and the increase in the number of its vibrations, from the Formula XL in page 113.

Table XXXV contains the *expansion of various substances* used in experiments and observations on the pendulum. It is extracted from a more copious list given by me at the end of my paper "On the Mercurial Compensation Pendulum" inserted in the *Memoirs of the Astronomical Society*, Vol I, pages 416—419. The numbers in the table contain the linear expansion of the body, for one degree of Fahrenheit's thermometer: if this number be denoted by e and the difference in the thermometer by t, the total expansion for t degrees of the thermometer will be $e\,t$. If l denote the length of the body before expansion, then will its length after the application of heat be $l' = l\,(1 + e\,t)$.

At the bottom of the table I have inserted the *cubical* expansion of Mercury and of Air. The expansion of mercury has been determined with the greatest accuracy by MM. Dulong and Petit. The *absolute* expansion is that which does not depend on the form of the vessel which contains it, nor on *its* expansion: whereas in the *apparent* expansion (in glass) these circumstances are taken into the computation *. The expansion of air is generally assumed as equal to $\frac{1}{480}$ of its bulk, for every degree of Fahrenheit's thermometer: but this applies more particularly to air rendered perfectly *dry* for the purpose of the experiments employed. The expansion of common atmospheric air, impregnated as it generally is with a certain degree of *moisture*, is supposed by M. La Place to be $\frac{1}{450}$ of its bulk. These are the two values stated in the table.

Table XXXVI is a new and very convenient table for computing, by means of logarithms, the *difference of altitudes with the barometer*. It is deduced from the Formula XXXVIII in page 111 †, by expanding the last term in M. La Place's formula, reducing the measures to English feet, and expressing the temperature by Fahrenheit's thermometer. Whence the difference of altitude between the two stations will be found equal to

$$x = 60345{\cdot}51 \left[1 + {\cdot}001111\,(t + t' - 64^\circ) \right]$$
$$\times \log.\text{ of} \left[\frac{\beta}{\beta'} \times \frac{1}{1 + {\cdot}0001\,(\tau - \tau')} \right] \times \left[1 + {\cdot}002695 \cos 2\,\phi \right]$$

In this Table, A denotes the logarithm of the first term here given, expressed in English feet; and C the logarithm of the last term: B denotes the logarithm of $1 + {\cdot}0001\,(\tau - \tau')$. In the formation of this table I have assumed the expan-

* The *relative vertical expansion* of mercury, in the glass vessel used for the bob of a pendulum (and which is the quantity that affects the rate of the clock) is found by deducting *twice the linear* expansion of the containing vessel from the *absolute* expansion of the mercury: and it may be assumed equal to ·00009.

† See the *correction* of this formula, amongst the *Errata*.

sion of air, for one degree of Fahrenheit, to be ·00222222, instead of the quantity ·00208333 assumed by M. Biot. The rule for the application of this table is given at the bottom thereof in page 183: and here it may be useful to remark that the heights of the barometers may be taken in any measure whatever; whether in English inches, French metres, or any other scale.—For an example of the use and application of this Table, see Problem XVI.

Table XXXVII will be found useful in converting hours, minutes and seconds, into the *decimal parts of a day:* an operation of frequent occurrence in computations relative to practical astronomy. Its use and application are sufficiently evident without further explanation.

Table XXXVIII will be found of similar use in converting the *sexagesimal divisions* (or the minutes and seconds) *of a degree*, into the decimal divisions of the same. This mode of dividing the *degree* has all the advantages of (and is much less liable to confusion and error than) the mode, adopted by the French, of dividing the *quadrant* into 100 parts: and I hope that some public-spirited individual will ere long be bold enough to print tables founded on this arrangement. The decimal division of the degree is very convenient in all tables (particularly those formed for the purposes of astronomy) where proportional parts are required: and it has many advantages over the ordinary sexagesimal division. But the French mode of dividing the *quadrant* into 100 parts, calling each of those parts a degree, and adopting the *same character* that is used in the sexagesimal notation, to express that degree (although its value is only $\frac{9}{10}$ths of that quantity) leads to endless confusion, and ought to be discountenanced and discontinued.

Table XXXIX serves to convert any given day into the *decimal part of a year* consisting of 365 days. In leap years we must add unity to the given date, if subsequent to February, in order to obtain the corresponding tabular date: and in such case, if great accuracy be required, the value thus found should be multiplied by ·997.

Table XL. The new division of the circle into 400 degrees, by the French, and the numerous and valuable tables that have been computed agreeably to that arrangement, render a table of this kind absolutely necessary for converting the centesimal division of the quadrant into the sexagesimal: and *vice versâ*. It is formed on the comparison stated in Table XLII: where it will be seen that each degree of the French division is equal to 54′ only of the sexagesimal division; and each minute of the French division equal to 32″,4 only of the sexagesimal. When the French first introduced the centesimal division of the quadrant, the new degrees were denominated *grades*, and were denoted by the small letter *g* placed above the number. Thus, 164 degrees were written 164^{g}. This method, so long as degrees only were concerned, and whilst the subdivisions of the degree were expressed decimally, could not lead to any confusion: but this mode of denoting the new quantities has been gradually discontinued, and at the present day most of the French mathematicians use the sexagesimal notation, not only for the new degrees, but likewise for the new minutes and seconds: thus introducing considerable confusion in the value of the quantities alluded to. In order to remedy this, in some measure, I have denoted the new degree by a small *square* character placed over it, instead of a round one. Perhaps a better plan would be to adopt the small Greek letters δ, μ, σ, to denote the degrees, minutes and seconds of the new division: thus 24 degrees 15 minutes and 37 seconds of the centesimal division of the quadrant would be written thus, $24^{\delta}.\ 15^{\mu}.\ 37^{\sigma}$. But much the best plan would be to discontinue the system altogether.

Table XLI serves to compare *Fahrenheit's thermometer* with Reaumur's and the Centesimal; and *vice versâ*. It is founded on the comparisons given in Table XLII: by means of which equations the present table may be extended to any number of degrees required. Its use and application will be evident on inspection.

Table XLII contains various comparisons (of frequent use in practice) relative to French and English measures, &c. In the *Base du Système Métrique*, Vol. 3, page 470, the French metre, which is made of platina, is stated to be equal to 39·3827 English inches: and it has been assumed as such by Professor Schumacher, in his *Sammlung von Hülfstafeln*, page 16. But this comparison was made when the two measures were both at the freezing point of the thermometer. This degree of temperature is the standard for the French scale: but the standard temperature for the English scale, which is made of brass, is 62° of Fahrenheit. Consequently the value given by Professor Schumacher ought to be corrected for the relative expansion of the metals, which will reduce it to 39·37079 inches, as given by Captain Kater in the *Phil. Trans.* for 1818, page 109; each scale being brought to its standard temperature.

By Act 5 Geo. IV, c. 74, Bird's scale of 1760 * is de-

* There is another scale made by Bird in 1758, called also Bird's Parliamentary standard, which differs +·00016 of an inch from Sir

clared to be the legal standard: and this scale differs so little (only +·00002 of an inch) from Sir George Shuckburgh's scale, that they may be considered as identical. And as most of the comparisons have been made with Sir G. Shuckburgh's scale, its measure is therefore here retained.

The old legal standard of France was the *Toise de Perou,* so called from its being used by the French Academicians in that country. It is formed of iron, and was made by M. Langlois in 1735, under the direction of M. Godin. By a comparison of this toise with a copy of Sir G. Shuckburgh's scale at the temperature of 56°,3 Fahr. it was found to be equal to 76·7394 English inches. See *Base du Syst. Mét.* Vol. 3, page 479. By a Report in the same work, page 433, made by the French Commissioners, it appears that the toise of Peru, at 61°,3 Fahr. is equal to 1·949036 metres. It is upon these authorities that I formed the comparison of the measures in the table.

In the comparison of the centesimal degrees of the quadrant, I have introduced a new method of denoting those degrees. It consists in annexing a small *square* character, in order to distinguish them from the ordinary sexagesimal degrees, with which they are frequently confounded. See the remarks I have already made on this subject in page 212.

The *length of the pendulum* is taken from Capt. Kater's last determination in the *Phil. Trans.* for 1819, page 415.

The *velocity of sound* is deduced from a comparison of the experiments which have been recently made by several

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George Shuckburgh's scale: and this must not be confounded with the scale of 1760. All these scales are made of brass.

distinguished philosophers: the mean of which is very near the value here assumed.

Table XLIII contains the *logarithms of various quantities* which occasionally occur in astronomical computations; and will prevent the necessity of referring to other works, when they are required for use. The last two logarithms should more properly have had the characteristic 9 affixed, instead of 1 and 3 respectively: as it is in general required to convert the divisions into *integers* of a similar denomination.

Table XLIV contains a comparison of the French and English barometrical measures: and will be found very convenient for reducing either French or English measures, to the contrary denomination. It is deduced from the assumption that the French metre, at the freezing point, is equal to 39·37079 English inches, at the temperature of 62° Fahrenheit: as already alluded to in page 213.

Table XLV will be useful in comparing the height of the quicksilver in barometers, where the tubes are of different diameters: and affords a correction which ought not to be rejected in any nice calculations.

PROBLEMS.

Problem I. To convert sidereal time into mean solar time: and *vice versâ*.

This problem is of frequent occurrence in practical astronomy, and is solved by the help of Tables II—VII in the present collection, which have been expressly formed for that purpose. Their application is as follows: let us make

M = the *mean solar* time at the place of observation

S = the corresponding *sidereal* time

Æ = the mean *right ascension* of the meridian (or the mean longitude of the sun, converted into time) at the preceding *mean* noon, at the place of observation

a = the acceleration of the fixed stars (as shown by Table VI) for the interval of time denoted by (S − Æ)

A = the acceleration (as shown by Table VII) for the time denoted by M

then we shall have

$$M = (S - Æ) - a$$
$$S = Æ + M + A$$

The mean right ascension of the meridian, at *mean* noon on any day in any year, as reckoned from the mean equinox, is found by Tables II and III: to which must be applied the equations deduced from Tables IV and V, in order to give the right ascension reckoned from the *apparent equinox* *.

Example. An eclipse of the first satellite of Jupiter was observed at Greenwich on Jan. 17, 1825 at $2^{h}. 19^{m}. 49^{s},0$ sidereal time by the clock. The clock however was too fast $59^{s},14$: consequently the correct sidereal time was $2^{h}. 18^{m}. 49^{s},86$. Required the corresponding mean solar time?

			h	m	s	☊
Table II	Jan. 17	=	19	43	4,885	2
Table III	1825	=		3	20,654	749
Table IV	Lunar nutation . .	=			+ 1,056	747†
Table V	Solar nutation . .	=			+ 0,062	
Mean Æ at preceding *mean* noon reckoned from *app.* equinox		=	19	46	26,657	
	S	=	2	18	49,860	
	(S − Æ)	=	6	32	23,203	
Table VI	− a	=	−	1	4,282	
Mean solar time required . .		=	6	31	18,921	

If the observed time had been mean solar time $6^{h}. 31^{m}. 18^{s},921$ and it were required to find the corre-

* All the ephemerides give the true Æ of the sun for *apparent* noon: but we may easily deduce therefrom the Æ for *mean* noon, if required, in the following manner. The equation of time being expressed in mean solar time, we must first convert it into sidereal time by Table VII: then change the sign and apply it to the Æ of the sun for apparent noon. The result is the true Æ of the sun for mean noon: provided the ephemeris has been correctly computed.

† The place of the moon's node on Jan. 1, 1825 is, by Table III, equal to 749, from which must be deducted the motion of the node from that day to Jan. 17, which, by Table II, is equal to 2: the difference (=747) is the argument for entering Table IV.

sponding sidereal time, the operation would have stood thus:

		h	m	s
	M =	6	31	18,921
as above found . .	Æ =	19	46	26,657
Table VII . . .	A =		1	4,282
Sidereal time required	=	2	18	49,860

When the place of observation is situated on a different meridian from that of Greenwich, the value of Æ as found by these tables must be increased or diminished agreeably to the rule given in page 196.

The mean solar time being ascertained, we may readily determine the corresponding *apparent time*, by applying the *equation of time*, as shown by an ephemeris, for the moment of observation *. Thus the equation of time on Jan. 17, 1825 at $6^h. 31^m. 18^s,92$ as given in the Nautical Almanac, was $10^m. 34^s,57$; which (being directed to be *added to apparent* time) must be *subtracted from mean* time: consequently the corresponding apparent time was $6^h. 20^m. 44^s,35$.

There is another very convenient, and equally correct mode of converting sidereal time into mean solar time, by the help of an ephemeris computed for the given place; which is as follows:

$$M = (S - Æ') - a + e$$

where Æ' denotes the apparent right ascension of the sun

* The equation of time is equal to the difference between the sun's *mean* longitude, converted into time, and the sun's *true* right ascension. It may be useful to remark that Table VIII in M. Delambre's *Tables of the Sun* (which is the same as Mr. Vince's XXIX) is inaccurate, and in some cases produces an error of $1^s,6$. Another and a more correct table is given in the *Con. des tems* for 1810, page 492.

at the preceding *apparent* noon, and e the equation of time at the same moment: both of which are given in the ephemeris. Take the example above given.

		h	m	s
	S =	2	18	49,86
By Nautical Almanac	Æ' =	19	56	57,70
		6	21	52,16
Table VI . . .	$-a$ =		− 1	2,58
		6	20	49,58
By Nautical Almanac .	e =		10	29,30
		6	31	18,88

This result ought to be exactly the same as the preceding: and the slight difference, which occurs, arises from the circumstance that the co-efficients of the two nutations, in Tables IV and V, are not precisely the same as those which are used in the computations of the Nautical Almanac.

Problem II. To determine the refraction of a heavenly body.

The tables of refraction are very numerous, and many celebrated mathematicians have devoted their time and abilities to the investigation of this intricate subject. The labours of Bradley, Bessel, Littrow, LaPlace, Carlini, Groombridge, Atkinson, Gauss, Young, Ivory and Brinkley, are too well known to require any comment. But, as it was necessary to make a selection, I have chosen the tables of the two latter authors: those of Dr. Young are given annually in the Nautical Almanac.

Example 1. The zenith distance of *α Aquilæ* was observed 71°. 26′. 0″ the barometer standing at 29·76 inches and the thermometer at 43° Fahr. Required the refraction?

By Mr. Ivory's tables

			logarithms
Table IX	Mean refraction	=	2·23609
Table X	Barometer 29·76 . . .	=	9·99651
Table X	Thermometer 43° . . .	=	0·00668
	Refraction = 2′ 53′,49	=	2·23928

By Dr. Brinkley's tables

Table XII	Thermometer 43° . . .	=	0·2965
	Barometer 29·76	=	1·4736
	Tangent 71°. 26′	=	0·4738
	Approximate Refraction = 2′ 55″,35	=	2·2439
Table XIII	. . . . − c = − 2 ,03		
	True refraction = 2 53 ,32		

If we denote the *apparent* zenith distance by Z^a and the *true* zenith distance by Z^t we shall have

$$Z^t = Z^a + r$$

where r is the computed refraction. Or, if we prefer expressing these values by means of *altitudes*, we shall have

$$A^t = A^a - r.$$

Example 2. The apparent zenith distance of α *Lyræ* was observed 87°. 42′. 10″: the barometer standing at 29·50 inches, and the thermometer at 35° Fahr. Required the refraction?

By Mr. Ivory's tables

			logarithms
Table IX	Mean refraction	=	3·00856
Table X	Barometer 29·5	=	9·99270
Table X	Thermometer 35° . . .	=	0·01444
	Approximate mean refraction = 17′ 16″,80	=	3·01570
Table XI	− ·606 × − 15 = + 9 ,09		
Table XI	+ 1·04 × − ·5 = − 0 ,52		
	True refraction = 17 25 ,37		

PROBLEM III. To determine the time from *single altitudes** of the sun, or a star: its declination, and also the latitude of the place being given.

The observed altitude or zenith distance of the sun or star must first be corrected for refraction, as in the last Problem †. Then, by means of Formula XV, in page 89, we obtain the logarithm of $\sin^2 \frac{1}{2} P$: and by the help of Table XV, we deduce from that logarithm the value of P, or the hour angle from the meridian.

In the case of the sun, this hour angle will be the *apparent* time ‡ from apparent noon *at the place of observation:* to which the equation of time must be applied, in order to deduce the *mean* solar time. But, in the case of a star, it will denote the distance, in time, of the star from the meridian; and which, being added to the right ascension of the star, if the observation be made to the westward

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* Single altitudes, or absolute altitudes, is a term given to observations of the altitude of the sun or a star, when made on the *same* side of the meridian: in opposition to *equal* altitudes, which are made on *both* sides of the meridian. See the next problem. It is not meant to imply thereby that *only one* altitude is taken; because in general there are *several:* and the usual method is to note down the *time* of each observation, and the *altitude* observed; and then to take the *mean* of the *times* and the *mean* of the *altitudes* as *one* observation.

