Klasa .... Rok ....

Oddział .... Półrocze ....

MARKA OCHRONNA

....

28

....

LIGA POMOCY PRZEMYSŁOWEJ

$$\int_{-\infty}^{+\infty} e^{-\frac{\nu \delta^2}{2}} d\delta = \sqrt{\frac{2\pi}{\nu}}$$

$\delta\sqrt{\frac{\nu}{2}} = x$

$$\frac{\int_0^\infty \delta e^{-\frac{\nu\delta^2}{2}} d\delta}{\int_0^\infty e^{-\frac{\nu\delta^2}{2}} d\delta} = \frac{\int_0^\infty x e^{-x^2} dx}{\sqrt{\frac{\nu}{2}}\int_0^\infty e^{-x^2} dx} = \frac{\frac{e^{-x^2}}{2}\Big/^0}{\sqrt{\frac{\nu}{2}}\ \frac{\pi}{2}} = \sqrt{\frac{2}{\nu\pi}}$$

$$h = \cancel{\frac{32}{3}} \cancel{\frac{\lambda^{+3}}{\pi^2}\cdot\varepsilon^2}\ \frac{32}{3}\frac{\pi^3}{\lambda}\varepsilon^2$$

$\varepsilon = 7{\cdot}2 . 10^{-3}$

$$\frac{32 . \cancel{27} . 500\,|\,10^{-6}}{\cancel{3} . 0{\cdot}00006} = \frac{16 . 10^{-3}}{0{\cdot}00016}$$

$$= 2{\cdot}7 . 10^2$$

$$\frac{32}{\cancel{3}}\ \frac{\cancel{70} . 10}{0{\cdot}00006}\left[0{\cdot}00029 . 2{\cdot}7 . 10^{-4}\right]^2$$

$8 . 10^{-8}$

$$= \frac{3{\cdot}2 . 10^2 . 64 . 10^{-16}}{6 . 10^{-5}} = \underline{\underline{3 . 10^{-8}}}$$

$0{\cdot}03 . 0{\cdot}434$

$0{\cdot}013$

$10\ km = 10.000^{00}$

$e^{-3 . 10^{-2}}$

$e^{-hx} = \frac{1}{2}$

$hx\ {}^{10}lg\,e = lg\,2$

$$x = \frac{lg\,2}{h\ {}^{10}lg\,e} = \frac{0{\cdot}301}{2{\cdot}7 . 0{\cdot}434}$$

$$\begin{array}{r} 6375 \\ 4314 \\ \hline 0689 \end{array} \qquad \begin{array}{r} 479 \\ -069 \\ \hline 410 \end{array}\ |\ 026$$

$p + a\rho^2 = RT\rho\,\varphi\left(\frac{\rho}{\rho_0}\right)$

$$1 + 2a\rho\frac{\partial\rho}{\partial p} = RT\left[\varphi + \frac{\rho}{\rho_0}\varphi'\right]\frac{\partial\rho}{\partial p} \quad \Big| \frac{\partial p}{\partial T}$$

$$2a\rho\frac{\partial\rho}{\partial T} = R\rho\varphi + RT\left[\varphi + \frac{\rho}{\rho_0}\varphi'\right]\frac{\partial\rho}{\partial T} \quad \Big| -\frac{\partial\rho}{\partial p}$$

$$\frac{\partial p}{\partial T} = -R\rho\varphi\,\frac{\partial\rho}{\partial p}$$

$$-\varphi\left(\frac{\rho}{\rho_0}\right) = \frac{1}{R\rho}\left(\frac{\frac{\partial\rho}{\partial T}}{\frac{\partial\rho}{\partial p}}\right) \qquad a = -\left(\frac{T\frac{\partial\rho}{\partial T}}{\rho^2\frac{\partial\rho}{\partial p}}\right)_{p=0}$$

Pentan p 59

0°	$\frac{1}{\rho}\frac{\partial\rho}{\partial p} =$	229 10^{-6}	146
20°		318	164
40°		416	
60°		486	
80		610	
100		714	00257

$$\frac{a_0}{a_{100}} = \frac{273}{373}\,\frac{257}{146}\,\frac{229}{714}\,\frac{1}{(1{\cdot}1935)^2} = \cancel{0.9201}$$

$\frac{273.146}{229}\cdot\frac{714}{257.373}$

4362, 1644, 8537, 4543 | 3598, 4099, 5717, 1536, 4950

4362, 4099, 3598, 2059, 7434, 4625 — 5717, 1644, 8537, 1530, 7434

~~2768~~ 4543, 4950, 9593

0·9106

~~Ø~~~~6~~~~46~~ 6
~~12~~~~7~~0

0014646 / ...6186 / .4825 / 0025657

14646 / 1239 / 536 / 16421

$v_{100} = v_0$ (1·1465 / ·309 / ·161 / 1·1935

165701 / 174 / 1·831

~~0·0014646~~ / 0·001028 / ~~178~~ / 256 / 0001384

1·1028 / 178 / 1·1206

Ithul $s = 0_4 770\,(1 + 0\cdot0_2 65701 + 0\cdot0_4 174\,t^2)$

$$\frac{a_0}{a_{100}} = \frac{1{\cdot}831}{\cancel{1{\cdot}82}}\cdot\frac{273}{373}\cdot\frac{1028}{1384}\cdot\frac{1}{(1{\cdot}1206)^2}$$

4362 / 0120 / ~~...~~ / 4482 / 2627 / 7109

~~1607~~ / 5717 / 1412 / 0988 / ~~0744~~ / 8117

~~3866~~ 0494 / 0988

7109 / 8117 / 8992

0·793

$$\cancel{}\; p + a\rho^2 = RT\, \varphi\left(\frac{v}{b}\right)$$

$$= RT\, \varphi\left(\frac{\rho}{\rho_0}\right)$$

$$a\rho^2 = RT\, \varphi\left(\frac{\rho}{\rho_0}\right)$$

$$\frac{\partial\rho}{\partial T} \qquad \frac{\partial\rho}{\partial p}$$

$$1 + 2a\rho \frac{\partial\rho}{\partial p} = RT\, \varphi'\left(\frac{\rho}{\rho_0}\right)\frac{1}{\rho_0}\frac{\partial\rho}{\partial p}$$

$$2a\rho \frac{\partial\rho}{\partial T} = R\varphi\left(\frac{\rho}{\rho_0}\right) + \frac{RT}{\rho_0}\varphi'\left(\frac{\rho}{\rho_0}\right)\frac{\partial\rho}{\partial T}$$

$$\frac{\partial\rho}{\partial T} = -R\varphi \cdot \frac{\partial\rho}{\partial p}$$

$$-\varphi\left(\frac{\rho}{\rho_0}\right) = \frac{1}{R}\,\frac{\frac{\partial\rho}{\partial T}}{\frac{\partial\rho}{\partial p}}$$

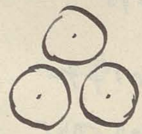
$$a = -\left(\frac{T\,\frac{\partial\rho}{\partial T}}{\rho^2\,\frac{\partial\rho}{\partial p}}\right)_{p=0}$$

$$\mu_v : \mu_{t_0,v} = \sqrt{t} : \sqrt{t_0}$$

$$\mu_{t_0,v} = \psi\left(\frac{\rho}{\rho_0}\right)$$

$u \qquad \kappa u$

$$\frac{u^2}{\frac{\partial u}{\partial z}} \qquad \frac{\kappa^2 u^2}{\kappa \frac{\partial u}{\partial z}} \qquad \sim \kappa$$



n

μ

$$F' = F\frac{n}{n}\cdot n^2$$

Gleiche Raumerfüllung & gl. Dimens.

$c \qquad \kappa c$

$u \qquad \kappa u$

$$\frac{\kappa^2 \quad \kappa^2 u^2}{\kappa^2 \quad \kappa\frac{\partial u}{\partial z}} \sim \kappa$$

$$mc^2 = MC^2$$

$$\mu = \frac{mc}{\delta^2}\,\psi\left(\frac{\rho}{\rho_0}\right) = \sqrt{mT}\,\psi(\cdot) = (mT)^{\frac{1}{2}}\,\psi\left(\frac{v}{b}\right)$$

$$\lg \mu = \tfrac{1}{2}\lg T + \lg \psi$$

$$\frac{1}{\mu}\frac{d\mu}{dT} = \frac{1}{2}\frac{1}{T} + \frac{1}{b}\frac{\psi'\left(\frac{v}{b}\right)}{\psi}\frac{dv}{dT} = \frac{1}{2}\frac{1}{T} + \boxed{\frac{\psi'\left(\frac{v}{b}\right)}{\psi\left(\frac{v}{b}\right)}\cdot\frac{v}{b}}\left(\frac{1}{v}\frac{dv}{dT}\right)$$

$\chi\left(\frac{v}{b}\right)$ $\qquad \alpha$

$$\varphi\left(\frac{v}{b}\right) = \frac{\mu}{\sqrt{mT}}$$

m = 200

	273			
Hg	−21°0	0·01868	251·0	
	0°	0·01688	273	
	16·7	0·01575	289·7	
	99°	0·01227	372	
	196·7	0·01018		
	340·1	0·008975		

lg = 06856, 03822, 00795

124° : 0·01171

154 : 0·01092

196·7 : 0·01018

2·46495
2·30103
4·76298

2·38149

5705
2610
4·8715
2·43575

19756
38149
81607

0889
43575
65315

Methylpropylketon	0°	0·00644
m = 86	100°	0·00238
Heptan	0°	00519
100	900	00214
Oktan	0°	0·00703
114	120°	0·002075
C_2H_5OH	0°	0·01770
46	70°	0·005045
Amylalk. (inakt)	0°	0·08532
88	130°	0·00386
Butylalk.	0°	0·05185
74	110	0·004545
Propylacetat	0°	0·00770
	100°	0·00250

$$\frac{1}{\mu}\frac{d\mu}{dT} = \frac{d}{dT}(\lg \mu) = \frac{^e\lg}{^{10}\lg}\frac{d}{dT}(^{10}\lg \mu)$$

−21·4°		2690
0°	~~1976~~	2274
16·70	1976	
99°	0889	
124°	~~07078~~	0686
154°	0382	
196·70	0 0175	

