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Application of composite indicators and nonparametric methods to evaluate and improve the efficiency of the technical universities

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Publicly funded universities, like commercial organizations are obliged to ensure their efficiency. This article presents a model to measure and assess the relative efficiency of technical universities. The analysis was performed using publically available data from 2011 for 18 universities using the Composite Indicators method and the SBM Data Envelopment Analysis model. Fourteen indicators for efficiency were defined in the five areas of the university performance: research, teaching, scientific staff development, quality of teaching processes and public funding. Inefficient units were identified, based on their calculated efficiency scores and the directions for change to allow them to reach greater efficiency were suggested. Methods used to assess efficiency allowed the combined effect of all relevant factors to be taken into account which described the basic operations of the university.

Keywords: economics of education, Data Envelopment Analysis, Composite Indicators, efficiency, technical universities.

The performance of institutions of higher education to satisfy statutory obligations for teaching and research is evaluated using various methods, e.g. rankings (*Ranking szkół wyższych*, 2013). Rankings, however, are more important for building institutional image than evaluation and improvement of teaching or research, as they do not directly reflect their efficiency. For research activity, the Ministry of Science and Higher Education (MSHE) carries out periodical parametrisation of institutions, the results of which – besides prestige building – have more measurable effects, as they translate into the value of research finance. Marginson (2014), who made a critical assessment of world higher education rankings (including the Shanghai Academic Ranking of World Universities, the so-called Shanghai List), has the opinion that comparison of universities should focus on the essence of their operation, rather than their reputation. Rankings based on reputation operate on the principle of a competitive game, which is an aim in itself and which does not contribute to improvement of teaching, the quality of research, or other services provided. Higher education financed mainly from the public purse should pay particular attention to

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efficient use of funds. Law on higher education in Poland determines general principles for division of the subsidy for public higher education, emphasising, however, that efficient spending of public funds is ensured with respect to the quality of teaching (Journal of Laws No. 84, item 455). This confirms the legitimacy of interest in efficiency measurement and evaluation in the institutions.

Operating conditions for higher education in Poland have changed radically since 1990, when state monopoly on the creation of higher education institutions was lifted. This caused rapid growth of the number of private institutions. At the start of academic year 2011/12, the study period for this work, there were 460 institutions in the Polish higher education system (HE), of which 328 were private and 132 public (GUS, 2012).

Dynamic development of higher education is currently hampered by demographics. Antonowicz and Gorlewski (2011) did not call their report *Demographic tsunami* without good reason – they presented the difficult situation which the Polish higher education system will soon face. In the academic year of 1990/1991, around 403 000 people studied at Polish Institutions, while the figure peaked at almost 1 954 000 in the record year 2005/2006. Since that time, the number of students has been gradually decreasing, down to 1 676 000 in 2012, of which 27.4% attended non-public institutions. In 2012, compared to 1995, potential candidate numbers for HE, nineteen-year-olds, fell by 24.3% (GUS, 2013). The observed decrease in demand for HE strengthens the argument for close inspection of the efficiency of the institutions.

Research in HE is often financed from public funds in areas which are of decisive importance for innovation in high-tech industries, e.g. pharmaceutical, chemical or electronic. The need to integrate research activity with teaching future personnel for industry is emphasised, as innovative solutions are only translated into economic benefits by the availability of suitably trained personnel, primarily, highly qualified graduates to contribute to economic growth (Mansfield, 1995; Salter and Martin 2001). This argument justifies attempts to evaluate relevant efficiency in universities of technology.

The main objective of this study, the results of which are presented here, was to measure and evaluate the efficiency of public technical higher education in Poland, to identify causes of inefficiency and to determine remedial actions. Based on the available statistical data, areas were identified that affected efficiency. Models were used, based on composite indicators (OECD, 2008) and using the nonparametric method Data Envelopment Analysis (DEA), developed by Charnes, Cooper and Rhodes (1978). DEA has many applications in efficiency studies of various types of institution. It enables formulation of recommendations leading towards greater institutional efficiency and appropriate remedial action.

Methods for measurement of relative efficiency

General concept for measurement

Measurement of efficiency is based on determination of the relation between input to and output from of given entity. One goal is to obtain the information necessary for optimisation of decision-making processes. Besides economic efficiency based on the criteria of costs, income or profit, allocative efficiency, also called Pareto–Koopmans efficiency is identified. In this type of allocation of resources, one entity cannot be improved without simultaneous deterioration of the situation of another. It is therefore referred to as Pareto efficient or Pareto optimal (Stiglitz, 2004), the allocative efficiency determines whether the existing combination of inputs and outputs is in good proportion. On this basis, Debreu (1951) and Farell (1957) defined the concept of technical efficiency as a relation between productivity of a given entity and the maximum productivity which could be obtained with given inputs and the technology used. An organisation is technically inefficient, if it consumes more input than necessary to obtain a given production level (or when, with given input, it operates below the limit of production capacity). Technically efficient entities are located at the efficient frontier, while inefficient ones fall below.

There is a connection between allocation and technical efficiency. An organisation may be technically efficient (on the frontier of production possibilities) and at the same time allocatively inefficient when a change to the combination of inputs and outputs could contribute to lowering its cost of operation. A situation in which an organisation is efficient both in allocation and technically is called overall efficiency or economic efficiency (Coelli, Rao, O'Donnell and Battese, 2005).

In efficiency studies, methods include: classical (index), parametric (econometric models) and nonparametric (mathematical programming; Guzik, 2009a). Classical methods are usually used to evaluate the relations between two factors. However, composite indicators (CIs) are also commonly used, which find broad application in the analysis of public policy (the principles of building composite indicators can be found in Appendix 1).

When analysis includes at least two types of input and one output (or vice-versa), parametric methods can be used which utilise the economic production function that determines the interdependence between input and output. In the case of multidimensionality (at least two types of inputs contributing to at least two outputs), parametric methods cannot be applied, as it is impossible to determine the input of one type to obtain specific outputs directly (Guzik, 2009b). In such cases, nonparametric methods are used, such as the DEA (the algorithm of the applied model can be found in Appendix 2).