† If the *sun* be the body observed, it must be corrected also for Parallax by Table XIV. And, since it is the *border* only of the sun that can be observed at one observation, we must reduce this to the *centre,* by applying its semidiameter, which may be found in any ephemeris: or by observing alternately the upper and lower border, and taking the mean of each *pair* of observations. We must also compute the declination for the *apparent* time of observation, *at the given place.*

‡ When the observation is made in the forenoon, this apparent time must be subtracted from 24^h in order to show the apparent time of the day.

of the meridian, or subtracted therefrom (increased by 24^h if necessary) if the observation be made to the eastward, will give the *sidereal* time of observation.

Example 1. On October 18, 1818 at $3^h. 6^m. 30^s,7$ P.M. mean solar time as shown by a clock, the zenith distance of the upper border of the sun (corrected for refraction and parallax) was found to be $75°. 0'. 8'',9$: the place of observation being situated in N. Lat. $52°. 13'. 26''$ and W. Long. $4^m. 49^s$ from Greenwich. What was the correct time of observation?

If we add the semidiameter of the sun on the given day ($= 16'. 6''$) to the zenith distance (corrected as above) of the observed border, we shall have the true zenith distance of the sun's centre equal to $75°. 16'. 15''$. We must next compute the sun's declination for the approximate *apparent* time of observation at the place. Let us suppose that the Nautical Almanac is made use of for these computations. The equation of time at apparent noon (or at $11^h. 45^m. 18^s,9$ mean solar time) at Greenwich, was $-14^m. 41^s,1$: which being added, with its sign changed*, to the mean solar time of observation, will give $3^h. 21^m$ for the approximate apparent time, at the place. But, as the computations are made from the Nautical Almanac, we must add the longitude from Greenwich; and compute the sun's declination for $3^h. 26^m$ *Greenwich* apparent time, with a daily variation of $21'. 51''$. Consequently the sun's declination at the time of observation was $-9°. 33'. 30''$. With these elements the computation will stand thus:

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* The equation of time is an equation which in an ephemeris is directed to be added to, or subtracted from, *apparent* time, in order to deduce the mean solar time: consequently we must *reverse the sign* when we wish to apply the equation to *mean solar* time, in order to deduce apparent time.

	° ′ ″		logarithms
L =	+ 52 13 26	cos =	+ 9·7871611
D =*	− 9 33 30	cos =	+ 9·9939285
(L − D) =	+ 61 46 56		+ 9·7810896
Z =	+ 75 16 15		
Z − (L − D) =	+ 13 22 19	sin ½ =	+ 9·0698112
Z + (L − D) =	+137 3 11	sin ½ =	+ 9·9687570
			+ 9·0385682
		as above =	+ 9·7810896
		sin² ½ P =	+ 9·2574786

		h m s	
By Table XV . sin² ½ P	=	3 21 22,7	
equation of time † . .	=	−14 42,7	
correct mean solar time .	=	3 6 40,0	at the place.
observed mean solar time ‡	=	3 6 30,7	at the place.

Consequently the clock was . . 9,3 too slow.

Example 2. On March 23, 1822, in N. Lat. 51°. 33′. 34″ I observed, to the westward of the meridian, the zenith distance of *Aldebaran* (after correcting for refraction) to be 68°. 2′. 21″ at 9^h. 24^m. 44^s,0 by a sidereal clock. What was the error of the clock at that moment?

On that day the apparent Æ of the star was 4^h. 25^m. 43^s,8 and its apparent Declination was + 16°. 8′. 44″; as shown by the Nautical Almanac. Consequently the operation will stand thus:

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* The declination of the sun being *south*, it is considered as a *negative* quantity in the computations.

† By Nautical Almanac at 3^h. 26^m. 17^s,7 from *Greenwich:* or at 3^h. 21^m. 22^s,7 + 4^m. 49^s.

‡ If a sidereal clock had been used in this observation, we must have reduced the sidereal time to mean solar time, in order to show the error of the clock.

	° ′ ″		logarithms
L =	+ 51 33 34	cos =	+ 9·7935825
D =	+ 16 8 44	cos =	+ 9·9825238
(L − D) =	+ 35 24 50		+ 9·7761063
Z =	+ 68 2 21		
Z − (L − D) =	+ 32 37 31	sin ½ =	+ 9·4485148
Z + (L − D) =	+ 103 27 11	sin ½ =	+ 9·8949038
			+ 9·3434186
			+ 9·7761063
		sin² ½ P =	+ 9·5673123

		h m s
By Table XV .	sin² ½ P =	4 59 21,7
	Æ =	4 25 43,8
correct sidereal time . .	=	9 25 5,5
observed sidereal time* .	=	9 24 44,0
error of the clock		− 21,5

It is scarcely necessary to remark that the most favourable opportunity, for determining the time from altitudes of the sun or a star, is when those bodies pass the *prime vertical*: since their motion in altitude is then the most rapid, and a slight error in the assumed latitude will not materially affect the result †.

In fact, in a fixed observatory, (or where the observer may be stationed at the same place for several successive

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* If a clock, showing mean solar time, had been used in this observation we must have reduced the mean solar time to sidereal time, in order to show the error of the clock.

† In the latitude of Greenwich, a star varies 9″,3 in altitude in one second of time, when on the prime vertical. The general expression for such variation is 15 cos Lat.

nights) if we observe the same stars we may very much abridge the computations by making the observations on the prime vertical; since *one* calculation made for each star will be sufficiently accurate for a long period: as I have shown more at length in the *Memoirs of the Astronomical Society*, Vol. 1, page 315.

Problem IV. To determine the time of noon or midnight, from *equal altitudes* of the sun*: the interval of time between the observations, the latitude of the place, the declination of the sun and its daily variation being given.

This problem is solved by means of the formula in page 92: where the value of the correction x is to be applied to the mean of the times at which the equal altitudes have been observed. The logarithms of A (which is always *minus*) and of B will be found in Table XVI, opposite the given interval. Then find by the Nautical Almanac, or any other ephemeris, the sun's declination at the time of noon on the given day and at the given place: which however need not be taken out to any great accuracy. Find also the double daily variation of the declination expressed in *seconds*; that is, the difference (in *seconds*) between the declination at the time of noon on the preceding day, and the declination at the time of noon on the following day†.

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* If we observe equal altitudes of a *star*, it is evident that half the interval of time elapsed will give immediately the time of the star passing the meridian, without any correction.

† The logarithms of these values, for every day in the year, are now given annually by M. Schumacher in his *Astron. Hülfstafeln.* They would form a valuable addition to our own national ephemeris; since they would be very useful in navigation, as well as in the observatory

This quantity is denoted by δ; and it must be considered *negative*, when the sun is proceeding towards the south pole *.

Example. On July 25, 1823, in N. Lat. 54°. 20′ at $8^h. 59^m. 4^s$ A.M. and at $3^h. 0^m. 40^s$ P.M. the sun had equal altitudes. Required the equation, or correction to be applied to the mean of those times in order to find the time of noon?

The interval of time being $6^h. 1^m. 36^s$, we have by Table XVI log. A = 7·7707 and log. B = 7·6187. And by the Nautical Almanac the declination of the sun, at noon on that day, was + 19°. 48′. 29″, and its double daily variation equal to − 25′. 29″ = − 1529″. The operation therefore will stand thus:

	logarithms		logarithms
$-A =$	$-\ 7{\cdot}7707$	$B =$	$+\ 7{\cdot}6187$
$\delta = -1529'' =$	$-\ 3{\cdot}1844$	$\delta = -1529'' =$	$-\ 3{\cdot}1844$
$\tan 54°.\ 20' =$	$+\ 0{\cdot}1441$	$\tan 19°.\ 48' =$	$+\ 9{\cdot}5300$
$+\ 12^s,57 =$	$+\ 1{\cdot}0992$	$-\ 2^s,15 =$	$-\ 0{\cdot}3331$

$$\text{Correction} = +\ 12^s,57 - 2^s,15 = +\ 10^s,43$$

This value, being added to the mean of the times of the observed altitudes, or $\frac{1}{2}(20^h. 59^m. 4^s + 27^h. 0^m. 40^s) = 23^h. 59^m. 52^s$, will give $0^h. 0^m. 2^s,43$ for the time at apparent noon.

This however denotes *apparent* time: and we must add thereto the equation of time, which on that day was upwards of 6 minutes. If the observations were made on the meridian of Greenwich the chronometer ought to show $0^h. 6^m. 7^s,2$; and consequently would denote that it was too slow by $6^m. 4^s,77$.

* That is, from the time of the summer solstice, to the time of the winter solstice; or from June 21 to Dec. 21. See the *Errata*.

Problem V. On the reduction to the Meridian.

In order to determine the meridional altitude of a heavenly body, it is usual, in large observatories furnished with a mural circle or quadrant, or a transit circle, to observe such body at the precise moment of its passing the meridian. But, where the observer is furnished with an altitude and azimuth instrument, or a repeating circle, this is not absolutely necessary; since, by means of Tables XVIII and XIX, we may render any number of observations made on each side of the meridian, and at a short distance therefrom, equal in accuracy to those which are made immediately at the moment of culmination. For this purpose, it is necessary to know the distance (in time) of the sun or star from the meridian at the moment of each observation: and, opposite to such given distance in time in Table XVIII, is stated the value which ought to be applied to the zenith distance observed. The sum of these values, divided by their number, and multiplied by $\frac{\cos L \,.\, \cos D}{\sin Z}$, will give the correction which ought to be applied to the *mean* of the zenith distances observed (corrected for refraction) in order to determine the true meridional zenith distance of the sun or star. Should greater accuracy however be required, we must take the second part of the reduction from Table XIX, the sum of which (divided also by the number of observations) must be multiplied by $\left(\frac{\cos L \,.\, \cos D}{\sin Z}\right)^2 \times \cot Z$. But this second correction is seldom necessary.

The expression for the zenith distance of a star, in terms of its declination and of the latitude of the place, will vary according as the observations are made to the south of the zenith, or to the north of the zenith; and, in this latter case, according as the observations are made above or below the pole *. These several values will be as follow:

$Z = L - D$ . . . if the obs. be made to the south
$Z = D - L$ . . . if to the north, *above* the pole
$Z = 180° - (L + D)$ if to the north, *below* the pole.

It should be observed here that, when the sun is the object observed, there is a further correction to be applied, which is shown in the formula in page 93: where E and W are expressed in minutes of time, considered as integers.

Example. On May 13, 1819 a set of 14 observations of Polaris, near the time of its *lower* culmination, was made with a repeating circle, at Shanklin in the Isle of Wight, on each side of the meridian; the mean of which gave the observed zenith distance, corrected for refraction, equal to $41°. 1'. 54'',1$. The latitude of the place is assumed equal to $50°. 37'. 23''$; and the apparent declination of Polaris is found by the Nautical Almanac to be $88°. 20'. 28'',87$. The observations were made with a chronometer showing mean solar time, having a *losing* rate equal to $1'',8$. Polaris passed the meridian at $9^h. 37^m. 32^s$ by the chronometer; and the respective observations were made at the several periods indicated in the second column of the following table. What was the correction to be applied to the mean of the zenith distances above mentioned, in order to reduce them to the meridian?

By taking the difference between the several times of observation and $9^h. 37^m. 32^s$ we obtain the values in the

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* M. Delambre has, in his *Astronomie*, considered only the case where the observations are made to the south of the zenith: and it was by following him too closely that I was led into the error of employing (L−D), instead of Z, in the Formula in page 93. See the list of *Errata*.

third column of the following table. Entering Table XVIII with these values, as arguments, we obtain the values set down in the last column: the sum of which being divided by 14 will give 490″,9.

No.	Time of Observation	Time from Meridian.	By Table XVIII
	h m s	m s	″
1	9 13 15	24 17	1156,8
2	9 16 40	20 52	854,3
3	9 21 21	16 11	514,0
4	9 33 55	3 37	25,7
5	9 36 44	0 48	1,3
6	9 39 55	2 23	11,2
7	9 42 27	4 55	47,5
8	9 45 40	8 8	129,9
9	9 48 56	11 24	255,1
10	9 51 53	14 21	404,2
11	9 54 40	17 8	576,1
12	9 56 48	19 16	728,4
13	9 59 40	22 8	961,1
14	10 2 20	24 48	1206,4
			14)6872,0
			490,9

The subsequent operation then will stand thus:

L =	50° 37′ 23″	cos =	+ 9·8023765
D =	88 20 29	cos =	+ 8·4615613
	138 57 52		+ 8·2639378
180° − (L + D) =	41 2 8	sin =	+ 9·8172528
		constant log. =	+ 8·4466850
		490,9 =	+ 2·6909930
		on account of mean solar time =	+ 0·0023715
		on account of rate of clock =	+ 0·0000181
		13″,806 = x =	+ 1·1400676

As the star was below the pole, at the time of observation, this correction must be *added* to the observed zenith distance; whence the true *meridional* zenith distance will be 41°. 1'. 54'',1 + 13'',81 = 41°. 2'. 7'',91; and the latitude = 180° − (Z + D) = 50°. 37'. 23'',22.

In observations with the repeating circle, for determining the latitude, it is necessary to attend to the *verticality of the circle;* since an inclination of the circle will cause a corresponding error in the results. But if the amount of the inclination i be known, we may ascertain the error e in the result, by means of the following equation:

$$e = \tfrac{1}{2} \sin 1''.\, i^2 . \cot Z.$$

We must also attend to the *position of the level*, and either bring it, by the proper screw, to its zero point, or take an account of the place of the bubble in the two opposite positions of the circle, and allow for the difference, according to the value of the divisions of the scale: prefixing the sign + or − according as either end of the bubble is *nearer to* or *farther from* the observer than the true zero point.

There are also some other circumstances to be attended to, in the management of this instrument, which are pointed out in the works expressly written on that subject.

Problem VI. To determine the latitude of a place.

The best mode of determining the latitude of a place, so as to be independent of the declination of the star observed, and also as free as possible from the errors of refraction, is by observations of a circumpolar star at the time of its upper *and* lower culmination. These observations may be made by means of a mural circle or quadrant, or a transit circle, at the precise moment of the passage of the star across the meridian, or they may be made by an altitude and azimuth circle, or a repeating circle, either in the same manner, or in the mode alluded to in the last problem. In either case, therefore, let Z denote the observed or deduced meridional zenith distance of the circumpolar star at its lower culmination, and ϱ its refraction at that point: also let Z′ denote the observed or deduced meridional zenith distance of the same star at its upper culmination, and ϱ' its refraction at that point. Then will the correct zenith distance of the pole, or the co-latitude (ψ) of the place, be

$$\psi = \tfrac{1}{2}(Z + Z') + \tfrac{1}{2}(\varrho + \varrho')$$

It is evident that the accurate determination of ψ will depend on the tables of refraction that are used in the computation: and there is no mode of rendering the problem free from this ambiguity.

If we take the case of the pole star as observed at Greenwich, its zenith distance at the upper culmination may be 36°. 55′ only, whereas at its lower culmination it may be 40°. 7′: and half the sum of the refractions at these points will differ according to the tables of refraction employed. This half sum (the barometer being at 30 inches, and Fahrenheit's thermometer at 50°) will be by

Bradley's Tables	= 46″,02
Bessel's Tables	= 46 ,58
Ivory's Tables	= 46 ,53
French Tables	= 46 ,52

Consequently a difference of half a second, at least, will take place, at that temperature and pressure, according as Bradley's or the other tables are made use of.

The next usual mode of determining the latitude of a place, is by means of meridional zenith distances of the sun, or a star (whether circumpolar or otherwise) whose declination is well known. The expression for the latitude will, in such cases, vary according as the observations are made to the south of the zenith, or to the north of the zenith: and in this latter case, according as the observations are made above or below the pole. Let L denote the latitude required, D the declination of the sun *, or the apparent declination of the star, and Z the observed meridional distance of the same (corrected for refraction in the case of a star, and for refraction *minus* parallax in the case of the sun *), then we shall have

$L = Z + D$ . . . if the obs. be made to the south
$L = D - Z$ . . . if to the north *above* the pole
$L = 180° - (Z + D)$ if to the north *below* the pole.

