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On the Practical Applicability of Stokes' Law of Resistance,
and the Modifications of it Required in Certain Cases.

by

Prof. N. S. Smoluchowski

[54]

dr. Intern. Congress of Math. Cambridge 1912
p. 10



estimated to cost about \$1000.00. I want to do all

the work myself & I want to do it all too.

Thank you, etc.

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On the Practical Applicability of Stokes' Law of Resistance, and the Modifications of it Required in Certain Cases.

By M. S. Smoluchowski, Ph.D., L.L.D., Professor of Physics at the University of Lemberg.

§1. Stokes' law for the resistance of a sphere in a viscous liquid rests, as is well known, on the fundamental assumptions:

- I. Slowness of motion, so that the inertia terms in the hydrodynamical equations may be neglected, in comparison with the effects of viscosity,
- II. Complete adhesion without slip, of the liquid to the sphere, this being considered as a rigid body,
- III. Unboundness of the liquid and immobility at infinity.

In the following I should like to contribute some remarks on this law with regard to certain cases of practical importance, where the underlying conditions are changed to some extent, which may be of some interest to those who are engaged with research work on subjects connected with Stokes' law.

First let us touch briefly the question of slipping, connected with the second of the above assumptions. Stokes' calculation can be generalized, by allowing the liquid to slip.

Mr. Ladd suggested I make another to Philadelphia last Saturday and
was invited to breakfast by Mr. Webster with
President and the members of his cabinet. President L. B. Thomas also had a talk with
and went to

Dear Friend Garrison in evening of Saturday night and with the
members of the Union League who want me to
do something at a meeting when Mr. Teller or another to speak. I
offered to speak with the members of the Union League who
first and urge Mr. L. B. to speak at the meeting when he stopped. I
had to go to a lecture
therefore I did not speak at the meeting. I
will do so however now if you desire it longer than two
days from today just let me know when you desire
the lecture and I will do so. However the lecture was over with the lecture
and will

so however the lecture will be given at the time and date
mentioned in circular of lecture and will be held with continuous note at
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along

(2)

the surface of the sphere, with a velocity proportional to the frictional force in tangential direction, [which in the case of a parallel laminar flow implies the surface condition: $\beta u = \mu \frac{\partial u}{\partial y}$].

In this case, as Dasset has shown, the simple law of Stokes has to be replaced by:

$$\cancel{F = 6\pi\mu R c} \quad F = 6\pi\mu R c \frac{\beta R + 2\mu}{\beta R + 3\mu} \quad \dots \dots \quad (1)$$

Thus the minimal value of the resistance, for the case of infinite slip ($\beta=0$), is two third of the maximal value for no slip ($\beta=\infty$).

Now it is generally assumed, on account of the ^{experimental} researches of Poenselle, Whitham, Comte, Zadenburg and others, that the slip of liquids along solid walls is negligibly small. Mr. Arnold's recent ^{measurements} prove, by the exact agreement with Stokes' law, that the coefficient of sliding friction β is certainly greater than 5.000 and probably greater than 50.000. [still greater values would result from the fact that even the resistance of electrolytic ions ^{correspond} ^(with well) by its order of magnitude to molecular diameters]

§2). On the other side ~~the liquid~~ ^{his} experiments, on bubbles of gas moving through liquid, gave the unexpected result that ~~the~~ the slip at clean ⁴ surfaces between gas and liquid is infinite, as the velocity turned out to

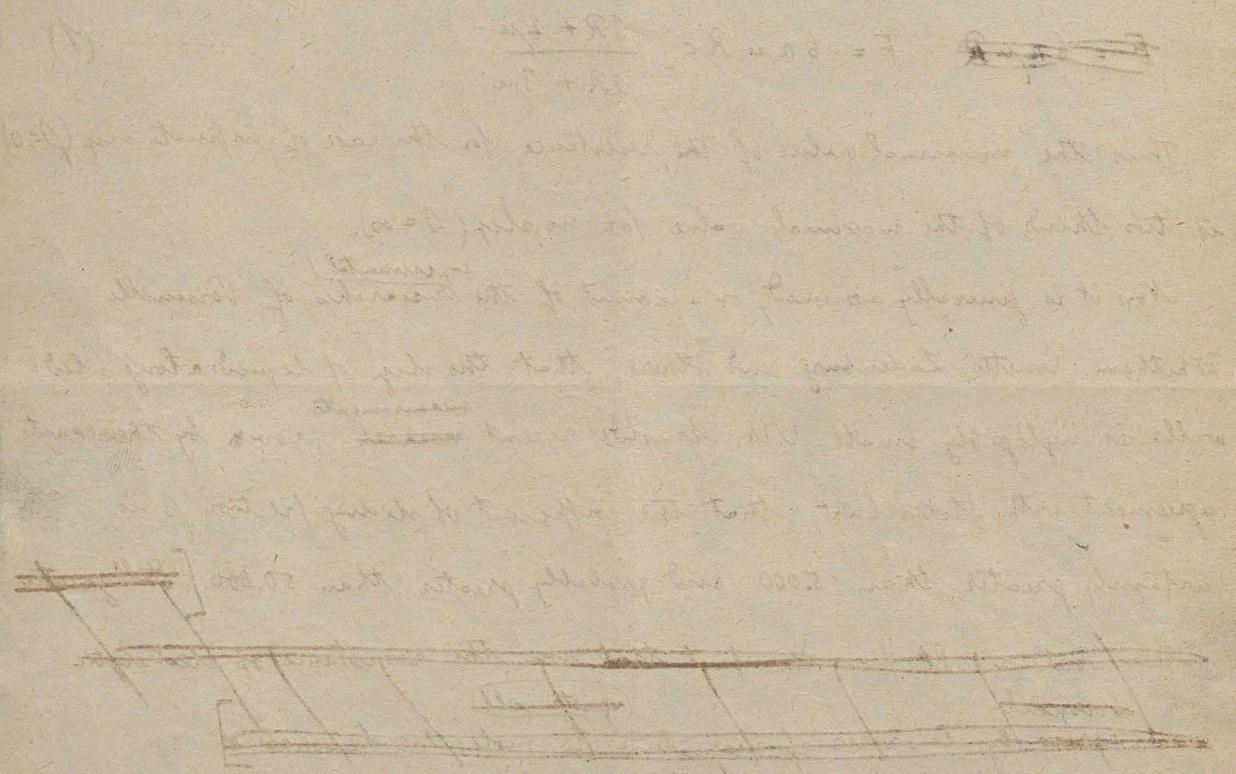
1) H. D. Arnold, Phil. Mag. 22 p. 755 (1911).

⁴ i.e. provided the surface be not contaminated with solid films.

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ports

described all of the passengers & sailors on the ship as it was at the
well known library & it was there that I often layed out and
had my books written upon it ~~and~~
it to go with him. Spent all night at hotel as was intended
and had a good nap.



got up with early morning bus 6007 with three passengers
made to me on the road down to the first bridge across
the river. And it was about as long as the bridge which was

about 22 ft. It cost 10/- Board 6/-
and three D. 1/- and a number of hours at leisure with

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great by 50 per cent.

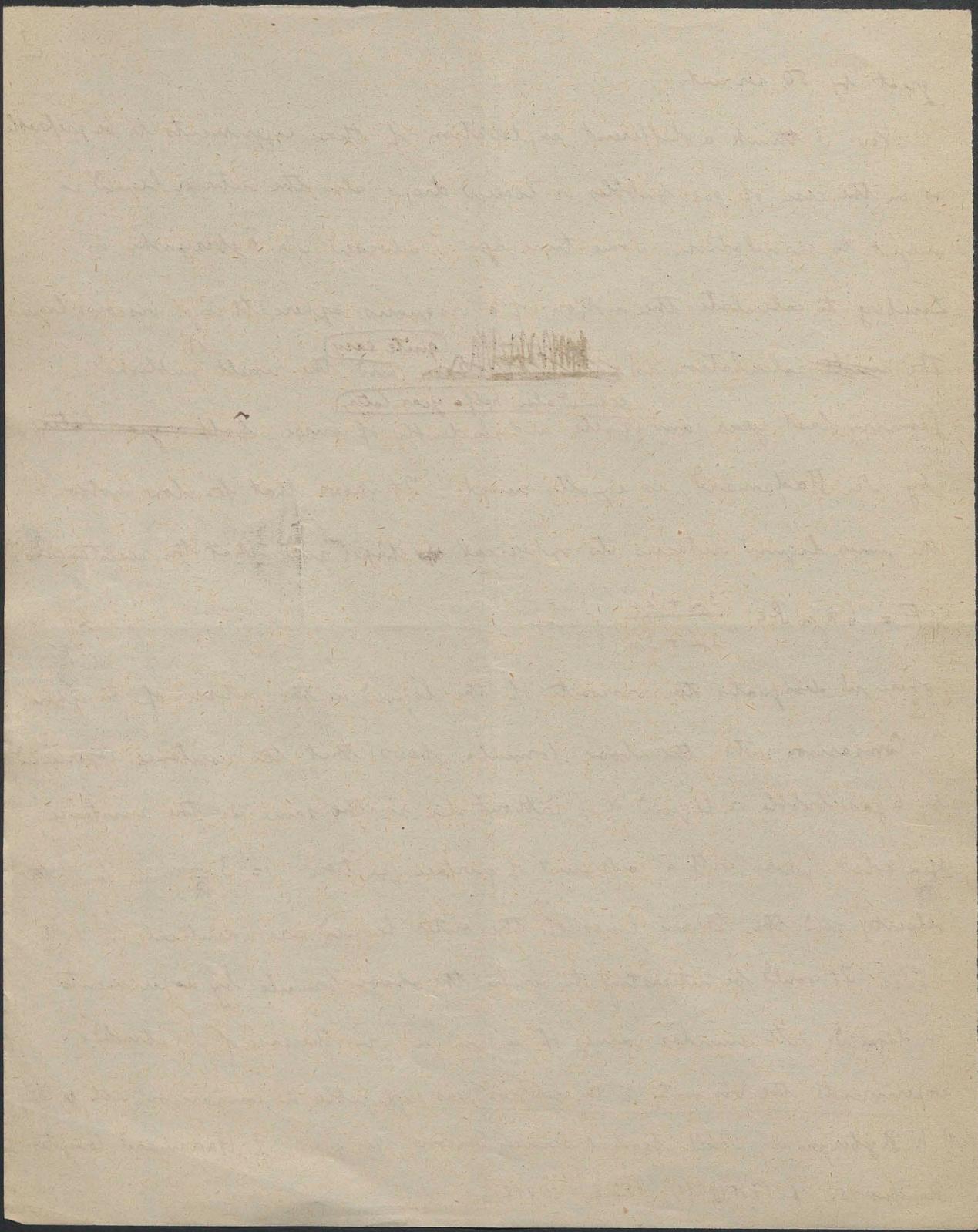
Now I think a different explanation of those experiments to be preferable, as in the case of gas bubbles or liquid drops also the interior liquid is subject to circulation. Some time ago I advised Mr. Rybczynski in Lemberg to calculate the motion of a viscous sphere through viscous liquid. The ~~whole~~ calculation is ~~quite easy~~ (and the result, published ¹⁾ ~~quite easy~~ ~~deduced also half a year later~~) January last year, and (quite independently of course, ~~half a year later~~) by M. Hadamard, is equally simple. It shows that for slow motion the inner liquid retains its spherical ~~shape~~ shape and that the resistance is:

$$F = 6\pi\mu R c \frac{3\mu' + 2\mu}{3\mu' + 3\mu} \quad \text{--- (2)}$$

where μ' designates the viscosity of the liquid in the interior of the sphere.

Comparison with the above formula shows that the resistance experienced by a gas bubble or liquid drop without slip is the same as the resistance of a solid sphere with a coefficient of surface friction $\beta = 3\mu'/R$; in fact the velocity and the stream lines of the outer liquid are identical in both cases. It would be interesting to verify the above formula by experiments on liquids with similar values of μ and μ' ; in the case of Mr. Arnold's experiments the viscosity in the interior was negligible in comparison with ~~the~~ the

¹⁾ W. Rybczynski, Bull. Acad. Scienc. Cracovie 1911 p. 40; J. Hadamard, Comptes Rendus 152, p. 1735 (1911); 153 p. 9912.



viscosity of the outer medium, which had the same effect as if the surface slip were infinite. So far his results too are explained without the assumption of surface slip.

§3) However, there is a case where the existence of surface slip has been proved beyond doubt: in rarefied gases. As is well known, the magnitude of the coefficient of slipping $\beta = \frac{\mu}{\rho}$ is, according to the kinetic theory and also to the old experiments of Knudsen and Warburg, roughly equal to the mean length of the free path of the gas molecules; therefore the phenomenon plays an important part even at ordinary pressures in the motion of very minute droplets, as in Millikan's experiments.

Unfortunately one cannot ~~it would seem natural~~ use formula (1) for this case, with substitution of the empirical value for β^3 , but such a procedure would give quite erroneous results, except for the case of comparatively small slip. For if the mean length λ is comparable with the dimensions of the moving sphere, the ordinary hydrodynamical equations cease to be valid altogether, since the implicit assumption underlying them, that the state of the gas is varying little for distances comparable with λ , is impaired.

Therefore also the interesting deduction of a corrected formula by Prof. E. Cunningham is not to be considered as a demonstration and Messrs. Knudsen and S. Weber may be right in trying to get closer approximation by empirical

J. E. Cunningham, Proc. Roy. Soc. A 83, p. 357 (1910)

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(5)

formulas.¹⁾ At any rate the formula proposed by Cunningham

$$\cancel{F = \frac{6\pi\mu R c}{\lambda} \left[1 + A \frac{\lambda}{R} \right]^{-1}}$$

serves remarkably well for interpolation, considering the experiments of the three authors and those of Mr. Mc. Keahan²⁾. It is preferable to write it in the form $F = 6\pi\mu R c \left[1 + \frac{B}{R\rho} \right]^{-1}$

where ρ is the density of the gas, as mistakes are easily involved by using the mean length of free path,^λ, which is a very indefinite term and really has no precise meaning.

For great rarefaction the resistance is proportional to the cross section of the sphere, and for this case the calculation can be carried out exactly, if the way is known, how the interaction between the surface of the sphere and the gas molecules takes place. If they rebound like elastic bodies, we get in accordance with Cunningham

$$F = \frac{4}{3} \sqrt{\frac{8}{\pi}} R^2 \rho c V$$

where V is the square root of the mean square of molecular velocity.

The empirical coefficient, as ^{calculated} ~~followed~~ from the experiments mentioned above, is considerably larger, it amounts to 1.65 (Knudsen and Weber) or 1.84 (Mr. Keahan). Mr. Keahan concludes that molecules are reflected from the surface of the sphere only in a normal direction; I think however ^(that) his theoretical formula is not quite exact.

¹⁾ M. Knudsen u. S. Weber, Ann. d. Phys. 36 p. 981 (1911).

²⁾ Mr. Keahan, Physik. Zeitsch. 12, p. 707 (1911).

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and the first of the year. I have

$\frac{1}{2} \text{ A.M.}$ 3 hours = 3 hours per A.

The last 18 hours, after all of the above was done, was
spent in the preparation of the final line of the report
 $\frac{1}{2} \text{ A.M.}$ 3 hours = 3 hours per A.

The time now is 10 o'clock at night. The report
is finished and it is now 10 o'clock. The work is done and
the time is 10 o'clock.

It is now 10 o'clock and the report is finished. The work
is done and the time is 10 o'clock.