Principles for selection of DEA model structure

The starting point for a study using DEA is the understanding of a process, determining the study objective and identification of the group of decision making units (DMUs) subject to evaluation with the set of variables to describe inputs and outputs (Cook, Tone and Zhu, 2014). Although DEA is flexible, some conditions must be met: the values of inputs and outputs must be above zero; a smaller number of inputs than outputs is preferred; selection of inputs, outputs and DMUs should reflect the objectives of the analysis (Cooper, Seiford and Tone, 2007).

The set of DMUs should be homogeneous or nearly homogeneous (to ensure that no entities differing in nature are compared) and the direction of preferences must be uniform, i.e. a growth in output, from the point of view of the goal of performance of the DMUs, must be evaluated as positive, while a growth in input, with constant outputs, must be evaluated as negative (Guzik, 2009a). A parameter which affects the discrimination power of the DEA is the relation between the number of evaluated DMUs and the total number of variables determining inputs and outputs. A rule of thumb has been suggested (Cook et al., 2014), that the number of DMUs analysed should be double the total number of inputs and outputs and three times that number for radial models. Such rules are not obligatory, nor do they have statistical grounds, but they simply result from practice.

Application efficiency study in HE

There are many examples of the use of the DEA to evaluate efficiency in HE. In the model for efficiency evaluation of Austrian HE (Leitner, Prikoszovits, Schaffhauser-Linzatti, Stowasser and Wagner, 2007), 2 types of input are used (number of teachers and lecture room space) and 12 outputs (funds from external sources, number of completed projects per employee, number of completed projects in the department and the numbers of: examinations, degree candidates, monographs, articles, reports, presentations, other publications and doctoral students). The authors of the study emphasised that DEA – besides calculation of efficiency and creation of a ranking – also enabled determination of directions for improvement of inefficient DMUs. In Australia (Abbott and Doucouliagos, 2004), DEA was used to evaluate research activity of universities. It was assumed that the output was a weighted publication index. The following inputs were included: income from research, number of academic staff and other employees, as well as the size of institution. The motivation for efficiency analysis in departments at a Taiwan university was significant reduction in government subsidy (Kao and Hung, 2008). The goal was to evaluate the efficiency of the use of resources on the basis of a model which included the following outputs: total teaching load, number of publications from the Science Citation Index and total grants obtained by employees. As inputs, the weighted number of teachers and administrative employees, the total operational costs and lecture room floor space were used. Directions for action were identified to improve the inefficient units through better use of resources. Kempkes and Pohl (2010), in a study on German universities, used the number of graduates and the value of research grants as outputs and numbers of technical and research personnel and current expenditure as inputs. They applied an output-oriented model, assuming that institutions financed from public funds without input control, so they needed to maximise outputs with given inputs.

In Poland, the most comprehensive study using DEA was an analysis of 59 public institutions in HE (Ćwiąkała-Małys, 2010). Several models were formulated, in which the following were treated as inputs: the number of employees (teaching and non-teaching staff), costs, fixed assets and teaching subsidy, while numbers of students and graduates were outputs. The inefficiency of the finance system for public HE was determined, indicating that the algorithm for subsidy distribution was not adequate for efficiency and several possibilities were proposed for its modification.

The method of composite indicators is also used to study the efficiency and develop HE rankings. An example is offered by the evaluation of efficiency and quality of Spanish HE (Murias, de Miguel and Rodriguez, 2008), which analysed all Spanish universities, with the exception of technical institutions (to maintain sample homogeneity). A similar evaluation of efficiency and quality, at the level of national HE, was performed for 16 countries, including Japan, Tunisia, Morocco and thirteen European countries (Zrelli and Hamida1, 2013). In both studies, the DEA model was used to establish the weights, in which one major input with a value of unity was adopted for all compared objects. The concept of *helmsman* was applied, which had been introduced by Koopmans when examining efficiency in decentralised decision-making systems. The concept assumes that each country has the tools at its disposal to enable macroeconomic policy and its results depend on one input only, the authorities making macroeconomic decisions, defined here as the *helmsman* (Koopmans 1951; Lovell 1995). The method of weighted indicators, in the form presented in Appendix 1, was applied to rank institutions on sub-indicators from three areas: research, teaching and environmental impact (Lukman, Krajnc and Glavic, 2010). Similar principles are applied to build the generally known world rankings of higher education institutions, such as the Shanghai Academic Ranking of World Universities, or the British Times Higher Education Supplement. However, their reliability is disputed, mainly due to excessive subjectivity in the selection of the applied model structure (Marginson, 2014; Paruolo, Saisana and Saltelli, 2013; Saisana, d'Hombres and Saltelli, 2011).

Selection of entities for the study

A group of higher technical education institutions was selected for the study, for reasons including that their courses are characterised by high cost-intensity, usually do not enjoy mass-appeal and their scientific work requires costly research infrastructure. This group of institutions meets the postulate of homogeneity (Guzik, 2009a), due to the fact that they are public, function on the basis of the same regulations, offer similar courses and similar research, aiming mainly at educating engineers and the development of science in technical fields. Lack of homogeneity of DMUs could lead to the fact that the results of efficiency evaluation would reflect differences in the operational environment rather than their actual inefficiency (Haas and Murphy, 2003).

At present, according to the classification of the MSHE, 18 public technical higher education institutions operate in Poland, in: Białystok (PB), Częstochowa (PCz), Gdańsk (PG), Koszalin (PK), Cracow (PKr), Lublin (PL), Łódź (PŁ), Opole (PO), Poznań (PP), Radom (PRa, since 11 September 2012 – Kazimierz Pułaski University of Technology and Humanities in Radom), Rzeszów (PRz), Silesia (PŚl), Kielce (PŚw), Warsaw (PW), Wrocław (PWr) and the AGH University of Science and Technology (AGH), West-Pomeranian University of Technology (ZUT, established in 2009 from merger of the Szczecin University of Technology and the Szczecin University of Agriculture) and the University of Technology and Humanities in Bielsko-Biała (ATH, formerly a branch of the Łódź University of Technology). They differ in size, founding date and origins of foundation. Also their areas of special interest differ, yet due to the dominant technological profile declared, they form a homogeneous group.