There is another mode of determining the latitude of a place, by means of observations of the altitude of the pole star *at any time of the day;* a method which is capable of great accuracy, and may frequently prove very convenient and useful. I have already treated this subject, more at length, in two papers inserted in the *Philosophical Magazine* for June and July 1822: and shall here merely refer to the formula, which is given in page 110 of the present work, for the mode of explaining the subject by the following example.

Example. At 4^h after the passage of the pole star, at its upper culmination, its altitude (corrected for refraction) was observed to be $50°. 47'. 43'',6$, and its apparent north polar distance was $1°. 38'$. What was the latitude of the place?

* Where the sun is observed and where great accuracy is required, its declination should be corrected on account of its *latitude.*

The operation will stand thus:

			logarithms.
$\Delta =$	1° 38′ 0″	tan $=$	$+$ 8·4550699
$P =$	60 0 0	cos $=$	$+$ 9·6989700
$a =$	0 49 0,6	tan $=$	$+$ 8·1540399
$a =$	as above	cos $=$	$+$ 9·9999559
$\Delta =$	as above	cos $=$	$+$ 9·9998235
			0·0001324
$A =$	50 47 43,6	sin $=$	$+$ 9·8892424
$(\phi + a) =$	50 49 0,7	sin $=$	9 8893748
$a =$	0 49 0,6		
Latitude $=$	50 0 0,1		

For a *fixed* observatory, these computations might be somewhat abridged, and rendered less liable to error, by determining the logarithms of the values of $(\frac{1}{2}\sin 1''.\tan\phi)$ and of $-(\frac{1}{2}\cot\phi + \tan\phi)\sin 1''$: which in the above case would be 4·4607741 and $-$4·8533647 respectively*. The subsequent process then, agreeably to the formula given in page 110, will be as follows:

$$-\Delta = -\ 3{\cdot}7693773$$
$$\cos P = +\ 9{\cdot}6989700$$
$$-\ 49'.\ 0'',0 = \alpha = -\ 3{\cdot}4683473$$
$$\Delta = +\ 3{\cdot}7693773$$
$$\sin P = +\ 9{\cdot}9375306$$
$$\Delta.\sin P = +\ 3{\cdot}7069079$$
$$\Delta^2.\sin^2 P = +\ 7{\cdot}4138158$$
$$\tfrac{1}{2}\sin 1''.\tan\phi = +\ 4{\cdot}4607741$$
$$+\ 1'.\ 14'',9 = \beta = +\ 1{\cdot}8745899$$
$$\alpha = -\ 3{\cdot}4683473$$
$$\gamma = -\ 4{\cdot}8533647$$
$$+\ 1'',6 = \alpha\beta\gamma = +\ 0{\cdot}1963019$$

* The latitude of the place is always known sufficiently near for the determination of these logarithms.

Consequently the latitude of the place will be

50°	47′	43″,6
	1	14,9
		1,6
50	49	0,1
−	49	0,0
50	0	0,1

There is still another mode of determining the latitude of a place, which is independent of the divisions of the instrument, and depends only on the apparent declination (D) of the star observed, and on the interval of sidereal time which has elapsed between the observations. This method (which I have also explained more at length in the *Philosophical Magazine* for May 1825) consists in placing the *axis* of the telescope of an altitude and azimuth instrument due north and south, so that the vertical circle should stand east and west, and thus twice cut the parallels of all the stars between the equator and the zenith. The observation of the two times T and T′ (at which the star passes the wire of the telescope in its diurnal revolution) will give the latitude (L) of the place from the following formula,

$$\cot L = \cot D . \cos \tfrac{1}{2} (T - T')$$

In this formula, (T − T′) denotes the *correct* interval of *sidereal* time elapsed between the observations of each star, expressed in *degrees* &c. So that if *mean solar* time be employed, we must multiply the interval by 1·0027379; or, which is the same thing, *add* the values found in Table VII, against the given interval.

It is evident that this method (like all the others, except that by means of circumpolar stars) depends on the accuracy of the apparent declination of the star observed: a small error in this point, however, will not materially affect the results. But, if this mode be adopted in geodetical operations, it is evident that we may obtain the *difference* in the latitude of two places *very exactly* and almost independent of the declination of the star. It is this circumstance that renders the method valuable in such investigations.

This method is indeed recommendable on account of its independence of any error in the instrument. If the collimation should not be sufficiently corrected, the cylinders of the axis should be unequal in their diameter, the telescope or the axis should bend &c &c, we shall still obtain a correct result, either by reversing the axis between the two operations, or by observing one day in one position and the next day in the other position of the axis, and taking the mean of the two. The success solely depends on the quality of the telescope, and the care employed in *levelling* the axis. It is scarcely necessary to add that observations of this kind should be made on stars that culminate near the zenith of the place.

PROBLEM VII. To determine the longitude of a place.

The method of determining the difference of longitude between two given points on the surface of the earth, which is one of the most difficult problems in practical astronomy, has long engaged the attention of various astronomers and mathematicians; and has been executed with more or less accuracy according to the means employed for that purpose. If the distance between the two observatories be not very great, their difference of meridian may be determined with considerable accuracy, by means of chronometers conveyed from one observatory to the other; or by means of signals previously agreed on. These methods have been practised very successfully on many recent occasions. But, where this is impracticable, we must have recourse to the observation of certain celestial phænomena for the solution of the problem: and for this purpose, five several and distinct methods have been proposed: 1° the eclipses of Jupiter's satellites: 2° eclipses of the moon: 3° eclipses of the sun: 4° occultations of the fixed stars: 5° the meridional transits of the moon, compared with certain stars previously agreed on.

The results deduced from the observations of the eclipses of Jupiter's satellites are, for obvious reasons, very unsatisfactory. The phænomena will, in fact, appear to take place at different moments of time, with different instruments and to different observers. Moreover, they are visible only in certain positions of the planet in its orbit; a circumstance which very much circumscribes the utility of the method.

The eclipses of the moon afford a still more unsatisfactory result: they occur but seldom in the course of a year, and the phænomena attending them cannot (on account of the indistinctness of the border of the earth's shadow) be observed with that degree of accuracy which the present state of astronomy requires for such purposes.

Eclipses of the sun are more certain in their deductions: but, they so rarely occur, and are at the same time so limited in extent, that they can seldom be brought in aid of the general solution of the problem. From September 1820 to November 1826, there is only one solar eclipse that will be visible in this country.

There remain therefore only the two other methods, on which the practical astronomer can safely and constantly depend. Of these, I am aware that occultations of the fixed stars by the moon have been long considered as affording the best means of determining the difference of longitude between two places: and, assuredly, the results deduced from such observations, made under favourable circumstances, have agreed with each other to a greater degree of accuracy, than those deduced by any of the preceding methods.

There are, however, many circumstances, attending the practical solution of the problem by this method, which tend to diminish the confidence which is reposed in the correctness of the theory. In the first place, it is necessary to know the apparent right ascension and declination of the star very exactly, on the day of observation; which, if the star is of inferior magnitude (and such being the most numerous, are the most likely to be occulted), may not be readily determined: for, we may not be able to find it in any catalogue; and, when found, we have to compute its precession, aberration, and nutation expressly for this purpose. In the second place, we have to calculate the parallax of the moon for the given moment of observation: and in this computation we must assume a given quantity for the compression of the earth; respecting which, astronomers are by no means agreed, and which will consequently give rise to various results, according to the view which each astronomer may take of the subject. Thirdly, this method is dependent on the accuracy of the lunar tables, not only as to the position of the moon and her horary motion, but also as to her horizontal parallax and semidiameter. Fourthly, the method is, in a great measure, dependent on a correct knowledge of the longitude and latitude of the place of observation. And lastly, the apparent border of the moon is so uneven (consisting of projecting mountains and hollow valleys) that we cannot always depend on the immersion or emersion having taken place at the exact distance from the moon's centre, as computed from the lunar tables.

The *meridional transits* of the moon, agreeably to the method about to be described, are free from all these objections: the observations are made with the greatest facility; the opportunities are of frequent occurrence; the absolute time is of no material consequence; the computations are by no means intricate or troublesome; and the results are (I believe) more to be relied on than by any of the preceding methods.

This method consists in merely observing, with a transit instrument, the *differences* of right ascension between the *border* of the moon, and certain fixed stars *previously agreed on**; which stars are so selected that they shall differ very little from the moon in declination. It is evident that this method is quite independent of the errors of the lunar tables, except as far as the horary motion of the moon (in right ascension) is concerned, and which, in the present case, may be depended on with sufficient confidence: that it does not involve any question as to the compression of the earth: that a knowledge of the correct position of the star is not at all required: and finally, that an error in the state of the clock, is of no consequence. Consequently, a vast mass of troublesome and unsatisfactory computation is avoided. Moreover, it is the only method that is *universal*, or, that may be adopted, at one and the same time, by persons in every habitable part of the globe: for, it is applicable to situations distant 180° in longitude from each other; and even *beyond* that if required.

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* Lists of such stars, called *moon culminating* stars, are now annually published.

It might indeed, at first sight, appear that the same results would be obtained, if we merely observed the correct *time* of the moon's transit, without any reference to the contiguous stars: but, a moment's reflection will convince us that, by referring the moon's border to the adjacent stars, we obviate all errors not only of the clock, but also in the position of the transit instrument.

For the solution of this problem, let us make,

t = the difference (in sidereal time) of the transit of the moon's *limb*, and of the star previously agreed on, at the observatory situated most *westerly;* which will be *positive* when the star precedes the moon, or when the Æ of the moon exceeds that of the star; but, on the contrary, *negative.*

τ = the similar difference, at the observatory situated most *easterly.*

$(t-\tau)$ = the true observed difference in the Æ of the moon's *limb*, for the time elapsed between the two observations *.

c = the *apparent* time (as shown at Greenwich †) of the culmination of the moon, at the *western* observatory.

x = the *apparent* time (as shown at Greenwich) of the culmination of the moon at the *eastern* observatory.

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* If more than one star has been observed at both observatories on any given night, t and τ must be taken equal to the *mean* of *all* the corresponding comparisons made at each observatory respectively.

† Or as shown at Paris, Berlin, Milan or any other place for which the ephemeris is calculated, from which the computations are made. And this must always be understood, when Greenwich is alluded to in this manner.

a = ☽'s right ascension, in *space* } computed for the time c.
d = ☽'s true declination*
r = ☽'s true radius, or semidiam.*

α = ☽'s right ascension, in *space* } computed for the time $\varkappa$.
δ = ☽'s true declination*
ϱ = ☽'s true radius, or semidiam.*

s = the length of the true solar day, expressed in seconds of time.

m = the moon's motion in Æ, in *half* that interval, expressed in seconds of space: See page 246.

χ = the *assumed* difference of longitudes in time: *plus* when west, and *minus* when east.

$(\chi + e)$ = the *correct* difference of longitudes.

Find the apparent times, c and $\varkappa$, of the moon's culmination, *to the nearest minute*†, in order to compute d, r and

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* The *true* declination and semidiameter of the moon, are such as they are supposed to be if *seen from the centre of the earth:* in opposition to the *apparent* declination and semidiameter, which some persons have erroneously imagined ought to be adopted.

It may be sufficient to observe here, once for all, that (with a view to prevent confusion) the quantities connected with the *eastern* observatory, are denoted by *Greek* letters: and that the similar quantities connected with the *western* observatory, are denoted by *Roman* letters.

† The apparent time of the moon's culmination at Greenwich, to the nearest minute, may be seen in the Nautical Almanac: and the apparent time (at Greenwich) of its culmination on any other meridian may thence be easily deduced. Or, if the sidereal time is known, we may determine the Greenwich apparent time very nearly, by subtracting therefrom the sun's right ascension at Greenwich at the preceding noon; and diminishing the interval by the acceleration of the fixed stars. Or, we shall have, in all cases sufficiently near for this purpose, the required interval $c - \varkappa = [\chi + (t - \tau)] \times \cdot 99727$: where it should be observed that $[\chi + (t - \tau)] \times \cdot 99727$ is equal to the time $\chi + (t - \tau)$ diminished by the acceleration of the fixed stars for that interval.

δ, ϱ, for those approximate times respectively*: and then make

$$\Delta = (t - \tau) \pm \frac{r}{15.\cos d} \pm \frac{\varrho}{15.\cos \delta}$$

which is the true observed difference in the Æ of the moon's *centre*, for the time elapsed between the two observations: where the *upper* sign is to be taken when the first (or western) border of the moon is observed; and the *lower* sign when the second (or eastern) border is observed †. Then, by assuming χ equal to the presumed difference of longitude, and knowing the apparent time (at Greenwich) at one of the observatories to the nearest minute, we may determine the required apparent time (at Greenwich) at the other observatory, by the following equation:

$$c = \varkappa + (\chi + \Delta)\,\frac{86400}{s}$$

Compute a and α for the respective times c and $\varkappa$ ‡; cor-

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* It may be useful here to remark that it is not necessary to determine with strict accuracy the *absolute* value of the moon's semidiameter at *both* observatories, in order to find the value of Δ: for, the values may be estimated (in most cases by inspection) in whole seconds only, for one observatory, and the correct *differences*, in the given interval, being added thereto, will give the proper values for the other observatory. With respect to the declination, it may be taken to the nearest ten or twenty seconds only.

† These expressions are the same as those which are used in my Paper on this subject, inserted in the 2nd volume of the *Memoirs of the Astronomical Society;* where this subject is treated more at length.

‡ It may here also be useful to remark that it is not necessary to determine with strict accuracy the *precise moment* of the apparent time of the transit of the moon at *both* observatories, for the purpose of determining a and α: for it will be sufficient to know the apparent time of the transit of the moon to the nearest minute only, for one observa-

recting the moon's motion, for third differences, if required. And the formula for the correction of the assumed difference of longitude will be

$$e = \Big[15\,\Delta - (a - \alpha) \Big] \frac{s}{2m}$$

which, being added to χ, will give $(\chi + e)$ for the true difference of meridians required.

It is evident that, if $15\,\Delta - (a - \alpha) = 0$, the value of χ has been assumed sufficiently accurate, and does not require correction. In fact, the difference will in general be very small: and, when this is not the case, we may justly suspect some error in the steps of the process.

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tory, and to find the correct *difference* of the apparent times, by means of the expression $(\chi + \Delta)\frac{86400}{s}$.

In fact, nothing more is required than to compute the true increase of the moon's Æ during this given *interval:* and for this purpose Dr. Brinkley has suggested a very convenient rule, which is given in the first number of the *Dublin Philosophical Journal.* This distinguished astronomer has there shown that $(a - \alpha)$, as far as *first differences* only are concerned, may be expressed by $(\chi + \Delta) . \Delta'' . \frac{2}{s}$: leaving the equation of *second* differences (and of the *third* differences, if required) to be applied in the usual manner.

Under this point of view, the problem admits of two cases: one where both the observations are made on the same side of noon or midnight; and the other where they are made on different sides. In the former case, the expression $(\chi + \Delta) . \Delta'' . \frac{2}{s}$, as far as *first differences* are concerned, will lead us to the correct solution, and will save much time and labour: but, in the latter case, it must be divided into two parts: viz.

$$(12^h - \varkappa) . \Delta'' . \frac{2}{s} \quad \text{and} \quad [\chi + \Delta - (12^h - \varkappa)] . \Delta''' . \frac{2}{s};$$

and consequently becomes more intricate. In these expressions Δ'' and Δ''' denote the successive *first differences* in the moon's motion in Æ; or the same quantities that are alluded to in page 103.

Example. On December 5, 1824, Lieut. Foster observed the differences in the culmination of the moon and of the two stars 62 and 95 *Tauri*, at Port Bowen; the station where the Expedition (for the discovery of a North West passage, under the command of Capt. Parry) passed the winter of 1824-25. Similar differences were observed also at Greenwich. These differences, in sidereal time, were respectively as follow:

	at Greenwich	at Port Bowen
62 *Tauri*	$\tau = + 9^m\ 45^s{,}58$	$t = + 24^m\ 53^s{,}98$
95 ——	$\tau = - 9\ \ 25{,}98$	$t = +\ 5\ \ 42{,}90$

what was the longitude of the place?

For the solution of this question, we must first assume an approximate longitude. Now it appears from some occultations of fixed stars, observed by Lieut. Foster, that the longitude might be considered as $5^h.\ 55^m.\ 39^s{,}5$ west from Greenwich: but, for the sake of round numbers, I shall assume it equal to $5^h.\ 55^m.\ 40^s$. The operation therefore will be as follows.

The mean of the two observations gives $t = + 15^m.\ 18^s{,}44$ and $\tau = + 0^m.\ 9^s{,}80$: consequently we have $(t - \tau) = + 15^m.\ 8^s{,}64$; which being added to $5^h.\ 55^m.\ 40^s$, and the sum diminished by the acceleration of the fixed stars during that interval, will give in round numbers $6^h.\ 10^m$ as an approximate value of the apparent time elapsed between the two culminations.