It is now 10 o'clock and the report is finished. The work
is done and the time is 10 o'clock.

It is now 10 o'clock and the report is finished. The work
is done and the time is 10 o'clock.

10 A.M. 6 hours

The day is now 10 o'clock and the report is finished. The work
is done and the time is 10 o'clock.

The day is now 10 o'clock and the report is finished. The work
is done and the time is 10 o'clock.

The day is now 10 o'clock and the report is finished. The work
is done and the time is 10 o'clock.

The day is now 10 o'clock and the report is finished. The work
is done and the time is 10 o'clock.

~~and at any rate his conclusion seems to me at variance with fundamental principles of the kinetic theory of gases~~
~~I think~~
that the experimental results are explained best by the view supported
~~especially those of~~
also by other researches of this kind, ~~by~~ Knudsen, that a solid surface acts in
~~whether with or without change of mean kinetic energy~~
scattering the impinging molecules irregularly in all directions. We shall
not go into these questions now, however, as they belong to the kinetic theory of
gases, not to hydrodynamics.

(§4). Now let us consider what modifications are required in Stokes' law
if the third of the ~~the~~ fundamental assumptions is impaired, the liquid
being limited by solid walls, or a greater number of similar spherical bodies
being contained in it.

In this case the linear form of the hydrodynamical equations makes
it possible to obtain their solution by a method of successive approximations,
analogous to ~~the~~ (used in the theory of electrostatic potential. It consists
in the ~~successive~~ superposition of solutions formed as if the fluid would extend
to infinity, but so chosen as to destroy the residual motion in ~~alternating parts~~
at the boundaries, ~~represented by solid walls~~ with increasing approximation.

This method was used first by H. Lorentz in order to determine the ~~change~~
~~of the~~ influence of an infinite plane wall on the ~~movement around the sphere~~
^{progressive} ^(of a)
and we shall refer to his formulas later on.¹⁾
He found that the resistance of the sphere is increased by a fraction amounting
to $\frac{q}{\rho} \frac{R}{a}$ for normal motion, $\frac{q}{\rho} \frac{R}{a}$ for parallel motion, if a denotes the distance
from the wall. Mr. Stock in Lemberg has extended the calculation for the second

¹⁾ H. A. Lorentz, Abhandlungen i. th. Physik I p. 23 (1906). In Müller's determinations
of the ~~the~~ ionic charge the increase of resistance due to the presence of the
condenser plates, may produce an increase of the order of one thousandth.

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case to the fourth order of approximation, & including terms with $(\frac{R}{a})^4$.¹⁾

In a somewhat similar way Ladenburg²⁾ calculated the resistance experienced by a sphere, when moving along the axis of an unlimited cylindrical tube, and his result, indicating an increase in comparison with the usual formulae of Stokes in the proportion of $1 : 1 + 2.4 \frac{R}{\rho}$, (where ρ = radius of the tube), has been verified with very satisfactory approximation by his own experiments and by those of Mr. Arnold.

§5). Now let us apply this method to the case, where a greater number of similar spheres are in motion, and extend a little further now an investigation which I had begun in a paper published last year.³⁾ Imagine a sphere of radius R , moving with the velocity c along the X axis, its centre being situated at the distance x from the origin. It would produce at the point P (with coordinates ξ, η, ζ) certain current velocities u_0, v_0, w_0 , of order $\frac{Rc}{x}$, defined by Stokes' equations, if the fluid be unlimited. But if we assume this point P to be the centre of a solid sphere of radius b , we have to superpose a fluid motion u_1, v_1, w_1 , chosen so as to annul the velocities of the primary motion at the points of this sphere and satisfying the conditions of rest for infinity.

¹⁾ J. Stock, Bull. Acad. Sciunc. Cracovic 1911 p. 18.

²⁾ R. Ladenburg, Ann. d. Phys. 23, p. 447 (1907).

³⁾ M. Smoluchowski, Bull. Acad. Sciunc. Cracovic 1911 p. 28

(2) the most common & most variable star studied at least
in the northern hemisphere is probably the star known to Chinese as
the Great Dipper. It is a group of stars forming the bowl of the
Great Bear which is visible all year round. It consists of 7 stars
of which the two stars at the end of the bowl are called the 'pointers' and these
point to the north star.

At present I am studying the star
known as the Southern Cross which is

the southern analog of the Great Bear. It consists of 5 stars of which
one star is very bright while the others are much less brilliant
and of lower magnitude. They form a cross shape which has
to be completed with the eye X star, a star at the bottom. It
has 2 stars at the top, one on either side of the central
bright star and 2 stars on either side of the bottom star.
The name of the star is derived from the fact that it
is visible throughout the year and is therefore called the
Southern Cross.

85. A star named after the Southern Cross is the star
known as the Southern Cross.

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This motion may be called the "reflected" motion; it can be found with any degree of approximation, by making use of the solution of the hydrodynamical equations given by Lamb, in form of a development in spherical harmonics. But as it is of order $\frac{Rc}{r}$ at the surface of the second sphere, which is its origin, it seems probable, *a priori*, that its magnitude at the first sphere will be of order $c(\frac{R}{r})^2$, and I have verified this as well as the following results by explicit calculation. Thus if we confine ourselves to terms of order $c(\frac{R}{r})^2$, we can apply a simplified method of evaluating the mutual influence of such spheres, by neglecting the difference between the velocity at the centre of the second sphere and ^{at} its surface.

That is to say: the sphere P, being at rest, is subjected to frictional forces:

$$X = 6\pi\mu R u_0$$

$$Y = 6\pi\mu R v_0$$

$$Z = 6\pi\mu R w_0$$

on account of the motion of the first sphere; on the other side, the moving sphere experiences a reaction by virtue of the ^{presence of the} sphere P, such as if this would execute simultaneously the three motions $-u_0, -v_0, -w_0$; the three current systems resulting therefrom, according to the usual formulae of Stokes, produce at the centre of the first sphere nine current components, giving rise to nine components of frictional force, to be calculated each according to Stokes' law of resistance.

If both spheres are in simultaneous motion, the mechanical effects ~~result~~^{are found} by superposition of the forces corresponding to the two cases where one of them

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and I am the first to thank you for your kind & considerate
words. I am sorry to say that I have not been able to
get away from my work & have not had time to go
out & see the country. I have however been able to
see some of the country around here & it is very
beautiful. The mountains are very high & the
valleys are deep & the streams are rapid &
the water is clear. The people are very friendly
and the country is very fertile. The climate is
very mild & the weather is very pleasant.
I hope to get away soon & see more of the
country. I will write again when I have
more time.

is moving and the other one at rest.

In this way an interesting conclusion is obtained for the case where both spheres are moving in parallel directions with equal velocity: then both are subjected to equal additional forces in the same direction, one component in the direction of motion, tending to diminish the resistance by the amount: $\frac{g}{2} \frac{R^2 n c}{r} [1 - \frac{3}{4} \frac{R}{r}]$, the other component along the line joining the centres, towards the sphere which is going ahead, of amount: $\frac{g}{2} \frac{R^2 n c \cos \theta}{r} [1 - \frac{g}{2} \frac{R}{r}]$.

[where θ is the angle between the line of centres and the direction of motion].

Thus two heavy spheres of this kind would sink faster than Stokes' law is indicating and besides, their path must be deflected from the vertical towards the line of centres by an angle ϵ defined by:

$$\sin \epsilon = \frac{3}{4} \frac{R}{r} \left[1 - \frac{3}{2} \frac{R}{r} \right] \sin \theta \cos \theta$$

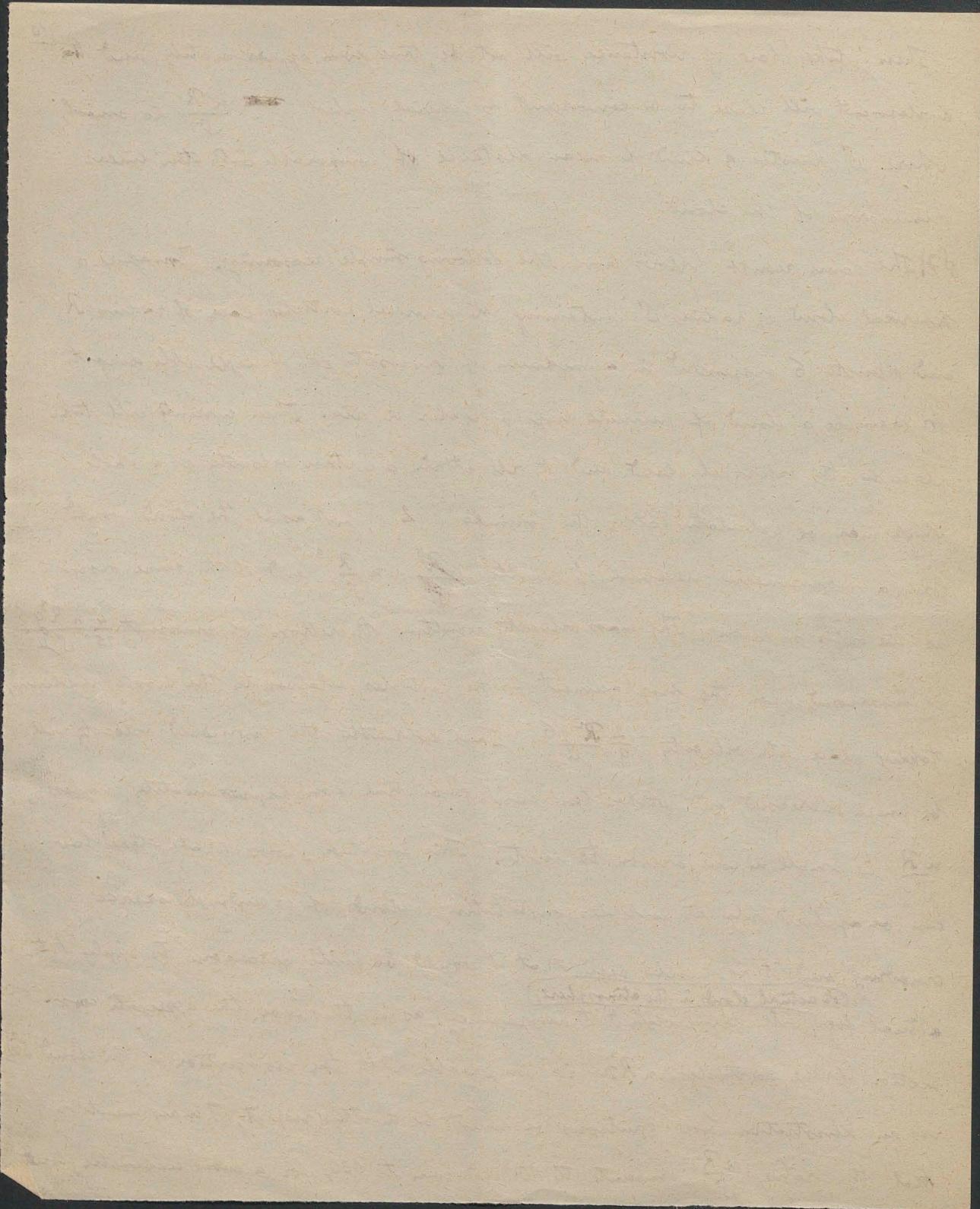
(6). Analogous methods are applicable to a greater assemblage of spheres. The motion results from superposition of simpler solutions, where one sphere is supposed moving and all the other ones ~~are~~ resting. Each of the component solutions ~~is~~ comprises the direct action, and for higher approximation also its "reflections."

Now if the parallel motion of n similar spheres ~~from~~ is considered, the resistance of each of them will be diminished by an expression ~~proceeding~~ proceeding after powers of R , the first term of which will be of the order of magnitude $n c R^2 \sum \frac{1}{n}$. We see that these developments would be divergent for an infinite number of spheres. It is evident that for instance an infinite row of spherical particles, at equal distances, would acquire infinite velocity, by virtue of their gravity, as also an infinite cylinder ^(infinite) would behave in the same way. This applies a fortiori to two dimensional assemblages.

and went with others ~~with~~ ^{about} them. Went out to camp ground and went
back to camp and talked with some men there until
about 11 o'clock. Then went to camp and had dinner. Went to bed about
12 o'clock. Went up at 2 o'clock. Went to bed again at 2:30.

Thus Stokes' law of resistance will not be true even approximately, and the development will cease to be convergent in general, unless $\frac{nR}{S}$ is small, where S denotes a kind of mean distance, ~~not~~ comparable with the linear dimensions of the cloud.

(2). The same result follows from the following simple reasoning. Imagine a spherical cloud of radius S , containing n spherical particles, each of radius R and density σ , suspended in a medium of viscosity μ , of negligible density, for example a cloud of minute drops of water in air. Then currents will take place in the spherical cloud and it will attain a certain velocity as a whole, which may be calculated after the formula (2), just as if the cloud would form a homogeneous medium of density ~~$n\sigma$~~ $n\left(\frac{R}{S}\right)^3$ and of the same viscosity as the outer medium. The mass velocity resulting therefrom, of amount $\frac{4}{15} \frac{nR^3 g}{S\mu}$ is superposed upon the displacement of the particles, relative to the moving medium, taking place with velocity $\frac{2}{9} \frac{R^2 g}{\mu}$. Thus evidently the downward velocity will be much increased, and Stokes' law cannot be true even approximately, unless $\frac{nR}{S}$ is small in comparison to unity. This condition shows that Stokes' law can be applied only to particles constituting clouds of exceedingly scarce crowding, and it is easily seen that it would be quite erroneous to apply it to actual clouds in the atmosphere, actual fogs, with diminished transparency [as in this case the aggregate cross-section of the particles nR^2 is comparable with the cross-section of the cloud S^2]. As an illustration how cautious we must be in this respect, I may mention that the ratio $\frac{nR}{S}$ amounts to 10 and even to 100, for a cubic centimeter cloud



(11)

as produced by J. J. Thomson and H. A. Wilson, in their experiments # on the determination of the ionic charge.

§8. What has been said, applies of course only to clouds moving in an otherwise unlimited medium. The conditions of motion are quite different for a cloud contained in a closed vessel. Prof. E. Cunningham has attempted to evaluate the order of magnitude of the correction ~~to Stokes'~~, to be applied to Stokes' law in this case. His estimate is founded on the supposition that each particle moves approximately in such a way, as if it were contained in a rigid spherical envelope, of radius comparable with half the distance to its next neighbours. Now this supposition does not seem quite evident, although we shall see that it leads to results of the right order.

~~that~~ We can calculate the resultant motion in quite an exact way, if we consider a homogeneous assemblage of equal spherical particles, moving all of them with the same velocity c in the direction of negative X , towards an infinite rigid wall, which we assume to be the plane Y_2 . In this case we see, by making use of H. A. Lorentz's calculation before alluded to, that a moving sphere x, y, z produces at a point ξ , situated on the axis of X , a velocity component

$$u = - \frac{3}{4} \frac{Rc}{n} \left[1 + \left(\frac{\xi-x}{n} \right)^2 \right] + \frac{3}{4} \frac{Rc}{\rho} \left[1 + \frac{x^2 + \xi^2}{\rho^2} + \frac{6x\xi(x+\xi)}{\rho^4} \right] \dots \dots \quad (3)$$

The first part of this expression, containing $n = \sqrt{(x-\xi)^2 + y^2 + z^2}$, is the component of direct motion, according to Stokes; the second part is the component caused by "reflection" at the plane Y_2 ; it contains the distance between the point ξ and

in w^h Durban is not a multi-cultural city, it is hardly to
expect such an open atmosphere
and there is no real unity, the various groups live very well.