~~2152~~
41·6 : 21·4 = 19·4
20·8 : 16·7 = 12·7
109 : 82·3 = 132·5
20·3 : 25 = 80
30 : 30 = 100
30·5 : 42·5 = 72·0

$$\frac{1}{\mu}\frac{d\mu}{dT} = 0·00110 \cdot 2·3026$$

23026

0·0025329

~~46 : 99 = 465~~ 66 : 200 = 33

$\frac{0033}{123} =$

3

Benzol 0° 0·009025 || $V_t = V_0(1 + 0·0_2 11\,7626\,t + 0·0_5 127755\,t^2 + 0·0_7 080648)$

78 80° 0·00327 || $(1 + 0·0_2 116\,t + 0·0_5 2226\,t^2)$

Antterain 0° 0·02284

88 160° 0·00314

~~9904~~ 1·8921   9554   10° 1·8921   8802

2·4362   1641   24518   1720

4·3283   7913   43439   7082

2·16415   0°   2·171195   10°

20° 1·8921   8122   30° 1·8921   7497

4669   1795   4814   1867

3590   6327   3735   5630

2·1795   2·18675

7913

7082   83   −79

653·2   55   29·5   6327   75·5   74·5 $\frac{20·5}{74·5}$ =   7118 / 8722 / 4396   = 275

73

5630   70

99°   ← – – →   22·75°

Hg   C6H6

0°	902·5	153·5
10°	759	110
20°	649	87
30°	562	70
40°	492	55
50°	437	

$\frac{1}{\mu}\frac{d\mu}{dt} = 0·00253$ 0·0150

0·0065 . 2·3

130

195

0·01495

9554	75	7
8802	68	
8122	63	6
7497	57	5
6920	52	
6405		

65·5   65 . 275 = $\frac{11}{4}$ 794

65

715 : 4 = 18

1·3590   7140
0645   3476
1·2115   0616
   115·2

7610
1·3570
3476
·0056   103

$\alpha = 0·000\,182_{15}$ || $\alpha = \frac{0·00116}{10} = 0·00126$

Dy 99°

5705
4295

0·00269

$\frac{1}{r}\frac{dr}{dt} = -0·00253$

$-\frac{1}{2}\frac{1}{T}$ 0·00134

~~0·00119~~

$-0·00387$

$\alpha =$ 0·000182

5877
2601
3276

213

C_6H_6 2275
275
29573

4709
5291

0·0033815

$-0·0150$
0·0017

2

$-0·0167$

0·00126

2227
1004
1223

13·2

10°	3872	5879			
20°	2947	4694			
30°	0·02266	3553	1753	1800	
40°	1780	2504	1823	0681	
50	1409	1489			
60	1136·5	0556	1958	8598	9597
70	926·5	9668	2024	7646	
80	762	8820	2085	6735	
90	2504	8017	2145	5872	
100	1820	7279	2204	5075	

Butylalk 453

Pt

1·8692
2·4814
4·3506

30° n

8692
4955
3647

5092
3784

5224
3916

5353
4045

5478
4170

5599
4291

5717
4409

6335
5345

0681

1489
1892
9597

6532

952
911
863
797

887
875

82·270

$\frac{20}{88} = 2·27$

0·0008375
4711
0013086
$-$ 252
0·00012834

0·00875
0·00128

1572

9420
3617
3037
-1072
1965

91·52
3010
4569
6731
4711

8304
4771
0940
4015
2520

4

$$\begin{array}{r} 2.0569 \\ 4362 \\ \hline 4.4931 \end{array} \quad \begin{array}{r} \cancel{8470} \\ -\cancel{24055} \\ \hline 60045 \end{array} \quad \begin{array}{r} 1.6628 \\ 2.5353 \\ \hline 4.1981 \end{array} \quad \begin{array}{r} 7028 \\ -09905 \\ \hline 60375 \end{array}$$

70°

C_2H_5OH

60° 1.6628, 2.5224, 4.1852 ; 7720 − 0926 = 6794

50° 6628, 5092, 1720 ; 8435 − 0860 = 7575

70°	6037	7
60°	6794	4
50°	7575	5

6532 76 78

$\frac{26.2}{77} = .4183$, 8865, 532 =34

63.40

$$\frac{d}{dt}\left(\frac{\mu}{\sqrt{mT}}\right) = \frac{1}{\mu}\frac{d\mu}{dt} - \frac{1}{2}\frac{1}{T} = 0.0077, 23$$

161, 161, 0.01771

120.00

$\frac{1}{50}$ 0.00104139, 9935, 2125, 0.001353

8021, 8941, 1010, 9972, 9935

6042, 2460, 4771, 3273, 212.5

2480, 1313, 1167

13.1

Amylalk. (88)
inact.

~~50~~°	01849	2669	22683	1400	154	
60	1443	1593	23345	9859	166	
70	1147	0597	2399	8198	1005	
80	923·5	9654	24615	7193	92	963
90	007575	8794	2522	6272	877	90 6532 $\frac{26}{90} = 29$ 874·0
100	627·5	7976	2581	5395	798	
110	529	7235	263885	4597	755	
120	4505	6507	26945	3842	725	
130	386	5806	2740	3117		

5092	5224	5353	5478	5599	5717	5832	5944	6053
9445	9445	9445	9445	9445	9448	9445	9445	9445
4537	4669	4798	4923	5044	5162	5277	5389	5498

42. 22 22
963 921 899 877
50 29

00913 .23
1826
2739

0·0210

0·00092410
0460
307

0·001277

9400 8800
7010 4771
4221 1299

6631 4870
460 307

0·0210
0·00128

3222
1072
215

164

Dimethyläthylcarbinol

$C_5H_{12}O$

16
12
60
88

50°	0·01457	1641	2268	9373	138
60°	10.775	0324	2334	7990	120
70°	·830	9191	2389	6792	107
80°	657·5	8179	2461	5718	100
90°	530	7243	2522	4721	93
100°	434	6375	2581	3794	

— 113·5

$\frac{26}{111} = 2·4$

72·40

70°	75°	80°	
113·5	107	103·5	100

110·2

112

0·0112 . 2·3
224
336
0·02576

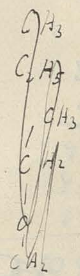
0·00106608
2554
222
0·001543

0·02576
0·00154

4109
1875
2234

16·7

8557	7194
2010	4771
2465	1498
4072	3463
2554	222



Br_2 56·41 0·00706 8488

80 273 − 2104

3294 6384

~~4~~45·6 : 447

9031 10·2

5177 46·19 0·00772 8876 651·2 148 = 1703

4208 272 2036 440 ~~6435~~ 3·36

3192 6840 5268

9031 35·86 0·00848 9284 478 460

5041 273 19655 104

4072 309 7318

9031 26 0·00934 9703

4900 299 1894 491 496

3931 ~~326~~ 7809 99

9031

4757

3788

56·41

− 3·36

53·05 0·0044

0·0010622 7243 4486

199 7010 4771

0·0012612 2735 4893

− 260 2988 4150

0·001235 199 260 366· 23

732

1098

8·42 8·42

$\frac{0·0044}{0·0012}$ = 3·66 !

$CHCl_3$ 8451 ↔ Butylalk.

− 2569 74

106·5

13

119·5

0777

4362

5139

Dimethylpiltylsulfonal (?)
88

0°	0·1418	1517	1995₅	9622	9613₅	264
10°	0·0786	8954	1973₅	6981	6972₅	236
20	464₂	6667	2049₅	4618	461	197
30	300	4771	2122₅	2649	264	154
40	204₄	3104	2193₅	0911	090₄	4
50°	145₇	1635	2262	9373	936₇	

524

2·8
39 – 216
43

$\frac{63}{225} = \frac{7}{25} = 2·8$

17·2°

9445 9445 9445 9445 | 9445 9445
~~4346 4502 4654 4800 | 4942 5079~~

~~3791~~ | ~~3947~~ | 4099 | 4245 | 4387 | 4524 |

4362 4518 4669 4814 4955 5092
9445 9445 9445 9445 9445 9445

3807 3963 4114 4259 4400 4537
1517 8954 6667 4771 3104 1635
1903₅ 1981₅ 2057 2129₅ 2200 2268₅

9613₅ | 6972₅ | 4610 | 2641₅ | 0904 | 9366₅

225

0·0225
0·00114

= ~~20~~

46

0·001066
~~67~~
12
0·00114

9010 4771
2355 4710
2465 1498

7830 0979
607 125

Ontyd...

0°	0·05185	7147	1903	5244	135	141
10°	3872	5879	1981	3898	126	
20	2947	4694	2057	2037	121	
30	2266	3552	2129	1423		

5628 148 38 428

$\frac{38}{140} = 2.7°$

0·0138
0·000845

1482 9432 1226

1399 9269 213

8692 4346 3038 7147 1519 5628

0008375 76 845

28. 27 56 136

76·3 27 326 489

375

Amylalk. ...