Proposed model of efficiency evaluation of technical universities

Complete data collection is a fundamental problem in analysis. Available public data from several sources were combined for the analysis. Citation and publication data were available from the Web of Science. Numbers of students, graduates, doctoral students, courses and figures for university staff were derived from basic data published by the Finance Department for Higher Education, MSHE (until 2009 annual data were published in book form, entitled *Higher Education, 2009. Basic data*). Financial data were obtained from the Official Journal of the MSHE, in which the subsidy to each university is published and the Polish Monitor B August 2012, where public university financial statements are available, including net incomes and incomes generated by the university. Analysis and evaluation of efficiency were carried out on the basis of data from 2011, for which full data were available in all the analysed areas. It was decided that, to maintain cohesion between the two applied models, the same indicator variables were used in the DEA model as in the CIs model. An additional argument for using the indicators is that their use eliminates the differences of scale of the compared entities. The analyses often do not use identical denominators for all indicators, which creates the advantage that they are independent of the size of entity, and this facilitates comparison (Hollingsworth and Smith, 2003).

For the purposes of the study, an initial set of 14 indicators was adopted, 8 of which had the nature of outputs (marked with R – higher values are evaluated as positive), and 6 treated as inputs (marked with N – lower values are evaluated as positive). Specific indicators have been applied to five basic processes: research activity, teaching activity, personnel development, ensuring teaching quality and financial management. The choice of this set was guided by the principles described by Tarantola and Mascherini (2009), and provide a set of good practices for creating composite indicators:

- the importance of the indicator in terms of the study objective: indicators must be relevant for decision-making processes and reflect the examined phenomenon;
- eliminating redundancies: when two indicators are redundant, it is advisable that only one should be selected, the one already used in other studies being preferable;
- correlation: when two indicators are strongly correlated, and they convey important information from the point of view of the study objective, they may be both incorporated into the final model;
- availability: the use of indicators available for all entities compared and obtainable from reliable databases is recommended.

The problem of making the decision to eliminate strongly correlated variables has also been raised by other authors. Decancq and Lugo (2010) claimed that correlation between variables at a level of 0.8 justifies including them, as long as they reflect important aspects of the situation described by the model. The choice of indicators for the model may always lead to controversy, due to the previously mentioned subjectivity in rankings (Paruolo, Saisana and Saltelli, 2013).

Table 1 presents a short description of indicators, along with the descriptive statistics and their interpretation (when the description of indicators includes number of students, it should be interpreted as total number of full-time and part-time students).

For research activity, two indicators were defined to characterise scientific outcomes for university employees at the international level (based on data from the Web of Science database): (R1) – number of citations per academic and (R2) – number of registered publications per academic. Exceptionally, indicators were calculated for that area over a five-year period (2007–2011), which is justified by the length of the research and publication cycle, which is practically never closed within one year (Leitner et al., 2007). The indicators are correlated at the level of 0.92, yet according to the principles described above, they were included in the model. The high correlation results from the fact that universities which publish many valuable publications are often cited. It should be emphasised that both the indicators (number of publications and citations) are for the same period, that is the indicator for citations refers preferentially to publications from years preceding 2007. From the point of view of evaluation of the research area, both indicators are crucial.

In the area of teaching activity, variable (N1) – ratio of BA degrees to the total number of first cycle degrees, in full and part-time courses, indirectly reflect the scale of studies on non-technical courses, that is, the value should be minimised from the viewpoint of the mission of universities of technology. Variable (R3) – ratio of full and part-time students on technical courses to the number of technical courses directly determines cost of teaching, as the cost per student decreases with increase in number of students on the course, so this indicator should be maximised. The results of N1 and R3 are correlated at a level of 0.66.

In the area of personnel development, variable (R4) – number of doctoral students per senior academic, reflects the involvement of potential supervisors in the process of acquiring doctoral qualifications, when the supervisor holds a postdoctoral degree, it indirectly influences when the obtain the title of professor. Variable (R5) – ratio of the number of postdoctoral degrees obtained to the number of assistant professors reflects the effectiveness of doctoral staff development. Then, variable (R6) – ratio of doctoral degrees to number of doctoral students and assistant lecturers is the measure of effectiveness for development for doctoral students and assistant lecturers. All these variables should be maximised. Variables R4 and R5 are correlated at the level

Table 1
Set of indicators used in the analysis

Area	Symbol	*M*	*SD*	Min	Max	Description
Research	R1	4.72	3.65	0.55	12.6	Citations per academic teacher.
	R2	1.50	0.71	0.34	2.72	Registered publications per academic teacher.
Teaching	N1	0.26	0.24	0.00	0.74	Ratio of BA awards to the total number of degrees.
	R3	349.90	161.40	144.10	692.70	Ratio of students of technical courses to the number of technical courses.
Development	R4	1.34	0.72	0.26	2.49	Doctoral students per senior academic staff member.
	R5	0.02	0.01	0.00	0.03	Ratio of obtained postdoctoral degrees to the number of assistant professors.
	R6	0.09	0.04	0.03	0.18	Ratio of obtained doctoral degrees to the number of assistant lecturers and doctoral students.
Quality	N2	5.80	1.48	3.51	8.73	Awards per supervisor.
	N3	0.67	0.15	0.38	1.06	Ratio of the number of courses offered part-time to the full-time courses.
	R7	0.12	0.06	0.00	0.23	Ratio of initiated doctoral procedures to the number of doctoral students.
Finance	N4	0.46	0.04	0.39	0.54	Ratio of non-teaching staff to all staff.
	N5	119.80	16.60	88.60	152.40	Subsidy per academic teacher [in thousands PLN].
	N6	0.59	0.08	0.46	0.74	Ratio of subsidy to the value of university's net revenue.
	R8	90.00	35.50	35.90	154.00	University's own revenue per academic teacher [in thousands PLN].

of 0.71, while correlation of the remaining variables falls within the range 0.17–0.20.