Today I had a day off. I went to the beach with
friends to investigate sea weed which is supposed to be
a good source of iodine. I collected some and will
try to extract iodine from it. I also went to the
supermarket to buy some food. I am still not used to
the supermarket system here. It is very different from
the one in India. I am still not used to the
supermarket system here. It is very different from

the one in India. I am still not used to the supermarket system here. It is very different from
the one in India. I am still not used to the supermarket system here. It is very different from
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The weather is still not better. The sun is still not
out every day. The weather is still not better. The sun is still not
out every day. The weather is still not better. The sun is still not
out every day. The weather is still not better. The sun is still not

$$\text{the reflected source } \rho = \sqrt{(x + \xi)^2 + y^2 + z^2}.$$

[12]

The terms with higher powers of $\frac{R}{r}$ have been neglected, as we confine ourselves to the first approximation. The total current produced in the point ξ by the motion of all the particles is equal to: $U = \sum u$, where the summation extends over all their values of x, y, z . Now we might think us ~~not~~ entitled to replace the summation by an integration, ^{is to be} considering that one particle corresponds to the space ΔA^3 , if A denotes ^(area of mean) distance between the particles. In this case the result would be very simple, for we should have:

$$U = \frac{1}{A^3} \iiint u \, dx \, dy \, dz$$

~~2~~ The integrals of the separate terms constituting u can be evaluated explicitly, if we extend them to a cylinder with $\sqrt{2}$ as basis, of height h and of radius R . Then we can use the well known expression for the potential of a disk in points of its axis, and expressions derivable from it by differentiation with respect to ξ , and by these means we find the unexpected result that the integral current U is zero, if we extend the summation to an infinite value of R .

But in reality U is not defined by integration but by summation. Evidently both operations lead to the same result for distant parts of the space, but not for those parts whose distance from the point ξ is comparable with the distances A between two particles. Therefore the resultant current U in points at a great distance (in comparison with A) from the wall will be given by:

$$U = \frac{3}{4} \frac{Rc}{A} \beta$$

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} ... (4)

$$\text{where } \beta = \frac{1}{A^2} \iiint \frac{1}{r} \left(1 + \frac{x^2}{r^2}\right) dx dy dz - \sum \frac{A}{r} \left(1 + \frac{x^2}{r^2}\right)$$

to be extended over a space great in comparison with A , is a purely numerical coefficient. ~~This~~

In order to evaluate β we must know how the particles are arranged. If we suppose ~~for instance~~ an arrangement in ~~a~~ rectangular order, we can get easily an approximate value by explicit calculation ^(and) by integrating over a cube of height H , constructed around the point ξ , which gives

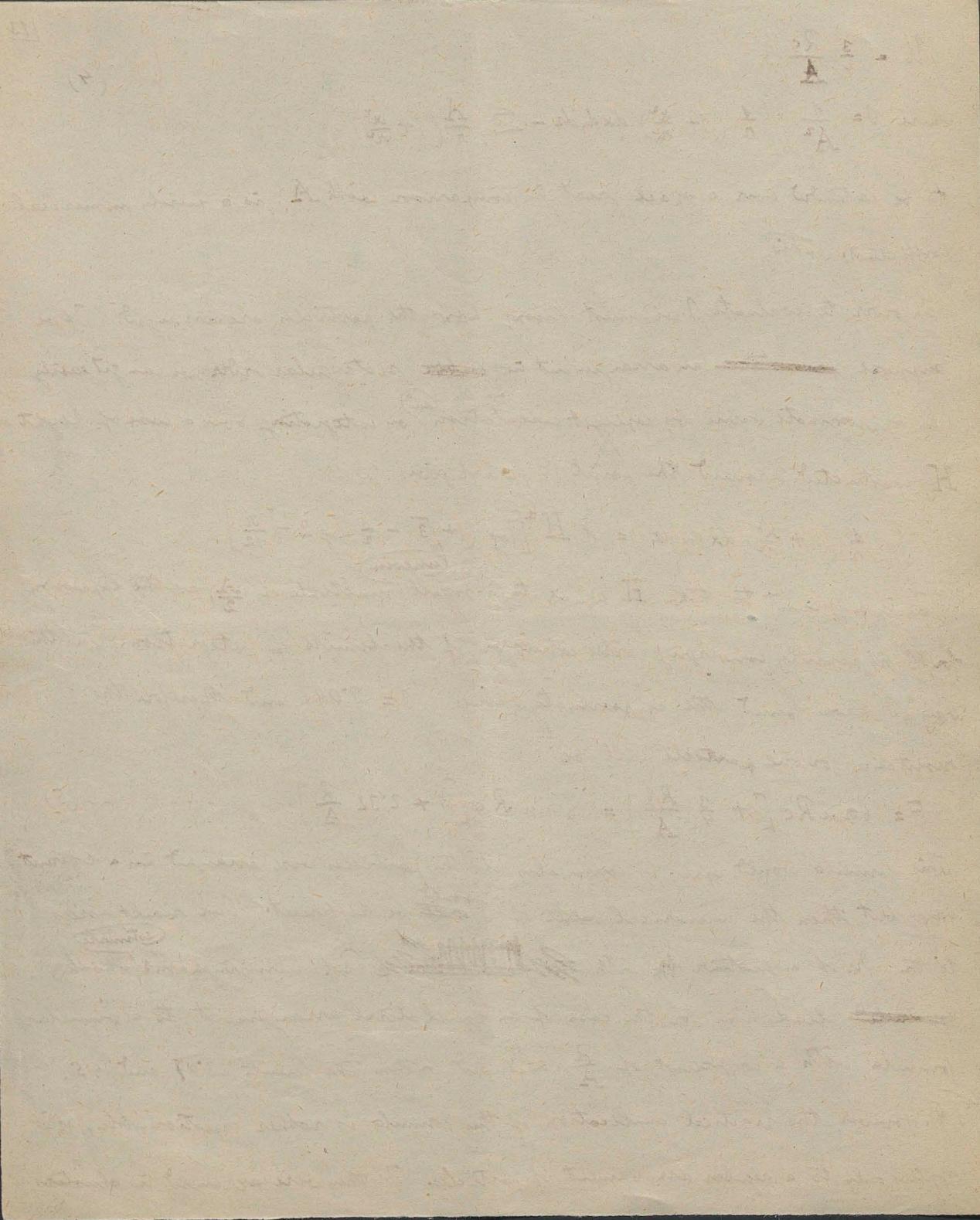
$$\iiint \frac{1}{r} \left(1 + \frac{x^2}{r^2}\right) dx dy dz = 8H^2 \left[\log(1 + \sqrt{3}) - \frac{1}{2} \log 2 - \frac{\pi}{72} \right]$$

It is sufficient to take H equal to a small multiple of $\frac{A}{2}$, as the expression for β is rapidly converging with extension of the limits of integration. In this way I have found the approximate value $\beta = 3.09$, and therefore the resistance for one particle will be

$$F = 6\pi\mu Rc \left[1 + \frac{3}{4} \frac{Rc}{A}\right] = 6\pi\mu Rc \left[1 + 2.32 \frac{R}{A}\right] \quad \dots \dots \dots (5)$$

This formula would apply, of course also if the particles were arranged in a different way, but then the numerical value of β ~~would~~ be different. Our result agrees to the order of magnitude ~~with~~ with ~~Prof.~~ Prof. Cunningham's, which ~~calculated~~ led him for the case of an equilateral arrangement to a similar formula, with a coefficient of $\frac{R}{A}$ included within the limits 3.67 and 4.5.

(§9). However, the practical application of this formula is rather questionable, as it applies only to a regular arrangement of particles. If they were arranged in clusters,



the correction might even become negative. It is interesting to note that ⁽¹⁴⁾ the average value of β , for a particle whose position relatively to the other ones is defined by pure accident, should be zero. That seems quite natural, as the average current of liquid U in the cross section must be zero. Thus it follows what we should not have expected at first sight, ^{and} that to this order of approximation Stokes' law ~~does~~ applies for the particles of (an actual) cloud, on an average with no correction whatever, of this order of magnitude.

The evaluation of the quadratic terms would be much more complicated of course, ^{and the question of the convergence of these developments} as not only the reaction of the ^{then} as then all possible kinds of single reflections (by any one sphere, ^{caused}) have to be taken into account.

The general result of our calculation shows at any rate that Stokes' law is undergoing but small corrections, if applied to the particles of a cloud filling a closed vessel. But it is important to note that things will change entirely, if the cloud is not ^{of} quite uniform density, or if it does not fill the whole empty space between the walls. Then as a rule convection currents will arise, ~~the velocity of which~~ of which in certain cases may be of predominant influence. Their velocity may be calculated approximately, by considering the medium as a homogeneous liquid, subjected to certain forces, the intensity of which per unit volume corresponds to the aggregate force ~~acting~~ acting on the particles contained in it.

Consider for instance an electrolyte in an electric field. If it is conducting in accordance with Ohm's law, the average electric density is zero and no currents will take place. But in bad liquid conductors, with deviations from Ohm's law

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convective currents may arise, which may influence also materially the apparent value of the conductivity. They have been observed long ago, for instance by Warburg¹⁾

Similar movements may be evident in ionized gases, and I think more attention ought to be paid to ^{them} ~~possible deviations from the normal ionic mobility, connected with Stokes' law, in this case than usually is done.~~

In experiments where the saturation current of strong radioactive material is observed between condenser plates wide apart²⁾, these phenomena may be of importance as producing an apparently greater mobility of the ions than under normal conditions.

(10). There is ^{another} application of the theoretical methods exposed above, which may be mentioned. Imagine a two dimensional infinite assembly of equal spherical particles, ^{distributed} uniformly over the plane $x = l$, whilst the plane $y = 2$ again may be supposed to be a rigid wall. Now let all these particles be moving along the plane in direction \vec{V} with the equal velocity c ; what motion will be produced in the surrounding liquid, what will be the resistance experienced by every particle?

According to Lorentz again the motion produced by a single sphere moving parallel to a fixed wall is, with neglect of higher powers of the ratio $\frac{R}{l}$, which we suppose to be a small quantity:

$$v = \frac{3}{4} \frac{Rc}{\pi} \left[1 + \left(\frac{y}{R} \right)^2 \right] - \frac{3}{4} \frac{Rc}{\pi} \left[1 + \left(\frac{y}{R} \right)^2 \right] - \frac{3}{2} \frac{Rcx(x+\xi)}{\theta^3} + \frac{9}{2} \frac{Rcx^2y^2(x+\xi)}{\theta^5}$$

1) Warburg. Wied. Ann. d. Phys. - -

2) F. inst.: Rutherford, Radioactivity p. 35, p. 84.

had good plants and enough to make a fine addition
and I hope we can do well with them at the
show which will be held at the University of
~~on~~
and a week before the show I hope to have the
new varieties of fruit and vegetables and flowers
and some old ones too. I hope to have a
good collection of all kinds of plants and flowers
and I hope to have a good time at the show.

Yours truly,
John C. Steward

according to Stokes,

where the first term is the direct current, while the remaining terms represent the ~~reflected~~ current reflected by the wall, just as in the former example.

[16]

We might also in this case calculate the resultant current ~~as~~ by forming $\sum v$ over all values of y and z , and derive therefrom the resistance of a single particle. But we shall confine ourselves to the following remarks.

In the extreme case where the particles are so crowded, as ~~to~~^{nearly} touch one another, a lamellar flow will take place in the liquid, between the fixed wall and the plane $x=l$, with a velocity $v = \frac{cx}{l}$, while on the other side of the plane $x=l$ the liquid will be dragged along by the moving particles with the constant velocity c . The frictional force per unit of surface of the plane $x=l$ is evidently equal to $\frac{mc}{l}$, therefore the resistance experienced by each particle is

$$F = \frac{\mu c A^2}{l}$$

which is much smaller than Stokes' law would indicate, as A is of the order of R , but the distance l is supposed to be of higher order.

Now consider the other extreme case, where the distances A between the particles are so great, that Stokes' law is approximately valid, which requires A to be of order l . Let us calculate the resultant motion of the liquid, for points at infinite distance from the wall ~~near~~ ($\xi = \infty$). For such points the summation mentioned above can be replaced by integration; besides we can put: $\frac{1}{r} - \frac{1}{\rho} = \frac{2l^2}{r^3}$,

$$\frac{1}{r^3} - \frac{1}{\rho^3} = \frac{6l^2}{r^5}; \text{ and thus we get}$$

$$V_\infty = \sum v = \frac{9\pi cl^2}{A^2} \iint \frac{y^2 dy dz}{(\xi^2 + y^2 + z^2)^{5/2}}$$

Brackets indicate the number of species found in each tree species
grouped by genus. The following is a list of the species of trees
which are known to grow in the State of Oregon in addition to
those which are known to grow in the State of Washington.
The following table shows the number of species of each tree species
which are known to grow in the State of Oregon. The following
table shows the number of species of each tree species
which are known to grow in the State of Washington.

$$A = \underline{\underline{A}}$$

The following table shows the number of species of each tree species
which are known to grow in the State of Oregon. The following
table shows the number of species of each tree species
which are known to grow in the State of Washington.

$$\frac{a}{x^2 + x^2 + 1} = A$$

This integral can be transformed by putting: $y = s \sin \varphi$, $z = s \cos \varphi$, $dy dz = s ds d\varphi$,
 and we get finally: $V_\infty = \frac{6 R l \pi c}{A^2}$

By comparing this with Stokes' law for the resistance F we have

$$V_\infty = \frac{F}{A^2} \frac{l}{\mu} \quad (\text{in both cases})$$

that means that the liquid at a great distance from the wall will be dragged along, in a parallel direction to it, with such a velocity as if the force corresponding to unit surface $\frac{F}{A^2}$ were distributed uniformly over the liquid, in a plane at a distance l from the fixed wall. This result, which can be generalized for a greater number of similar layers, seems natural enough, if the distances between the particles are small in comparison ^{with} their distance from the wall, so that the liquid can be considered as if forming a homogeneous medium, but we see it remains true for particles widely apart. Without going into further details, I may only mention that this result has an important bearing on the theory of electric endosmose, which I am going to will be explained elsewhere with full details.