88

0°	0·08532	9310	1903	7407	160·6	12 — 154
	6000	7782	1981	5801	148·2	10
	4341	6376	2057	4319	138·9	
	3206	5059	2129	2930		

524

$\frac{56}{152} = 3·7$

13·7°

$\frac{0·0150}{0·00094} = 16·0$

32 48 36·8

0·0009241
72
76
939

1367	2734
4221	1303
2010	4771
8598	8808
724	760

5092	5224	5353	5478	5599
9542	9542	9542	9542	9542
4634	4766	4895	5020	5141

$(C_2H_5)_2O$

$\frac{\frac{28}{58}}{\frac{16}{74}}$

0^0 ·00286	4564	1527	3037	51·5	
2585	4125	1605	2520	50	20°
0·00 2345	3701	1680₅	2020	51	
212	3263	1753	1510		

$\frac{0.00505}{0.00165} = \cancel{3.06}$ 6.18 7.04

0·001480
140
32
0·00165

8692	8692	8692	8692
4362	4518	4669	4814
3054	3210	3361	3506

$0.0_2\,148026$ $0.0_5\,3503$ $0.0_7\,2701$

Propylj.ide 109·5

60	487	6964	3758	3206	43·8
70	456	6590	3822₅	2768	42·6
80	4195	6227	388₅	2342 − 202	51
90	387	5877	394₅₅	1832	78₅ 65 57
100	0·00359	5551	4504₅	1047	

$\frac{19}{66} = 32$ 86·8°

2·2292	2292	2292	2292	2292
2·5224	5353	5478	5599	5717
7516	7645	7770	7891	9009

$0.0_2\,10276$ $0.0_5\,1866$ $-0.0_7\,0005$

$\frac{0.0055}{0.00135} = \cancel{4.07}$ 9·36

Äthylsulfid 90

$\frac{\frac{14}{62}}{76}$

50	0·00331	5198	2317	2881	44·3
60	3035	4821	2383	2438	43
70	279	4456	2447₅	2008	41·9
80	257	4099	2510	1589	41·2
90	237	3747	2570₅	1177	

$0.0_2\,11964$ $0.0_5\,18065$ $0.0_7\,07882$

$\frac{1.2}{42} = 0.03$ 69·7°

$\frac{0.00420}{0.00156} = \cancel{2.69}$ 5.38 7 8 6.19

Propionsäureanhydrid 130

90	0·004715	6735	3369	3372	47		
100	4295	6330	3428	290	38		
110	399	6010	3485₅	252	50	46	
	3595	5557	3541₅	202	42		
	331	5198	3596	160	39	120°	
	306	4857	3649₅	121	38		
	284	4533	3701	83	37	$\frac{0·0046}{146}$ =	
	2635	4208	3752	46			

0·0010911 $0·0_5 038295$ $0·0_7 06515$

9·25

Para-Xylol 106

				296	45	
70	377	5763	2803			
80	345	5378	2865₅	251	42	91·7°
90	317	5011	2926	209	42	
100	292	4654	2985	167	40	
110	270	4314	3042₅	127	38	
120	2505	3988	3098₅	89	37	
130	233	3674	3153	52		

0·0097013 $0·0_5 08714$ $0·0_7 05287$

$\frac{0·0042}{0·0100}$ = 4·2

9·66

1139	1139	1139	1139	1139	1139	1139	1139
5599	5717	5832	5944	6053	6160	6263	6365
6738	6856	6971	7083	7192	7299	7402	7504

0253	0253	0253	0253	0253	0253	0253
5353	5478	5599	5717	5832	5944	6053
5606	5731	5852	5970	6085	6197	6306

0792 1584
3010 4771
5832 8139
9634 4494

9192 2814_5

0.0010 91
92
281
1464

4.10^4

.6628
1644
4984

9624 9248
3010 4771
9402 7232
2036 1251

1598

1334

0.0097013
~~2006~~
1598
1334
0.009995

Hg $\beta = 3{\cdot}85 \cdot 10^{-6}$

$\alpha = 0{\cdot}0001817$

C_6H_6

16	82
14·8	75
16	90
17·9	92
15·4	87
301·5	26 : 5 = 52

16·02 $\beta = 85{\cdot}2 \cdot 10^{-6}$

$\alpha = 0{\cdot}00123$

$a_{Hg} = \dfrac{273 \cdot \cancel{0{\cdot}000}1817}{(13{\cdot}6)^2 \quad 3{\cdot}85}$

4362
2594
6956 — 2670
8525 — 5855
8431 — 8525
8525
9906 — 2576

a = 97·86

$a\rho^2 = 18100$ atm

$\dfrac{289 \cdot \cancel{0{\cdot}00}123}{8{\cdot}52 \cdot 10^{-5} \; (0{\cdot}863)^2}$

4609 — 9304
0899 — ~~9360~~
5508 — 8720
8024 — 8024
7484 — 6204

a = 5602

4173 atm

~~0·880~~
124.16
72
1·0192
880
−17
863

$\varphi = \dfrac{a\rho}{RT}\,\mu$

9906 — 5705
1335 — + 3010
1241 — ~~8715~~
8715 — 8715
2526

$0{\cdot}01789 \cdot 40000$

7156

5602
9335
4937
3631
4306

273
22·75
295·75

12.22
24
24
1·0264
208
208
858

4710
8921
3631

5000

$0{\cdot}135 \cdot 78^2$

8921
7842
1303
9145

φ dla temp. odpow. (wielk. ρ)
z równ. $\varphi = \dfrac{a\rho\mu}{RT}$

8213

Calb OH $\quad$ 63·40 $\quad$ 9

$\rho = 0·0_4 870 (1 + 0·0_2 3177\, t + 0·0_4 550\, t^2)$

24
16
6
46

8021 $\quad$ 6042
5021 $\quad$ 7404
3042 $\quad$ 3446

~~1~~2015 $\quad$ 2211
2211

$\beta = 1·4226 \,.\, 0·0_4 870$

$\alpha =$

80625

1530
9868
1398

$\beta = 0·0_3 1380$ $\quad$ $\alpha = 0·001353$ (1313)

9212 $\quad$ 4624
8021 $\quad$ 6042
0·07233 $\quad$ 0666
656

$a = \dfrac{3364 \,.\, \cancel{0·00}1353}{(0·7522)^2 \; \cancel{0·000}138}$

0·05288 $\quad$ 80625
116 $\quad$ 05404
0·05404 $\quad$ 07522

$\varphi = \dfrac{13·53 \qquad .46}{1·38 \,.\, 0·7522 \,.\, \cancel{}}$

1313 $\quad$ 1399
6628 $\quad$ 8763
7941 $\quad$ 0162
0162
7779

$\varphi = 59·7$

	Hg	C_6H_6	C_2H_5OH
krit. T		288·5	243·6
π		47·89	62·76
d		0·3045	0·288
			0·00713
v			

$$\eta_1 : \eta_2 = \frac{\sqrt{\mu_1 T_1}}{\sqrt[3]{l_1^2}} : \frac{\sqrt{\mu_2 T_2}}{\sqrt[3]{l_2}}$$

C_6H_6 22·75

C_2H_5OH 63·40
273
336·40

243·6 288·5
273 273
516·6 561·5

5268
7494
2762
−7131
5631

3657
273
92·7

Br T_K = 302·2
Amylalk. 348·0
Äther 195
Äthylsulfid 284·67
ParaXylol 344·4

344·4 284·7 91·7
273 273 ~~273~~
617·4 557·7 364·7
5371 7544 9624
4362 4362 4362
9733 9906 3986
8901
2892
−9733
3159

~~207~~

5619
7464
3083
7906
5177

3294
273
56·4

377 32
325 28
317 25 — 26.5 $\frac{26.0}{313}$ $\frac{1}{\mu}\frac{d\mu}{dt} \sim \frac{1}{617.4}$
292 22
270
250.5 19.5
491
364 37
331 33
27.5
303.5
24.5 $\frac{23.3}{279}$ $\sim \frac{1}{557.7}$
279 23.2
22
257
237 20

4150	3674	4150	3674
7906	7464	−4955	−4456
2056	1138	9195	9218
−4955	−4456	831	835
7101	7682		
513	586		

$\frac{1}{\mu}\frac{d\mu}{dt}$

$\rho =$

CO_2:

5	0·922	0·000925	9661 · 04375	9224
10	0·875	852	9304 · 04765 = 8828	
15	864	784	8943 − 05145 = 8429	
20	827	712	8525 − 0552 = 7973	
25	783	625		
29		539		

40 − 40

12
32
44

273
15
288

4594 − 0435 = 1029; 4518 + 6435 = 0953; 4669 + 6435 = 1104; 4440 + 6435 = 0875

$(C_2H_5)_2O$: 74

15·8	0·002502				
29·2	2180				
47·2	1870	2718	1873	0845	43·9
63·5	1626	2111	1980s	0130	46·4
78·7	1413	1501	2076s	9425 (8828)	42·6
100·4	1177	0708	2207	8501	

273
373·4

8692 − 5722 = 4414; 8692; 78·7 + 273 = 351·7; 63·5 + 273 = 336·5; 47·2 + 273 = 320·2; 5461 − 8692 = 4153; 8692 − 5269 = 3961; 5054 − ~~2732~~ 3886; 5054 − 8692 = 3746

$(C_2H_5)_2O$: $v = v_0 (1 + 0·0_2 1349 + 0·0_5 6554 t^2 - 0·0_7 3449 t^3 + 0·0_9 3377 t^4)$

~~214~~ 924; 9657 − 3765 = 6292; $\frac{705}{15·2}$; 8482 + 1818 = 6664; $\frac{715}{16·3}$; 8543 − 2122 = 6421; $\frac{32·7}{42·6}$ = 5145 − 6294 = 8851; 767

92·7

$$\alpha_{10} = \frac{27}{5.913} \quad \frac{31}{5.880} \Bigg\} \quad \frac{29}{5.900} = \frac{58}{9000} = 0{\cdot}00644$$

9671	9342	9013	8684
1300	8165	5377	5285
0971	7507	4390	3969
1·1251	5632	2748	2494

1·1251
563
249
1·2063
− 275
1·1788

3010	4771	6021
9671	9342	9013
8165	5377	5285
0846	9490	0319
1215	8892	1076

α = 0·001349
1215
1076
0·003640
− 889
0·002751

4393
0714
3679

$\alpha = 233^{0{\cdot}0_2}$

$$\frac{0080}{00644} \; (\times 2{\cdot}3) \qquad \frac{00426}{00233} \; (\times 2{\cdot}3)$$

= 9031 / 8089 / 0942 — 6294 / 3674 / 2620

1·24 — 1·83

$$\mu = \sqrt{m\theta}\ \psi\left(\frac{v}{b}\right)$$

$$\frac{d\mu}{d\theta} = \frac{1}{2}\sqrt{\frac{m}{\theta}}\ \psi + \frac{\sqrt{m\theta}}{\cancel{\theta}}\ \psi'\ \frac{1}{v}\frac{dv}{d\theta}\cdot\frac{v}{b}$$

$$\frac{d\mu}{dp} = \sqrt{m\theta}\ \psi'\ \frac{1}{v}\frac{dv}{dp}\ \frac{v}{b}$$

$$\frac{d\mu}{d\theta} = \frac{1}{2}\frac{\mu}{\theta} + \frac{d\mu}{dp}\cdot\frac{\frac{1}{v}\frac{dv}{d\theta}}{\frac{1}{v}\frac{dv}{dp}}$$

$$\frac{1}{\mu}\frac{d\mu}{d\theta} = \frac{1}{2\theta} + \frac{1}{\mu}\frac{d\mu}{dp}\cdot\frac{\frac{1}{v}\frac{dv}{d\theta}}{\frac{1}{v}\frac{dv}{dp}}$$

Experimentell $\quad v = v_0(1 + 0{\cdot}0_2 09003 + 0{\cdot}0_5 19595\, t - 0{\cdot}0_7 044998\, t^2)$

$$\alpha_{20} = 0{\cdot}0_2 0973$$

μ Temperatur

20°	0.01461	1647	8353	68.43	~~50~~ 34.9	7	12
50°	968	9859	0141	1033	363	14	
80°	716	8549	1451	1396 5			

$$\frac{1}{\mu} = a + bt$$

$$-\frac{1}{\mu^2}\frac{d\mu}{dt} = b = 34.2$$

3340
1647
6987

$$\frac{1}{\mu}\frac{d\mu}{dt} = b\mu = \frac{0.4997}{30}$$

$$= 0.01666$$

7° 0.001668 ~~275~~

15° 0.001625 27

20° 0.00160 $= \frac{1}{\mu}\frac{d\mu}{dp}$ (1 atm)

$$\alpha_{20} = 0.000973$$

$$\beta = 79.12 \cdot 10^{-6}$$

$0.000973 \cdot 0.0016$

0.00000791

4362
2010
7372
2628

0.001831

1831

2041
9881
1922
− 8984
2938
15

0.01967

− 0.0197
+ 0.0018
−0.0179

−0.0167 || −0.0179 ||

" beob " ber.