By the Nautical Almanac it appears that the moon's *centre* passed the meridian at Greenwich at $11^h.\ 35^m$: consequently the moon's *first limb* passed at $11^h.\ 34^m$, at Greenwich; and at $17^h.\ 44^m$ (Greenwich time) at Port Bowen. With these approximate values we find the declination and semidiameter of the moon, at those respective periods, as follow:

	at $11^h\ 34^m$		at $17^h\ 44^m$
$\varrho =$	$0°\ 15'\ 42''$	$r =$	$0°\ 15'\ 44'',39$
$\delta =$	$23\quad 39\quad 20$	$d =$	$23\quad 53\quad 30$

whence we find

$$\Delta = +\,15^m.\,8^s,64 + \tfrac{1}{15}(17'.\,12'',88 - 17'.\,8'',41) = 15^m.8^s,938$$

and the correct value of c, for the subsequent computations, will be

$$c = 11^h.34^m + (5^h.55^m.40^s + 15^m.8^s,938)\tfrac{86400}{86622} = 17^h.\,43^m.\,41^s,67$$

The moon's true right ascension must now be calculated for the apparent times $\varkappa = 11^h.\,34^m$ and $c = 17^h.\,43^m.\,41^s,67$ *: whence we have

$$\begin{aligned} a &= 69°\ 53'\ 49'',21 \\ \alpha &= 66\quad 6\quad 29,93 \\ (a-\alpha) &= 3\quad 47\quad 19,28 \\ 15\,\Delta &= 3\quad 47\quad 14,07 \\ 15\,\Delta - (a-\alpha) &= -\ 5,21 \end{aligned}$$

This remainder, being multiplied by $\frac{s}{2m}\left(=\frac{24^h.\,4^m.\,22^s}{2\times 7°.24'.42''}\right)$ $= 1{\cdot}62$, will give $e = -\ 8^s,44$: and the correct longitude will be $(\chi + e) = 5^h.\ 55^m.\ 31^s,56$ †.

Should the value of $15\,\Delta - (a-\alpha)$ be considerable, it will show either that there is some error in the computation, or that the value of χ has not been assumed sufficiently near. In the latter case, we must diminish e by the acceleration of the fixed stars during that interval, and apply the result to c as a new value for the computation of a. Thus $-\,(8^s,44 - 0^s,02) = -\ 8^s,42$ being added to c, will give $17^h.\ 43^m.\ 33^s,25$ as the correct time for which a

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* Or, at once, for the given *interval;* agreeably to the method proposed by Dr. Brinkley, as stated in the preceding note.

† From a mean of 21 eclipses of Jupiter's satellites, the longitude was found to be $5^h.\ 55^m.\ 29^s$.

should have been computed: and the result would then have been *

$$\begin{array}{rrrr} a = & 69^\circ & 53' & 44'',00 \\ \alpha = & 66 & 6 & 29\,,93 \\ \hline (a-\alpha) = & 3 & 47 & 14\,,07 \\ 15\,\Delta = & 3 & 47 & 14\,,07 \end{array}$$

For the convenience of those persons who make use of this method of solution, I have computed the following table of the value of $\frac{s}{2m}$: the argument of which is $m =$ the moon's motion in Æ in 12 true solar hours; or the quantity which is actually employed, as the *first difference*, in computing the moon's place, for c or $\varkappa$, as the case may be. The value of s is, in this table, assumed to be equal to $24^h. 4^m$.

Argument.	$\frac{s}{2m}$	diff.	Argument.	$\frac{s}{2m}$	diff.
5° 0′	2·4066		6° 30′	1·8513	
		·1145			·0686
5 15	2·2921		6 45	1·7827	
		·1042			·0637
5 30	2·1879		7 0	1·7190	
		·0951			·0592
5 45	2·0928		7 15	1·6598	
		·0873			·0554
6 0	2·0055		7 30	1·6044	
		·0802			·0517
6 15	1·9253		7 45	1·5527	
		·0740			·0485
6 30	1·8513		8 0	1·5042	

The following table will also be convenient for determining the logarithm of the value of $\frac{86400}{s}$. The argument

* All these values of a and α have been corrected for *third* differences: which diminish the value of $a - \alpha$ about 0″,5.

is the increase in the sun's Æ in 24^h; which is found from an ephemeris, by taking the difference in the Æ of the sun for two successive days.

Argument	Logarithm of $\frac{86400}{s}$	Diff. for seconds
m s 3 30	9·9989457	s 1 = ·050
3 40	9·9988955	2 = ·100
3 50	9·9988454	3 = ·150
4 0	9·9987953	4 = ·200
4 10	9·9987451	5 = ·250
4 20	9·9986950	&c &c
4 30	9·9986449	

PROBLEM VIII. To determine the *apparent* obliquity of the ecliptic, from observations of the sun made near the time of the solstices: and thence the *mean* obliquity at the beginning of the year.

The observations necessary for the determination of this problem are always made a few days previous and subsequent to the day of the solstice; and consist of meridional observations of the sun's altitude, or zenith distance, which must afterwards be cleared of refraction and parallax. These observations are made either at the time of the sun's passing the meridian, or a short time before and after that period, and reduced thereto by the methods already explained in Problem V. The declination of the sun's centre at the time of observation being thus deduced from the formula $D = (L - Z)$, we may determine the correction that ought to be applied thereto, in order to express the obliquity of the ecliptic, from Formula XX in page 94, since the true value of the obliquity is always *very nearly* known. When the right ascension and declination are determined by the same instrument, we may make use of No. I. But in other cases we shall find the correction sufficiently near as follows: viz.

$$x = 13'',6347\,\delta^2 - 0'',00054\,\delta^4$$

where δ denotes the distance of the sun's true longitude from the solstice, at the time of observation, expressed in *degrees* and decimal parts of a degree; and which may be found by an ephemeris.

Example. On June 25, 1812, by an observation of the sun on the meridian at Greenwich, it was found that the zenith distance of the centre, cleared of refraction and parallax, was 28°. 4′. 1″: what was the correction which ought to be applied to the declination of the sun, in order to deduce the *apparent* obliquity of the ecliptic at that time: and what was the *mean* obliquity at the beginning of the year?

The assumed latitude of the place being 51°. 28′. 40″ we have the declination at the time of observation equal to 51°. 28′. 40″ − 28°. 4′. 1″ = 23°. 24′. 39″. By the Nautical Almanac, the true longitude of the sun on that day, at noon, was 93°. 40′. 33″; consequently by Table XXXVIII $\delta = 3°{\cdot}6758$: and the operation will stand thus:

	logarithms.
$3{\cdot}6758 =$	$+\ 0{\cdot}5653519$
$(3{\cdot}6758)^2 =$	$+\ 1{\cdot}1307038$
$13{\cdot}6347 =$	$+\ 1{\cdot}1346456$
$+\ 3'.\ 4'',225 =$	$+\ 2{\cdot}2653494$
$(3{\cdot}6758)^4 =$	$+\ 2{\cdot}2614076$
$-\ {\cdot}00054 =$	$-\ 6{\cdot}7323938$
$-\ 0'',099 =$	$-\ 8{\cdot}9938014$

and the correction will be $3'. 4'',23 - 0'',10 = 3'. 4'',13$: whence the apparent obliquity at the solstice will be $23°. 27'. 43'',13$. In this computation no notice has been taken of the latitude of the sun, which must be computed from the solar tables, and applied with a contrary sign, to the apparent obliquity above deduced. In the present case, the latitude of the sun is $+ 1'',00$: consequently the correct apparent obliquity is $23°. 27'. 42'',13$.

The apparent obliquity being thus obtained we may readily deduce the mean obliquity at the beginning of the year, by the help of Tables XXI and XXII. The mean place of the moon's node being 423, we have in Table XX, opposite thereto, $- 8'',21$; and in Table XXII against the given date we have $- 0'',76$. But those Tables having been formed for the purpose of determining the apparent obliquity from the mean obliquity, the quantities must be applied with a *contrary sign*, in order to obtain the mean obliquity from the apparent: consequently we have, at the *beginning of the year*, the

$$\text{mean obliquity} = 23°. 27'. 51'',10.$$

Problem IX. To determine the apparent equinox, from observations of the sun made near the time of the equinoxes.

Observations similar to those alluded to in the preceding problem, made a few days previous and subsequent to the equinox, will enable us to determine the precise time at which the sun is at that point. For the declination of the sun being found from the observed zenith distance as therein stated, we must correct the same for the latitude of the sun, in the following manner. Let D be the declination, deduced as above, and let D′ be the true declination: then we shall have

$$D' = D - l.\frac{\cos\omega}{\cos D}$$

where l denotes the latitude of the sun (*minus* when south) and ω the apparent obliquity of the ecliptic. The true declination of the sun being found, we may determine the longitude by the following formula:

$$\sin \odot = \frac{\sin D'}{\sin \omega}$$

which being compared with the tables of the sun, will show if there is any error *. As the determination of the equinoxes depends on the correctness of the latitude of the place, it is desirable that the observations should be repeated at the opposite equinoxes, in order that the errors may destroy one another.

Problem X. To determine the correct place of the moon from an ephemeris, by means of differences.

The place of the moon is usually given in an ephemeris for apparent noon and for apparent midnight: but her motion is so variable that her place cannot be accurately determined for any intermediate time without the help of *second*, and sometimes of *third* differences. In the annual volumes of the Nautical Almanac there is given a *table of second differences*, by the help of which this problem is usually solved. But as third differences may sometimes be wanted, I have adapted the whole to logarithmic computation, by means of Table XXVI. In order to use this table, we must find the first, second and third differences,

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* As the declination, in these cases, is always a small quantity, we have the longitude of the sun with considerable accuracy; even if the obliquity is not well determined.

in the manner pointed out in the Nautical Almanac: and to the logarithms of those differences (taking the *mean* of the two second differences) add the respective logarithms in the Table. The natural numbers thence resulting (due regard being had to the signs) will be the total correction to be applied to the moon's place in the usual way.

Example. What will be the true right ascension and declination of the moon on March 3, 1828 at $7^h. 10^m$ P.M. apparent time at Greenwich?

By proceeding agreeably to the instructions in the Nautical Almanac, we have

For the Right Ascension

		1st diff.	2nd diff.	3rd diff.
Mar. 2. Midn. =	175° 38′ 34″			
		+ 5° 59′ 40″		
—— 3. Noon =	181 38 14		+ 4′ 10″	
		+ 6 3 50		+ 58″
—— 3. Midn. =	187 42 4		+ 5 8	
		+ 6 8 58		
—— 4. Noon =	193 51 2			

For the Declination

		1st diff.	2nd diff.	3rd diff.
Mar. 2. Midn. =	−0° 56′ 11″			
		− 2° 2′ 57″		
—— 3. Noon =	−2 59 8		+ 1′ 5″	
		− 2 1 52		+ 1′ 24″
—— 3. Midn. =	−5 1 0		+ 2 29	
		− 1 59 23		
—— 4. Noon =	−7 0 23			

The operation will consequently stand thus:

For Right Ascension	logarithms.	For Declination	logarithms.
+ 6° 3′ 50″ =	+ 4·3390537	− 2° 1′ 52″ =	− 3·8640362
Tabular factor =	+ 9·7761360*	factor =	+ 9·7761360
+ 3° 37′ 17″,4 =	+ 4·1151897	− 1° 12′ 46″,9 =	− 3·6401722

* This factor has been computed from the expression $\frac{7^h. 10^m}{12^h. 0^m}$ agreeably to what has been stated in the note to page 205.

	logarithms.		logarithms.
+ 4′ 39″ =	+ 2·4456	+ 1′ 47″ =	+ 2·0294
Tabular factor =	− 9·0802	factor =	− 9·0802
− 33″,6 =	− 1·5258	− 12″,9 =	− 1·1096
+ 58″ =	+ 1·7634	+ 1′ 24″ =	+ 1·9243
Tabular factor =	− 7·5908	factor =	− 7·5908
− 0″,2 =	− 9·3542	− 0″,3 =	− 9·5151

The correction in Æ will therefore be + 3°. 36′. 43″,6, and the correction in declination − 1°. 13′. 0″,1 : consequently the true place of the moon will be as follows:

Æ = 181° 38′ 14″ + 3° 36′ 43″,6 = 185° 14′ 57″,6
D = −2 59 8 − 1 13 0 ,1 = −4 12 8 ,1

Problem XI. To determine the moon's parallax.

The parallax of the moon differs at every point of the earth's surface. No general tables therefore can be given adapted to the situation of every observer. The latitude of the place, the position of the moon (not only in her orbit, but as seen from the earth), the hour of the day, and the assumed compression of the earth, are so many varying elements in the computation, that I have always found it much less laborious to calculate the values from the formulæ, than to make use of any tables not computed for the exact place of observation.

The most convenient formulæ are those given in pages 98—100: and in the computation of occultations I prefer calculating the parallax in *right ascension* and *declination*, to that of *longitude* and *latitude;* not only because the latter involves the computation of the nonagesimal (a tedious and useless operation), but also because the positions of the stars are given in right ascension and declination in the catalogues, and therefore require no further conversion, when considered as in conjunction with the moon. In the computation of solar eclipses, either method may be adopted. But, whichever mode is pursued, the method of *series* will be found the most convenient; and not so liable to error as the other methods: or, at least, any mistake of the pen is soon detected, and may be easily rectified without disturbing any material part of the process. If we wish for a near approximation only, we may stop at the first term of the series: and, in no case need we extend it beyond the third term.

Example. Let the *reduced* latitude of the place be 48°. 39′. 50″; the horizontal parallax of the moon, *at that latitude*, 54′. 2″,5 *; the horary angle at the pole (or the correct sidereal time minus the moon's true right ascension) converted into degrees &c, equal to 58°. 43′. 50″; and the true declination of the moon at that time, + 4°. 49′. 44″. What should be the parallax of the moon in right ascension and declination?

This is the same example as that given by M. Delambre in his *Astronomie*, Vol. I, page 376 and 379; and the solution, by means of the series No. 4 in Formulæ XXVI and XXVII, will be as follows:

---

* The *reduced* latitude is equal to the observed latitude *minus* the *angle of the vertical*, which angle is determined by Formula XXI in page 95, or may be seen in Table XXIV, if the compression is assumed equal to $\frac{1}{300}$. And the horizontal parallax *at the place* is determined by multiplying the horizontal parallax at the equator by $(1 - a.\sin^2 L)$ as stated in page 95: or it may be found by adding the logarithm of the horizontal parallax at the equator to the logarithm which in Table XXIV is set against the given latitude, if the compression be assumed equal to $\frac{1}{300}$.

For the parallax in Right Ascension

		logarithms.
$p = 0° 54' 2'',5$	sin	$= +8{\cdot}1964369$
$L = 48\ 39\ 50$	cos	$= +9{\cdot}8198564$
		$+8{\cdot}0162933$
$D = 4\ 49\ 44$	cos	$= +9{\cdot}9984557$
$\sin P = 58° 43' 50'' = +9{\cdot}9318319$	a	$= +8{\cdot}0178376$
$\sin 2P = 117\ 27\ 40 = +9{\cdot}9480822$	a^2	$= +6{\cdot}0356752$
$\sin 3P = 176\ 11\ 30 = +8{\cdot}8222925$	a^3	$= +4{\cdot}0535128$

a	$= +8{\cdot}0178376$
$\sin P$	$= +9{\cdot}9318319$
comp. $\sin 1''$	$= +5{\cdot}3144251$
$+30' 36'',939$	$= +3{\cdot}2640946$
a^2	$= +6{\cdot}0356752$
$\sin 2P$	$= +9{\cdot}9480822$
comp. $\sin 2''$	$= +5{\cdot}0133951$
$+9'',935$	$= +0{\cdot}9971525$
a^3	$= +4{\cdot}0535128$
$\sin 3P$	$= +8{\cdot}8222925$
comp. $\sin 3''$	$= +4{\cdot}8373039$
$+0'',005$	$= +7{\cdot}7131092$

whence, since the quantities are all positive, the parallax in right ascension is $+\ 30'.\ 46'',879$ *.

If we denote the *apparent* right ascension of the moon by Ꭿ^a, and her *true* right ascension by Ꭿ^t, we shall have

$$Ꭿ^a = Ꭿ^t + \Pi$$

where Π is the computed parallax in right ascension. This parallax will be a *positive* quantity when the moon is on

* We here see that the first term gives a very near approximation, and that the third term is almost insensible.

the *west* side of the meridian: but when she is on the *east* side, it will be *negative;* because P will then be negative.