(§11). I may conclude with ^{a brief} ~~a remark~~ about the influence of the inertia terms in the hydrodynamical equations (assumption I), which have been neglected as well in Stokes' original calculations as in the above reasoning. It is well known that this neglect is justified only, if the ratio $\frac{R c}{\mu}$ is small in comparison to unity. But it has been proved by Oseen ¹⁾ in ~~an interesting~~ paper, commented upon ~~by~~ by Lamb, that the solution given by Stokes is defective, even if this

¹⁾ Oseen, Actas f. mat. astr. physik 6 (1911); H. Lamb, Phil. Mag. 46 p. (1911)

$$\frac{S \cdot A}{A} = S \text{ : surface area}$$

area of a rectangle is equal to the product of its length and width

$$\frac{L \cdot W}{A} = L \text{ : width}$$

area of a rectangle is equal to the product of its length and width
length times width is also called perimeter and is equal to twice the sum of the length and width of a rectangle

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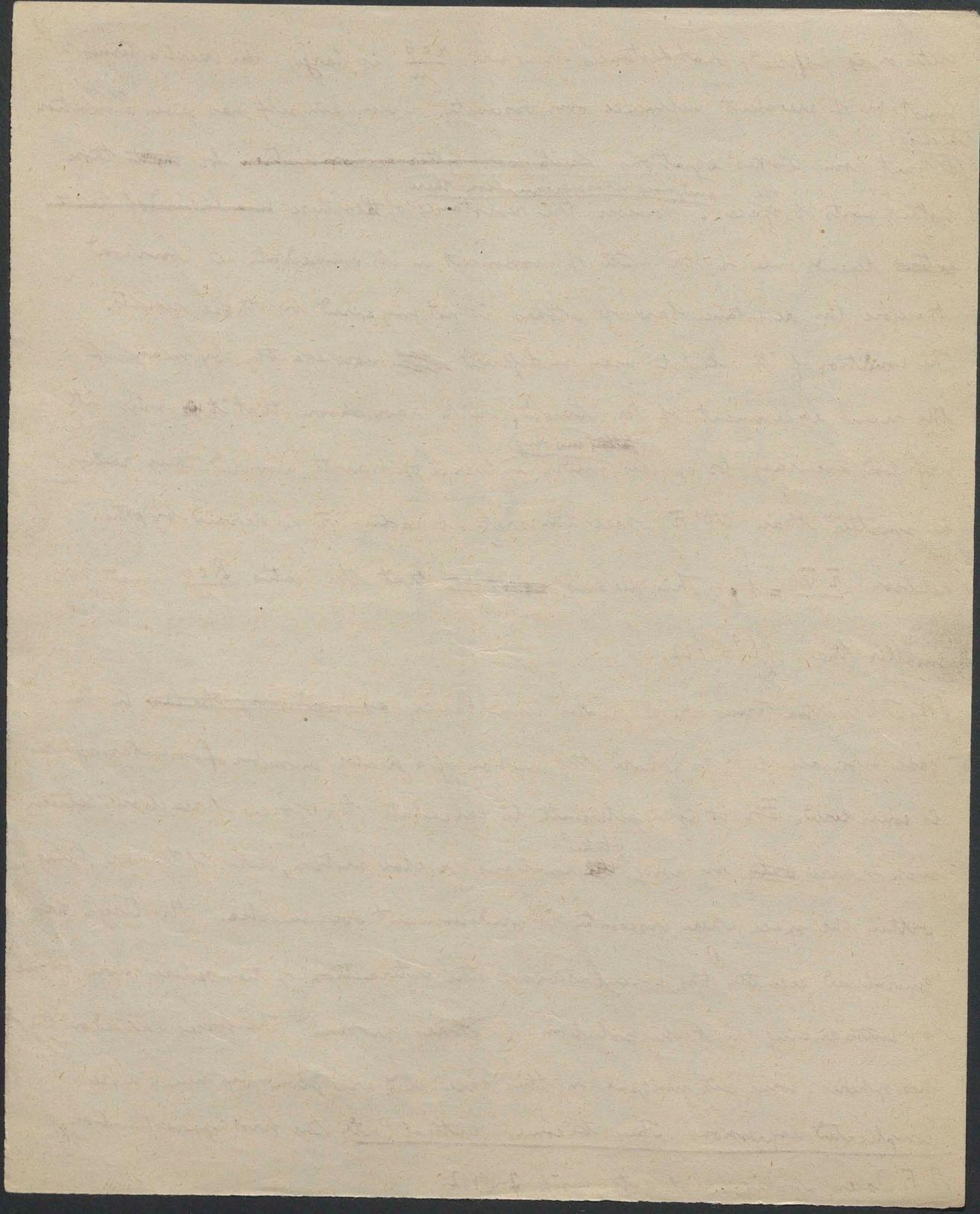
area of a rectangle is equal to the product of its length and width
length times width is also called perimeter and is equal to twice the sum of the length and width of a rectangle

18

condition is fulfilled; for at distances r where $\frac{rc^6}{\mu}$ is large, the inertia terms must be of prevalent influence over viscosity. Oseen himself has given a solution which is different from Stokes' equations which gives better approximation for those distant parts of space. However, the resistance of the sphere in liquid of finite extent depends only on the state of movement in its immediate neighbourhood, therefore the resistance law of Stokes is not impaired by those results. The condition of its validity may be defined more exactly by means of the recent experiments of Mr. Arnold, which have shown that it holds with very good accuracy for spheres moving under influence of gravity, provided their radius is smaller than $0.6 \bar{r}$, where the critical radius \bar{r} is defined by the relation $\frac{\bar{r} c^6}{\mu} = 1$. This means that the ratio $\frac{R c^6}{\mu}$ must be smaller than $(0.6)^3 = 0.22$.

§12. The inertia terms are of greater importance, ~~as regards~~ than in the case before alluded to, where the motion of a greater number of similar spheres is considered. For it is legitimate to calculate the forces of reaction between such spheres ~~by~~ by using ^{Stokes'} equations for slow motion, only if they are lying within the space where viscosity is predominant over inertia. Mr. Oseen has generalized recently¹⁾ the calculation of the interaction of two spheres given by me, by introducing in it his solution of Stokes' problem. The forces exerted on the two spheres come out unequal in this case and are given by much more complicated expressions. They become identical with the first approximation.

¹⁾ F. Oseen, Arkiv f. mat. astr. fysik 7 (1912)



(19)

given by me, if the distance r between the spheres ^{two} satisfies the condition
that $\frac{rcb}{2u}$ is small. Mr. Osun thinks this to be a great restriction
on the validity of those formulae for experimental purposes, but he ~~himself~~ omits
the factor 6 in the above expression. We satisfy ourselves easily that for
instance in the case of waterdrops in air, as in Mr. J. J. Thomson's and
H. A. Wilson's condensation experiments, r is of the order of several centimeters;
in Perrin's experiments on the validity of Stokes' law for the particles of emulsions
it would amount to hundred of meters. It is also sufficiently great for
direct experiments, when highly viscous liquids are used, as Zadenburg did in his
laboratory research. Ordinary hydraulic experiments, with water and spheres of
a size to be handled conveniently, are excluded of course when Stokes' law or
any of those modifications are in question.
(One might try to apply)

Osun's method of approximate correction for inertia ~~will be applied~~ also to
the other cases treated above, but it will imply rather cumbersome calculations
and besides, for movements in closed vessels it will be generally of lesser importance
than in a liquid extending to infinity.

~~Done at Fort Ticonderoga~~

rkp. 9353

~~Some~~ Remarks on Stokes's Law and Some Applications of it.

~~On the Practical Applicability of Stokes's Law~~

(~~the practical applicability of~~
On Stokes's Law (~~and~~ ~~the corrections~~ required in certain cases.
~~of Resistance~~) ~~and applications~~)

~~A brief slip and the force exerted between~~

parts

~~The Surface Slip~~ Stokes's Law for the resistance of a sphere in a viscous medium has recently acquired greater importance for modern physics than any other result of theoretical hydrodynamics, especially by its application to the determination of the electronic charge and to theory of Brownian movement.

But the limits of its applicability are not always ~~clearly~~ ^{realized}; they are

thus

~~So the question of the limits of its applicability and of the corrections to be introduced in certain
cases is very actual only, and I should like to add some contributions to these developments.~~

The calculation

this subject

Stokes ~~assumes~~ rests, as is well known, on the fundamental assumptions

~~in the hydrodynamic equations~~

I. That the motion is so slow that the inertia terms ~~may~~ may be neglected

II. That the liquid is adhering completely, without slipping, ^{to} ~~to~~ the surface of the sphere, ~~they consider~~ ^{this} as a rigid body.

III. That the liquid is unlimited and at rest at infinity.

The first condition ~~restricts~~ imposes the most important restriction on the validity ^{of S-} ~~as it requires the radius~~ of the sphere being small in comparison to the value $\bar{r} = \frac{\mu}{\rho v}$, where μ is the viscosity, ρ the density of the liquid, v the velocity of the sphere. ~~The careful experimental study of D. Smold,~~ published recently in the Phil Mag., shows that the ^{law of} resistance is applicable provided the radius be ~~less than~~ $r = 0.6 \bar{r}$ ~~but the adhesion law does not hold~~ ~~which would imply for v~~

Now as the critical radius for a water drop in air moving under influence of its gravity is $\bar{r} = 5 \cdot 10^{-3}$ cm we see that this condition is certainly satisfied for the minute droplets of diam. ^{which} ~~the~~ ^{influence} experiments of Thomas Wilson ^{and to the great number made with} ^{using it in gaseous} ~~and by Thomson, Weber, Reichenauer~~ to determine the ion charge, and ~~for Perrin~~ ^{and with maximum} ~~conducted experiments on~~ ^{suspensions of} ~~the~~ ^{in water} ~~of the particles of a given size~~. ~~that~~

On the other side ~~it has been proved that the solution given by Stokes is defective for~~ ^{in very strong} ~~for spheres of any diameter if the motion occurs~~ ^{at} ~~in distances exceeding the critical radius is considered.~~ ^{is bounded by the place} ^{in parts of the space} ^{when the inertia terms are of greatest influence.}

~~Osman himself has shown that the case~~ However, Stokes's law ^{of resistance} remains approximately valid, under the above conditions and it is not necessary in so far ~~as resistance~~ to replace the simple relation of Stokes by the rather ingenious but complicated method ^{indicated by Osman and Lamb.}

^{which give a better expression of the hydrodynamic problem}
^{for great distances from the sphere.}

*Dobut
Lamb Jr²*

and common name, a number of which are of great interest.

*especially about the Atlantic coast, and the following species
have been noted from the coast of Demerara.*

Leptoscarus *leptoscarus* *leptoscarus*
Leptoscarus *leptoscarus* *leptoscarus*

(notable & difficult to find in the sand on the beach)
Leptoscarus *leptoscarus* *leptoscarus*

* Phil Ry. 22, 755, 1911

* Bull Sc. Crac. 1911 p. 40 (January)

* C.R. 152, p. 1735 (1911)

In the following [I shall not go into further consideration of the influence of inertia terms, but] I should like to call attention ~~to~~ ^{now of the above named} to the conditions.

~~If instead of the~~ It is easy to ^{generalise} ~~introduce~~ in Stokes calculation (the general supposition that the liquid relatively to ~~st~~ the sphere is moving along its surface with $v = \beta F$ ^{surface layer of}) a tangential velocity ~~is~~ proportional to the frictional force, which in the case of a parallel lamellar flow assumes the form $\beta v = \mu \frac{dx}{dy}$

In this case as (Stokes and Dassit have shown, the ^{simple} law of Stokes has to be replaced by

$$F = 6\pi\mu ac \frac{\beta a + 2\mu}{\beta a + 3\mu}$$

~~Thus the frictional resistance is diminished by~~ ^{would be,} ~~slipping~~ ^{surface slip and the other minimal value, for the case of infinite slip, $\beta=0$ ^{with infinite surface friction} two thirds of the maximal value, for no slip.}

Now ~~it is generally assumed~~ that the slip of liquids at solid ~~walls~~ walls is negligibly small; ^{on account of the results of Poinsot, Reichen, Courant, Lederberg}

~~Dr. Smold's recent research~~ proves, ^{by the method of D. Smold} that the coefficient of sliding friction β is certainly greater than 5000 and probably greater than 50,000.

~~According to Dr. Smold~~, I think it even probable that the coefficient β may be of the order 10^6 ^{would result} still greater values ^{experienced by}

from the fact that even the resistance of electrolytic ions agrees ~~with~~ in its order of magnitude with Stokes law.

On the other side (^{concluded from his} Dr. Smold ~~not yet published~~ ^{clearly} results) that the slip at ^{the} ~~the~~ interface between gas and liquid

~~for bubbles of gas moving through liquid~~

is infinite, ^{provided the surface is not contaminated with liquid film} ~~that was not an~~ ^{except in the case where solid films are formed on the surface.} ^[Explanation of Dr. Smold experiments]

Now I think a different interpretation preferable, as in the case of gas bubbles or ~~the~~ liquid drops also the interior fluid is ~~passing~~ ^{subject to circulation.} ~~which is not~~

Some time ago I advised Mr. Rybczynski in Lemberg to calculate the motion of a viscous sphere through viscous liquid. (The result ~~has been~~ published ^{January} last year and which has been ~~published~~)

The calculation is surprisingly ~~easy~~ ^{easy} deduced ⁱⁿ half a year later quite independently by M. Hadamard, is equally simple. It shows that in the case of slow motion the inner liquid retains its spherical shape and that the resistance ~~of~~ ^{of} ~~the~~ ^{inner} ~~outer~~ ^{inner} ~~outer~~ is

$$F = 6\pi\mu' a c \frac{3\mu' + 2\mu}{3\mu' + 3\mu}$$

where μ' designates the viscosity of the liquid ^{of which} the moving sphere is composed. ^(or liquid drop without slip)

Comparison with the above formula shows that the resistance ^{of} ~~of~~ a gas bubble is the same as the resistance of a solid sphere with a coefficient of surface friction $\beta = \frac{3\mu'}{a}$; ~~and~~ in fact the stream lines and velocity of the outer liquid are identical in the two cases. In the case of Dr. Smold the experiments the coefficient μ' was negligibly small - compared with μ which had the same effect as if there were an infinite slip at the surface. It would be interesting to verify the ~~exact~~ formula

(I should like to add)

In the following some contributions ~~will~~ may be added to the one, discussed above of the validity of the law of resistance to the discussion of its validity in part
and of the corrections to be applied for practical purposes
and besides to ~~and~~ and besides ~~with~~ hydrodynamical problems connected with it. It will be
seen to point to some interesting

by experiments on liquid drops ~~moving~~ in a liquid medium of similar viscosity; the total film forming so easily certain fluid masses may be a serious obstacle to movement of this kind [3] 23

Thus it seems that real slipping does not occur except in one case where
so far

Thus also Dr. Knudsen's results are explained without the assumption of deep surface slip.

However ~~This is a case of~~ ^(the existence of) surface slip has been proved ~~to~~ and its magnitude determined by beyond doubt by the old experiments of Knudsen ^{in rarefied gases, since the old experiments of K. S. L.} by direct experiments and where its order of magnitude has been explained by theory:

As you will know have shown ^(the magnitude of the coefficient of slip $\mu = \frac{\mu}{\beta}$ is, according to the Knudsen theory and also to the old experiments of K. S. L., about approximately equal to the order of the mean length of the free path of the gas molecules. Therefore the phenomenon of slipping is to be considered minute) play an important part even at ordinary pressures (in the motion of particles whose dimensions are smaller than ^{dimensions} ^{finites} and especially in the laboratory of R. Kuhn's) as in Trillat's experiments.

~~Now Dr. Cunningham has undertaken to deduce by aid of the Knudsen theory a theoretical formula for the resistance~~

Now it would seem natural to use formula () for this case, with substitution of the empirical value for μ , but that would not be such a procedure would give ~~entirely~~ quite erroneous results, except for the case of a comparatively small ~~amount of~~ slipping. For if the mean length λ is comparable with the dimensions of the moving sphere the validity of the hydrodynamic equations for viscous motion is imperceptible altogether, ^{since the condition} ^{application} ^{now} ^{implies} ^{underlying} assumption underlying them, that the state of the gas is ^{varying little} ^{undergoing small} ^{varying little} ^{over distances comparable with λ} , is impaired.

Dr. Cunningham has undertaken to deduce by aid of the Knudsen theory a theoretical formula for this case. The general form of his formula $F = \frac{6\pi \rho a^2}{1 + A^2}$, [where A is a numerical coefficient depending on the way how the resistance between the molecules before their impact on the surface of the sphere, and on the way how the mean length of free path λ is defined] is in remarkable good agreement with the experiments of Dr. Kuhn and Knudsen. However, ^{His} the theoretical deduction is not really a demonstration, and I think Dr. Kuhn is right in trying to get closer approximation by empirical different formulas. However the general agreement of Dr. Cunningham's formula proposed by him with the experiments of Dr. Kuhn and Knudsen is remarkable.