$\eta = \eta_0 (1 + \alpha p)$

	CO_2	$(C_2H_5)_2O$	C_6H_6
	(25.1°)	(20°)	(20°)
$a \cdot 10^6 =$	7470	730	930 ~~638 (Quincke)~~
$\beta =$	0.003040	0.000~~173~~ 167	0.000091

638 Quincke
92 Rö
82 Jäger
90 Amagat
75 De Metz
90 Pagliani

0.0916 .585
5265
58
20
6385

0.001654
$\alpha =$ 0.001248

$\frac{0.000730 \cdot 0.001654}{0.000\cancel{14}}$

$\frac{\cancel{0.000}930 \cdot \cancel{0.00}1248}{\cancel{0.0000}91}$

8633
2186
0819
− 2227
8592

9685
0962
0647
− 9590
1057

4669
5331
201
2321

1707

0.007232
171
− 0.00552

0.01275
171
0.01104

0.0150
~~0.0~~

C_6H_6

5° 0.00286
10° 0.002585
20° 0.002345
30° 0.00212

$\left.\begin{array}{l}\frac{240}{2465} \\ \frac{225}{2232}\end{array}\right\} = 0.0098$

$C_4H_{10}O$

6·7	0·002668	4262	227	: 5·12	444
11·81	2532	4035	215	5·31	405
17·12	2410	3820	180	4·68	385
21·8	2312	3640	151	3·65	395
25·45	2233	3489	275	6·59	418
32·04	2096	3214			

3560	3324	2553	1790	4393
7093	7251	6702	5623	8189
6467	6673	5851	5967	6204

0°	0·009025	1435	110	0414	
100	759	118	704	8476	156
				1938	
20°	649	87	87	9395	— 150
30	562	70	6055	7822	144
40	492			1573	

Glycerin

2·8	42·2	6253	3747	237
8·1	25·2	4014	5986	397
14·3	13·9	1430	8570	719s
20·3	7·78	8910	1090	1285
26·5	4·94	6937	3063	2024

3500
7243
6257

4123
7924
6199

2930
7924
5006

2239 : 5·3 = 422
2584 : 6·2 417
2520 : 6·0 420
1973 : 6·2 317

Amylalkohol.

$u = a e^{\alpha t + \beta t^2}$

0·11129	04650	1758	119
7425	8707	1639	126
5091	7068	1513	118
3593	5555	1395	103
2606	4160	1292	
1935	2868	1189	103
1472	1679	1083	106
1147	0596	1010	73
909	9586	923	87
725	8663	845	78
608	7818	785	60
505	7033	708	77
429	6325		

$$a = -\left(\frac{T \frac{\partial p}{\partial T}}{p^2 \frac{\partial p}{\partial p}} \right)_{t=0}$$

$(C_5H_{11})_2O$: $\beta_0 = 0·000146$ $\beta_{35} = 0·$ ~~1403~~ 2277 || $\alpha_0 = 0·00148$

$$\frac{T \frac{1}{v_0}\frac{\partial v}{\partial T}}{\frac{1}{v_0}\frac{\partial v}{\partial p}} \left(\frac{v}{v_0}\right)^2 v_0^2$$

			2010	4771	
5441	0882	6323	5441	0882	~~6323~~
1703	5445	4314	5445	4314	
7144	6327	0637	3896	9967	
1·0518	429	116	245	9925	

1·0518
43
12
1·0573

0·0014803
245
99
0·001825

$$a = \frac{1}{\rho_0^2}\left\{\frac{273 .\ 0·00148}{0·000146}\right\} \quad \left\| \quad \left\{\frac{308 .\ 0·001825 .\ 1·057}{\cancel{1·057}\ 0·0002277}\right\}\right.$$

4362
1703
6065
− 1644
4421

= 2770

4886
2613
0241
7740
− 3573
4167

= 2610

$$p + \frac{a}{v^2} = \frac{R\theta\sqrt{2}}{v - v_0}$$

$$1 - \frac{2a}{v^3}\frac{\partial v}{\partial p} = -\frac{R\theta\sqrt{2}}{(v-v_0)^2}\frac{\partial v}{\partial p}$$

$$-\frac{2a}{v^3}\frac{\partial v}{\partial \theta} = \frac{R\sqrt{2}}{v-v_0} - \frac{R\theta\sqrt{2}}{(v-v_0)^2}\frac{\partial v}{\partial \theta} \qquad \|$$

$$\boxed{\frac{\partial v}{\partial \theta} = -\frac{R\sqrt{2}}{v-v_0}\frac{\partial v}{\partial p}}$$

$$1 = \frac{\partial v}{\partial p}\left[\frac{2a}{v^3} - \frac{R\theta\sqrt{2}}{(v-v_0)^2}\right] = -\frac{\frac{\partial v}{\partial p}\frac{R\sqrt{2}}{v-v_0}}{\frac{\partial v}{\partial \theta}}$$

$$-\frac{R\sqrt{2}}{v-v_0} = \left[\frac{2a}{v^3} - \frac{R\theta\sqrt{2}}{(v-v_0)^2}\right]\frac{\partial v}{\partial \theta}$$

$$-\frac{2}{v}\frac{\partial v}{\partial \theta}\frac{R\theta\sqrt{2}}{v-v_0} = \frac{R\sqrt{2}}{v-v_0} - \frac{R\theta\sqrt{2}}{(v-v_0)^2}\frac{\partial v}{\partial \theta} \qquad \frac{a}{v^2} = \frac{R\sqrt{2}}{\frac{\partial v}{\partial \theta}}$$

$$-\frac{2}{v}\frac{\partial v}{\partial \theta}\theta = 1 - \frac{\theta}{v-v_0}\frac{\partial v}{\partial \theta}$$

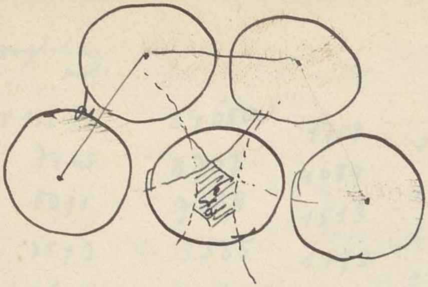
$$\theta\frac{1}{v}\frac{\partial v}{\partial \theta} = \frac{1}{\frac{1}{v-v_0} - \frac{2}{v}} = \frac{v(v-v_0)}{v-2(v-v_0)} = \frac{(v-v_0)}{2v_0 - v} \neq \frac{v-v_0}{v_0}$$

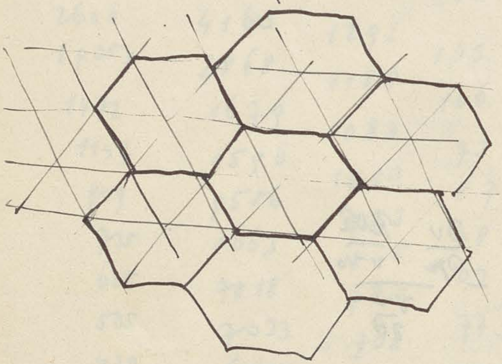
Benzol $\alpha_{20} = \frac{0.00125 \cdot 293}{0.36} = \frac{v-v_0}{v_0}$

$$\frac{1}{v-v_0} \neq \frac{1}{\theta\frac{\partial v}{\partial \theta}}$$

$$\frac{\partial v}{\partial \theta} = -\frac{R\sqrt{2}}{\theta\frac{\partial v}{\partial \theta}}\frac{\partial v}{\partial p}$$

$$\frac{\partial v}{\partial p} = -\left(\frac{\partial v}{\partial \theta}\right)^2\frac{\theta}{R\sqrt{2}}$$





$$p+\frac{a}{v^2}=\frac{RT}{v_a}\left(\frac{a}{\frac{d}{2}}\right)^2$$

$$v_a : v=\left(\frac{a}{\frac{d}{2}}\right)^3$$

$$p+\frac{a}{v^2}=\frac{RT}{v}\,\frac{d}{2a}$$

$$=\cancel{\frac{RT}{v}\,\frac{d}{2(d-2a)}}$$

$$v_0 : v=(\sigma : d)^3$$

$$=(d-2a)^3 : d^3$$

$$\frac{v_0}{v}=1-\frac{3a}{d}$$

$$\frac{3a}{d}=1-\frac{v_0}{v}$$

$$p+\frac{a}{v^2}=\frac{3}{2}\frac{RT}{v}\,\frac{1}{\left(1-\frac{v_0}{v}\right)}=\frac{3}{2}\,\frac{RT}{v-v_0}$$

$$v-v_0=\frac{\frac{3}{2}RT}{p+\frac{a}{v^2}}$$

$$\cancel{\frac{\partial v}{\partial p}=\frac{\frac{3}{2}RT}{\left(p+\frac{a}{v^2}\right)^2}\left(1-\frac{2a}{v^3}\frac{\partial v}{\partial p}\right)}$$