For quality, three indicators were defined. Variable (N2) – number of degrees from full--time and part-time studies per academic (doctors or above), should be minimised, since the number of candidates per supervisor should be as low as possible to ensure efficiency and high quality of the award. Variable (N3) – ratio of the number of part-time courses to the number of full time courses reflects the scale for paid study. Part-time studies require less commitment and students may not dedicate the same amount of time to their work compared with those full-time – this reflects in the quality of obtained effects and so this variable should be minimised. Variable (R7) – the ratio of the number of transfers to the final phase of doctoral studies to the number of doctoral students and assistant lecturers describes mainly the effectiveness of the scheme for doctoral study. In the four-year cycle of the studies, the phase leading to award of the doctoral degree is started after the second year. The variable should be maximised, as the available statistics indicate low effectiveness. Correlation of these variables falls in the range 0.24–0.58.

In the financial area, four variables were included to determine the profile of costs and sources of financing, as well as the effectiveness in obtaining funds from external sources. Variable (N4) – ratio of the number of non-teaching staff to all staff indirectly determines the costs for administration and technical staff, and should be minimised. Variable (N5) – value of subsidy per academic teacher indirectly reflects the burden of salaries, which are the dominant cost for an institution, thus the variable should be minimised. Variable (N6) – ratio of subsidy to net revenue, reflects an institution's ability to obtain revenue from external sources. Then, variable (R8) – an institution's own revenue per academic teacher, indirectly reflects the role of employees in obtaining funds, e.g. in the form of grants. Correlation of the variables should be in the range of 0.24–0.70.

Measurement of efficiency and its interpretation

For each of the five areas, following the methodology described in Appendix 1, composite indicators were calculated, CIs. Then, one aggregate indicator projecting all analysed areas was calculated. Identical weights were adopted in calculations for all sub-indicators, which is one of the more frequently encountered aggregation methods (Paruolo, Saisana and Saltelli, 2013). Table 2 presents the results.

Universities are ranked according to their overall composite indicators CI_t, calculated from five sub-indicators: CI_b – for research, CI_d – teaching, CI_r – development, CI_j – quality and CI_f – finance. Such calculations determine a university's ranking with respect to each major area. The method uses composite indicators for rankings, but it may also allow identification of institutional strengths and weaknesses. However, it is difficult to quantify specific directions of change, to improve their positions by improvement in efficiency. A qualitative analysis can be performed, e.g. for the University of Technology and Humanities, occupying the last place in the rankings, it may be interpreted that finance is its strength, where it occupies the first place. It results mainly from the fact that the university has the least non-teaching staff (39%) and consequentially lower administrative costs, and also the lowest indicator of subsidy per academic, which is a direct result from one of the highest proportions of part-time students, and the high revenue to which they contribute. Wrocław University of Technology, despite occupying first place in the total ranking was ranked 10th in the finance area. This results from the relatively high subsidy per academic and a high proportion of non-teaching personnel (49%).

Application of the DEA method allows extension of the analysis beyond development of the ranking by suggesting quantitative changes for each factor taken in the efficiency analysis. To incorporate all mentioned areas, the most representative variables for each were chosen. From the set of 14 variables, 9 were selected (2 inputs and 7 outputs). When selecting variables, the principle that compared units should not number fewer than twice the total number of variables incorporated into the model and that each area should be represented was respected. In accordance with the earlier description, the variables are: N3, N6, R1, R2, R3, R4, R5, R7, and R8. Correctness of the selection of variables was verified by calculation of a composite indicator for the 9 variables. A satisfactory agreement between the rankings was obtained from the two sets of variables, presented in Table 3 (the correlation coefficient for rankings from 14 and 9 variables is 0.98).

Table 2
Ranking of technical universities based on the composite indicators

University	Place in the ranking on the basis of composite indicators for five separated areas					Total indicator CI_t	
	Research CI_b	Teaching CI_d	Development CI_r	Quality CI_j	Finance CI_f	Value	Rating
Wrocław University of Technology	2	1	3	1	10	7.99	1
Warsaw University of Technology	1	2	2	6	5	7.84	2
AGH University of Science and Technology	5	3	4	3	1	7.59	3
Gdańsk University of Technology	3	4	1	5	15	7.25	4
Poznań University of Technology	6	5	5	12	2	6.74	5
Łódź University of Technology	4	10	6	4	8	6.70	6
Silesian University of Technology	8	8	8	7	7	6.23	7
West-Pomeranian University of Technology	7	12	7	2	18	5.85	8
Cracow University of Technology	13	6	14	9	4	5.39	9
Lublin University of Technology	9	7	10	11	11	5.36	10
Częstochowa University of Technology	12	14	9	8	6	5.22	11
Rzeszów University of Technology	10	11	15	10	9	4.70	12
Białystok University of Technology	11	13	13	13	12	4.39	13
Kielce University of Technology	16	9	18	17	13	3.76	14
Opole University of Technology	14	15	11	14	14	3.67	15
Radom University of Technology	18	16	12	16	17	3.16	16
Koszalin University of Technology	15	17	16	15	16	3.06	17
University of Technology and Humanities of Bielsko-Biała	17	18	17	18	3	2.80	18

Calculations for efficiency were carried out by the use of the output-oriented DEA--SBM model with variable returns to scale, whose description can be found in Appendix 2. Calculations were performed with the use of the DEA Solver LV(3) software. In three cases, zero output values occurred in the data describing universities. This concerned Radom and Kielce Universities of Technology, where nobody had obtained a postdoctoral degree in the study year (output R5), while, additionally, no new transfer leading to the award of a doctoral degree was initiated at the latter (output R7). The values were changed into small positive values (0.0001), which enabled inclusion of those universities in the analysis. Table 3 presents the obtained results of efficiency and the resultant ranking, along with comparison with two rankings created on the basis of adopted indicators. In addition, reference sets are presented for inefficient universities that show how they should use the results of efficient universities to improve their efficiency. The most efficient university, Wrocław University of Technology, is included in the

Table 3
Results of efficiency measurement and comparison of rankings of universities of technology