Cunningham $F = \frac{6\pi \rho a^2}{1 + A^2}$ ^{interpretation formula which agrees} it is a very simple and ^{consequently} well with the exp. of Kuhn and Knudsen.

But it is certainly preferable to write it in the form $F = 6\pi \rho a^2 \left[1 + \frac{B}{a^2} \right]$ where ρ is the ^{density} of the gas, as mistakes are easily involved by ^{using} ^{indefinite} of the mean length of free path λ , which is a very indefinite term and really has no precise meaning.

It must be taken exactly what the ^{resistance} ^{term} ^{is}. For great simplification the ^{resistance} ^{is proportional} to the cross-section of the sphere and for this case the calculation can be carried out ^{quite} exactly if the way is

* This was in Yangtze to the μ .

Archie f mott & off. 6,
Lands Thib May

Archie ... 7 N 33(1912)

14
The third of the above named conditions is imposed, namely,

Now let us examine what modifications are required in Stokes' law, if (the ~~largest~~ viscous medium is) limited by ~~solid~~ walls, or if a greater number of similar spherical bodies are contained in it.

In this case the linear form of the hydrodynamical equations makes it possible to attain their solution by a method of successive approximations, consisting in superposition of

additions to those in the theory of electromagnetism

It consists in the superposition of additions ~~which~~ ^{such as} to destroy the residual motion at the parts of the boundaries represented by solid walls. ~~These are formed as if the fluid would extend to infinity.~~

In this way H. A. Lorentz has calculated the correction of the first order to be applied to the resistance of a sphere ^{moving} (in a normal or in a parallel direction to it).

in the neighbourhood of an infinite plane wall, ^{concerning the function} ^{of R in wave, $\frac{1}{16}$ of wave length.}

M. Stock in Zemberg has extended the calculation to the fourth order of approximation (the fourth power of the ratio $\frac{R}{a}$, when R is the radius of the sphere, a its distance from the wall)

$$\text{His result is } F = 6\pi\mu R \cdot \left(\frac{1}{1 - \frac{9}{16} \frac{R}{a}} - \left(\frac{R}{2a} \right)^3 \left(1 + \frac{9R}{16a} \right) \right)$$

In the case of motion normal to the plane wall the correction is approximately twice as great

In a similar way it has been found by Zemberg that in the case of a sphere moving along the axis of cylindrical tube of radius p the resistance ~~is~~ ^{calculated after Stokes} increased in the ratio

$$1 : 1 + 2\frac{4}{5} \frac{R}{p}$$

and by adding the correction deduced by H. A. Lorentz also the influence of the ^{rigid} bottom of the tube can be taken into account.

Zemberg's improved formulae ^{for so as} ^{the condition of zero velocity, $\frac{R}{p}$ order, $\frac{1}{16}$ of wave length.} take into account both the influence of the cylindrical walls and of the plane bottom and this ^{his own very elaborate experiments of} Zemberg's formula.

It has been verified with some ^{accuracy} quite satisfactory approximation by Zemberg's own experiments and by those of Smoluchowski.

The paper published last year I had

Now I should like to point to some interesting results, concerning the motion of a finite number of similar spheres ~~I will give you I have published~~ and I may be allowed to extend these investigations still further now.

Imagine a sphere of radius a ^{moving with the velocity c in the direction of x} ~~the axis of rotation of the body~~ ^{on this axis} from the origin. If ^{of an unextended} the fluid is unextended ^{length of the wave $\frac{R}{c}$,} this would produce at the point P

(with coordinates $\{x, y, z\}$) certain current velocities u, v, w (defined by the well known Stokes' equations). But if we assume a ^(this point is the centre of rotation) sphere of radius b situated at that point, we have to superpose a fluid motion u_1, v_1, w_1 ^{and} satisfying the condition of rest for infinity and ~~which~~

~~chosen so as to annul the velocities of the primary motion at the points of this sphere.~~

and I have written down what I can get at present. I have not been able to get much information from the ~~new~~ old manager, as he has been here only about 6 months & has not had time to learn much. He says that nothing happened to the ~~new~~ old manager after he left, but that he was not told what it was.

and I have written this as well as the following notes by my calculation.

~~He said he left because the new manager was not a good man. He said he~~ ~~left because the new manager was not a good man. He said he~~

left because he was not allowed to do his work the way he wanted to do it.

He said he left because he was not allowed to do his work the way he wanted to do it.

He said he left because he was not allowed to do his work the way he wanted to do it.

He said he left because he was not allowed to do his work the way he wanted to do it.

He said he left because he was not allowed to do his work the way he wanted to do it.

$$\left[\frac{1}{10} + \frac{1}{10} \left(\frac{1}{10} - \frac{1}{10} \right) \right] \times 100 = 20\%$$

~~He said he left because he was not allowed to do his work the way he wanted to do it.~~

~~He said he left because he was not allowed to do his work the way he wanted to do it.~~

~~He said he left because he was not allowed to do his work the way he wanted to do it.~~

~~He said he left because he was not allowed to do his work the way he wanted to do it.~~

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~~He said he left because he was not allowed to do his work the way he wanted to do it.~~

~~He said he left because he was not allowed to do his work the way he wanted to do it.~~

may be called the motion reflected; it)

This motion can be found fairly with any degree of approximation by making use of the solution of the hydrodynamical equations given by Lamb in ~~the~~ spherical harmonics. [5] 25

In point at a greater distance of P it is about

Similarly it ~~must~~ be of the order of magnitude of $\frac{c^2}{R}$ at the surface of the sphere b , which is its centre of origin. It seems ~~as~~ probable a priori that it will be of the order $\frac{c^2}{R}$ at distances ^{second} from P or compare ourselves to.

Thus ~~we see that~~ ^{source?} ~~so far as terms of higher order than c^2/R are neglected,~~ we can apply

a simple method of evaluating the mutual influence of such spheres, ~~since we can identify the velocity at points of their surfaces with the velocity which would~~ ^{between} by neglecting the difference of the values of the ~~velocity at the centre of the second sphere~~ ^{the points of intersection} at the surface.

That is to say; the effect of motion v_0, v_0, w_0 at the

front sphere experiences a reaction by virtue of the presence of the sphere b ~~which~~, ~~such~~ ~~from Stokes~~ (as if the sphere b would execute simultaneously the three motions $-u_0, -v_0, -w_0$) ^{according to the} and give rise the three current systems resulting therefrom (~~in the usual formulae of Stokes~~) produce at the centre of the first sphere nine current components and give rise to nine components of frictional force, to be calculated ^{and} according to ^{the law of} the resistance.

the ~~second sphere~~ sphere b being at rest, is subjected to frictional forces

$$X = 6 \pi \mu a v_0$$

$$Y = 6 \pi \mu a v_0$$

$$D = 6 \pi \mu a w_0$$

on account of the motion of a , on the other side

If both spheres are in simultaneous motion the effects result ~~by~~ ^{balanced} superposition of the forces corresponding to the cases where one ~~of~~ ^{two} of them is moving and the other at rest.

In this way an interesting conclusion is obtained for the case when both spheres are moving in the parallel lines with equal velocity: then ~~the~~ ^{one component in the direction} both are subjected to equal forces: ~~this resistance of motion is diminished~~ ^{tending to diminish the resistance} (by the amount $\frac{q}{2} \frac{2\pi \mu c}{R} \left[1 - \frac{3}{4} \frac{a}{R} \right]$)

The other component along the line joining the centres and directed from back to front sphere radius vector from the ~~following~~ sphere to the ~~preceding~~ centre of the sphere which follows to the plane which goes

$$\text{of amount: } \frac{q}{2} \frac{2\pi \mu c \cos \theta}{R} \left[1 - \frac{q}{4} \frac{a}{R} \right]$$

(where θ is the angle between the line of centres and the direction of motion)

Analogous methods are to be applied for a greater number of spheres. The motion is ~~not~~ ^{is} ~~repeated~~ ^{for the} by superposition of ~~particular~~ singular solutions where one sphere is supposed moving and all the others at rest. Each of the component solutions ^{results from the direct action and its reflections at the} ~~other~~ spheres.

and it was found out when you wrote me the first time of my failure with
the same kind of powder in that you were unable to get a good
~~explosion~~

So consider this & you will be able to get a good explosion by the same
processes. If this is difficult to do, why don't you try to change
the gunpowder to something else? I have never had any trouble
with gunpowder, but if you can't get a good explosion, then
you may have to change (it) and the only way to do this is to
try different kinds of gunpowder.

* M. Th. P. I f²³ (1906)

* 27 (This increase of resistance ~~ought to be~~ ^{is} taken into account in M. Th. P. I's determinations of the time charge; it may produce an increase of the order of one thousandth.)

* Bull. A. C. 1911 f. 18 (1911)

This note is made to answer the question of whether a gunpowder gun
can be used with the dynamite charge. There is no reason why not, and
(with a slight ~~hesitation~~) repeat you can and you have nothing to worry about.
The powder gun is not very dangerous, and we have the following table:
which shows the velocity of the gunpowder gun and the velocity of the
dynamite gun.

* Bull. A. C. 1911 f. 23, 447 (1907)

and having a lighter gun has less recoil & is safer.

all right

all right

all right

has nothing to do with the gunpowder gun.

and all is well again. Now the gunpowder gun is no longer hot.

* Dynamite. Leipzig 1906 (Part)

The author of this note is unknown. He writes as follows:
"I think we find ~~nothing~~ with dynamite gun, because * Bull. A. C. 1911 f. 28
[257] the gunpowder gun is not so good as the dynamite gun." (Note: 257 is a reference to the note above.)

and it is not good to have the gunpowder gun, as you have to wait
until it is cool before you can use it, and otherwise
there is no good gunpowder gun.

[257] See note 2; turned to

Author of note 2 has written to me and stated that it is only
when there is a gunpowder gun that you can't get a good explosion,
but if you have a gunpowder gun, it is not good, except when you fire it quickly
and then it is not good. In this case it is not good, ~~but~~ ^{but} when you fire it quickly

Now if the parallel motion of ~~n~~^{similar} spheres is considered, the resistance of each of them will be diminished by an expression involving after powers of $\frac{n}{S}$ and ~~with~~ the first term of which will be of the order of magnitude $\mu c^2 \sum \frac{1}{R}$

Thus Stokes law of resistance will not be (even) approximately, and the development will cease to be convergent, in general unless $\frac{n}{S}$ is small, where S denotes a kind of mean distance comparable with the linear dimensions of the cloud of spheres.

The same result follows ~~from~~ by the following simple reasoning. Imagine a spherical cloud of radius S , containing n spherical particles of radius a and density δ , suspended in a medium of viscosity μ , ~~and~~ density, say ~~that of water~~ a cloud of minute drops of water in air, for instance. Then ~~the spherical cloud will move with a velocity~~ currents will take place in the cloud just as if the cloud would form a homogeneous medium of density $\frac{n\delta^3}{S^3}$ and of viscosity μ as the outer medium, namely

$\frac{4}{15} \cdot \frac{3}{9} \frac{\delta n a^3 g}{S^3 \mu}$; this mass ~~velocity~~ is superposed upon the relative displacement of the particles relative to the moving medium. (Thus evidently Stokes' law cannot be true even approximately taking place with rate $\frac{2}{9} \frac{\delta a^3 g}{S^3 \mu}$) This condition shows that Stokes law ~~can~~ can be applied unless $\frac{n}{S}$ is a small (~~insufficient to unity~~). This condition shows that Stokes law ~~can~~ can be applied only ~~in the case of~~ to particles constituting ~~the~~ clouds of exceedingly rare distribution transparent consistency. It is easily seen that it is quite erroneous to apply it ~~in the case of~~ ^{of the particle} scarce crowding ~~with diminished transparency~~ [as in this case the aggregate cross section $n a^2 \pi$ is ~~not~~ comparable with the cross section of the cloud πS^2]. (For illustration) I may mention that the ratio $\frac{n}{S}$ ~~is~~ amounts to 10 and even 100 for ~~a~~ a cubic centimeter cloud produced by JJ Thomson and A. Wilson in their experiments on determination of the ionic charge.

Of course what has been said above applies only to clouds in an otherwise unlimited medium. The conditions of motion ~~in a cloud vessel~~ ^{for a cloud contained} are quite different.

Mr. E. Cunningham has attempted to evaluate ^{the order of magnitude of} (the condition of ~~it~~) ^{approximated} to be applied to this case. ~~He makes~~ His ^{estimate} calculation is founded on the supposition that each particle moves approximately in such a way as if it were contained in a rigid spherical envelope of radius comparable with the distance to its next neighbors. Now this supposition does not seem quite evident although ~~it~~ or shall that it leads to a result of the right order ~~result~~ ^{agreed} ~~by no means~~ evident to me and I think it leads to an erroneous result.

^{But if} We can calculate the resulting motion in quite an exact way, if we consider ~~an ensemble~~ ^{homogeneous} of equal spherical particles ~~moving~~ moving all of them with the same velocity c in the direction of negative X toward an infinite rigid wall which we assume to be the plane $Y2$.

I have now completed all the calculations for the case of a rectangular arrangement of particles. The results are as follows:—
 The total force of attraction between two particles of radius R and separation a is given by the formula

$$F = \frac{6\pi \mu e c}{R} \left[1 + 2.32 \frac{a}{R} \right]$$

 where μ is the density of the particles, e is the charge per unit area, and c is the constant of proportionality.
 The potential energy of a system of n particles is given by the formula

$$U = \frac{1}{2} \sum_{i,j} F_{ij} r_{ij}$$

 where r_{ij} is the distance between the i -th and j -th particles.
 The work required to move a particle from a point P to a point Q is given by the formula

$$W = \int_{P}^{Q} U dr$$

 where dr is the differential displacement along the path of the particle.
 The total work required to move all the particles from their initial positions to their final positions is given by the formula

$$W_{total} = \sum_i W_i$$

 where W_i is the work required to move the i -th particle from its initial position to its final position.
 The total energy of the system is given by the formula

$$E = \frac{1}{2} \sum_{i,j} F_{ij} r_{ij}$$

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In order to calculate F we must know how the particles are arranged. This is done

If we suppose for instance arrangement in cubic order, we can get the value by explicit calculation, for a cube of length H by the formula

$$F = \frac{6\pi \mu e c}{H^2} \left[\log(1+1/3) - \frac{1}{2} \log 2 - \frac{\pi}{12} \right]$$

It is sufficient to take H equal to a small multiple of R as the expression for F is rapidly converging with extension of the limits of integration. In this way I have found the approximate value $F = 3.09$

whence the resistance of one particle will be

$$6\pi \mu e c \left[1 + 2.32 \frac{a}{R} \right]$$

The minimum ^{constant} absolute force for an equilibrium arrangement is $6\pi \mu e c \left[1 + 3.67 \frac{a}{R} \right]$ this is of the ^{approximate} order of magnitude.