$\lim p=\infty$:

$$\cancel{\frac{\partial v}{\partial p}=-\frac{\frac{3}{2}RT}{p^2}}$$

$$l(v-v_0)=l\left(\tfrac{3}{2}RT\right)-l\left(p+\tfrac{a}{v^2}\right)$$

$$\frac{1}{v-v_0}\frac{\partial v}{\partial p}=-\frac{1}{p+\frac{a}{v^2}}=-\frac{v-v_0}{\frac{3}{2}RT}\left(1-\frac{2a}{v^3}\frac{\partial v}{\partial p}\right)$$

$$\frac{\partial v}{\partial p}=-\frac{(v-v_0)^2}{\frac{3}{2}RT}=\frac{(v-v_0)}{\left(p+\frac{a}{v^2}\right)}$$

$$=\frac{\frac{3}{2}RT}{\left(p+\frac{a}{v^2}\right)^2}$$

v_0 bedeutet nicht den thatsächlich von d. Mol. eingenom. Raum, sondern den Raum d. Körpers bei grösster Compression.

$$\frac{\partial v}{\partial p} = \frac{1}{\frac{2a}{v^3} - \frac{3}{2}\frac{R\theta}{(v-v_0)^2}} = \frac{1}{\frac{2a}{v^3} - \left(p + \frac{a}{v^2}\right)^2 \cdot \frac{2}{3R\theta}}$$

$p_0 = 0.736$

$$R\theta = \frac{28}{74 \cdot 0.00125}$$

1.8692
0.0972 − 3
0.9664 − 2

1.4472
−0.9664 +2
2.4808

$\cancel{\frac{2a}{v^2}}$ $\frac{a}{v^2} = 2770$

~~3.4425~~
0.8669 − 1
9010
3.6104

6.8850
9010
71860
2.9579
42281

4771
2.4808
~~19963~~

0.00007793 || w rzeczywistości
Amagat
→ 0.000107

16910
4077
12833

1083
8917

2770
3000
5770

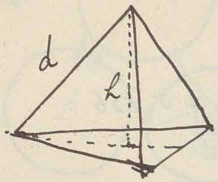
7612
5224
9010
8234
9579
8655

73400
408
−69320

8408
1592

0.00001443

→ 0.000032



$h = d\sqrt{\frac{2}{3}}$

Vollständige Bewegungsfreiheit für horizontale Schichtenverschiebung wenn

$h \geqq 6$

Gar keine Bewegungsfreiheit wenn

$h \leqq \frac{6}{2}\sqrt{3}$

$\cancel{> \frac{d}{6} > \sqrt{\frac{3}{2}}}$ Vollständig Freih.: $\frac{d}{6} > \sqrt{\frac{3}{2}} = 1.225$ $\qquad \frac{v}{v_0} > 1.84$

Mittelfreih.: $\sqrt{\frac{3}{2}} > \frac{d}{6} > \frac{\sqrt{3}}{2} \cdot \sqrt{\frac{3}{2}}$

$\sqrt{\frac{3}{2}} > \frac{d}{6} > \frac{3}{2\sqrt{2}}$

Unfreih.: $\frac{d}{6} < \frac{3}{2\sqrt{2}} = 1.061$ $\qquad \frac{v}{v_0} < 1.193$

Fester Körper^{2}: $\frac{d}{6} = 1.061$.

$v_0 : v = 6^3 : d^3$

4771
3010
1761
08805
26415

4771
3010
1505
4515
0256
0768

$$\frac{1}{\rho}\frac{\partial \rho}{\partial \theta} = \frac{R\varphi}{2a\rho - RT\left(\varphi + \frac{\rho}{\rho_0}\varphi'\right)}$$

$$= \frac{R\varphi}{2RT\varphi - \frac{2p}{\rho} - RT\varphi - RT\frac{\rho}{\rho_0}\varphi'}$$

$$= \frac{R\varphi}{RT\left(\varphi - \frac{\rho}{\rho_0}\varphi'\right) - \frac{2p}{\rho}}$$

dla $p = 0$:

$$\frac{1}{\rho}\frac{\partial \rho}{\partial \theta} = \frac{\varphi}{T\left(\varphi - \frac{\rho}{\rho_0}\varphi'\right)} = \frac{1}{\theta}\,\chi\left(\frac{\rho}{\rho_0}\right)$$

Wartości p odpowiadające sobie

	Hg	99°	$\alpha = 0{\cdot}000182$	!
	C_6H_6	22·7°	$\alpha = 0{\cdot}00126$	!
	C_2H_5OH	63·40	$0{\cdot}00135$	
	Dimethyläthylcarbinol	17·2°	$\alpha = 0{\cdot}00114$	
	Amylalk.	13·70	$0{\cdot}00094$	
	Butylalk	2·70	~~0·000845~~	
	$(C_2H_5)_2O$	20°	$\alpha = 0{\cdot}00165$	
	Paraxylol	92°	$\alpha = 0{\cdot}00100$	

$$\frac{\partial \rho}{\partial p} = \frac{1}{RT(\varphi + \frac{\rho}{\rho_0}\varphi') - 2a\rho} = \frac{1}{RT(\varphi + \frac{\rho}{\rho_0}\varphi' - 2\varphi) - 2\frac{p}{\rho}}$$

$$= \frac{-1}{RT(\varphi - \frac{\rho}{\rho_0}\varphi') + \frac{2p}{\rho}}$$

$$\left(\frac{1}{\rho}\frac{\partial \rho}{\partial p}\right)_{p=0} = -\frac{1}{RT\rho(\varphi - \frac{\rho}{\rho_0}\varphi')}$$

$$\frac{\alpha}{\rho} = -\varphi R\rho$$

$$\mu = (n) m \overbrace{(\lambda + \sigma)}^{d} \quad (n) = C\rho/ \quad = \frac{Nc}{3} \quad \text{Alagesen}$$

$$p + \frac{a}{\rho^2} = (n)\, m c = R\theta\rho\varphi = \frac{3}{2}\frac{R\theta}{v - v_0}$$

$$\mu = \left(\frac{3}{2}\frac{R\theta}{v - v_0}\right)\frac{d}{c}$$

$$c = \sqrt{\frac{\theta}{m}} \cdot ge$$

Jäger
Fortschr. p. 121

$$d = \sqrt[3]{\frac{m}{\rho}} \cdots \qquad d^3 = \frac{1}{\rho \underset{\text{anzahl in 1 gr}}{N}} = \frac{m}{\rho A} \qquad N = \frac{A}{m}$$

$$\mu = \frac{3}{2}\frac{R\theta}{v - v_0}\sqrt[3]{\frac{m}{\rho}}\sqrt{\frac{m}{\theta}}\ \text{Ch} = \cancel{\ldots}$$

$$\cancel{\ldots} \sim \frac{\rho^2}{\sqrt{\theta} \cdot \rho^{1/3}}$$

$$\frac{1}{\mu}\frac{d\mu}{d\theta} = \frac{5}{3}\rho^{2/3}\frac{d\rho}{d\theta} - \frac{1}{2\theta} \qquad = -\frac{5}{3}\rho^{5/3} \cdot \alpha - \frac{1}{2\theta} \qquad !!!

$\cancel{\eta = \sqrt{m\theta}\,\psi\left(\frac{p}{p_0}\right)}$

$$\left(\frac{1}{\eta}\frac{\partial\eta}{\partial t}\right)_{v=\text{const}} = \frac{1}{\eta}\frac{d\eta}{dt} - \frac{1}{\eta}\frac{\partial\eta}{\partial p}\frac{\alpha}{\beta}$$

$$\cancel{= \left(\frac{1}{\eta}\frac{\partial\eta}{\partial\theta}\left(\frac{\partial\sigma}{\partial t}\right)_{v=\text{const}}\right)}$$

$$\eta = \frac{\sqrt{m\theta}}{\sigma^2}\,\psi\left(\frac{v}{b}\right)$$

$$b \sim \sigma^3$$

$$\frac{1}{b}\frac{\partial b}{\partial\theta} = 3\,\frac{1}{\sigma}\frac{\partial\sigma}{\partial\theta}$$

$$\cancel{\frac{1}{\eta}\frac{\partial\eta}{\partial\theta} = -\frac{2}{\sigma} + \frac{\psi'}{\psi}\frac{1}{b}\frac{\partial v}{\partial\sigma}}$$

$$\left(\frac{1}{\eta}\frac{\partial\eta}{\partial\theta}\right)_v = \frac{1}{2\theta} - \frac{2}{\sigma}\frac{\partial\sigma}{\partial\theta} - \frac{\psi'}{\psi}\frac{v}{b^2}\frac{\partial b}{\partial\theta}$$

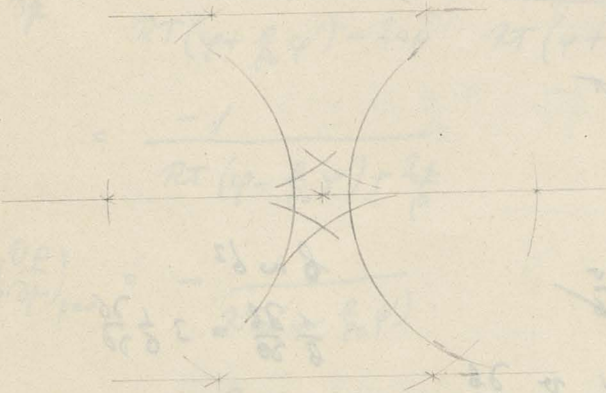
$$= \frac{1}{2\theta} - \frac{1}{\sigma}\frac{\partial\sigma}{\partial\theta}\left[2 + 3\frac{\psi'}{\psi}\cdot\frac{v}{b}\right]$$