University	Rankings acc. to the value of CI_t		DEA-SBM efficiency calculations		
	14 variables	9 variables	Rating	Score	Reference set
Wrocław University of Technology	1	1	1	1	
Warsaw University of Technology	2	2	2	1	
AGH University of Science and Technology	3	4	3	1	
Gdańsk University of Technology	4	3	4	1	
Łódź University of Technology	6	5	5	0.738	PWr (0.639), PW (0.361)
Poznań University of Technology	5	6	6	0.650	PWr (0.879), PW (0.121)
West-Pomeranian University of Technology	8	8	7	0.616	AGH (0.764), PW (0.236)
Silesian University of Technology	7	7	8	0.611	PWr (0.710), PW (0.290)
Częstochowa University of Technology	11	9	9	0.485	PWr (1.0)
Lublin University of Technology	10	11	10	0.466	PWr (1.0)
Cracow University of Technology	9	12	11	0.435	PWr (1.0)
Białystok University of Technology	13	13	12	0.416	PW (0.775), PWr (0.225)
Rzeszów University of Technology	12	10	13	0.411	PWr (1.0)
Opole University of Technology	15	15	14	0.303	PW (1.0)
Koszalin University of Technology	17	17	15	0.235	PW (1.0)
University of Technology and Humanities of Bielsko-Biała	18	16	16	0.178	PWr (1.0)
Kielce University of Technology	14	14	17	0.022	PW (0.889), AGH (0.111)
Radom University of Technology	16	18	18	0.003	PWr (1.0)

reference set 10 times, and in six cases it is the only reference for inefficient universities. Warsaw University of Technology occurred 8 times, in two cases was the only reference for inefficient universities, whereas the AGH University of Science and Technology occurs in the reference set in two cases. Gdańsk University of Technology, despite the result of efficiency equal to one, did not find itself in the reference set, while the phenomenon is referred to as efficiency by default (Tauer, Fried and Fry, 2007). The values provided in brackets are intensity variables, identifying the share of technology of efficient entities, which has to be applied for an inefficient university to attain full efficiency.

Sensitivity analysis was performed to verify the reliability of the obtained results from the model. Sensitivity analysis provides a measure of sources of uncertainty, such as: the data normalisation method, the weighting scheme, the aggregation system, inclusion and exclusion of sub-indicators and imputation of missing data (Cherchye et al., 2006; Mascherini and Manca, 2009). For the model proposed in the article, the analysis was limited to outlier identification, performed by subsequent exclusion inefficient universities from the set and observation of the impact on the places of the other universities in the ranking. As a result of the calculations performed, it was determined that the movements in the ranking were not greater than one place, so reliability of the results is assumed adequate.

Application of DEA to improve the performance

Table 4 presents the results of projection of inefficient universities onto best practice frontier, made up by four fully efficient universities. According to the DEA definition of efficiency, it means that inefficient

Table 4
Projection of inefficient universities onto the best practice frontier (in %)

University	Value and direction of change						
	R1	R2	R3	R4	R5	R7	R8
Łódź University of Technology	40.2	16.8	129.7	6.0	10.0	39.1	6.4
Poznań University of Technology	37.8	56.6	58.9	10.8	7.7	186.2	18.7
West-Pomeranian University of Technology	38.1	16.4	197.2	14.4	0.0	5.9	163.7
Silesian University of Technology	149.9	67.4	58.6	26.3	54.7	72.2	16.1
Częstochowa University of Technology	229.2	95.6	241.7	14.3	34.3	66.7	60.9
Lublin University of Technology	187.7	78.7	107.8	217.6	95.6	70.4	44.2
Cracow University of Technology	197.1	210.0	72.2	135.4	175.8	85.1	32.9
Białystok University of Technology	386.0	70.9	131.0	97.8	182.6	0.0	114.6
Rzeszów University of Technology	167.7	102.8	77.0	500.2	45.8	78.7	32.3
Opole University of Technology	805.2	160.7	173.9	131.8	139.3	8.5	188.0
Koszalin University of Technology	806.6	210.9	310.8	116.3	418.1	91.4	328.1
University of Technology and Humanities of Bielsko-Biała	937.8	480.0	303.8	718.4	128.2	556.2	108.2
Kielce University of Technology	999.9	215.3	70.3	107.7	999.9	44.3	95.5
Radom University of Technology	999.9	702.8	321.1	867.0	999.9	999.9	189.0

universities will achieve efficiency equal to one and have zero input surpluses and zero output shortages. Introduction of changes at the universities, in compliance with the values provided would make them attain similar efficiency to the four best universities.

Using the percentage values of the desired changes, based on source data, the absolute values were determined for the indicators. In the case of output R5 (ratio of the number of awarded postdoctoral degrees to the number of assistant professors), calculation of the absolute number of postdoctoral degrees which should be awarded during a year was performed. The value of 999.9% in the table results from the zero values (as already described) for output R5 for two universities: Radom University of Technology and Kielce University of Technology. The results are presented in Figure 1.

At the two universities which did not award any postdoctoral degrees in the year of study, 6 and 4 academic teachers should have obtained their postdoctoral degrees per year. The difference results from there being almost twice as many doctoral staff at Radom than Kielce University of Technology. According to the criteria, the West-Pomeranian University of Technology operates inefficiently, whereas their process of attaining the postdoctoral degree runs correctly. The highest increase is required for Silesian University of Technology (from 19 to 29), since the university has a similar number of doctoral staff compared with the three best scoring, while the number of postdoctoral degrees obtained is lower by more than 30%.

As regards academic activity, as measured by citations (R1) and publications (R2) per academic, all inefficient universities should aim to raise R1 to around 10 and R2 to 2.5. Figure 2 illustrates the changes necessary for output R1.

Poznań, Łódź and West-Pomeranian Universities of Technology should achieve this number of citations within five years, 2–3 times more than in the years 2007–2011. For remaining institutions, the quality of academic papers needs to be improved for a greater number of citations. For instance, Opole and Koszalin Univesities of Technology need to multiply this result almost tenfold. The requirements for increasing publications per academic are similar (R2).