The result agrees to the order of magnitude with the

conclusion by his reasoning that the value of the constant c must be included between the limits

$$\frac{1}{12} \pi R^2 \mu e c$$
 and $3.67 \mu e c$

we can apply H.A. Lorentz formula before added to and the result as follows:

In this case ~~a moving~~ a spherical particle whose coordinates are x, y, z is producing by its motion a velocity β component at a point ~~illustrated~~ situated thusly 27

$$u = -\frac{3}{4} \frac{\alpha c}{r} \left[1 + \frac{(\xi-x)^2}{r^2} \right] + \frac{3}{4} \frac{\alpha c}{\rho} \left[1 + \frac{x+\xi^2}{\rho^2} + \frac{6x\xi(x+\xi)}{\rho^4} \right]$$

here r is the distance between the two points $\sqrt{(x-\xi)^2 + y^2 + z^2}$ and ρ is the distance between the reflected source and the point in question $\rho = \sqrt{(x+\xi)^2 + y^2 + z^2}$.

The first part of this expression is the component of direct motion ~~according to the~~ according to the usual formula of Stokes; the second part is the component of reflection ~~caused by~~ at the plane V_2 ; according to H.A. Lorentz and ~~containing~~ ρ is the distance between the point ξ and the reflected source.

$$\rho = \sqrt{(x+\xi)^2 + y^2 + z^2}$$

Higher powers of $\frac{\alpha}{r}$ have been neglected, as we confine ourselves to the first approximation.

The total current produced in the point ξ by the motion of all particles is equal to the ~~sum~~

~~U =~~ $\sum u$ where the sum is to be extended over all values of x, y, z . Now we might ~~it justifies~~ think us entitled to replace the summation by an integration, considering that one particle corresponds to the space R^3 , (if R denotes the distance between the particles). In this case the result would be very simple, for we ~~should~~ have

$$U = \frac{1}{R^3} \iiint u \, dx \, dy \, dz$$

and the integrals ~~can be evaluated~~ ^{especially} if we ~~had~~ extend them to a cylinder with V_2 as bases, of height h and of ~~radius~~ radius G ; ~~and if we make~~ use of the well known expression for the potential of a disk in points of its axis, and of similar expressions derivable from it by differentiation with respect to ξ ; we find the unexpected result that the integral current U is zero, if we extend the summation to an infinite value of G .

But it does not follow that the ~~it~~ But in reality U is not defined by integration but by summation. Evidently ~~both~~ both operations lead to the same result for distant parts of the space but not for those parts whose distance ^{from the point ξ} is comparable with the distances ^(R) between two particles. The difference is of the order $\frac{1}{R}$. Therefore the resultant current U in ~~points~~ points at a great distance (in comparison with R) from the wall will begin

by: $U = +\frac{3}{4} \frac{\alpha c}{R} \beta$

where $\beta = \left(\sum \frac{R}{r} \left(1 + \frac{x^2}{r^2} \right) - \frac{1}{R} \right) \iiint \frac{1}{r} \left(1 + \frac{x^2}{r^2} \right) dx \, dy \, dz$

[to be extended over a space great in comparison with R]

is a numerical factor. The evaluation of β is rather cumbersome. This current is directed ~~along the positive X~~ along the positive X and has the effect there to increase the resistance.

I have found by ~~an~~ explicit numerical calculation ~~for~~ for a cube of $5R$ ~~the~~ the provisional value: $\beta = 2.09$, whence ~~the~~ the resistance of the particle will be which may be compared with D. Cunningham's value $6\pi \mu a [1 + 3.67 \frac{a}{R}]$ $6\pi \mu a [1 + 3.67 \frac{a}{R}] + 2.32 \frac{a}{R}$

We see that these developments would be divergent for infinite number of spheres. It is evident (for instance) that an infinite row of similar spheres ~~separately~~ would acquire infinite velocity by virtue of their gravity, as ~~spheres~~^{or them} also an infinite cylinder would behave in the same way. A portion this applies to two-dimensional infinite assemblies.

$$F = \frac{F_0}{2} \cos \left[\pi \left(1 + \frac{3\ell}{4} \frac{2}{R} \right) \right]$$

18.

The same formula (7) would apply of course also if the particles were arranged in a different way but
 (1) then the numerical value of F will be different
 — of course it must be independent of the arrangement of the particles in order to validate the theory or have considered only the terms with the first power of $\frac{\ell}{R}$; if we like to include quadratic terms, it would be necessary to consider how the transmitted motion is modified by the presence of any other particle, but if not, it would be necessary to take into account the reflections of the motion of a particle at another particle and also the reaction of the motion of the particle on the motion of all other particles. This is done by reflection at all other particles ought to be taken into account.

28

The general result of

the calculation agrees with the approximate evaluation given by A. Cunningham, it shows

that the (law of Stokes) is undergoing but comparatively small corrections if applied to the particles of a (cloud filling) cloud vessel. But it is important to note that things will go on quite differently if the cloud is not uniform density, ~~and~~ filling the whole empty space between the walls. Then as a rule convection currents will take place, ~~the~~ the velocity of which is superposed on the movement of the particles, and which in certain cases may be of predominant influence. These currents may be calculated approximately by considering the medium as a homogeneous liquid ~~subjected~~ subjected to certain forces, the intensity of which (^{per unit volume}) correspond to the aggregate force acting on the particles ~~and~~ contained ~~in~~ in it.

Consider for instance an electrolyte ~~conducting~~ in an electric field. It is conducting in accordance with Ohm's law, the average electric density is zero and no currents will take place. But in bad liquid conductors ~~with~~ with deviations from Ohm's law ~~the conditions for convection currents~~ may arise. They have been studied for instance by Warburg.

which may influence materially also the conductivity

Similar movements may be produced in ionized gases and I think more attention ought to be paid to the possible deviations from Stokes law in this case than usually is done. If the number of ions is small and if the dimensions of the vessels filled with gas are small and if the ionization is weak, no appreciable effect will be produced of course, but in experiments where the saturation current of strong radioactive material is observed between condenser plates ~~at great~~ wide apart, these ^{mobility of the ions} phenomena may be of importance as producing an apparently greater conductivity than ~~it would be~~ under normal circumstances.

(*) F. inst. Rutherford Reddick 7. 84, p. 35

There is one more application of the theoretical methods mentioned above which may be mentioned.

Imagine a two dimensional infinite assemblage of equal spherical particles, arranged (in a ~~square~~^{explained} quadratic order?) in the plane ~~parallel to~~^{parallel} the V_2 plane ~~which~~ again may be supposed to be ~~a~~ fixed wall $x = l$

Now let all these particles be moving in the direction \vec{V} parallel to V with the velocity c ; what motion will be produced in the liquid, what will be the resistance ~~experienced by every particle~~^{of each particle} to the movement of

The method of explicit calculation

~~Method~~ The explicit calculation can be effected by means of the formula defining after Scandrigg (Lorentz) the motion produced by a sphere of radius a moving with velocity c parallel to a fixed wall

is, with neglect of higher powers of $\frac{a}{l}$:
which we suppose a small quantity

$$v = \frac{3}{4} R c \left(\frac{1}{r_1} - \frac{1}{r_1^3} \right) - \frac{3}{4} R c \frac{1}{r_1} \left[1 + \frac{(x+l)^2}{r_1^2} \right] - \frac{3}{2} R c x \frac{(x+l)}{r_1^3} + \frac{9}{2} \frac{R c x (x+l)^2}{r_1^5}$$

where the first term is the direct current, according to Stokes, while the terms containing r_1 ~~represent~~^{represent} the reflected current reflected by the wall just as in the former example.

We might calculate the resultant current in the point ξ by ~~summation~~^{also this case} of the forming Σv over all values of x and derive therefrom the resistance of a single particle. But it shall content ourselves with the following remarks.

The resultant motion must depend on the ratio of the radius of each particle to their near distance.

In the extreme case where the particles are so crowded as to touch one another, ~~the motion of~~^{the motion of} a lamellar flow will take place in the liquid between the fixed wall and the plane $x = l$ with velocity $v = \frac{cx}{l}$, while ~~no~~ on the other side of the plane $x = l$ the liquid will move with the constant velocity c .

The frictional force per unit of surface of the plane $x = l$ is evidently equal to $\frac{\mu c}{l}$; therefore the resistance experienced by each particle is

$$F = \frac{\mu c R^2}{l} \quad \text{it is much smaller of course than the Stokes would indicate instead of the Stokes resistance } 6\pi\mu c a \text{ where } a \text{ is supposed to be}$$

Now consider the other extreme case where the distances between the particles are so great that Stokes' law is approximately valid, the condition will be that R be of order l .

So we have $F = 6\pi\mu c a$

On the other side let us calculate the resultant motion for a point at great distance from the wall. For such points the summation mentioned above can be replaced by integration, ~~but~~ besides the difference $\frac{1}{r} - \frac{1}{r_1}$ ~~can be developed~~ ^{can be developed} ~~put equal to~~ ^{put equal to} $= \frac{2l\xi}{r^3}$; $\frac{1}{r^3} - \frac{1}{r_1^3} = \frac{6l\xi}{r^5}$, thus the above expression simplifies into:

$$T = \sum v = \frac{3}{4} R c \left(\frac{2l\xi}{r^3} + \frac{6l\xi^2}{r^5} \right) - \frac{3}{2} R c \frac{l\xi}{r^3} + \frac{9}{2} R c \frac{l\xi^2}{r^5}$$

$$= 9 R c \int \frac{y^2}{r^5} dy dr \quad \frac{9 R c l \xi}{R^2} \int \frac{y^2}{\sqrt{y^2 + 4l^2}} dy dr$$

The integral can be transformed by putting $y = \rho \sin \varphi$, $z = \rho \cos \varphi$, $dy dr = \rho d\rho d\varphi$ into $\int \frac{\rho^3 \sin^2 \varphi d\rho}{\sqrt{\rho^2 + 4l^2}} = \pi \int \frac{\rho^3 d\rho}{\sqrt{\rho^2 + 4l^2}} = \frac{2\pi l^4}{3}$ and we get $T = \frac{6a l \pi c}{R^2}$

of chromosomes for arrangement in an equatorial

However,

Practical application of this formula is rather questionable
answer for

(3) It is interesting (to note) that the average value of β for a particle whose position ~~is at~~^{of a spherical particle} is ~~assumed~~^{relatively to the other particles} is ~~defined by~~^{defined by} according to the particles of a cloud) thus it follows that to this order of approximation Stokes law would apply on an average without any correction.

that seems quite natural as the average current \bar{U} in the cross section must be zero and

By comparing this expression with F we get

$$V_\infty = \frac{F}{R^2} \frac{l}{\mu}$$

that means that the liquid at a great distance from the wall will be dragged along with such a velocity as if the force corresponding to unit surface $\frac{F}{R^2}$ would act ~~not~~ not on a row of particles but on a plane distant by l from the fixed wall. Now if we suppose a greater number of similar layers we see that generally under condition of approximate the without going into further details, I may only mention that this result has an important bearing on the theory of electric induction which I am going to explain elsewhere with full details.

~~Now~~ I may conclude with some remarks about the influence of the inertia terms, which are neglected as well in Stokes' original formula as in the preceding calculations. ^{Ones} ^{will then} ^{in the hydrodynamical equations} It is well known that this neglection is ~~not~~ justified only if the ratio $\frac{rv_0}{\mu}$ ~~is~~ is small in comparison to unity. ^{recent experiments} ~~which have~~ ^{shown that Stokes' law} The ~~recent experiments~~ study of Dr Arnold ~~has shown that Stokes' law~~ is valid if the radius of a sphere ^{for moving} under influence of gravity, provided their radius R is smaller than $0.6 \frac{\mu}{v_0}$, where the critical radius R is defined by $\frac{rv_0}{\mu} = 1$; ~~it is to say by the value~~ this means as much that the ratio $\frac{rv_0}{\mu}$ must be smaller than $(0.6)^3 = 0.22$.

^{in an arbitrary paper} But it has been proved by Oseen that the hydrodynamical solution given by Stokes is defective even if this criterion is fulfilled; for at distances r where $\frac{rv_0}{\mu}$ is large, the inertia terms must be of predominant influence. ~~Stokes~~ ^{Oseen} ~~himself~~ ^{has given} a solution, different from Stokes' equation, which gives better approximation for those distant parts of the space. However, the resistance of a sphere in ~~the infinite~~ ^{of infinite extent} liquid depends only on the state of movement in its immediate neighbourhood, therefore the resistance law of Stokes is not impaired by those results, provided the criterion mentioned above is fulfilled. The conditions of its validity may be ^{defined} expressed still more exactly ^{by means} ~~on account of~~

The inertia terms are of greater importance, as modifying the law of resistance in the case of ~~stokes~~ before alluded to where ^{the motion of} a greater number of similar spheres is considered. In it is legitimate to ^{use} the use of Stokes' equations for slow motion will be legitimate only if ^{it can} ~~it is~~ ^{is} ~~wanted that~~ forces of reaction between such spheres can be calculated by ~~is~~ ^{is} ~~that~~ ^{that} if they are lying within the space where viscosity is predominant over inertia. Mr. Oseen ^{recently} ~~has~~ ^{generalized} the calculation of ^{resistive} the interaction of two spheres, given by me, by introducing his solution of the Stokes' problem. ~~He~~ The result is much more complicated of course and the forces exerted on the two spheres come out unequal in this case and are given by much more complicated expressions. They ^{become} ~~are~~ ^{the first approximation} ~~the~~ ^{criterion} ~~given by me if~~ the distance R between spheres satisfies the condition that $\frac{Rv_0}{2\mu}$ is small. ^{for experimental purposes} Mr. Oseen thinks this to be a great restriction on the validity of those expressions, but he omits ^{inevitably} the factor 6 in the above expression. We satisfy ourselves easily that for instance

various difficulties with experiments of this kind ~~with~~ may arise on account of solid surface films as Dr Arnold has shown so easily perturbed at ~~at~~ quiet surfaces.

It is not difficult to see that the action of impulsion to projectiles ~~is~~ of all kinds is increased in presence of a surface film. Now if we ~~are~~ go to consider ~~the~~ effect of a surface film on the motion of a projectile it is necessary to take into account the effect of the surface film on the motion of the projectile itself. This is done by the method of successive approximations. Now if we ~~are~~ go to consider the effect of a surface film on the motion of a projectile it is necessary to take into account the effect of the surface film on the motion of the projectile itself. This is done by the method of successive approximations.

and it is found that it is much the same as the ~~as~~ case of a smooth surface. The effect of a surface film on the motion of a projectile is to give it a small retardation which is proportional to the speed of the projectile.

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Nuttall Phys Rev 32, 349
Dr Kuhn 1910

Nuttall Ph. 2, 11, 1097 (1911)

Dr K. Ph. 12, 707 (1911)

R. Kunder & S. Weber Ann Ph. 56, 981, 1911

Dr R. S. 83, 357 (1910)

known how the interaction between gas molecules and solid wall takes place. If the molecules rebound like elastic spheres we should get ~~in~~ in accordance with Cunningham:

$$F = \frac{4}{3} \sqrt{\frac{8}{\pi}} \alpha n p c V \quad \text{where } c \text{ is the square root of the mean square of molecular velocity}$$

The empirical numerical coefficient as following from Dr Kuhn's ~~experiments~~ experiments is considerably larger, it amounts to ~~1.65~~ (Kunder and S. Weber) according to Dr Kuhn

instead of 1.23 as would follow for elastic impacts. Dr Kuhn concludes that molecules are reflected from the surface of a sphere ~~only~~ only in normal direction; I think however that his theoretical formula is ~~not quite correct~~ and requires a little correction and that

See also Reinemann Verh D. Ph. 5, 12, 1025 (1910)

We shall not go into these questions ~~here~~, however, as they belong to the kinetic theory of gases.

these experiments are quite in accordance with the results supported by other ~~researches~~ other researches by Barby and Kunder that a solid surface acts in scattering the impinging molecules irregularly in all directions.