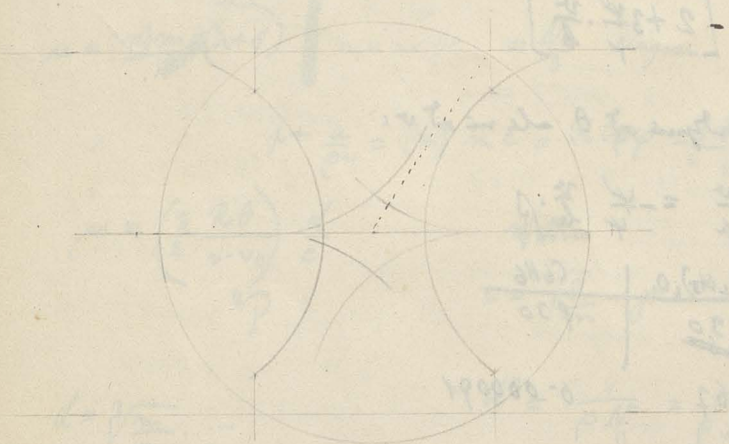
Pod założeniem, że σ zależy jedynie od θ, ale nie od v:

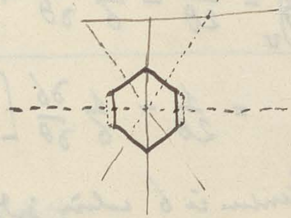
$$\left(\frac{1}{\eta}\frac{\partial\eta}{\partial p}\right)_{t=\text{const}} = \frac{\psi'}{\psi}\frac{1}{b}\frac{\partial v}{\partial p} = -\frac{\psi'}{\psi}\frac{v}{b}\cdot\beta$$

	$(C_2H_5)_2O$	C_6H_6
$\frac{1}{\eta}\frac{\partial\eta}{\partial p} =$	0·000730	930
$\beta =$	0·000167	0·000091
$\frac{1}{\eta}\frac{d\eta}{dt} =$	−0·0098	−0·0150
$\frac{\alpha}{\beta}\frac{1}{\eta}\frac{\partial\eta}{\partial p} =$	0·00723	0·0127
$\frac{1}{\eta}\frac{\partial\eta}{\partial t} =$	−0·00157	−0·0023
zamiast	+0·00171	+0·00171
	328	401
$\frac{1}{\sigma}\frac{\partial\sigma}{\partial\theta} =$	−0·0003	−0·00015

$\frac{\psi'}{\psi}\frac{v}{b} = -4{\cdot}3 \quad \| -10{\cdot}2$







Aluminium (Pontschow)

274) −250° 0° 250° 500° 625

0·1428 0·209 0·238 274 308

3·84 8·44

186
79
265

274
√696
3·84

8·22
26

18

Sb Dulong Naccari

120 −133° 15° 100 200 300

0·0462 0·0489 0·0503 520 537

(N. Petersen)

Se 0−100 0−300

91 0·0425 0·0506

Pb −90° 15° 100 200 300

207 0·0294 0·02993 310 324 338

B −40° 27° 126 233

11 0·1915 238 307 366

Cd −133° 21° 300°

112 0·0498 0·0551 0·0617

Cr −200 −100 0° 200 400 600

52·1 0·0666 0·0898 0·104 118 133 187

Fe Schmitz Nichol Pionchon

55·9 −760 60° 157 250 350 860 1100

0·0890 0·119 0·1275 176 324 218 199

Ge	50°	220°				
72.5	0.0737	0.0757				
Au	50°	0.0316				
197						
Ir	−84°	50	700			
193	0.0282	0.0323	0.0401			
K	−25°	0.1662				
39.2						
				Tilden		
Co	−84°	57°	83	282	322	
59	0.0822	103	109	121	123	
						Abondan
				250	400	500
				145	184	204
C	−50.5°	10.7	140	247	607	985
Graphit 12	0.0635	113	222	303	441	459

Pt	-84°	59°	500	1000	1500
195	0·0293	324	377	409	461

Prędkość, z którą się kurczy przerwana błona [illegible]

$$\left.\begin{aligned} m\frac{d^2x}{dt^2} &= \alpha \\ m &= \delta\rho x \end{aligned}\right\} \qquad x\frac{d^2x}{dt^2} = \frac{\alpha}{\delta\rho}$$

$$m\frac{d^2x}{dt^2} = -kx - ax^3$$

$$\frac{m}{2}\left(\frac{dx}{dt}\right)^2 = c - \frac{k x^2}{2} - \frac{a}{4}x^4 = c - U_x$$

$$\frac{\sqrt{m}\, dx}{\sqrt{c - kx^2 - \frac{ax^4}{2}}} = dt$$

$$\frac{1}{\sqrt{c - kx^2 - \frac{ax^4}{2}}} = \frac{1}{\sqrt{c-kx^2}}\left[1 - \frac{ax^4}{2(c-kx^2)}\right]^{-\frac{1}{2}}$$

$$= \cancel{\sqrt{c-kx^2}} - \frac{ax^4}{4\sqrt{c-kx^2}}$$

$$= \frac{1}{\sqrt{c-kx^2}} + \frac{ax^4}{4\sqrt{c-kx^2}^{\,3}}$$

$$\sqrt{c - kx^2 - \frac{ax^4}{2}} = \sqrt{c-kx^2} - \frac{ax^4}{4\sqrt{c-kx^2}}$$

$$\frac{1}{t}\int U_x\, dt = \frac{1}{t}\left[U_x t - \int dt\, t\, \frac{\partial U}{\partial x}\frac{\partial x}{\partial t}\right]$$

$$\cancel{\frac{dx}{\sqrt{1-\alpha x^2 - \beta x^4}} = \frac{dx}{\sqrt{\left(1+\frac{\alpha^2}{4\beta}\right)^2 - \left(\frac{\alpha}{2\sqrt{\beta}} + x^2\sqrt{\beta}\right)^2}}}$$

$$= 2$$

$$= \frac{1}{2t}\int \frac{c - kx^2 - \frac{ax^4}{4}}{\sqrt{c - kx^2 - \frac{ax^4}{2}}}\sqrt{m}\, dx$$

$$= \frac{\sqrt{m}}{2t}\int_0 \sqrt{c - kx^2 - \frac{ax^4}{2}}\, dx$$

$$c - kx^2 - \frac{ax^4}{4} = 0$$

$$\cancel{\int \frac{dx}{\sqrt{c-kx^2}} = \frac{1}{\sqrt{k}} \arcsin x\sqrt{\frac{k}{c}}}$$

$$\int \frac{dx}{\sqrt{1-\alpha^2x^2}} = \frac{1}{\alpha}\arcsin \alpha x$$

$$-\int \frac{\alpha x^2}{\sqrt{}^{\,3}}\, dx = -\frac{1}{\alpha^2}\arcsin \alpha x + \frac{1}{\alpha}\frac{x}{\sqrt{1-\alpha^2x^2}}$$

$$\int \sqrt{1-\alpha^2x^2}\, dx = \int\frac{1}{\sqrt{}} - \alpha\int\frac{x^2}{\sqrt{}}\, dx = x\sqrt{1-\alpha^2x^2} + \int\frac{x^2\alpha^2}{\sqrt{}}$$

$$\int\frac{\alpha^2x^2}{\sqrt{}} = \frac{1}{2}\left[\frac{1}{\alpha}\arcsin \alpha x - x\sqrt{1-\alpha^2x^2}\right]$$

$$\cancel{J = \frac{1}{\alpha}\arcsin \alpha x}$$

$$J = \frac{1}{\alpha}\arcsin \alpha x + \alpha\frac{\partial J}{\partial \alpha}$$

$$\int\sqrt{1 - \dots} = \frac{1}{2}\left[\frac{1}{\alpha}\arcsin \alpha x + x\sqrt{1-\alpha^2x^2}\right]$$

$$\frac{1}{2}\left(\frac{\alpha}{\alpha\sqrt{1-\dots}} + \sqrt{1-\dots} - \frac{\alpha^2x^2}{\sqrt{}} - \right] = \frac{1}{2}\left(\frac{1-\alpha^2x^2 + 1 - \alpha^2 x^2}{\sqrt{}}\right)$$

$$\frac{d}{dx}\left(x^3\sqrt{1-\alpha^2x^2}\right) = 3x^2\sqrt{1-\alpha^2x^2} - \alpha^2x^2\sqrt{1-\alpha^2x^2}$$

$$\frac{d}{dx}\left(x\sqrt{1-\alpha^2x^2}\right) = \sqrt{1-\alpha^2x^2} - \alpha^2x^2\sqrt{1-\alpha^2x^2} \Big| 3$$

$$\frac{d}{dx}(\alpha^2x^3+3x)\sqrt{1-\alpha^2x^2} = 3\sqrt{1-\alpha^2x^2} - \alpha^4x^4\sqrt{1-\alpha^2x^2}$$

$$\alpha^4\int x^4\sqrt{1-\alpha^2x^2}\,dx = \frac{3}{2}\left[\frac{1}{\alpha}\arcsin\alpha x + x\sqrt{1-\alpha^2x^2}\right] - (\alpha^2x^3+3x)\sqrt{1-\alpha^2x^2}$$

$$\int x^4\sqrt{1-\alpha^2x^2}\,dx = \frac{3}{2\alpha^5}\arcsin\alpha x - \left(\frac{x^3}{\alpha^2}+\frac{3x}{2\alpha^4}\right)\sqrt{1-\alpha^2x^2}$$

$$\cancel{\sqrt{c-kx^2-\frac{\alpha x^4}{2}} = \left[(c-kx^2)\left(1-\frac{\alpha x^4}{2(c-kx^2)}\right)\right]^{\frac{1}{2}} =}$$

$$\frac{d}{dx}\frac{x^3}{\sqrt{\ }} = \frac{3x^2}{\sqrt{\ }} + \frac{\alpha^2x^4}{\sqrt{\ }}$$

$$\frac{d}{dx}\frac{x}{\sqrt{1-\alpha^2x^2}} = \frac{1}{\sqrt{\ }} + \frac{\alpha^2x^2}{\sqrt{\ }}$$

$$\alpha^4\int\frac{x^4}{\sqrt{1-\alpha^2x^2}}dx - 3\int\frac{dx}{\sqrt{\ }} = \frac{\alpha^2x^3}{\sqrt{\ }} - \frac{3x}{\sqrt{\ }}$$

$$= x\frac{\alpha^2x^2-1}{\sqrt{\ }} = -x\sqrt{1-\alpha^2x^2} - \frac{2x}{\sqrt{1-\alpha^2x^2}}$$

$$\alpha^4\int\frac{x^4}{\sqrt{\ }} = -\frac{3}{\alpha}\arcsin\alpha x + x\sqrt{1-\alpha^2x^2} + \frac{2x}{\sqrt{\ }}$$

$$\frac{3}{\sqrt{1-\alpha^2x^2}} - \sqrt{1-\alpha^2x^2} + \frac{\alpha^2x^2}{\sqrt{\ }} = \frac{\cancel{3-1}+\alpha^2x^2+\alpha^2x^2}{\sqrt{\ }} - \frac{2\alpha^2x^2}{\sqrt{\ }^3}$$

$$-\frac{2}{\sqrt{\ }} + \quad = \frac{2\alpha^2x^2}{\sqrt{\ }^3}\left[1-(1-\alpha^2x^2)\right]$$