For the parallax in Declination

		logarithms.
* $\cos(P + \frac{1}{2}\Pi)$ =	58° 59′ 13″	= +9·7120040
$\cot L$ =	48 39 50	= +9·9443044
		+9·6563084
$\cos \frac{1}{2}\Pi$ =	0 15 23	= +9·9999957
$\cot b$ =	65 37 8	= +9·6563127
$\sin p$ =	0 54 2,5	= +8·1964369
$\sin L$ =	48 39 50	= +9·8755520
		+8·0719889
$\sin b$ =	65 37 8	= +9·9594325

$\sin(b-D)$ =	60° 47′ 24″	= +9·9409331	c =	+8·1125564
$\sin 2(b-D)$ =	121 34 48	= +9.9303936	c^2 =	+6·2251128
$\sin 3(b-D)$ =	182 22 12	= −8·6165019	c^3 =	+4·3376692

c	= +8·1125564
$\sin(b-D)$	= +9·9409331
comp. sin 1″	= +5·3144251
+ 38′ 53″,005	= +3·3679146
c^2	= +6·2251128
$\sin 2(b-D)$	= +9·9303936
comp. sin 2″	= +5·0133951
+ 14″,754	= +1·1689015
c^3	= +4·3376692
$-\sin 3(b-D)$	= −8·6165019
comp. sin 3″	= +4·8373039
− 0″,006	= −7·7914750

whence the parallax in declination will be 39′. 7″,753.

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* This logarithm is taken out erroneously in M. Delambre's example; which will account for the slight difference in our results. It may be

If we denote the *apparent* declination of the moon by D^a and her *true* declination by D^t, we shall have

$$D^a = D^t - \varpi$$

where ϖ is the computed parallax in declination. It should be remarked that when the declination of the moon is *south*, D is a *negative* quantity, and must be treated as such in the algebraic operations: consequently ϖ will, in such case, *increase* the declination.

Problem XII. To determine the aberration, and lunar and solar nutation of a star, by the general tables of M. Gauss.

These tables are given in pages 174—177: and, in order to show their use and application, take the following

Example. What is the amount of aberration and nutation, in Right Ascension and Declination, of *Aldebaran* on May 11, 1827, at noon?

The right ascension of this star, expressed in degrees &c, is 66°. 30′, and its Declination + 16°. 9′: and we have, on that day, ⊙ = 49°. 59′, and ☊ = 224°. 10′. By referring therefore to the formula in page 207, the operation will be as follows:

---

here useful to state that astronomers formerly used to compute the parallax in declination and latitude without employing the value of $\frac{1}{2}$ II: which certainly rendered their formulæ more convenient. But, by neglecting this quantity, we may cause an error of 8 or 10 seconds in the parallax in declination and latitude. The parallax in declination cannot be correctly known without previously computing the parallax in right ascension: a similar remark holds good with respect to the parallax in latitude and longitude.

For Right Ascension

	$\odot = +\ 49^\circ\ 59'$		
by Table XXVIII	$A = +\ \ 2\ \ 25$		
	$-\ Æ = -\ 66\ \ 30$		logarithms.
	$-\ 14\ \ \ 6$	cos =	$+\ 9{\cdot}9867$
	$D = +\ 16\ \ \ 9$	sec =	$+\ 0{\cdot}0175$
by Table XXVIII		$-\ a =$	$-\ 1{\cdot}2918$
	Aberration = $-\ 19'',77$	=	$-\ 1{\cdot}2960$

	☊ $= +\ 224\ \ 10$		
by Table XXX .	$B = -\ \ \ 8\ \ 17$		
	$-\ Æ = -\ \ 66\ \ 30$		
	$+\ 149\ \ 23$	cos =	$-\ 9{\cdot}9348$
	$+\ \ 16\ \ \ 9$	tan =	$+\ 9{\cdot}4618$
		$-\ b =$	$-\ 0{\cdot}9315$
	$+\ \ 2'',12$	=	$+\ 0{\cdot}3281$
by Table XXX	$c = +\ 11\ ,52$		
Solar Nut. by Table XXXI	$\begin{cases} -\ \ 1\ ,01 \\ -\ \ 0\ ,09 \end{cases}$		
	Nutation = $+\ 12\ ,54$		

Consequently the correction for aberration and nutation in Right Ascension is $-\ 19'',77 + 12'',54 = -\ 7'',43$.

For Declination

			logarithms.
$(\odot + A - Æ)$	$= -\ 14^\circ\ 6'$	sin =	$-\ 9{\cdot}3867$
	$D = +\ 16\ \ 9$	sin =	$+\ 9{\cdot}4443$
		$-\ a =$	$-\ 1{\cdot}2918$
	$+\ 1'',33$	=	$+\ 0{\cdot}1228$
by Table XXIX . .	$\begin{cases} -\ 1\ ,63 \\ -\ 3\ ,34 \end{cases}$		
	Aberration = $-\ 3\ ,64$		

$$
\begin{array}{rcl}
 & & \text{logarithms.} \\
\sin 149^\circ\, 23' & = & +\ 9{\cdot}7070 \\
-\ b & = & -\ 0{\cdot}9315 \\
\hline
-\ 4'',35 & = & -\ 0{\cdot}6385
\end{array}
$$

by Table XXXI . . . . $-\ 0\,,26$

Nutation $= -\ 4\,,61$

Consequently the correction for aberration and nutation in Declination is $-\ 3'',64 - 4'',61 = -\ 8'',25$.

PROBLEM XIII. For determining the corrections to be applied to observations with the transit instrument.

It is well known that observations with the transit instrument are subject to three principal errors arising from the three following sources: viz. 1° from a deviation of the instrument in azimuth; 2° from an inclination of the axis; and 3° from an error in the line of collimation. There is also the error of the clock, which is usually the most important. Formula XXXV in page 108, shows the whole of the corrections to be applied to the observed time of the transit of a star, in order to obtain the true right ascension. The values of a, b, c, there given, are best determined by the methods pointed out in Formula XXXVI, and which are detailed more at length by M. Littrow in the *Memoirs of the Astron. Soc.* Vol. I, page 273. For a fixed observatory, it would be convenient to have a table made of the value of $\frac{\sin(\phi - \delta)}{\cos\delta}$, of $\frac{\cos(\phi - \delta)}{\cos\delta}$ and of $\frac{1}{\cos\delta}$ for every degree of declination: by means of which the corrections, to be applied to the observed transit would be seen almost on inspection, and rendered less liable to error. Such a table of the value of $\frac{\sin(\phi - \delta)}{\cos\delta}$ would also facilitate very much the method of determining the amount of the deviation in azimuth by means of a high and low star. For if we assume the tabular value of $\frac{\sin(\phi - \delta)}{\cos\delta} = n$, and the tabular value of another star, whose declination is δ', equal $\frac{\sin(\phi - \delta')}{\cos\delta'} = n'$, we shall have $\frac{1}{n - n'} = \frac{\cos\delta'.\cos\delta}{\cos\phi.\sin(\delta' - \delta)}$: which is the factor adopted in the formula in page 109 for determining the amount of the deviation in azimuth.

Example. On May 19, 1822, the transit of Sirius was observed at $6^h. 37^m. 55^s,97$ by the clock, at Breda in Hungary, situated in N. Lat. 47°. 29′. 44″: the clock being too fast $36^s,55$; and the three principal errors of the transit instrument being $a = -0^s,77$ *, $b = -0^s,11$, and $c = -0^s,16$. What are the corrections to be applied to the observed time, in order to get the true right ascension of the star?

The declination of Sirius being − 16°. 28′ 40″, we have $(\phi - \delta)$ = 63°. 58′. 24″, and the operation will stand thus †:

			logarithms.
sin 63° 58′	=	+	9·9535
cos 16 29	=	+	9·9818
		+	9·9717
a = − ·77	=	−	9·8865
− ·72	=	−	9·8582

---

* The error in azimuth (which is one of the most frequent errors) may be deduced either from a circumpolar star, or from a high and low star; as stated in page 109. See the list of *Errata.*

† The declinations, and the differences of latitude and declination, may be taken out to the nearest *minute* only: and *four* places of logarithms will be sufficient.

$$\begin{aligned}
\cos 63^\circ\ 58' &= +\ 9{\cdot}6424 \\
\cos 16\ \ 29 &= +\ \underline{9{\cdot}9818} \\
&\ +\ 9{\cdot}6606 \\
b = -\ {\cdot}11 &= -\ \underline{9{\cdot}0414} \\
-\ {\cdot}05 &= -\ 8{\cdot}7020
\end{aligned}$$

$$\begin{aligned}
\text{comp.}\ \cos 16^\circ\ 28'\ 40'' &= +\ 0{\cdot}0182 \\
c = -\ {\cdot}16 &= -\ \underline{9{\cdot}2041} \\
-\ {\cdot}17 &= -\ 9{\cdot}2223
\end{aligned}$$

Consequently the apparent right ascension of the star will be

$6^h\ 37^m\ 55^s{,}97 - 36^s{,}55 - 0^s{,}72 - 0^s{,}05 - 0^s{,}17 = 6^h\ 37^m\ 18^s{,}48.$

Problem XIV. To compute a table of Altitudes and Azimuths.

In all observatories, where an altitude and azimuth instrument is used, it is extremely desirable to have either a general table of altitudes and azimuths, or a table adapted to some particular stars; in order that we may be able to find such stars at any given hour of the day. The best mode of forming a table of this kind, for a fixed observatory, is by means of Formula XIV in page 88: since $\frac{\cos \frac{1}{2}(\psi \backsim \Delta)}{\cos \frac{1}{2}(\psi + \Delta)}$ and $\frac{\sin \frac{1}{2}(\psi \backsim \Delta)}{\sin \frac{1}{2}(\psi + \Delta)}$ are constant quantities for each star; and the only variable quantity for the azimuths will be $\cot \frac{1}{2}$ P.

Thus, assuming the north polar distance of *Polaris* to be 1°. 38′, and the colatitude of the place 38°. 31′. 20″, we have the logarithms of the following quantities: viz.

$$\frac{\cos \frac{1}{2}(\psi \backsim \Delta)}{\cos \frac{1}{2}(\psi + \Delta)} = \frac{\cos 18^\circ\ 26'\ 40''}{\cos 20\ \ 4\ \ 40} \begin{array}{l} = +\ 9{\cdot}9770973 \\ = +\ 9{\cdot}9727708 \end{array}$$

logarithms.

$$\text{constant} = +\ 0{\cdot}0043265$$

$$\frac{\sin \frac{1}{2}(\psi \backsim \Delta)}{\sin \frac{1}{2}(\psi + \Delta)} = \frac{\sin 18^\circ\ 26'\ 40''}{\sin 20\ \ 4\ \ 40} \begin{array}{l} = +\ 9{\cdot}5002159 \\ = +\ 9{\cdot}5356680 \end{array}$$

$$\text{constant} = +\ 9{\cdot}9645479$$

Each of these constant logarithms, being added to the logarithm of cot $\frac{1}{2}$ P, (where P must be taken equal to such intervals as may be required) will give the logarithms of the tangents of the *sum* and *difference* of two arcs A and V: whence we obtain the value of A alone *.

The value of the azimuthal angle being thus found, we may deduce therefrom the zenith distance of the star, by means of the other formula in page 88.

Example. The colatitude of the place, and the north polar distance of the star being as already stated, what are its altitude and azimuth at the distance of $3^h.\ 15^m$ from the meridian?

The arithmetical operations, for the solution of this question, will stand thus:

$$P = 3^h\ 15^m = 48^\circ\ 45'\ \ 0''$$

logarithms.

$$\tfrac{1}{2}P = 24\ \ 22\ \ 30 \qquad \cot = +\ 0{\cdot}3438116$$

$$\text{constant} = +\ 0{\cdot}0043265$$

$$\tfrac{1}{2}(A + V) = 65^\circ\ 50'\ 20''\ \tan = +\ 0{\cdot}3481381$$

$$\text{as above} \qquad \cot = +\ 0{\cdot}3438116$$

$$\text{constant} = +\ 9{\cdot}9645479$$

$$\tfrac{1}{2}(A \backsim V) = 63^\circ\ 49'\ 11''\ \tan = +\ 0{\cdot}3083595$$

Consequently $65^\circ.\ 50'.\ 20'' - 63^\circ.\ 49'.\ 11'' = 2^\circ.\ 1'.\ 9''$ will be the *azimuth*.

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* It should have been stated at the bottom of page 88 that, when ψ is greater than Δ, the *difference* of the segments will be equal to A, and the *sum* of them equal to V. See the *Errata*.

		logarithms.
$\Delta = 1^\circ\ 38'\ 0''$	sin = +	8·4548934
$P = 48\ 45\ 0$	sin = +	9·8761253
		8·3310187
$A = 2\ 1\ 11$	sin = +	8·5470791
$Z = 37\ 26\ 55$	sin =	9·7839396

and 90° − *37°. 26′. 55″ = 52°. 33′. 5″* will be the *altitude* at that hour angle.

PROBLEM XV. To compute the right ascension and declination of a heavenly body, from its longitude and latitude: and *vice versa.*

In the case of the *sun*, this problem is solved by means of Formula XII in page 86: and in case of the *moon* or a *star*, we must have recourse to Formula XIII in page 87. In all these cases the obliquity of the ecliptic is presumed to be known.

Example 1. The sun's longitude on Oct. 7, 1825, was 193°. 54′. 39″, and the apparent obliquity of the ecliptic 23°. 27′. 43″: what were his right ascension and declination?

		logarithms.
$\odot = 193\ 54\ 39$	tan = +	9·3938833
$\omega = 23\ 27\ 43$	cos = +	9·9625231
$Æ = \begin{cases} 192\ 48\ 1 \\ 12^h.\ 51^m.\ 12^s{,}07 \end{cases}$	tan = +	9·3564064

$\odot = 193^\circ\ 54'\ 39''$	sin = −	9·3809554
$\omega = 23\ 27\ 43$	sin = +	9·6000357
$D = -5\ 29\ 34$	sin = −	8·9809911

Example 2. On the same day, the moon's longitude at noon was 131°. 46′. 33″, and her latitude − 4°. 19′. 8″: what were the right ascension and declination?

					logarithms.
$L = +$	$131°\ 46'\ 33''$			$\sin = +$	9·8725974
$l = -$	4 19 8			$\cot = -$	1·1219270
$a = -$	5 46 57			$\cot = -$	0·9945244
$\omega = +$	23 27 43				
$(a+\omega) = +$	17 40 46			$\cos = +$	9·9789883
$L = +$	131 46 33			$\tan = -$	0·0489810
				$-$	0·0279693
$a = -$	5 46 57			$\cos = +$	9·9977845
Æ =	{ 46 59 22			$\tan = -$	0·0301848
	{ 133 0 38				
Æ $= +$	133 0 38			$\sin = +$	9·8640528
$(a + \omega) = +$	17 40 46			$\tan = +$	9·5034440
$D = +$	13 7 12			$\tan = +$	9·3674968

The right ascension is always in the same quadrant as the longitude: and the rule of the signs will indicate whether the declination be north or south.

Problem XVI. To determine the height of mountains by means of the barometer.

For the determination of this problem, observations of the barometer and thermometer must be made at the *foot* of the mountain, at the same time that corresponding observations are made at the *top* of the same. The *difference* in the height of the stations of the two observers may then be determined by means of Table XXXVI in page 183. agreeably to the rule there given.

Example. M. Humboldt made the following observations on the mountain of Guanaxuato in Mexico, in latitude 21°: viz.

	upper station	lower station
Therm. in open air . . .	$t' = 70 \cdot 4$	$t = 77 \cdot 6$
Therm. to barometer . .	$\tau' = 70 \cdot 4$	$\tau = 77 \cdot 6$
Barometer	$\beta' = 23 \cdot 66$	$\beta = 30 \cdot 05$

what was the difference in the height of the two stations?

By referring to the formula at the bottom of Table XXXVI, the operation will stand thus:

$$\begin{array}{rl} B = & 0 \cdot 00031 \\ \log \beta' = & 1 \cdot 37401 \\ \hline & 1 \cdot 37432 \\ \log \beta = & 1 \cdot 47784 \\ \hline D = & 0 \cdot 10352 \end{array} \qquad \begin{array}{rl} \log = & 9 \cdot 01502 \\ c = & 0 \cdot 00087 \\ A = & 4 \cdot 81940 \\ \hline 6843 \cdot 7 = & 3 \cdot 83529 \end{array}$$

Consequently the difference in the altitudes of the two stations was 6843·7 English feet. This differs from the values given by the tables of M. Oltmanns and M. Biot: but the variation arises from the slight difference in the coefficients employed.