W

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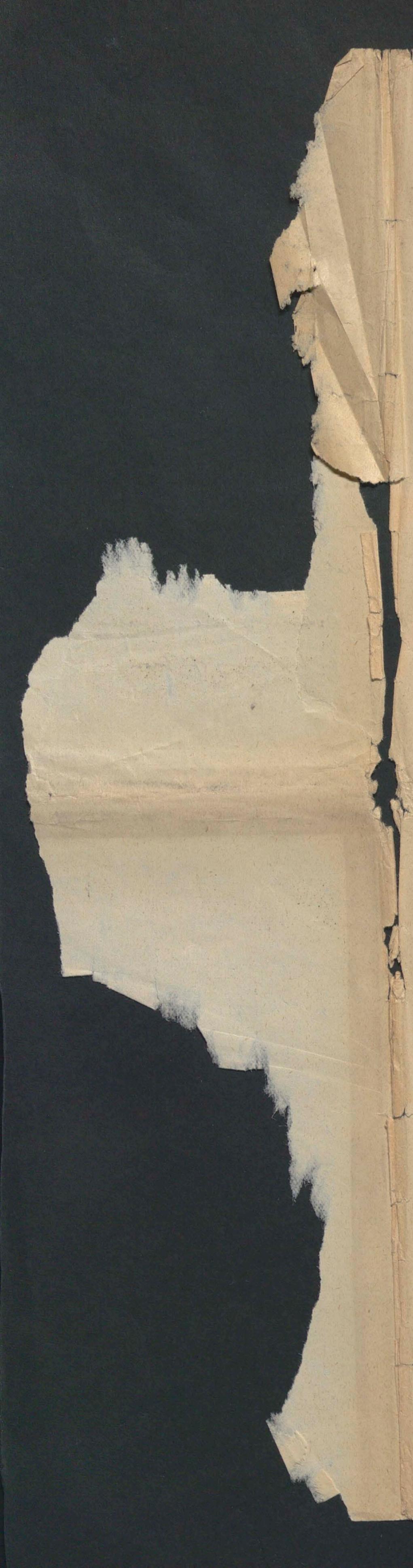
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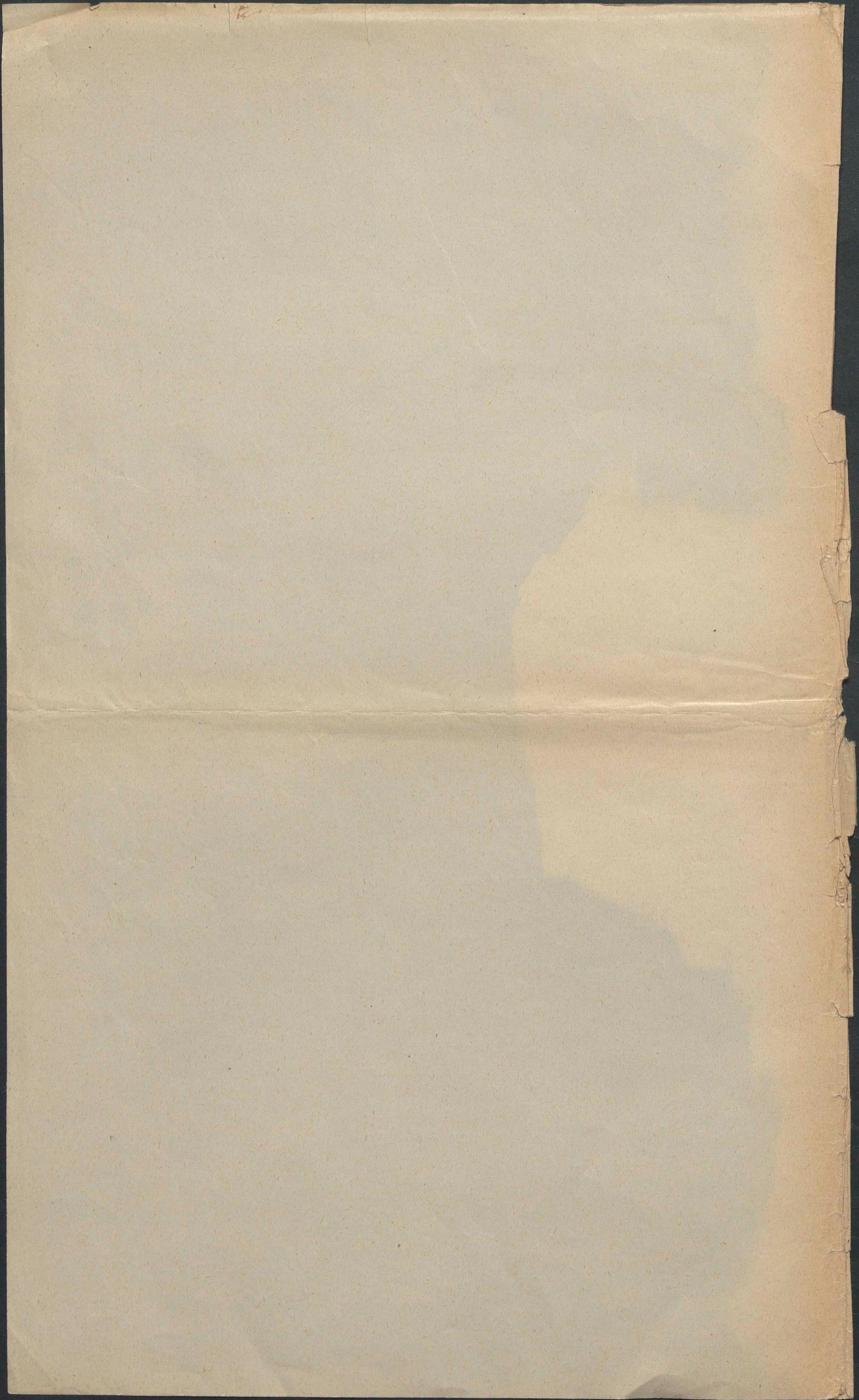
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ratio of the two.

Ordinary hydrodynamic experiments

Stokes law or any of those modifications





L7

On the Practical Applicability of Stokes' Law of Resistance, 33

and the Modifications of it Required in Certain Cases.

by

M. S. Smoluchowski, Ph.D., LL.D. Professor of Physics at the University
of Lemberg.

(§1). Stokes' ~~well known~~ law for the resistance of a sphere ~~rests~~, as is well known, on the fundamental assumptions:

- I. Slowness of motion, so that the inertia terms in the hydrodynamical equations may be neglected, in comparison with the effects of viscosity,
- II. Complete adhesion, ^(without slip) of the liquid to the sphere, this being considered as a rigid body,
- III. Unboundedness of the liquid and immobility at infinity.

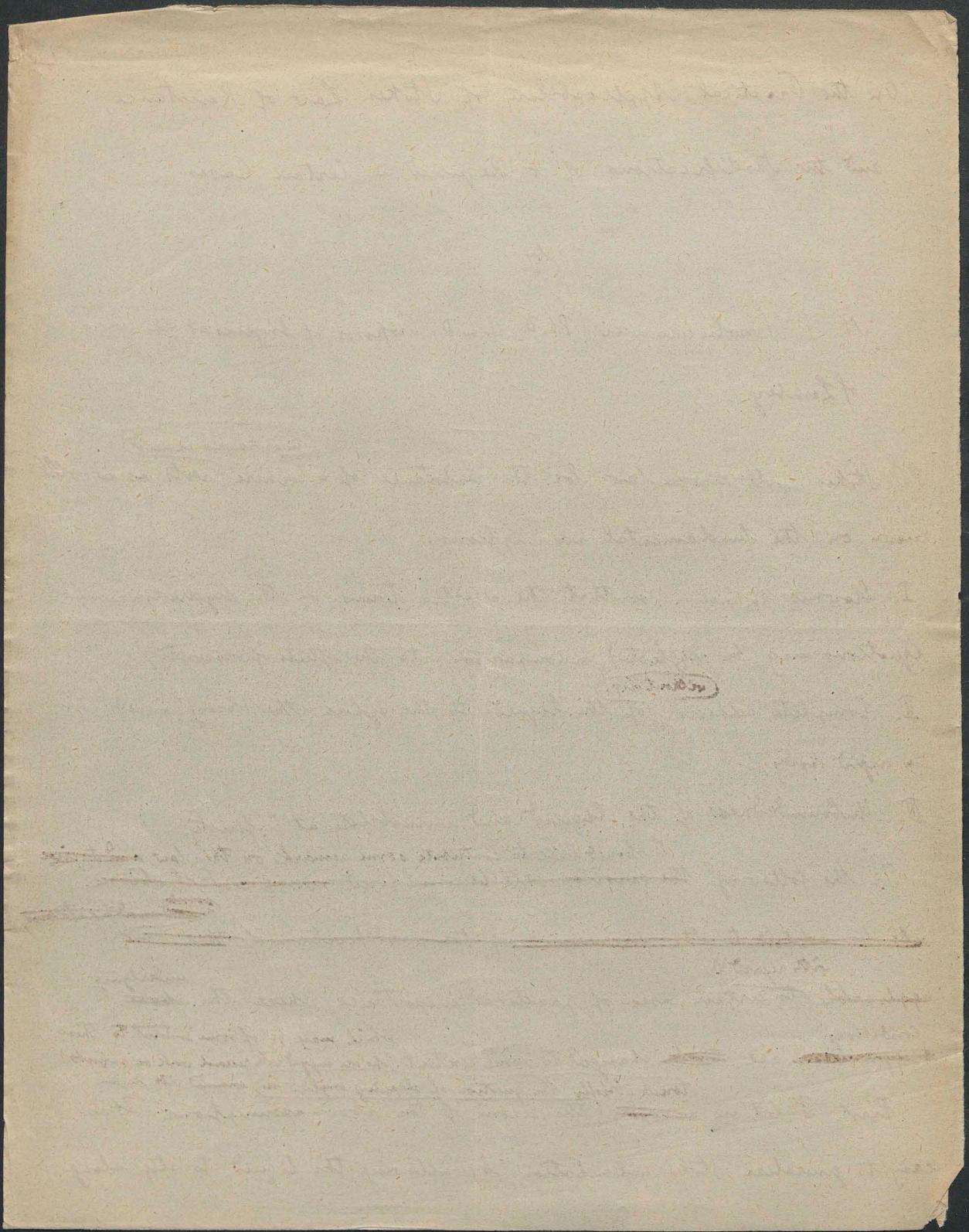
I should like to contribute some remarks on this law ~~and to give~~
~~the question will be considered~~ some contributions

~~will be added to the discussion of the modifications of this law,~~
~~with regard to~~
applicable to certain cases of practical importance, where the ~~underlying~~
~~conditions~~

~~simplifications~~ are ~~not~~ changed to some extent. who engaged in research work on subjects
which may be of some interest to those

First let us consider the second of the above assumptions. It is
touch briefly the question of slipping, implied in connected with the

easy to generalize Stokes' calculation, by allowing the liquid to slip along



case to the fourth order of approximation (including terms with $(\frac{R}{a})^4$).¹⁾ 17
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In a somewhat similar way Zedenburg calculated the ~~coefficient~~²⁾ of resistance experienced by a sphere, when moving along the axis of an unlimited cylindrical tube, and his result, indicating an increase in comparison with the usual formula of Stokes in the proportion of $1: 1 + 2\frac{1}{4} \frac{R}{\rho}$, (where ρ = radius of tube), has been verified with very satisfactory approximation by his own experiments and by those of Arnold. ³⁾ R. Zedenburg, Ann. d. Phys. 23, 447 (1907).

[§5] In a paper published last year³⁾ I have pointed to some interesting results concerning the motion of a greater number of similar spheres and I may be allowed to extend these investigations a little further now. Imagine a sphere of radius R moving with the velocity c ~~in the direction of~~^{along the} X axis, its centre being situated at the distance x from the origin. It would produce at the point P (with coordinates $\{x, y, z\}$) certain current velocities u_0, v_0, w_0 , of order $\frac{Rc}{r}$, defined by Stokes' equations, if the fluid be unlimited. But if we assume this point P to be the centre of a solid sphere of radius R_0 , we have to superpose a fluid motion u_1, v_1, w_1 , chosen so as to annul the velocities of the primary motion at the points of this sphere and satisfying the condition of rest for infinity.
3) M. Smoluchowski, Bull. Acad. Scienc. Cracovie 1911 p. 28.

3) J. Stock, Bull. Acad. Scienc. Cracovie 1911 p. 18. In Hellican's determinations of the ionic charge this increase of resistance, arising from the presence of the condenser plates, may produce an increase of the order of one thousandth.

Now let us apply this method to the case ~~where~~^{an investigation} where a greater number of similar
spheres is in motion, ~~a subject about~~^{a little (written now)} I have treated in a paper published
and extend in this way ~~an investigation which I had begun~~

$$\rho u = \mu \frac{\partial u}{\partial y}$$

$$\beta = \frac{\mu}{f} \quad \parallel \mu = c\lambda$$

Lamb:

$$\cancel{F = 6\gamma u c R} \quad \frac{1 + 4\left(\frac{R}{3n}\right) + 6\left(\frac{R}{3n}\right)^2}{\left(1 + \frac{3n}{R}\right)^2}$$

$$F = \underbrace{\frac{4}{3} \sqrt{\frac{E}{3n}}}_{= 1.23} R \gamma p c V \text{ short } = \frac{6\gamma u c}{1 + 1.05 \frac{R}{n}}$$

$$= \frac{6\gamma u c}{1 + 1.2 \frac{R}{n}} \text{ Ne Kehler (all directions)} = \frac{6}{3} \sqrt{\frac{E}{3n}} \dots = 1.535$$

$$= \frac{6\gamma u c}{1 + 1.05 \frac{R}{n}} \quad " \quad (\text{normal}) = 1.76$$

$$= \frac{6\gamma u c}{1 + 1.00 \frac{R}{n}} \quad \text{empiric.} = 1.84$$

$$\text{Siml. : all directions} = \cancel{R \gamma p c V} \cdot \frac{12}{9} \sqrt{\frac{E}{3n}} = 1.78 \\ (\text{very unchanged})$$

$$\text{all directions} = \frac{16}{9} \sqrt{\frac{E}{3n}} \dots = 1.64$$

$$\text{approx. Koeffiz.} = 1.65$$

$$u = \frac{3}{4} c \left\{ \frac{R}{r^2} + \frac{R_{xy}}{r^3} \right\} + \frac{1}{4} c \left(\frac{R^3}{r^3} - \frac{R^3_{xy}}{r^5} \right)$$

$$v = \frac{3}{4} c \left\{ \frac{R_{xy}}{r^3} - \frac{R^3_{xy}}{r^5} \right\}$$

$$w = \frac{3}{4} c \left\{ \frac{R_{x2}}{r^3} - \frac{R^3_{x2}}{r^5} \right\}$$

$$\text{Bsp. (7)} \quad u = \frac{3cR}{r^2} \left[1 + \left(\frac{r-x}{R} \right)^2 \right]$$