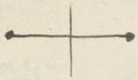
$$\int_0^{\frac{1}{\alpha}}\frac{x^4\,dx}{\sqrt{1-\alpha^2x^2}} = -\frac{3}{\alpha^5}\arcsin\alpha x + \frac{x}{\alpha^4}\sqrt{1-\alpha^2x^2} + \frac{2x}{\alpha^4\sqrt{\cdots}}$$

$$= \frac{3\pi}{2\alpha^5}$$

$$\infty$$

$$\overline{U}_{xa} < \overline{U}_{x0}$$

$$m\frac{d^2x}{dt^2} = -k(x-x_0) - f(x) = -\frac{\partial U}{\partial x}$$



$$-\overline{U} = \overline{\frac{mc^2}{2}}$$

~~jinak $f(x) = $ konst~~

~~lab $f(x) = f(x_0) + (x-x_0) f'(x_0)$~~

$$\frac{m}{2}\left(\frac{dx}{dt}\right)^2 = \underbrace{-\frac{k}{2}(x-x_0)^2 - \int f(x)\,dx}_{-U \quad +c}$$

$$= -\frac{k}{2}(x-x_0^2) - \left[x f(x_0) + \frac{(x-x_0)^2}{2} f'(x_0)\right] + c$$

$$\overline{U} = -\frac{1}{t}\int \frac{m}{2}\left(\frac{dx}{dt}\right)^2 dt$$

$$= -\frac{1}{t}\int U\,dt = -\frac{1}{t}\int U \frac{dx\sqrt{\frac{m}{2}}}{\sqrt{c-U}}$$

$$\overline{\frac{m}{2}\left(\frac{dx}{dt}\right)^2} = c - \overline{U} = \theta$$

$$c + \frac{1}{t}\int_0^c U \frac{dx\sqrt{\frac{m}{2}}}{\sqrt{c-U}} = \theta = c + \frac{\int_0^c U \frac{dx}{\sqrt{c-U}}}{\int_0^c \frac{dx}{\sqrt{c-U}}} =$$

$$\frac{m}{2} b^2\alpha^2 = -\frac{k}{2}a^2 - (x_0+a) f(x_0) - \frac{a^2}{2} f'(x_0) + c$$

$$c = \frac{m}{2} b^2\alpha^2 + \frac{k+f'(x_0)}{2} a^2 + (x_0+a) f(x_0)$$

$$-\overline{U} = \overline{\frac{m}{2}\left(\frac{dx}{dt}\right)^2} - c$$

$$\frac{m}{2}\left(\frac{d\xi}{dt}\right)^2 = -\frac{k}{2}\xi^2 - \xi f(x_0) + \frac{\xi^2}{2} f'(x) + c'$$

$$x - x_0 = \xi = a + b\sin\alpha t$$

$$-mb\alpha^2 \sin\alpha t = -ka - kb\sin\alpha t - f(x_0) - (a + b\sin\alpha t) f'(x_0)$$

$$-mb\alpha^2 = -kb - b f'(x_0) \qquad ka + f(x_0) + a f'(x_0) = 0$$

$$\alpha = \sqrt{\frac{k - f'(x_0)}{m}} \qquad a = -\frac{f(x_0)}{k + f'(x_0)}$$

$$\left(\frac{d\xi}{dt}\right)^2 = b^2\alpha^2\cos^2\alpha t$$

$$\overline{\frac{m}{2}\left(\frac{d\xi}{dt}\right)^2} = \frac{m b^2\alpha^2}{4} = \frac{b^2}{4}\left[k - f'(x_0)\right] = \kappa\theta$$

$$\cancel{\frac{m}{2} b^2\alpha^2\cos^2\alpha t = [a^2 + b^2\sin^2\alpha t + 2ab\sin\alpha t] - [a + b\sin\alpha t] f(x_0) + c'}$$

$$\cancel{-\frac{k + f'(x_1)}{2}}$$

$t = 0$:

$$\frac{m}{2} b^2\alpha^2 = -\frac{k + f'(x_0)}{2} a^2 - a f(x_0) + c'$$

$$\overline{U} = \frac{m b^2\alpha^2}{4} + \frac{f(x_0)^2}{2(k + f'(x_0))} + x_0 f(x_0)$$

W razie jeśli $f(x_0) = f'(x_0) = 0$ $\qquad \overline{U} = \overline{L} = \kappa\theta$

W przeciwnym razie różnica

którego $f(x_0) > 0$ (siła przyciąg.) $\qquad$ może być dodatnia lub ujemna

$f'(x_0) < 0$ ale maleje z odl.

ale $\frac{\partial U}{\partial \theta}$ niezmienione

$$m\frac{d^2x}{dt^2} = -C + kx$$

Kropelka elipsoidalna

$ds_1 : ds_2$ $\qquad \frac{x^2}{a^2}+\frac{y^2}{a^2}=1$

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1 \qquad \frac{y\,dy}{x\,dx} = -\frac{b^2}{a^2}$$

$$ds = dx\sqrt{1+\left(\frac{dy}{dx}\right)^2}$$

$$= dx\sqrt{1+\left(\frac{b^2x}{a^2y}\right)^2}$$

$$F = 4\pi\int_0^a x\,dx\sqrt{1+\frac{b^4x^2}{a^4y^2}}$$

$$= 4\pi\int_0^b y\,dy\,\frac{a^2}{b^2}\sqrt{1+\frac{b^2}{a^4y^2}\left(\cancel{a}b^2-a^2y^2\right)}$$

$$= 4\pi\frac{a^{\cancel{2}}}{b^2}\int \cancel{y}\,dy\sqrt{a^2y^2+\cancel{a}b^4-\cancel{a}b^2y^2} = \frac{4\pi a}{b^2}\int dy\sqrt{b^4+(a^2-b^2)y^2}$$

$$= 4\pi a\int dy\sqrt{1+\frac{a^2-b^2}{b^4}y^2}$$

$$\int\sqrt{1+\alpha x^2}\,dx = x\sqrt{1+\alpha x^2} - \underbrace{\int\frac{\alpha x^2}{\sqrt{1+\alpha x^2}}\,dx}_{+\int\left\{\frac{1}{\sqrt{1+\alpha x^2}}-\sqrt{1+\alpha x^2}\right\}dx}$$

$$\cancel{\frac{\sqrt{1+\alpha x^2}}{2} - \frac{\alpha x^2}{2\sqrt{}} + \frac{1}{2\sqrt{\alpha}}\,\frac{\sqrt{\alpha}}{\sqrt{1-\alpha x^2}}}$$

$$= \frac{x\sqrt{1+\alpha x^2}}{2} + \cancel{\frac{1}{2\sqrt{\alpha}}\arcsin\sqrt{\alpha}x}$$

$$+ \frac{1}{2\sqrt{\alpha}}\lg\left(x\sqrt{\alpha}+\sqrt{1+\alpha x^2}\right)$$

$$= 4\pi a\left\{\frac{\cancel{A}b\sqrt{1+\varepsilon^2}}{2}+\frac{b}{2\varepsilon}\lg\left(\cancel{b}\varepsilon+\sqrt{1+\varepsilon^2}\right)\right\}$$

$$= \cancel{4}2\pi ab\left\{1+\frac{\varepsilon^2}{2}+\frac{1}{\varepsilon}\left(\varepsilon+\frac{\varepsilon^3}{2}\cancel{A}-\frac{\varepsilon^3}{2}\right)\right\}$$

$F \neq 4\pi a b \left(1 + \frac{a^2-b^2}{6b^2}\right)$?

$\frac{a}{b} = \kappa$

$= 4 ab\pi = 4 \kappa b^2 \pi = 4\pi\kappa \left(\frac{c}{\kappa}\right)^{2/3} = 4\pi c^{2/3} \kappa^{1/3}$

$a b^2 = c = \kappa b^3$

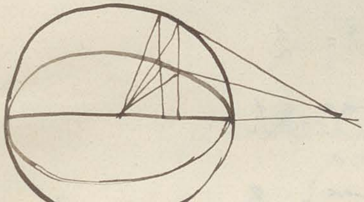
$\kappa = 1 + \alpha$

$F = \frac{4\pi c}{b}$

$\Delta F = 4\pi c^{2/3} (\kappa^{1/3} - 1)$

$= \frac{F_0 \alpha}{3}$

=



$\cancel{\xi \cdot \eta =}$

$\eta^2 = \xi\, r \cos\varphi = r^2 \sin^2\varphi$

$\xi = \frac{r \sin^2\varphi}{\cos\varphi}$

$ds : r\,d\varphi = \sqrt{\xi^2 + \alpha^2 \eta^2} : \sqrt{\xi^2 + \eta^2}$

$= \sqrt{\frac{\sin^4\varphi}{\cos^2\varphi} + \alpha^2 \sin^4\varphi} : \sqrt{\frac{\sin^4\varphi}{\cos^2\varphi} + \sin^4\varphi}$

$= \sqrt{1 + \alpha^2 \cos^2\varphi} : \sqrt{1 + \cos^2\varphi}$

Według Einsteina p. 374:

$$\sqrt{\overline{x^2}} = \sqrt{\frac{\int_{-\infty}^{+\infty} x^2 e^{-\overset{=h}{\left(\frac{N}{RT}\right)}U} dx}{\int e^{-\underset{=h}{\left(\frac{N}{RT}\right)}U} dx}}$$

$v = q\,x$

Gaz w walcowatym naczyniu, tłok równoważący ciężarem swoim ciśnienie gazu

$P = pq$

$U = pq\,x$

$$\frac{\int_0^\infty x^2 e^{-\overset{\alpha}{(hpq)}x} dx}{\int_0^\infty e^{-\overset{\alpha}{(hpq)}x} dx} = \frac{\frac{2}{\alpha^3}}{\frac{1}{\alpha}} = \frac{2}{\alpha^2}$$