APPENDIX.

January 1829.

APPENDIX.

The preceding work has now been printed about two years, and several copies have, during that interval, been in the hands of various astronomers, both in this country and on the continent. My object in thus distributing them was not only to ascertain what errors might be discovered in the Tables and Formulæ, prior to a more general circulation of the work (as I was anxious to submit it to the public as faultless as possible), but likewise to obtain from those astronomers, who have honoured me with the perusal of it, their opinion as to the propriety or advantage of enlarging it by the addition of other Tables and Formulæ, that might be considered of general use, and as coming within the original intention which I had in view.

The result has been a variety of hints and suggestions for the improvement of the work: some of which are made available by means of the present Appendix; but others must remain for future consideration and adoption, as they could not well be incorporated with their corresponding subjects, in the proper places. The *new* Tables and Formulæ, that have been introduced, are the following.

Table XVI in page 150 was originally given for the equation of equal altitudes of the sun, in order to find the time of *noon* only. This was founded on the supposition that the observations are made (as is usually the case) before noon and after noon of the *same* day. But it frequently happens that these observations are interrupted by the weather: nevertheless, it oftentimes occurs that we may be able to unite a series of observations made on the preceding afternoon, with a series made on the following morning, and thus deduce the time of *midnight;* which in many cases will be found extremely convenient and useful. This Table therefore has been enlarged with that view, by means of the additional pages 153* to 158*. I am indebted to Dr. Tiarks not only for this suggestion, but also for the Table itself, which is printed from his manuscript. This addition has rendered it necessary to reprint the Formula (XVIII) in page 92; and to cancel the leaf containing the old one.

An easy and expeditious mode of comparing the French and English measures, within the ordinary range of the *barometer*, has frequently been found useful and requisite. I have therefore inserted a new table for that purpose; being Table XLIV in pages 194, &c. This has rendered it necessary to reprint not only Table XLIII, but also the Explanation in page 215, and to cancel the former leaves accordingly.

Table IV has been reprinted in consequence of my having inadvertently omitted to include in the computation, the value depending on *sin* 2 ☊. And Table XXXIV has been reprinted in consequence of the detection of several errors, arising principally from the adoption of a false logarithm. The leaves therefore containing these Tables must be cancelled, and the new ones inserted instead. I have considered this the most convenient and expeditious mode of rectifying the error: less troublesome to the reader, and less unsightly to the work itself, than indicating in the list of errata the corrections to be made with the pen.

Amongst the Formulæ I have inserted three new ones. One for computing from the ephemeris the *true* distance of the moon from a star; being XLVII in page 120: and another, being XLVIII in page 119*, for computing the same from observation. This latter is Borda's method of working a *lunar*, as it is technically called: and I believe will be found to be the most simple and correct of all the various modes of solving this useful problem. The Formula XLIX in page 120* contains the rule of the *signs* in the trigonometrical analysis. The introduction of these new Formulæ has rendered it necessary to reprint page 119, and to cancel the old one; as well as to reprint a portion of the *Table of Contents*, in order to accommodate it to these improvements.

I have also taken this opportunity of reprinting the Formula XXXVIII in page 111; as there appear to have been some unaccountable errors made in copying it out for the press.

Such are the Additions which I have thought it necessary to make to the preceding part of the work. I shall now allude to some other points which will more properly form a portion of this Appendix.

New Solar Tables.

In consequence of the singular discordancies between the place of the sun, as computed from the best solar tables, and its true place as actually observed and pointed out by Mr. South in the *Phil. Trans.* for 1827, the attention of various astronomers has been directed to that subject, with a view to a solution of the difficulty. Amongst these, Professor Airy and Professor Bessel have most distinguished themselves by their very laborious and minute examination of all the points that bear on the subject: and the result of this severe and rigid inquiry has led to the proposal and adoption of various corrections in the solar tables, which it is presumed will tend to remove the discrepancies hitherto observed. The following are some of the conclusions drawn from M. Bessel's investigations, as inserted in Schumacher's *Astron. Nach.* No. 133 and 134.

	At mean noon at Greenwich on Jan. 1, 1801.
Mean longitude of the sun	280° 39′ 13″,17
Longitude of the perigee	279 31 9 ,91
Eccentricity	·0167918226
Mass of Venus	$\frac{1}{401847}$
Mass of Mars	$\frac{1}{2680337}$
Sidereal revolution of the sun	. 365.256374417 = 365^d 6^h 9^m 10^s,75
Tropical revolution of the sun	. 365.242220013 = 365 5 48 47 ,81

The principal corrections therefore to be applied to the solar tables of Delambre will arise from the alteration in the epoch of the mean longitude and in the longitude of the perigee: the former of which is increased 2″,65 *, and the latter 65″.

In order to accommodate Table III to these corrections, we must add

$$0^s,177 + y \times 0^s,0084$$

to the respective values there given. Thus the year 1830 will be 2^m 31^s,105 instead of 2^m 30^s,676.

Mr. Airy makes these corrections equal to +5″,061 for the epoch 1821.5, and +46″,3 for the perigee: each measured from the equinoctial point adopted by Mr. Pond in 1826. At the same time he states that the greatest equa-

* The proper quantity is 2″,90: but M. Bessel makes it 2″,65 only, because he proposes to take the constant of aberration 20″,25 instead of 20″,00, as assumed by Delambre.

tion of the centre ought to be diminished by 0″,84; the mass of Venus reduced in the proportion of 9 to 8 nearly; and the mass of Mars in the proportion of 22 to 15 nearly. He considers the irregularity in the motion of the perigee, and of the equation of the centre, as depending on a new expression which he has introduced, involving the longitudes of the Earth and of Venus; the period of which is 240 years. See *Phil. Trans.* for 1828, Part. I. Mr. Airy's corrections for 1829 are given in the Supplement to the Nautical Almanac for that year.

Other new Elements.

Since the preceding pages were printed, M. Bessel has also re-investigated the subject of the *precession of the equinoxes*, and has deduced the following results, which slightly differ from those inserted in page 104.

$$P = 50'',37572 - \&c.$$
$$p = 50\,,21129 + \&c.$$
$$m = 46\,,02824 + \&c.$$
$$n = 20\,,06442 - \&c.$$

It is from these values that Table XXVII was computed: with this exception, that the value of n was erroneously taken by M. Bessel equal to 20″,06175.

According to the latest observations of Professor Struve, with the great Dorpat telescope, he makes the axis of the poles of *Jupiter* to the diameter of the equator, as 35″,538 to 38″,327: whence the compression is = ·0728 or $\frac{1}{13\cdot71}$.

The same distinguished astronomer makes the diameters of *Jupiter's satellites* to be as under: viz.

$$I = 1'',015$$
$$II = 0\,,911$$
$$III = 1\,,488$$
$$IV = 1\,,273$$

Professor Struve has also given us the following measurements of *Saturn's ring*, reduced to the mean distance of the planet: viz.

Outer diameter of the outer ring . . .	=	40″,095
Inner diameter of ditto	=	35 ,289
Outer diameter of the inner ring . . .	=	34 ,475
Inner diameter of ditto	=	26 ,668
Equatorial diameter of Saturn	=	17 ,991

Whence we obtain the

Breadth of the outer ring	2″,403
——— of the space between	0 ,407
——— of the inner ring	3 ,903
Distance of the ring from Saturn	4 ,339
Equatorial radius of Saturn	8 ,995

See the results of M. Struve's measurements of Jupiter and Saturn in Schumacher's *Ast. Nach.* No. 139: which are more correct than those in No. 97.

He makes the *inclination* of the ring to the plane of the ecliptic equal to 28° 5′ 54″.

It has been recently stated by many of the continental astronomers that the body of Saturn is not exactly equidistant from the ring: and some late observations seem to confirm the opinion. The difference however is scarcely perceptible even with the most powerful telescopes.

Aberration of a Planet.

There is a very simple formula, for determining the aberration of a planet, given by Delambre in his *Astronomie*, vol. 3, page 106; which, in consequence of the great improvements recently introduced into the Berlin Ephemeris (to which I shall more particularly allude in the sequel), admits of a very ready and easy solution. Let us make

Δ = the distance of the planet from the earth.

μ = the daily geocentric motion of the planet, expressed in seconds of space, either in Right Ascension or Longitude, Declination or Latitude: *minus* (in the first two cases) when the motion is *retrograde*, and also (in the last two cases) when the motion is towards the *south*.

t = the time which light takes to come from the sun to the earth, expressed in minutes: assumed = 8·263.

then will the aberration (a) be equal to

$$a = -\frac{\mu t}{1440} \times \Delta = -\,{\cdot}0057382 \times \Delta \mu$$

Now, since the logarithm of ·0057382 is equal to 7·7587753, and as the logarithm of Δ is given in the new Berlin Ephemeris, we have only to seek for the logarithm of μ in the tables: and the sum of the three logarithms will give the logarithm of the aberration required.

The logarithm of Δ will also enable us to determine the *parallax of the planet*, since we have only to deduct it from the logarithm of the sun's mean parallax (=0·9333658), and the remainder will be the logarithm of the parallax of the planet.

And as the *semidiameter* is always in a constant ratio to the parallax, we may likewise deduce this quantity by means of a similar constant for each planet.

Augmentation of the Moon's semidiameter.

Mr. Henderson of Edinburgh, a gentleman well known for his zeal in the cause of science, and for his excellent judgment and skill in the various branches of astronomy, has pointed out an error in Formula XXVIII, page 101, into which I have fallen, by following too closely the steps of Delambre *. It appears that, in the first Formula here alluded to, the quantity $+ \frac{s^3}{2}(\cdot000017767)^2$ has been omitted by almost every writer on the subject, except it be Mayer. Its value varies from 0″,11 to 0″,16 according to the diameter of the moon. It is indeed alluded to by Delambre in his explanation of Table XLIV of *Bürg's* Lunar Tables; and he remarks that the semidiameter of the moon, seen at the horizon, will be greater by this quantity than when seen from the centre of the earth. But the Table is formed without this assumption: so that where great accuracy is required we must add this quantity to the tabular values. There can be no good reason why this expression should in future be neglected. The reader will supply the omission in Table XXV of the present collection, by annexing the proper constant to the head of each column.

Circum-meridian observations of the sun.

In Formula XIX, page 93, for the *Reduction to the meridian*, it is well known that when the sun is the object observed we must take into account the change of declination during the interval of the observations. This may sometimes prove troublesome; but, by the help of the following short table, a great deal of the labour may be saved. This table merely shows the change in declination, in one minute of time, corresponding to any given *daily* change. M. Cerquero prefers taking the sun's declination for the

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* I am also indebted to Mr. Henderson for the detection of some other errors; and for his numerous and valuable suggestions for the improvement of the work.

mean of the times of the observation: which, although it appears more simple, is in fact the same thing.

Daily change	Change in 1 min.	Daily change	Change in 1 min.	Daily change	Change in 1 min.	Daily change	Change in 1 min.
′	″	′	″	′	″	′	″
1	0,042	7	0.292	13	0,542	19	0,792
2	·083	8	·333	14	·583	20	.833
3	·125	9	·375	15	·625	21	·875
4	·167	10	·417	16	·667	22	·917
5	·208	11	·458	17	·708	23	·958
6	·250	12	·500	18	·750	24	1·000

M. Gauss has suggested another mode. It is well known that the sun does not always attain its greatest altitude when exactly on the meridian, but (with the exception of the two days of the solstices) either a little before or a little after that moment of time, according to its position in the ecliptic. In fact, it will vary in different countries: and the analytical expression for the distance (in seconds of time) from the true meridian will be

$$\Delta = \delta \times 15{\cdot}27924 \times \frac{\sin(L - D)}{\cos L . \cos D}$$

the characters denoting the same quantities as in Formula XIX, page 93, above alluded to *. Now M. Gauss proposes that the observations should be reduced, not to the true meridian, but to the time of the sun's greatest altitude. The value of Δ has therefore been computed and

* See *Bohnenberger's Anleitung zur geographiscen Ortsbestimmung*, page 275; where this subject is treated with all the accuracy and minuteness which distinguishes that excellent work.

published by M. Schumacher for every tenth day of the year, and for every degree of latitude from 36° to 60° both inclusive. Consequently all that is required in this plan is to apply the value of Δ to the time of the sun's meridian passage, and to reduce the observations to the time shown by the sum of these values, instead of the time of the meridional passage. This method is certainly very ingenious and simple, but it appears to be slightly incorrect, as Dr. Tiarks has shown in the *Phil. Mag.* for September 1828, page 182.

*Nautical Almanac.**

There is probably nothing that contributes so much to the progress and improvement of astronomy,—nothing that tends in so great a degree to keep alive a spirit of enquiry and research in this science,—as the annual publication of a correct and comprehensive Ephemeris; containing, in a concise and tabulated form, all the motions of all the heavenly bodies, computed from the best elements that can be obtained. It was this feeling which induced the Government of this country in 1765 to establish the Nautical Almanac, and to cause it to be published under its own authority: the good effects of a similarly authorized national Ephemeris having been experienced in several of the neighbouring states. The work was consequently placed, by a special act of parliament, under the direction of the Board of Longitude, then recently established, and the first volume appeared in the year 1767.

In the infancy of the science (for the *present system* of astronomy is of no very ancient date) the public were satisfied

* Some of the points alluded to in this article were arranged in the shape of a letter, and inserted in the *Times* newspaper of Nov. 19, 1828.

with the meagre details thus given in the Nautical Almanac: a work which was perhaps sufficiently well adapted to the wants of astronomers at the time of its establishment, but which falls far short of what is *now* required. New discoveries and new modes of observing—a more refined analysis and more improved instruments—have given rise to new wants and to new claims: so that what might be well suited to the last century is no longer tolerable in the present one. Many of the states on the continent have long seen this, and have improved their national ephemeris accordingly: and this improvement has most unaccountably been in the inverse ratio of their interest in navigation and nautical astronomy, which it is said these publications were originally and principally intended to promote *. Indeed, it has been pertinaciously maintained by some persons, that the Nautical Almanac was originally established *and is now continued* for this *sole* purpose: whilst others consider that it ought to partake more fully of (what its title imports) an *astronomical* ephemeris. But it will not be difficult to show that it is not adapted either for one or the other (at least to that extent which the present state either of navigation or astronomy demands): and that it is a constant charge upon the nation, without any equivalent advantage to science.

That it is not intended *solely* for the navigator is evident from an inspection of its contents. For, of what use are the eclipses of Jupiter's satellites to the sailor? How is he benefited by knowing the places of Mercury, and the Georgium Sidus; planets which are seldom seen even on shore? What does he want of the apparent places of *sixty* principal stars? And as to the positions of the Sun and

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* Witness the ephemerides of Coimbra, Berlin, and (*proh pudor!*) Milan.

Moon, if they are required merely to work out a *lunar* (as it is technically called), or to determine the latitude, a much more concise form might be adopted. So that the most ample demands of the *mere navigator* (from the humble skipper to the noble admiral) might be supplied from the Nautical Almanac in a sixpenny pamphlet.

But if it is intended *solely* for navigation, or if its object be the promotion of navigation *at all*, surely it ought to contain *all* the requisite facilities for determining the problems necessary at sea. Why then are the distances of the moon from the *planets* omitted? and why do we not see a list of all the *occultations* that will occur? The ephemeris of Coimbra has, for many years past, contained the lunar distances of the planets and many of the occultations: and the *little* state of Denmark (so *great* however in works of science), well knowing the importance of the subject, annually publishes a similar list of such distances, together with the position of the planets for every day in the year, at the Hydrographer's office at Copenhagen. As to occultations of the fixed stars by the moon, it has now been long discovered that they may be observed with a common telescope *at sea* (even from the unsteady deck of a vessel), down to the sixth magnitude; as may be easily verified by any one that will take the pains to look out for them. They are the most perfect of all lunar distances: and it is sufficiently well known that they afford the *best means* of determining the longitude.

Again, if the Nautical Almanac really has the advancement of navigation in view, why does it not contain a more enlarged ephemeris of the places of the four principal planets (Venus, Mars, Jupiter, and Saturn) for every day in the year, instead of the almost useless summary which it now exhibits: so that their accurate positions may be as ready for immediate use, when required, as those of the Sun and Moon? For these stars (and particularly Venus) can frequently be seen in broad daylight, and their altitudes consequently taken on the meridian when unfavourable circumstances prevent an observation of the sun or moon: an instance of which lately occurred in one of the American packets, where an observation of Venus *on the meridian* (soon after the passage of the sun, which was unfortunately obscured at the time) was the means of determining the latitude of the ship. An ephemeris, pretending to be for the use of nautical men, should contain *every thing* that can at all diminish the labour of computation at sea, or that will at all tend to help an enlightened sailor in pursuing his adventurous and doubtful path across the trackless ocean. He is frequently placed in situations of great difficulty, where *every* means, that can be made available for relieving him, ought to be *ready at hand.* It is but a small return we make to him for the perils he encounters.