Figure 3 illustrates the extent of university potential in terms of the number of doctoral students per senior academic (R4). Łódź, Poznań, Silesian, Częstochowa and

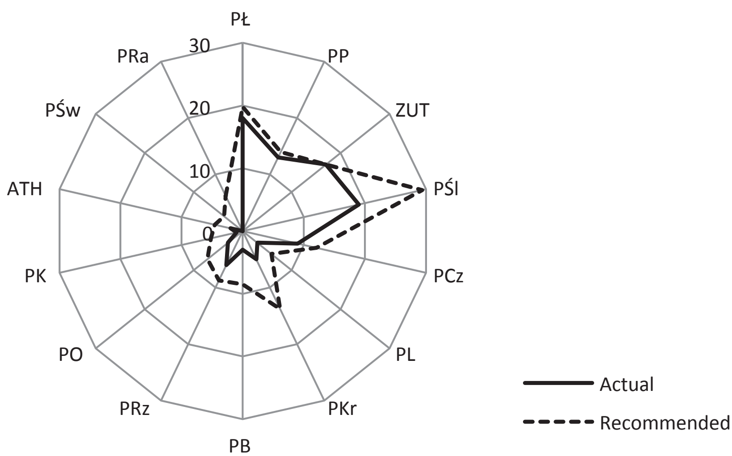

Figure 1. Number of postdoctoral degrees awarded during the year: actual and recommended over that period for full efficiency.

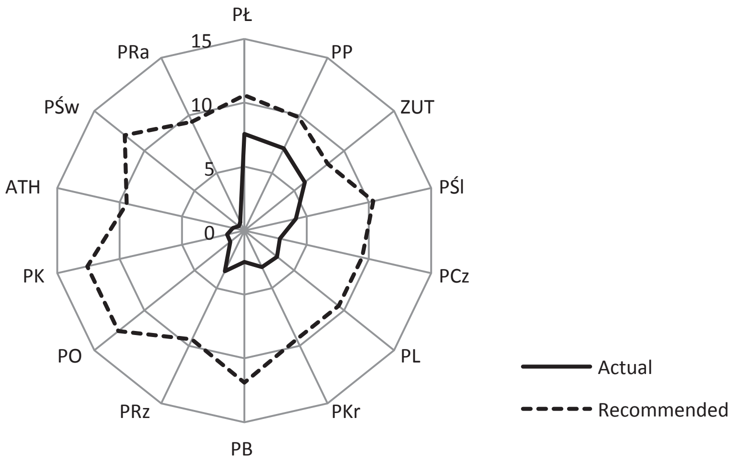

Figure 2. Number of citations per academic: actual and recommended for full efficiency.

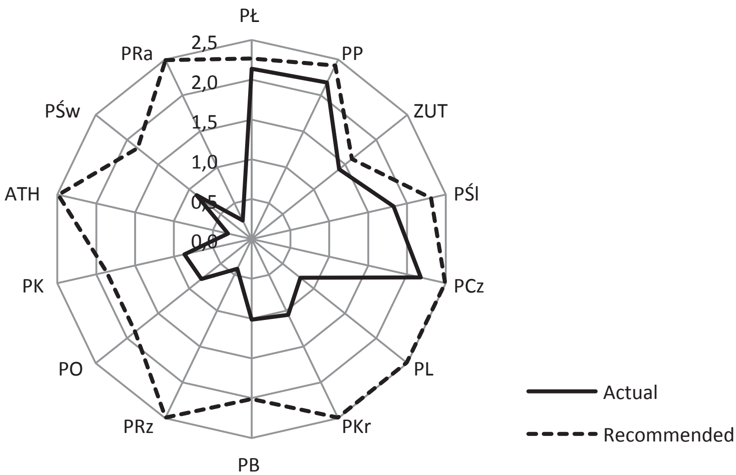

Figure 3. Number of doctoral students per senior academic: actual and recommended for full efficiency.

West-Pomeranian Universities of Technology exploit their potential well and the required change is small. Rzeszów and Radom Universities of Technology and the University of Technology and Humanities should increase the number of doctoral students per supervisor several times over. This diversity is partly determined by the academic authorisations that specific universities hold.

At all less efficient universities, the number of students per course is lower than it should be. A high number of courses makes the university look more attractive educationally, but from the economic point of view, efficiency of the teaching provision is negatively affected. For the three efficient and four inefficient universities selected, the basic characteristics of the problem are presented (Table 5).

At large universities operating at full efficiency, a proper relation between the number of students and the number of courses is manifest. Assuming a 2.5-year study cycle as the average (3–3.5 years for first cycle studies and 1.5–2 years for second cycle studies), around 250–300 students study at efficient

Table 5
Basic characteristics concerning teaching at selected universities

University	Efficiency	Number of students	Number of teachers	Number of full-/part- time courses	Number of full-/part- time technical courses	Proportion of technical courses	Students per course in 2011	Students per course recommended
Wrocław University of Technology	1	33 775	1 933	29/13	25/12	0.88	804	
Warsaw University of Technology	1	33 125	2 187	28/19	23/15	0.80	705	
AGH University of Science and Technology	1	34 248	2 154	35/20	28/17	0.82	623	
West-Pomeranian University of Technology	0.558	12 940	1 079	37/23	31/17	0.80	215	642
Koszalin University of Technology	0.240	9 244	524	24/20	15/12	0.61	210	804
University of Technology and Humanities of Bielsko-Biała	0.185	7 282	399	17/18	9/9	0.51	208	804
Radom University of Technology	0.003	8 125	508	27/18	9/8	0.38	180	804

universities per year of study of each course, while at inefficient universities the figure is around 80. To illustrate the difference in costs, a hypothetical group of 240 students studying at two universities was assumed. At one, the whole group attends one course and three at the other, 80 per course. Assuming that there are 300 hours of lectures in the curriculum for each course in an academic year, this could be most generally interpreted that in the second case, with the same number of students, three times as many teaching hours are performed in the latter case. Thus, the cost of lectures per student is three times higher.

The numbers of courses offered by Radom and Koszalin compare to those at Wrocław and Warsaw Universities of Technology, but with a quarter of the students and four times as many academic teachers. This must also affect the quality of the education provided. The least efficient universities are also more developed for part-time studies, as demonstrated by the number of courses and the ratio of part-time to the total number of students: for efficient universities, within the range of 17–24%, and for inefficient ones (apart from the ZUT) from 33% to 37%. The proportion of technical courses to the total number of courses (total full--time plus part-time) is important in determination of the inefficiency of a university. At inefficient institutions, technical courses are dominant, around 80–88%, while they do not exceed 40% at the least efficient Radom University of Technology. It is also worth noting the proportion of students on technical courses to the total number of students, within the range of 91–95% at efficient universities and 44–51% at the less efficient ones.