$$v = \frac{3}{4} c \frac{R (x-R) \eta}{r^3}$$

$$w = \frac{3}{4} c \frac{R (x-R) \zeta}{r^3}$$

2000 - 1000 - 500 - 200 - 100 - 50 - 25 - 10 - 5 - 2 - 1

1000 - 500 - 200 - 100 - 50 - 25 - 10 - 5 - 2 - 1

500 - 200 - 100 - 50 - 25 - 10 - 5 - 2 - 1

200 - 100 - 50 - 25 - 10 - 5 - 2 - 1

100 - 50 - 25 - 10 - 5 - 2 - 1

50 - 25 - 10 - 5 - 2 - 1

25 - 10 - 5 - 2 - 1

10 - 5 - 2 - 1

5 - 2 - 1

2 - 1

1

$m \frac{d^2x}{dt^2} = -6\pi \mu a \frac{dx}{dt} + X =$ force complémentaire qui maintient l'oscillation
 *~~elle~~ multiplié par a que sans elle la résistance visqueuse
 finirait par arrêter

$$\frac{m}{2} \frac{d^2(x^2)}{dt^2} - m \left(\frac{dx}{dt} \right)^2 = -3\pi \mu a \frac{d(x^2)}{dt^3} + X_x$$

$$\frac{d(x^2)}{dt} = 2x \frac{dx}{dt}$$

$$\frac{d^2(x^2)}{dt^2} = 2 \left(\frac{dx}{dt} \right)^2 + 2x \frac{d^2x}{dt^2}$$

grand nombre de points (dots et jumons) }
 la moyenne des signes }

la moyenne du terme X_x est évidemment nulle, à cause de

d'irregularité des actions complémentaires X , et il vient,

$$\text{en posant } \alpha_2 = \frac{d(x^2)}{dt^3}$$

$$\frac{m \alpha_2^2}{3} = \beta = R \theta \rho$$

$$\frac{m \alpha_2^2}{3} = R \theta \frac{\rho}{n} = \frac{R \theta \rho}{N}$$

$$\frac{m}{2} \frac{d^2x}{dt^2} + 3\pi \mu a \alpha_2 = \frac{RT}{N}$$

$$x = \frac{RT}{N} \frac{1}{3\pi \mu a} + C e^{-\frac{6\pi \mu a t}{m}}$$

régresse progressivement au bout d'un temps de l'ordre $\frac{m}{6\pi \mu a} (= 10^{-8})$

$$\therefore \frac{d(x^2)}{dt} = \frac{RT}{N} \frac{1}{3\pi \mu a}$$

d'où, pour un intervalle de temps t

$$\therefore \bar{x}^2 - \bar{x}_0^2 = \frac{RT}{N} \frac{1}{3\pi \mu a} t$$

le déplacement de l'axe patrule est donné par :

$$x = x_0 + \Delta x$$

et comme les déplacements sont intérieurs positif et négatif.

~~$$\Delta x = \bar{x} - \bar{x}_0 = \frac{RT}{N} \frac{1}{3\pi \mu a} t$$~~

Le moyen de fréquency approche plus l'ini. mais tout de même le déplacement n'est pas aussi grand que lorsque le diamètre n'est pas pris dans le sens longitudinal de l'oscillation plus facile à connaître et pour lesquels l'application de la formule de Stokes qui néglige la densité directe du liquide est plus légitime.

$$\Delta = c \sqrt{\frac{2m}{\hbar^2 n R}} = \frac{c \sqrt{m}}{\sqrt{3} \sqrt{\hbar^2 n R}}$$

$$\frac{8}{3\sqrt{3}} = \sqrt{\frac{64}{27}}$$

$$v e^{-v} + 2 v \frac{e^{-v}}{1!} + 3 v^2 \frac{e^{-v}}{2!} + \dots - v =$$

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$$v e^{-v} \left[1 + \underbrace{\frac{v}{1!} + \frac{v^2}{2!} + \dots}_{+v} \right]$$

~~$$(0-v)^2 e^{-v} + (1-v)v e^{-v} + (2-v)^2 \frac{v^2 e^{-v}}{2!} + \dots$$~~

$$= \cancel{v e^{-v} \left[1 + \frac{v}{1!} + \frac{v^2}{2!} + \dots \right]} \\ e^{-v} \left\{ 0^2 + 1^2 + \frac{2^2}{2!} v^2 + \frac{3^2}{3!} v^3 + \frac{4^2}{4!} v^4 + \dots \right\} \\ - 2v(0 + v + \frac{2v^2}{2!} + \frac{3v^3}{3!} + \dots) \\ + v^2(0 + v + \frac{v^2}{2!} + \dots)$$

~~$$\text{Ans} \quad 0^2 + v + \frac{2v^2}{1!} + \frac{3v^3}{2!} + \frac{4v^4}{3!} + \dots$$~~

~~$$\text{Ans} \quad \left(\text{Fehler} \right) \quad \frac{v^3}{2!} + \frac{v^4}{3!} + \dots = e$$~~

~~$$\Rightarrow \text{Fehler} = 290 \cdot 10^9$$~~

$$x = \cancel{2\Delta x} + \cancel{\Delta x}$$

$$\mathcal{Z}(x_0 + \Delta x) - \mathcal{Z}(x_0)$$

$$\mathcal{Z}(x_0) - \mathcal{Z}(x_0)$$

laminar
turbulent
Laminar

$$\frac{7.21}{0.24}$$

25

$$\int_{T_0}^0 \int_{T_0}^0 d\Omega(x_0) dx_0 = - \int_{T_0}^0 \int_{T_0}^0 \frac{dx_0}{(x_0)} = - \int_{T_0}^0 \int_{T_0}^0 \frac{dx_0}{x_0} = - \int_{T_0}^0 \int_{T_0}^0 \frac{dx_0}{x_0} = - \int_{T_0}^0 \int_{T_0}^0 \frac{dx_0}{x_0}$$

0 1 2 3 4

$$2 - \frac{N}{H\theta} A$$

$$A = a \varepsilon^2$$

38

$$dW = \text{Kont.} \cdot e^{-\frac{N}{H\theta} a \varepsilon^2} d\varepsilon$$

$$\bar{\varepsilon} = \frac{\int_{\varepsilon^2}^{\infty} e^{-\rho \varepsilon^2} d\varepsilon}{\int e^{-\rho \varepsilon^2} d\varepsilon} = \frac{1}{\sqrt{\rho}}$$

$$a \bar{\varepsilon}^2 = \frac{1}{2} \frac{H\theta}{N}$$

$$m \frac{n c^2}{3} = R\theta$$

$$H = 2 \cdot 3 \cdot 10^9$$

$$= \frac{1}{3} \frac{mc^2}{2}$$

$$\frac{mc^2}{3} = \frac{H\theta}{N}$$

$$R = \frac{8 \cdot 3 \cdot 10^7}{f_{ext}}$$

$$= 10^{16} \text{ J}$$

$$N = 6 \cdot 10^{23}$$

$$H = 8 \cdot 10$$

$$H = \frac{10^6 \cdot 2}{0.00009}$$

$$= \frac{10^6}{0.00009} = 8 \cdot 10^9$$

$$28 \cdot 10^9 \cdot \frac{10^6}{0.00009} = 28 \cdot 10^{16}$$

$$= 28 \cdot 10 \cdot \frac{2 \cdot 3 \cdot 10^9}{6 \cdot 10^{23}} = 28 \cdot 10 \cdot \frac{2 \cdot 3 \cdot 10^9}{6 \cdot 10^{23}}$$

Knowing

$$T = 290 \cdot 10^9$$

35-A

A
35
A

d

100
100
100
100

100
100
100
100

100
100
100
100

100
100
100
100

100
100
100
100

$$\frac{4\pi v^2 e^{-\frac{v}{\alpha}}}{\alpha^2 \sqrt{\pi}} \quad \frac{\sin \varphi d\varphi}{2} \quad \underbrace{v \cos \varphi}_{(v_0 \cos \varphi + u)}$$

moments remaining

$$v^2 = u^2 + v_0^2 + 2uv_0 \cos \varphi$$

$$\frac{1}{2} \int (v_0 \cos \varphi + u)(v_0 \cos \varphi + u) \sin \varphi d\varphi + \dots$$

Takich gęzys w sferze okladowej jest 2 dwoj. strzałek, z których momentu kątowe momentu:

$$\int \frac{\sin \varphi d\varphi}{2} \frac{dS \cos \varphi \cdot v \cdot n \cdot v \cos \varphi}{2} = v \int_0^{\frac{\pi}{2}} \frac{\sin \varphi \cos^2 \varphi d\varphi}{2} = \frac{2v}{3}$$

zatem ~~że~~ momentu kątowego:

$$\frac{\int_0^{\frac{\pi}{2}} \sin \theta \cos \theta d\theta \cdot \left(\text{wart } \frac{2v}{3}\right)}{\int_0^{\frac{\pi}{2}} \sin \theta \cos^2 \theta d\theta} = \frac{4}{9} v$$

torzowy skokowy \Rightarrow
skokowy momentu kątowego

$$\text{zatem prędkość } (v + \frac{4}{9} v) \cos \varphi \quad \text{wsp. położenia ośrodku} \quad \frac{13}{9} \quad \frac{13}{9} \cdot \frac{4}{9} \cdot \frac{1}{32}$$

$$15 \cdot \frac{9}{13} = 13.5 : 13 = 1.00 \quad \frac{1}{4}$$

1.23. 13

360

1599.9 =

6969

1781

wzór na gęzys taki moga się wyrazić całkowicie i gęzys ołówka dając po prostu prędkość wentylatora bez uwzględnienia momentu kątowego

$$\frac{\int 4\pi v^2 e^{-\frac{v}{\alpha}} dv}{\int 4\pi v^2 e^{-\frac{v}{\alpha}} dv} = \frac{2\alpha}{\sqrt{\pi}} \quad \text{wzór na gęzys taki}$$

SKK dla emuluji: taki dla
wentylatora = 1

$$\int_0^{\infty} \frac{\sin \varphi d\varphi}{2} (v_0 + u \cos \varphi)(v_0 \cos \varphi + u) + \frac{4}{9} \frac{2\alpha}{\sqrt{\pi}} \int_0^{\infty} \frac{\sin \varphi d\varphi}{2} (v_0 \cos \varphi + u)$$

u

$$\frac{U v_0}{\pi} (1 + \frac{1}{3})$$

$$\frac{4}{3} U v_0 + \frac{8}{9} \alpha U$$

$$\text{wzór na } 15 \cdot \frac{3}{4} = 1.125 \frac{1}{4} \\ = 1.64$$

$$U \left[\frac{4}{3} \frac{\int v^3}{\int v^2} + \frac{8\alpha}{9\sqrt{\pi}} \right] = U \left(\frac{8}{3} + \frac{8}{9} \right) \frac{\alpha}{\sqrt{\pi}} = U \frac{8}{3} \left(1 + \frac{1}{3} \right) \frac{\alpha}{\sqrt{\pi}} \\ = U_c \frac{32}{9} \sqrt{\frac{2}{32}} \\ = U_c \frac{16}{9} \sqrt{\frac{8}{32}}$$

$$\frac{1}{n} - \frac{1}{p} = \frac{2\pi}{n^3}$$

$$\sum \frac{1}{n^3} = \int_0^\infty \frac{\sin y}{y^2} dy = \frac{2}{\pi^2}$$

$$= \frac{(2\pi)^2}{\pi^3} \cdot 3 \cdot 3$$

$$= 15 \cdot 2 \frac{\pi^2}{\pi^3}$$

$$4\pi a = \frac{q_i - q_o}{2a} = \frac{2(q_i - 0)}{2a} = \frac{q_i}{a}$$

$$q_i = 4\pi a \alpha$$

$$= \frac{4\pi a}{\pi^2} \quad \text{oder zu } S \ll a$$

$$2 \frac{2}{\pi^2} \left[1 + \frac{1}{4} + \frac{1}{9} + \dots \right] = \frac{6 \cdot 6}{\pi^2}$$

$$S = \sum_{x=1}^{\infty} \frac{1}{x^2}$$

$$1 + 2 + 2^2 + 2^3 + \dots = \frac{1}{1-2} \quad \left| \begin{array}{l} [1 - e^{i\varphi} + e^{i2\varphi} + \dots] \\ [1 - e^{i\varphi}]^2 + e^{i2\varphi} \end{array} \right| =$$

$$1 + e^{i\varphi} + e^{i2\varphi} + \dots = \frac{1 - e^{i\varphi}}{2(1 - e^{i\varphi})} = \frac{1}{2}$$

$$+ i [e^{i\varphi} + e^{i2\varphi} + \dots] = i \frac{\sin \varphi}{2(1 - e^{i\varphi})} = \frac{i \cdot \operatorname{ctg} \frac{\varphi}{2}}{2}$$

$$\int dx [2e^{i\varphi x} + \sin 2\varphi x + \dots] = \frac{1}{2} \int \operatorname{ctg} \frac{\varphi x}{2} dx$$

$$1 + e^{-x} + e^{-2x} + \dots = \frac{1}{1 - e^{-x}}$$

$$\int_0^\infty dx [1 + e^{-\alpha x} + e^{-2\alpha x} + \dots] = \frac{1}{1 - e^{-\alpha x}} - 1$$

$$\frac{1}{\alpha} + \frac{1}{2\alpha} + \frac{1}{3\alpha} + \dots = \int \frac{e^{-\alpha x}}{1 - e^{-\alpha x}} dx$$

$$1 + \frac{1}{2}$$

$$\left. \begin{aligned} & \frac{4\pi}{\pi^2} \frac{q_i}{2a} + \frac{4\pi}{\pi^2} \frac{q_i}{2a} - \dots \end{aligned} \right\} \sqrt{a} u_1 = Y$$

$$\left. \begin{aligned} & \frac{4\pi}{\pi^2} \frac{q_i}{2a} + \frac{4\pi}{\pi^2} \frac{q_i}{2a} - \dots \end{aligned} \right\} \sqrt{a} u_2 = X$$

$$\frac{4\pi}{\pi^2} \frac{q_i}{2a} + \frac{4\pi}{\pi^2} \frac{q_i}{2a} - \dots = 1$$

- 16 M 126 Dachwurz Wiesen-Pfl. wächst, Mayr 90
18 D. S. w. Runkelwurz + Dattel. Tolkedorf
20 Rorippa s. rorabum Jansson
255 Nastur. rotla
216 Dicella Park

$$\lambda_{\text{met}} = -\theta \varphi$$

$$\lambda_{\text{stated}} = -\theta \frac{\varphi_0}{2}$$

$$\sqrt{\bar{\rho}_v} = \sqrt{\frac{RT}{N\theta}}$$

$$\varphi = 10^4 = 10^4 \cdot 60, 60, 60 = 36000$$

$$= 20''$$

~~1 mm reprezentuje skala~~
0,5 m

$$\theta = 5 \cdot 10^{-6} (\text{CSS})$$

$$= \frac{\pi r^4 E}{4(1+\nu)l} + \frac{r^4 E}{2l}$$

brzeg $l = 100$

$$10^{-3} = r^4 E$$

$$E = \cancel{2 \cdot 10^{12}} \quad \text{dla plastyk}$$

$$= 0,5 \cdot 10^{12} \quad (\text{kwas})$$

$$r^4 = \frac{10^{-3}}{0,5 \cdot 10^{12}} = 2 \cdot 10^{-15} = 20 \cdot 10^{-16}$$

$$r = < 2 \cdot 10^{-4} = 0,002 \text{ mm}$$

$$\text{wytrzymałość na rozciąganie } \text{nr. } 9 \cdot 10^9 = 30 \cdot 4 \cdot 10^8 \cdot 10^9 \\ = 1200 \text{ dygn}$$

$$\text{brzeg } r = 2 \cdot 10^{-4} \text{ mm}$$

$$= 1 \text{ gram}$$

otrzymałby się wytrzymałość = 0,01 gram

a wychylnie 60°