But, if the Nautical Almanac does not contain all that is requisite for the *navigator*, how much less does it supply the wants of the *astronomer;* and how vain are its pretensions to the title of an *Astronomical Ephemeris.* It will perhaps be scarcely credible to future ages, that for a period of thirty years after the discovery of *four new planets* in our system, *not the least notice* whatever was taken of any one of them, in a work pretending to show the motions of the celestial bodies: so that no astronomer could ever tell in what part of the heavens to look for them, or make any observations to perfect their theory. And as to any knowledge he could obtain of them, they might as well be blotted out of the creation. It has been said, in excuse, that there are no accurate tables of their motions: still, imperfect tables are better than none at all, and it is so much the more necessary to get them observed. Besides, I much doubt whether the tables of the moon and many of the planets were more correct, at the commencement of the Nautical Almanac, than the tables of the minor planets are at the present day. And it is fortunate for us that the same paltry and miserable excuse was not allowed to succeed in those times *. It is well known that it does not contain all, *nor nearly all*, the information that astronomers now require: and what it pretends to give, it does not state in that simple and correct manner which their uses demand. It ought to contain the places of *all* the planets, and notices of *all* such phenomena as the interests of astronomy require should be generally observed: and above all should be discarded altogether the absurd and useless mode of adapting the values to *apparent time;* which seems, indeed, to be retained for no other purpose than to give the practical astronomer the trouble of converting them back again into *mean solar time* before he can make use of them. But, these and various other improvements and additions have been so fully pointed out and so frequently insisted on by others as well as by myself, that it would be useless to repeat the subject here †. It is well known that about eleven or twelve years ago, the work had got into such bad repute that Mr. Croker is represented to have stated in the House of Commons that "it was become a *bye-word*

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* Dr. Maskelyne did not reason in this manner when the Georgium Sidus was discovered. The place of *that* planet is regularly given.

† See Mr. South's *Practical Observations on the Nautical Almanac;* and my *Remarks on the present defective state of the Nautical Almanac:* both published in 1822. This latter pamphlet was written as an *Answer* to some Remarks published in Brande's *Journal of Science*, wherein the writer attempted to vindicate the present state of the Nautical Almanac.

amongst the literati of Europe:" and a new (and rather expensive) Board of Longitude was consequently established, under whose auspices and direction the Nautical Almanac was in future to appear: it being anticipated that this additional expense would be attended with equivalent advantages. But the event has disappointed all our expectations: and Mr. Croker, after a trial of ten years, finding his new Board of Longitude of little or no use in promoting the objects for which it was instituted, brought in a bill (during the last session of Parliament) for its dissolution: and it now ceases to exist.

During the period, however, that the Nautical Almanac was under the direction of the late Board of Longitude, the defective state of that national work was frequently brought before them, not only by private individuals, but also by the Council of the Royal Society; and the proposed improvements were supported by the active and scientific members of that Board. But, unfortunately, whenever that learned body was assembled together to discuss these matters, some invisible and *Bœotian* influence was sure to paralyze all their proceedings; so that little or no permanent benefit has yet resulted from their efforts. And thus it ever will be with so heterogeneous a body as that which composed this assemblage of persons. The dissolution of *such* a Board was "devoutly to be wished."

It is true that some slight attempts (like angels' visits, "short and far between") were occasionally made at improvement; as it was impossible for the Board to shut their ears altogether to the universal complaints that were made. And it is probable, that to the active interference of some of the more scientific members we are to attribute the two recent SUPPLEMENTS to the Nautical Almanac, for 1828 and 1829; the Almanacs for those years having been previously published. It is now rumoured, however, that since the abolition of the Board, these Supplements are to be discontinued: and there seems to be some ground for the suspicion, when we see the Nautical Almanacs for 1830 and 1831 subsequently appear without their expected appendages: for, surely it never could be intended that the Supplement should be constantly two years in arrear of the original work. The plain course would have been (whether any further improvements be intended, or not) to have *incorporated* them together, for the convenience of those who consult them. But, *much more* than what has been here done is required at the present day. For, so rapid have been the strides, within these few years, both in practical and theoretical astronomy, that nothing short of a remodelling of the whole work, adapted to the present improved state of the science, will satisfy the increased and increasing demands of the modern astronomer.

Whatever may be the future intentions of the Government, however, it must be evident to the most common observer that the alterations and improvements here suggested would not be altogether for the benefit of astronomy alone, since they bear very powerfully on navigation also. Many voyages of discovery and scientific research have lately been made, and many are still in a state of progress, conducted by men of high scientific attainments, who are an honour to the country that employs them, and who have the proud and enviable satisfaction of knowing that, after having triumphed in war, they can also serve her in the no less brilliant walks of peace. In fact, there probably never was a period when the Royal Navy of Great Britain could boast of so many officers so devoted to science, and so proud of promoting its objects. Many of these, *I know*, lament the present defective state of the Nautical Almanac, and the *necessity* of referring to *foreign* ephemerides for what *ought* to be contained in our own *. Surely it is of some importance to foster and keep alive this laudable spirit in our navy, and to afford them every means for multiplying observations, which in many cases may be absolutely necessary for the safety of their vessels; and which, at all events, must inevitably tend to the promotion not only of astronomy but also of geography, hydrography, and navigation.

Besides, it frequently happens that, during these expeditions, a temporary landing is made at places either wholly uninhabited, or whose positions are but very badly determined. It is therefore desirable that every facility should be given for obtaining the longitude and latitude of such places in the most expeditious and correct manner; otherwise one great object of the voyage is lost: and the more these means are multiplied, the more likely are we to obtain a favourable result to our enquiries.

This subject in fact is of so much importance in a national point of view, whether we consider it in its relation to the safety of our navy or the scientific honour of the country, that I trust the subject will attract the particular and serious attention of the Government. Indeed, it might be a fit subject of enquiry, in either House of Parliament,

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* Witness the remarkable fact mentioned by Mr. South, that Capt. Smyth (whilst employed by the Admiralty in surveying the coasts in the Mediterranean) was obliged to refer to *foreign* ephemerides for information which was *not to be found* in the Nautical Almanac. To which may be added, that when the Expedition to the North Pole sailed in 1824, a Society furnished Capt. Parry with their copy of the *moon-culminating stars* (published by the Danish Government) for the purpose of making observations for more effectually determining the longitude of such places as he might visit in his adventurous voyage.

whether the funds appropriated (from the *public purse*) towards the formation and superintendance of the Nautical Almanac might not be made more effective than they now are: whether a much better work, at a much less expense, might not be produced: and whether in fact it might not even be made a source of revenue. The annual sale of the Nautical Almanac is about 7000: but the combined sale of all the other almanacs is nearly a MILLION copies: and many of these (*risum teneatis*) are not much inferior to the *present state* of the Nautical Almanac. It is, I fear, too generally supposed that those popular works are composed by men who live in garrets, and who pander to the ridiculous follies and absurd prejudices of the vulgar. This however is not the fact. The superintendants of some of those almanacs are men of high character and superior attainments; who are not only desirous of improving the works placed under their direction, by introducing therein a variety of new scientific and astronomical subjects, but also of removing the rubbish which annually disfigures some of their volumes. But they have the *insuperable* prejudices of the vulgar to encounter: and after an ineffectual attempt at such a reformation, they have been obliged to abandon it for the present, or, at least, to satisfy themselves with a *gradual* improvement *. Nevertheless the competition between these annual productions is so great, that each is

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* About nine or ten years ago the editors of Moore's Almanac began this attempt by discarding the monthly column containing the moon's supposed influence on the several members of the human body; and, as an experiment, to ascertain the feeling of the public on the occasion, printed at first only one hundred thousand copies. But the omission was soon detected, and nearly the whole edition was returned on their hands, and they were obliged to reprint the favourite column. The

striving for improvement, and I believe with as much effect as the disadvantages, under which they labour, will allow. Even in their present state, however, I consider some of these works (and particularly *White's Ephemeris*) as superior, in many respects, to the Nautical Almanac: since they contain much information which our boasted national work does not (and *will not*) afford: and in some of them it is, moreover, proposed to insert, in the next and subsequent volumes, the places of the four new planets ! ! !

I may be told, perhaps, that all these works are indebted to the Nautical Almanac for a *great portion* of the astronomical information they contain. This may be partly true: but it is also equally true that they contain much that *is not* (but *ought to be*) in the Nautical Almanac. And I am assured that the superintendant of *White's Ephemeris* has long laid it down as a practical maxim never to take for granted any thing which he meets with in the Nautical Almanac. His first step is to collate most cautiously the Nautical Almanac, with the Connaissance des tems and the Berlin and Coimbra ephemerides, as well as with other similar continental publications when he can procure them: and, in every case of serious discrepancy to institute an independent computation. This is the proper mode of proceeding, in order to insure accuracy: and I think there is no doubt but that the Stationers' Company (or some other respectable body) would also gladly undertake the computation and printing of the Nautical Almanac (provided the copyright were secured to them) not only *free of every expense*

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total *annual* sale of this work by the Stationers' Company is nearly *half a million* copies; besides pirated editions of about one hundred thousand copies; and two reprints of it in France,—one at Boulogne and the other at Paris ! ! !

to Government, but even subject to the stamp duty, from which it is at present exempt. Let it but fairly enter into *equal* competition with other productions of a similar nature, and we should in all probability have a work of a much superior kind to the present one, the number of its copies would be increased, and instead of being an unnecessary and useless expense (and a *disgrace* to the nation), it might thus become a source of annual income, and a means of improving the science of astronomy. If the Nautical Almanac were made what it *ought* to be, and such as the situation of this country demands, there is no doubt but that its sale might be considerably increased. It is known that the American booksellers (who *reprint* that work in the United States) correspond with the German astronomers for the supply of additional matter, to be inserted in the annual volumes. And what is the consequence? *One* bookseller alone (and there are *several* who reprint the work) sells upwards of twelve thousand copies! I believe the *total* sale of the Nautical Almanac, in *this* country, never amounted to seven thousand copies.

Hitherto I have said nothing of the *accuracy* of the Nautical Almanac; and, as far as *arithmetical calculation* goes, I am ready to accede to the computers their due meed of praise for diligence and attention. But, there is an attention of a much superior kind required, in order "to attain "the *highest possible* degree of accuracy *," (and belonging rather to the *directors* of the Nautical Almanac than to the *computers*,) which I am not so readily disposed to grant. I allude not only to the *choice of the Tables* from which those computations are made, and to the *corrections*

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* See the annual Advertisement prefixed to all the late Nautical Almanacs.

which ought from time to time to be applied, even to the best tables, in order to adapt them to the improving state of the science; but also to the *communication of that information* to those who consult the work. This plan was rigidly pursued by the late Dr. Maskelyne, when the Nautical Almanac was under his superintendence; the *best tables* were constantly sought after and adopted; and *additional equations* were supplied whenever subsequent investigations warranted such a measure. A minute account of the changes thus made, and incorporated with the computations, *was always given* in the Preface to each almanac. Such was the conduct of the *practical* superintendant; who well knew the *use of instruments*, and the true value and application of *correct and convenient tables;* and who employed his splendid abilities in aiding the enquiries not only of the astronomer, but also of the *seaman*, in *every branch* of the science: bearing in mind the well known apophthegm of Bacon, that "knowledge is power," and that it furnishes us with an increasing fund, on which we may at any time draw, in case of need. And who, moreover, *was not paid* for this branch of his labours.

Let us now see whether we bettered our condition when it was placed in other hands, and under a new direction: and subject, moreover, to *a charge of £300 a-year for its superintendance* *. I fear not:—for, at the very threshold

* I wish it to be understood that I have no *personal* allusions in view, in any part of these remarks: it is the *system* only which I attack, without reference to the individuals, who stand too high, both in character and abilities, to be affected by any observations that I may make. By the act of Parliament, constituting the late Board of Longitude, it was declared to be "highly expedient to the *interests of navigation*, and the "*honour of the country*, that the Nautical Almanac should be accurately "computed, compared, and published," and "that some person of com-

of our enquiries we find all hopes of improvement withered in the bud; since we are gravely told, year after year, in the Advertisement prefixed to the Nautical Almanac, that as far as the existing tables of the "sun and moon have "been examined, they appear to be already *sufficiently ac-* "*curate* for *every* purpose of practical astronomy." Why, so far from this being the case, there is *not one* purpose of practical astronomy for which the tables of the sun (setting aside those of the moon) are "sufficiently accurate:" and if the directors of the Nautical Almanac had ever condescended to look through a transit instrument (even of ordinary construction), they would readily have been convinced of the fact; and would soon have learned that they are not "sufficiently accurate" even to regulate a common chronometer. Indeed, the truth itself is tacitly acknowledged in the Supplement for 1829, where they have inserted Professor Airy's *table of corrections* of the solar tables for every fifth day of the year, from the recent investigations of that profound mathematician: thus virtually contradicting the bold assertion which they had so incautiously and so repeatedly made.

As to the tables of the *planetary* motions which have been

"petent skill and ability should be nominated by the Admiralty for su- "perintending, under the direction of the said Board *in general*, and "the Astronomer Royal *in particular*, the due and correct publication of "the Nautical Almanac." Under this *triple* responsibility the work has hitherto been, year after year, smuggled into the world, like an illegitimate child, without much regard either to the *interests of navigation* or the *honour of the country;* and each party has consequently been ashamed to own their offspring. It is now rumoured, however, that a new arrangement is about to be made: and, if so, it may be a fit subject of enquiry in the House of Commons whether that arrangement is likely to tend to a better result than the last.

employed, they are said to be "*chiefly* those which are "printed in the third volume of Professor Vince's Astro-"nomy, with the omission only of *some equations* which do "not materially affect the results." But, why this ambiguity and mystery? Do not the directors know precisely *which* planets have been computed from Vince's tables, and which *not?* And what *equations* have been *retained* and which *rejected?* And why are we not to be made acquainted with the fact? It is well known that M. Bouvard has published tables of Jupiter, Saturn, and Uranus, much more recent than those of Vince: and yet no allusion whatever is made to these tables; nor is it known whether any of them have ever been placed in the hands of the computers. Besides, who can doubt the propriety and even the *necessity* of stating *distinctly* the tables and authorities depended on in *every* calculation in the Nautical Almanac: and that, *not loosely*, but with *express notice* of any *equations* omitted in their use, and the *corrections* made in them. Not to do this is not only to deprive ourselves of the valuable consequences which could not but result from a *repetition and verification of the calculations* by other persons, who (from a peculiar turn of mind) delight in such computations, but likewise to *destroy all confidence* which such unreserved publicity is calculated to inspire. As to the positions of the *planets* however, as inserted in the Nautical Almanac, they are given in such a rough manner (to the nearest minute in time only) and for such long intervals, that "for any pur-"pose of practical astronomy," they might just as well have been computed from the Tables of Halley, or even the Rudolphine Tables of the sixteenth century.

The public indeed were, at one time, led to imagine that the *lunar distances from the planets* were about to be inserted in the Nautical Almanac: as they were *repeatedly* told in the Preface to that work, that "whether any advantage "would be gained from the insertion of the moon's distance "from Jupiter, must depend on the precision of the tables "of that planet [*whose* tables? Vince's or Bouvard's?]: a "point which is *expected* to be *very shortly* determined from "*the most accurate observations.*" But, this *expectation*, like that of the Mountain in labour, terminated in a much more ridiculous way. For after amusing the public, *for seven years*, with this idle tale, the printer appears to have been ordered to erase the paragraph silently from the Advertisement; and thus vanished all at once every trace of the "short expectancy," the "accurate observations," and the "precision of the tables:" so that even the *mouse* did not appear to give a colour to this septennial parturition *.