The above remarks are not applicable for the West-Pomeranian University of Technology, where, although inefficient, all parameters are close to those of the efficient institutions other than number of students per course. Improvement in teaching efficiency is possible, not by increasing the student numbers, but by reducing the provision of courses.

A weak characteristic of all doctoral programmes is demonstrated by the low numbers transferring into the concluding phase of study in relation to the total number of students. At Wrocław University of Technology, the indicator shows 23% and 20% at the AGH University of Science and Technology. Good results are also recorded at the following inefficient Universities of Technology: West-Pomeranian and Opole, while the University of Technology and Humanities has one transfer pending per 28 doctoral students, and at Radom University of Technology there were none (per 34 students – in the years 2009–2010, 4 transfers were initiated at the University of Technology and Humanities and none at Radom University of Technology). Figure 4 illustrates this indicator for inefficient universities.

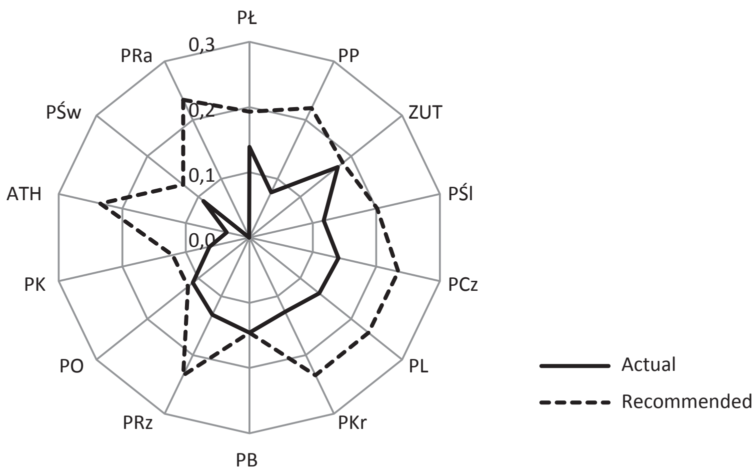

Figure 4. Ratio of transfer to the final doctoral study phase to doctoral student numbers: actual and recommended for full efficiency.

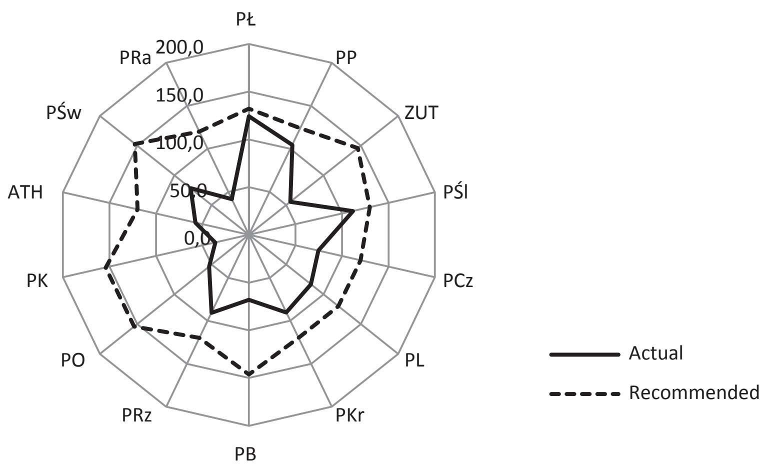

Figure 5. A University's own revenue per academic teacher: actual and recommended for full efficiency [in thousands PLN].

Public universities are mainly financed by subsidy, but budget resources are not adequate for their maintenance. Better universities obtain funds from grants and industry, while those operating at lesser efficiency do not conduct research at an appropriate level and have no such opportunity. All universities additionally obtain funds from paid forms of studies, although the scale of part-time studies is greater at inefficient universities, as already indicated and illustrated in Figure 5.

Summary

Efficiency at universities of technology was measured and evaluated in a way in which results could identify potential directions for change and which could allow less efficient universities to operate at full efficiency. This approach is referred to as projection of the inefficient universities onto the best practice frontier, as determined by the best institutions. Evaluation of efficiency with nonparametric methods should not be restricted to the creation of rankings, as the fundamental goal should be to identify causes of inefficiency and identification of changes to improve the situation.

An important issue in this type of research is the selection of an appropriate model and its verification before final evaluation. The use of radial models is straightforward, but it should be remembered that they allow zero weights for inputs and outputs, so in many cases, even with several variables, the result can be influenced by their small number. This means that not all factors influencing efficiency are represented in the calculation of the efficiency scores.

Apart from ranking, the method permitted identification of sources of inefficiency. The model incorporated the total influence of processes found in higher education. The mission to teach technical subjects was also taken into account. In 2012, Radom University of Technology changed its name to the University of Technology and Humanities, which in the light of the results is fully justified. It seems that similar changes might be introduced in other, less efficient universities of technology and this would reflect their teaching profiles more accurately.

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Appendix 1: Composite indicator method

Composite indicators enable aggregation of many sub-indicators into one measure which makes it possible to compare many objects. They integrate large volumes of information in a clear and understandable format, which is easy to interpret for the recipients (Shen et al., 2011). They find application both in creating policies and also for making decisions of operational significance. The methodology of creating CIs was developed by the OECD (2008), and the following are examples of its use: Human Development Index (HDI; Despotis, 2005) or the model of evaluation of Technological Capabilities (Filippetti and Peyrache, 2011). Similar principles are applied to the building of all types of rankings, including rankings in higher education, e.g. the annual ranking of the *Perspektywy* magazine.

Construction of CIs requires determination of factors which influence the studied phenomenon and creation of a composite indicator must be preceded by normalisation of input data. The Min-Max method can be used in normalisation, which in its basic version brings the values of all variables into the range [0, 1].

$$I_{i\_norm} = \frac{I_i - I_{min}}{I_{max} - I_{min}} \quad (1)$$

where:

I_{i_norm} – the normalised value of the i^{th} element of the vector of sub-indicators;

I_i – the value of the i^{th} element of the vector of sub-indicators;

I_{min} – the minimum value of the sub-indicator;

I_{max} – the maximum value of the sub-indicator.

For the purposes of the analysis, the way of normalisation was modified to maintain a uniform direction of preferences (Mohamad and Said, 2011). For sub-indicators, whose higher values are seen as positive (outputs), the following transformation was adopted:

$$I_{i\_norm} = \frac{9(I_i - I_{min})}{I_{max} - I_{min}} + 1 \quad (2)$$

For indicators whose lower values of sub--indicators are seen as positive (inputs), the following transformation was adopted:

$$I_{i\_norm} = \frac{9(I_{max} - I_i)}{I_{max} - I_{min}} + 1 \quad (3)$$

Normalised values belong to the range [1, 10]. The composite indicator is the weighted sum of normalised sub-indicators:

$$CI_r = \sum_{q=1}^{Q} w_q I_{qr} \quad (4)$$

where:

CI_r – the value of the composite indicator for the r^{th} object;

w_q – the weight of the q^{th} sub-indicator;

I_{qr} – the value of the normalised q^{th} sub-indicator for the r^{th} object for: $r = 1, \ldots, R$ – R is the number of objects incorporated into the analysis (the number of compared universities) and $q = 1, \ldots, Q$ – Q is the number of sub-indicators (total number of outputs and inputs).

Weights are usually adopted on the basis of expert opinion (Shwartz, Burgess and Berlowitz, 2009), but identical weights for all factors are adopted in many applications (e.g. Despotis, 2005; Manca, Governatori and Mascherini, 2010).

Appendix 2: Nonparametric DEA method

The nonparametric DEA method uses linear programming, which does not incorporate the impact of random factors or measurement error, and does not require determination of the functional relationship between inputs and outputs, nor the weights to be assigned to specific inputs and outputs. The optimum weights are calculated on the basis of data, rather than established subjectively (Cooper, Seiford and Tone, 2007).

Studying efficiency by the use of the DEA involves the determination of reference objects and comparison with the remaining ones. Thus, it is determined that the DEA verifies the relative efficiency of objects, called decision making units (DMUs). Units are decision making when they can influence the levels of incurred inputs and obtained outputs (Domagała, 2007).

The most general models used in the DEA may be divided into two groups: radial and non-radial. The two basic, most often used radial models, CCR (from the names of the authors: Charnes, Cooper and Rhodes) with constant returns to scale and BCC (from the names of the authors: Banker, Charnes and Cooper) with variable returns to scale, permit calculation of technical efficiency, pure technical efficiency and scale efficiency (Cooper et al., 2007). They evaluate radial (proportional) efficiency, but do not incorporate inputs surpluses and outputs shortages (so-called slacks).

According to the definition of the DEA, functioning of a DMU is fully (100%) efficient only when both the efficiency score is equal to one and the input surpluses and output shortages are equal to zero. It is possible to apply non-radial models, which enable incorporation of slack into the calculation of efficiency (Cooper, Seiford and Zhu, 2011). In the article, an efficiency measure based on slacks (Slack Based Measure, SBM) was used, which assumes values from the range [0, 1], eliminating non-zero slacks of inputs and outputs (Cooper et al., 2007; Tone, 2001).

The DEA-SBM model, when calculating an efficiency score, directly incorporates slacks of inputs and outputs (Cooper et al., 2011). Efficiency for $DMU_o = (\boldsymbol{x}_o, \boldsymbol{y}_o)$ according to an output-oriented SBM model, with variable returns to scale, is defined as follows:

$$\frac{1}{\rho_o^*} = \max_{\lambda,s^-,s^+} 1 + \frac{1}{s}\sum_{r=1}^{s}\frac{s_r^+}{y_{ro}} \quad (5)$$

under conditions:

$$x_{io} - \sum_{j=1}^{n} x_{ij}\lambda_j + s_i^- \quad (i = 1, \ldots, m)$$
$$y_{ro} - \sum_{j=1}^{n} y_{rj}\lambda_j + s_r^+ \quad (r = 1, \ldots, s) \quad (6)$$

$$\lambda_j \geq 0(\forall j) \;\; s_r^- \geq 0(\forall i) \;\; s_r^+ \geq 0(\forall r) \quad (7)$$

$$\sum_{j=1}^{n} \lambda_j = 1 \quad (8)$$

where:

$\boldsymbol{x}_j$, $\boldsymbol{y}_j$ – vectors of inputs and outputs of DMU_j for $j = 1,\ldots,n$;
n – number of DMUs;
s_i^- – input surpluses for $i = 1,\ldots,m$;
m – number of inputs;
s_r^+ – output shortages for $r = 1,\ldots,s$;
s – number of outputs;
λ_j – intensity variable for DMU_j.

It is required that inputs and outputs have values higher than zero. $DMU_o = (\boldsymbol{x}_o, \boldsymbol{y}_o)$ is efficient when $\rho_0^* = 1$, which means that outputs shortages are equal to zero, while inputs surpluses may be non-zero.

The SBM model requires that all values of outputs and inputs be higher than zero. In the case of the occurrence of zero values, one of the ways to meet the condition is to substitute the zero values with a very small positive number (Cooper et al., 2011), which prevents the need to remove the DMUs from analysis.

It is emphasised (Cooper et al., 2007), that one of the main goals of the DEA study is projection of inefficient DMUs onto the production frontier, when the inputs are treated as resources necessary for production of outputs. Although the DEA has a strong link to production theory, it is also used for comparative analysis (benchmarking). In this case, efficient DMUs, defined by the DEA, do not create a "production frontier", but lead to the development of "best practice frontier." In this case, inputs are not treated as resources needed to obtain specific outputs. Particular variables are classified as inputs if their lower values, from the point of view of the study objective, are seen as positive, and as outputs if their higher values are seen as positive (Cook, Tone and Zhu, 2014).

When selecting the model, it is important to determine its orientation. It depends on whether the study objective is to reduce inputs or focus on maintaining at least the current level of output, the so-called input orientation, or on maximisation of outputs to maintain at least the current level of inputs, output orientation (Cooper et al., 2007).

Usefulness of the DEA method is confirmed by its application in various domains, such as the banking, health care or education sectors. The basic objective of such studies is to identify sources of inefficiency, creating DMU rankings, evaluation of efficiency of courses and policies, as well as creation of quantitative grounds for relocation of resources, etc. (Liu, Lu, Lu and Lin, 2013).