It perhaps will hardly be credited that M. Schumacher, a few years ago, offered his *lunar distances from the planets* (which are published by the Danish Government) to the late Board of Longitude, for circulation with the Nautical Almanac: proposing to put the titles of the columns in English, and simply requiring that the Board should pay for the paper and printing. *But they declined the offer!!!*

If I were disposed to swell this list of complaints, I might

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* The Board of Admiralty, like the late Board of Longitude (for the Supplement for 1829 has come out under *their* authority), still

"Keep the word of promise to our ear,
"And break it in our hope:"

for, in the Advertisement to that volume we find that the parallaxes and logarithmic distances of the principal planets are given, because "the "*navigator* may have occasion to employ them in the determination of "his longitude by *their observed distances from the moon*, should that me- "thod be found sufficiently exact to be relied on." That is, these values are given *now*, in order to be used at the end of another official gestation of seven years.

enter into a number of minute inaccuracies that ought not to appear in a work of this kind, on which so much money is annually *wasted:* such as marking *invisible* occultations and eclipses, as if they were *visible*, and *vice versâ:*—giving the mean places of the stars, in one part of the work, *different* from what they are in another part:—inserting new tables without any explanation of their use and application:—omitting in leap year the 29th of February in the apparent places of the stars:—stating the mean places of the stars to be "deduced from the *latest* observations that "have been made up to the present time," although the very next line informs us that "from some late observations "[i. e. later than the latest] there is reason to conclude "that the right ascensions should be diminished one-tenth "of a second:"—with other things of a like kind; which, although they may appear trifling to the general reader, or to the "mere navigator," show great inattention to the arrangement of the work, and destroy that confidence and authority to which it *ought* to be entitled amongst astronomers, as published under the direction and sanction of the Government.

But I shall not pursue any further this tiresome and disgusting appeal:—an appeal which has been so repeatedly urged by others, as well as by myself, to so little purpose. Long and *loud* have been the complaints; and enough has been said to show the necessity of a reformation, if there existed a disposition to adopt it. But this country, once so distinguished in the science of astronomy, so celebrated for its artists and the superiority of their instruments, and at the same time so jealous of the productions and claims of others, now views with apathy (as far, at least, as the Government is concerned) the rapid advances that are making in the neighbouring states; and which have already left us *far behind* in *every* branch of the science, and in *every* art connected with its practical application. We see around us, on the continent, *unrivalled* artists in horology, in optics (including the art of manufacturing the most beautiful glass for optical purposes), and in the making and dividing of every kind of astronomical and geodesical instruments, and whose works are sought for in every observatory: we observe the most profound researches carrying on, both in theory and practice: we see the greatest activity in all their observatories: and, though last not least, they are now supplied with an Ephemeris *which ought to put England to shame.* These facts are too notorious to be denied: but, if those in power cannot feel for the honour of the country, or will not exert themselves for its support, little can be expected from those in a more humble station: and the evil must be left to find its own remedy.

It has been justly stated by a distinguished and much lamented philosopher of this country, that "there are some "sounds inaudible to certain ears:" and so it would seem in the present instance. There *is* a sound however, though not so *loud*, that *will* ultimately be heard,—"the still small voice" of time and reason (in common parlance yclept *the march of intellect*),—which, sooner or later, must and will bring about the reformation so repeatedly insisted on, and so anxiously desired by every friend of science. And with this "sure and certain hope" (for, come it must, at last) I now proceed to a more pleasing subject.

New Astronomical Ephemeris.

Many of my readers are probably aware that, within these few months, there has appeared at Berlin one of the most useful publications in practical astronomy that has ever yet been formed. It is an astronomical ephemeris arranged in an entirely new manner, computed on an entirely new principle, and every way adapted to the present advanced state of this important science.

It will be recollected that, for the last fifty years, the celebrated Professor Bode conducted the Berlin ephemeris, under the title of the *Astronomische Jahrbuch*, with great credit to himself, and with great advantage to astronomy. This work, inferior to none on the subject, contained annually a vast variety of valuable information, which would probably have perished, had it not been for the interest and zeal which Bode took in every thing regarding astronomy Yet, notwithstanding the rapid strides which the science has made on the continent, little or no alteration was made in the usual columns of this publication during Bode's lifetime: but, on his death, M. Encke, who has been appointed to succeed him, determined on re-modelling the work altogether, and on adapting it to the increased and increasing demands of the astronomer. With this view he has abandoned the plan of publishing the voluminous Appendix thereto, which has generally been filled with matter that more properly belongs to a periodical journal; and which will now be transferred to the pages of Professor Schumacher's very valuable *Astronomische Nachrichten*: whilst the monthly columns of the Ephemeris will be consequently enlarged without any additional expense to the reader. On the other hand, Professor Schumacher will in future discontinue his annual *Hülfstafeln*; which will henceforth form part of M. Encke's work above alluded to. This mutual exchange and arrangement, which commences with the present volume for 1830, will be highly advantageous and convenient to the practical astronomer; who will thus have, in one volume, all the daily information he requires for the use of his observatory.

One principal and great improvement in this ephemeris is the introduction of *mean solar* time into *all* the computations, instead of *apparent* time, as hitherto adopted in every other ephemeris, except it be that of Coimbra. The articles in the ephemeris are also *much better arranged:* and the computations are more extensive, and *carried to a much greater degree of minuteness*, than has hitherto been done in any other similar work. Many *new* subjects are likewise introduced: such as *the places of the four new planets*—a list of *occultations*—a list of *moon-culminating stars*—and a variety of other important and convenient tables which are of constant use to the practical astronomer. This however will be more fully seen, by a more minute description of the work in question, which is as follows.

The ephemeris of the *Sun* is, for each month, divided into two pages; one of which is devoted to *apparent* noon, and the other to *mean* noon. The former page contains, besides the days of the month and the days of the week, the mean time (to *two* places of decimals in the seconds), the right ascension of the sun (to *two* places of decimals), and its declination (to *one* place of decimals), together with the equation of time (to *two* places of decimals), and the logarithm of the double daily variation in the declination,—a quantity extremely useful in determining the time from altitudes of the sun. The latter page contains the right ascension of the meridian (to *two* places of decimals), the longitude of the sun (to *one* place of decimals), its latitude (to *two* places of decimals), the logarithm of the radius vector (to *seven* places of decimals), and the semi-diameter of the sun (to *two* places of decimals); together with, not only the days of the month, but likewise the number of days elapsed from the commencement of the year.

The ephemeris of the *Moon* is also divided into two parts; but as the computations are made for every twelve hours, each month occupies four pages. These contain the moon's longitude, latitude, right-ascension, declination, parallax, and semi-diameter, (each to *one* place of decimals,) for *mean* noon, and mean midnight. There is also given the mean time of the moon's upper and *lower* culmination, (to the *tenth* of a minute in time), as well as her right ascension and declination (to the tenth of a minute in space); together with the time of her rising and setting, the time of her changes, and the time when she is in perigee or apogee.

At the end of this joint ephemeris of the sun and moon, there is given for every *tenth* day of the year, the apparent obliquity of the ecliptic, the parallax of the sun, the aberration, and the equation of the equinoctial points (each to *two* places of decimals); together with the place of the moon's node (to the nearest *tenth* of a minute).

Then follows an ephemeris of each of the *Planets*, including the four newly discovered ones. The places of Mercury and Venus are computed for mean time at *noon* for every *second* day, and the remaining planets for mean time at *midnight* for every *fourth* day of the year. The columns contain the heliocentric longitude and latitude of the seven principal planets (to *one* place of decimals in the seconds), the geocentric right ascension (to *two* places of decimals), and the geocentric declination (to *one* place of decimals); the radius vector, and the logarithm of the distance from the earth (each to *seven* places of decimals); together with the time of their rising, setting, and passing the meridian. The computations of the four newly discovered planets are not so minute, except at the time of their opposition; for which period a separate ephemeris is given of the position of the planet for *every day*.

We have next an ephemeris of the time of the eclipses of *Jupiter's satellites* (to *one* place of decimals); to which is subjoined (for *each* satellite) a table for computing with the greatest accuracy, not only the configurations at any moment, but also the position of the satellite with respect to Jupiter at the time of its immersion or emersion. At the end of these tables, we are presented with another ephemeris (computed for every fortieth day) of the apparent position and magnitude of Saturn's ring.

After this comes a table of the mean places (for 1830) of 45 principal stars; the right ascensions to *three* places of decimals, and the declinations to *two* places of decimals. From these are computed and given for every tenth day of the year, the apparent places of the same stars (to *two* places of decimals), with their differences. And we have also the apparent places, for *every* day in the year, of α and δ *Ursæ Minoris.* To the whole of which are annexed formulæ for determining the amount of the diurnal aberration. Following these is given a table of the constants A, B, C, D, for every tenth day of the year, for the purpose of determining the apparent places of any other stars. It should however be remarked, that these letters do not indicate precisely the same quantities as are so designated in the catalogue of the Astronomical Society: and it should also be noted, that the numbers are adapted to *sidereal* time. There is however another table subjoined, for the use of those who are disposed to adopt *mean solar* time in these computations.

Next follows a particular account of all the solar and lunar *Eclipses* that will happen in the course of the year; together with all the necessary elements for computing them. This is followed by three pages of the *principal phænomena* of the planets: such as the time of their perigee or apogee, their perihelion or aphelion, their greatest elongation, their greatest latitude, their conjunction and opposition, their passing the nodes, their greatest brilliancy, their proximity to the moon, and occultation thereby, &c.

Then follows a list of *moon-culminating stars*, occupying seventeen pages; and (which is equally valuable) a list of the *occultations* of all the stars down to the 7th magnitude inclusive, that will take place in the course of the year; wherein the mean time of the immersion and emersion of the star (to the nearest tenth of a minute) is given, as well as the angle from the vertex of the moon at which the phænomenon will take place. To this list is subjoined some auxiliary tables for computing the occultation more minutely, if required.

To the whole is annexed an Appendix, giving an account of the mode in which all the computations are made, and the tables from which they are derived. By this excellent plan, the observer can at any time verify any of the calculations, and detect any error which he may have cause to suspect. The *names of the computers* also are given, which must materially tend to insure the accuracy of the work.

Such is the substance of the publication here alluded to, which has just reached this country, and which does so much credit to its distinguished conductor. It should be hailed as the harbinger of a general improvement in the mode of arranging and forming the ephemerides of different nations. And although it is mortifying to reflect that this country cannot (or will not) maintain its pre-eminence in these and other scientific subjects, yet we should be grateful for information wherever it can be found, and hope that we may be able eventually to emulate the splendid example which has thus been set us. M. Encke, disdaining the trammels of former and less enlightened times, and relying on his own excellent judgement and abilities, has nobly and boldly struck out a new path for himself, which there can be no doubt will soon be followed by every nation pretending to encourage the science of astronomy.

Indeed the present work may be considered as forming a new æra in the science; and something might now be done to place astronomy (as it ought to be) on a better footing in this country. And since œconomy is the order of the day, and has in fact been publicly declared to be one of the causes of the dissolution of the late Board of Longitude, why should we not follow up that system, by getting rid also of the whole of the expense incurred in forming the *Nautical Almanac*. This work is sold in the shops at *five shillings;* and it is said that the publisher is allowed sixpence for every copy that is sold. This, I am aware, is a less profit than usual for publications in general: but here the publisher runs no risk, and incurs no expense, or even outlay of money: I believe he does not even advertise the work. Let us suppose, however, that he is allowed a shilling for every copy: there will then remain four shillings for each copy, as a return to Government towards defraying the cost and charges on the work. This will raise (supposing the whole 7000 copies to be sold) the sum of £1400. The charge of printing and paper (even in the present expensive way in which it is got up) cannot amount to so much as £600: and there would then be left a sum of upwards of £800 to defray the charge for computations; which I presume does not amount to any thing like that sum.

But, by a *little* attention to œconomy in paper and printing, and by a *great deal* of improvement in the work itself, I have no doubt the sale might be increased to ten, or even twenty thousand copies: in which case, the *profit* would be manifest and considerable. And if the computers of the Berlin Ephemeris could be induced, for an adequate remuneration, to adapt their calculations to the meridian of Greenwich, when they are computing their own ephemeris, a further and very considerable portion of the charge which now attaches to the Nautical Almanac would be saved: and we might thus have an excellent work, at a very moderate expense, which would be both a saving and an advantage to the nation.

ERRATA.

Page	Line	
18	22	The quantity of the precession there stated is taken from Laplace: it will be found to differ from that deduced by M. Bessel, as given in page 104.
39	29	for ·0011 read ·011.
46	13	for 6793·39108 read 6798·279.
52		against 1833 for 0·36 read 9·36.
67		last line but one for 25. 12. 0 read $25^d\ 12^h\ 0^m$.
87	17	after "Latitude" add "(*minus* when South):" and at the bottom add the following note: viz. The Longitude and Right Ascension will always be in the *same* quadrant, unless $(a + \omega)$ exceeds 90°: in which case if L is in the $\left\{\begin{array}{c}\text{first}\\ \text{second}\end{array}\right\}$ quadrant, Æ is in the $\left\{\begin{array}{c}\text{fourth}\\ \text{third}\end{array}\right\}$ quadrant: and *vice versa*.
88	6, 7 and 8	for (A – V) read $(A \backsim V)$, and add as a note at the bottom of the page "When ψ is greater than Δ, the *difference* of the segments is equal to A, and the *sum* of them equal to V." See the note to page 261.
89	5	for Æ + P read Æ ± P. And add as a note "See page 222 and 223."
—	14	after Pole, insert "Which must be divided by 15 in order to reduce it to *time*."

Page Line

93 *passim* for (L — D) read Z. And add at the bottom "Z = the meridional zenith distance of the star." See pages 202 and 228.

— 12 after clock, dele "which angle will change its sign after the meridional passage of the star."

— 19 and 20 E and W should be expressed in *minutes* of time considered as integers.

95 5 for $+ \frac{a^2}{\sin 2''}$ read $- \frac{a^2}{\sin 2''}$.

101 5 See the additions to this formula in the Appendix, page 273.

— 9 for 0″,15 read 0″,16.

103 the second differences should be d' d''.

— 11 for $(h - 24)$ read $(h - 6)$.

104 See the Appendix, page 271.

108 22 The error in collimation should have been defined thus: $c =$ the error in collimation, *plus* when the circle described by the optical axis of the telescope falls to the east, and *minus* when it falls to the west.

109 16 for "the *second* star" read "the most *northern* star." These are not Professor Littrow's words, but it is evident that by adopting this language the result will lead us to the proper *sign* to be employed for correcting the error in azimuth; which, as it now stands, will be ambiguous.

110 12 for $\gamma = + (\frac{1}{2} \cot \phi$ &c) read $\gamma = - (\frac{1}{2} \cot \phi$ &c).

114 at the bottom insert "$e =$ the tabular expansion (for 1° Fahr.) of the metal, of which the pendulum is composed."

115 4 for "Ellipticity" read "Eccentricity."

131 at the bottom insert "If great accuracy is required, the time should be further corrected by $- 0^s,0127 \sin 2$ ☽: where ☽ denotes the moon's true longitude."

Page	Line	
132	1	The title of Table VI would be more properly thus, "for converting *intervals* of sidereal time into *inter-* "*vals* of mean solar time." A similar correction may be made to Table VII.
137	16	for 2,993 read 2,933.
146	3	for ·700865 read ·700861.
153	last line	for "15″ sin Lat." read "15″ cos Lat."
172	opposite $5^h\ 10^m$	for 7·4230 read 7·4530.
173	last line	for 20,94720 read 20,04720. See however the Appendix, page 271.
186	5	for 331 read 332.
—	16	for 446 read 447.
—	21	for 046 read 047.
187	9	for 846 read 847.
200	18	for "when *decreasing*" read "when the sun is proceeding towards the south."
203	3	add as a note "When r is *negative*, we must take the arithmetical complement of the logarithm denoted by ·000010053 r."
207	30	for $-\,b\,.\cos(\odot + B - Æ)\tan D$ read $-\,b\,.\cos(☊ + B - Æ)\tan D + c$.
208	3	for $-\,b\,.\sin(\odot + B - Æ)$ read $-\,b\,.\sin(☊ + B - Æ)$
209	18	for "contain" read "express."
218	22	for "the Æ" read "the Æ of the meridian."
—	26	for "the true Æ of the sun for mean noon" read "the "correct Æ of the meridian for mean noon, reckoned "from the apparent equinox."
226	15	dele "(which is always *minus*)".
227	17	The logarithm of the *sine* of 19° 48′ has been erroneously taken instead of the *tangent*: which slightly affects the result.
228		As a note to this Problem, it may be remarked that the observations of altitude ought to be discontinued

Page Line

when the change of altitude in one second of time amounts to one second of space.

234 7 and 9 prefix the sign + to the logarithms.

— It may be proper here to state, that it was omitted to be mentioned, that in the example for a *fixed* observatory the latitude was assumed equal to 50° 0′ 10″, in order to show that an error even of 10″ in the assumed latitude, would not make any difference in the result.

242 3 for $\pm \frac{\varrho}{15 \cos \delta}$ read $\mp \frac{\varrho}{15 \cos \delta}$.

245 8 for $\frac{86400}{86622}$ read $\frac{86400}{86662}$.

263 14—16 this remark belongs to Example 1: see the erratum to page 87.

THE END.

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