$\cancel{\int x^2 e^{-\alpha x} dx = \frac{e^{-\alpha x}}{\alpha} x^2 - 2\int x e^{-\alpha x} dx}$

$$\int_0^\infty e^{-\alpha x} = \frac{1}{\alpha}$$

$$\int_0^\infty x e^{-\alpha x} = \frac{1}{\alpha^2}$$

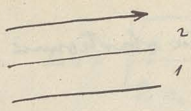
$$\int x^2 e^{-\alpha x} = \frac{2}{\alpha^3}$$

$$\sqrt{\overline{x}^2} = \frac{\sqrt{2}}{\alpha}$$

$$\Delta x = \frac{\sqrt{2}}{hpq}$$

$$\frac{\Delta x}{x} = \frac{\sqrt{2}}{phv}$$

$\int \mu \left(\frac{du}{dy}\right)^2 dy$



~~$\mu_1 du$~~ $u_1 = \alpha_1 y$

$u_2 = \alpha_2 y + \beta$

$\frac{\alpha_1}{2} = \frac{\alpha_2}{2} + \beta$

$u_2 = \alpha_2 y + \frac{\alpha_1 - \alpha_2}{2}$

$\alpha_2 + \frac{\alpha_1 - \alpha_2}{2} = u$

$\frac{\alpha_2 + \alpha_1}{2} = u$

$\mu_1 \alpha_1 = \mu_2 \alpha_2$

$\mu_1 (2u - \alpha_2) = \mu_2 \alpha_2$

$\alpha_2 = \frac{2\mu_1 u}{\mu_1 + \mu_2} \qquad \alpha_1 = \frac{2\mu_2 u}{\mu_1 + \mu_2}$

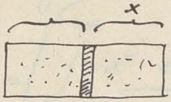
$\beta = \frac{(\mu_2 - \mu_1) u}{\mu_1 + \mu_2}$

$\int \mu_1 \alpha_1^2 + \mu_2 \alpha_2^2$

$\frac{1}{2} \frac{4u^2}{(\mu_1 + \mu_2)^2} (\mu_1 \mu_2^2 + \mu_2 \mu_1^2) = 2u^2 \frac{\mu_1 \mu_2}{\mu_1 + \mu_2}$

$\frac{2u^2 (\mu - d\mu)(\mu + d\mu)}{2\mu} = u^2 \frac{\mu^2 - d\mu^2}{\mu}$ [illegible] rzędu wyższego

I). Tłok w naczyniu walcowatem zamkniętem obustronnie, położony w samym środku



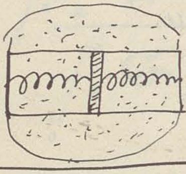
byłby wykonywać ruchy na prawo i lewo

średnio odległego z pozycji równowagi?

II). Zastępujemy gaz z jednej strony przez sprężynę działającą siłą proporc. do x.

$px = \text{const.}$ (izoterm.?)
lub ogólnie do odpowiedniej ... dla gazu x

III). Tak samo z drugiej strony, a gazowi zostawiamy wolny przestrzeń

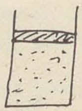


Przypadek III da się obliczyć według Einsteina

Czy I i III są identyczne?

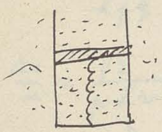
Albo lepiej?

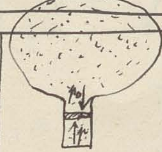
I). Tłok o ciężarze $= pq$



odgraniczający szczelnie dolny gaz

III).





I). Tłok z przyczepioną siłą $(p_0 - p_{v_0})$ równoważącą dokładnie ciśnienie

Wtedy każda pozycja równie możliwa $dW_1 = A\,dv$

II). Tłok bez tych sił przyczepionych

Wtedy $dW_2 : dW_1 = 1 : e^{-hU}$

$$dW_2 = A\,dv\,e^{-hU} = A\,dv\,e^{-h\int(p_0 - p_{v_0})dv}$$

zawartość x można oznaczyć: $V = vq$

długość taka że na cm² przekroju objętość v

Siła przy przesunięciu o dv: $= 2\,\frac{dp}{dv}\cdot q$

$$\frac{p\mu}{\rho T} = R$$

$$\sqrt{\overline{v^2}} = \sqrt{\frac{RT}{2N\frac{dp}{dv}q}}$$

$$\frac{N}{RT} = h$$

$$\frac{10^6 \cdot 30}{273 \cdot 0.0013} = 10^8$$

~~$N =$~~

$$N = 4 \cdot 10^{23}$$

$$\sqrt{\frac{p\mu\frac{T}{}}{\rho T\, 2N\frac{dp}{dv}q}} = \sqrt{\frac{p}{2n_0\frac{dp}{dv}q}}$$

$$qv = V$$

$$= \sqrt{\frac{pv}{2n_0 V\frac{dp}{dv}}}$$

$p = \frac{RT}{v}$ $\qquad \Delta v = \sqrt{\frac{pv \cdot v^2}{2 n_0 V\, RT}}$ ~~itd~~

$\frac{dp}{dv} = - \frac{RT}{v^2}$ $\qquad \frac{\Delta v}{v} = \frac{1}{\sqrt{2 n_0 V}} = \frac{1}{\sqrt{2\nu}}$

podobnie jak w teorii Boltzmann-Einsteina $\frac{1}{\sqrt{2\nu\pi}}$

(różnica w $\overline{\Delta v}$ i $\sqrt{\overline{\Delta v^2}}$)

Ogólnie zatem

$$\frac{\Delta v}{v} = \sqrt{\frac{RT}{2 v v^2 \frac{dp}{dv}}} = \sqrt{\frac{\frac{p}{v}\frac{dv}{dp}}{2\nu}}$$

Odwrotnie argumentując:

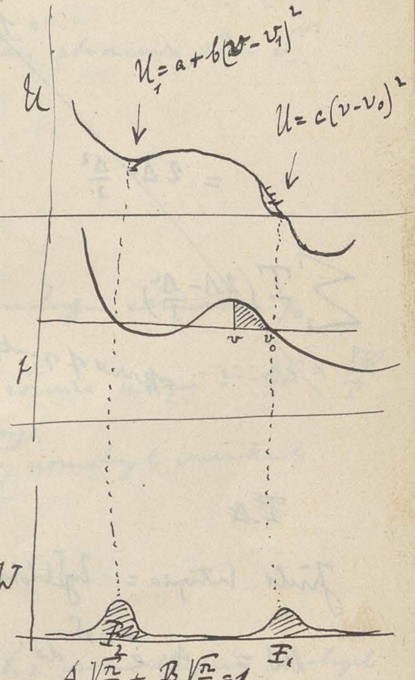
prawdop. zboczenia dv : $\quad A e^{-\left(\frac{N}{RT}\frac{dp}{dv} q\right) v^2} dv$

Sprężyna, która substytuowana należy zamiast p idealnie

$f = q(p_v - p_{v_0})$

Energia potencj. $\quad U = q\left[\int_{v_0}^{v} p_v\, dv - \cancel{x}\, p_{v_0}(v - v_0)\right]$ $\underbrace{\qquad}_{= \text{powierzchnia}}$

$dW = A e^{+hU} dv \;\Big\|\; = q\left[dv \left(\frac{\partial U}{\partial v}\right) + \frac{dv^2}{2}\left(\frac{\partial^2 U}{\partial v^2}\right) \dots\right]_{v_0}$
$= q\left[dv \cdot \left(p_v + \int_{v_0} \frac{\partial p}{\partial v} dv - p_0\right) \dots\right] = \left(\frac{\partial p}{\partial v}\right)_0 dv^2 + \dots$
$\frac{dv^2}{2}\, 2 \frac{\partial p}{\partial v_0} + \int_{v_0} \dots$



$A\sqrt{\frac{\pi}{\alpha}} + B\sqrt{\frac{\pi}{\beta}} = 1$

W obu miejscach krzywa prawdop. W ma charakter

krzywej ~~xxxx~~ $A e^{-\alpha v^2} dv$ $\qquad B e^{-\beta v^2} dv$

cała powierzchnia $\quad \int_{-\infty}^{+\infty} A e^{-\alpha v^2} dv = A\sqrt{\frac{\pi}{\alpha}}$ $\qquad$ przyczem $\alpha = \frac{hU}{(\Delta v)^2}$

~~$\Phi_1 = \Phi_2$ jeśli $U_1 = U_2$~~ $\qquad$ więc powierzchnie $\Phi_1 = \Phi_2$

jeśli $U = 0$

Rayleigh I p. 108 $\qquad$ IV p. 400

$$\mathrm{T}\ 3\,\frac{K'-K}{K'+2K} \qquad K=n^2 \qquad n=n_0(1+\Delta)$$

$$d(U+I)+Ap\,dv = c_v dT + AT\left(\frac{\partial p}{\partial T}\right)dv$$

$$\frac{\partial U}{\partial v}+\frac{\partial L}{\partial v}+Ap = AT\left(\frac{\partial p}{\partial T}\right)$$

$$\cancel{\frac{\partial U}{\partial T}}+\frac{\partial L}{\partial T} = c$$

$$3\,\frac{n^2-n_0^2}{n^2+2n_0^2} = \frac{2\Delta+\Delta^2}{1+\frac{2}{3}\Delta+\frac{\Delta^2}{3}}$$

$(1+x)^{-1} = 1-x+x^2-$

$$= 2\Delta\underbrace{\left(1+\frac{\Delta}{2}\right)\left(1+\frac{2\Delta}{3}+\frac{\Delta^2}{3}\right)^{-1}}$$

$$\underbrace{1+\frac{\Delta}{2}-\frac{2}{3}\Delta\left(1+\frac{\Delta}{2}\right)-\frac{\Delta^2}{3}\left(1+\frac{\Delta}{2}\right)+\left(\frac{2\Delta}{3}\right)^2\left(1+\frac{\Delta}{2}\right)}$$

$$=1+\frac{\Delta}{2}-\frac{2\Delta}{3}-\frac{\Delta^2}{3}-\frac{\Delta^2}{3}+\frac{4\Delta^2}{9}$$

$$=1-\frac{\Delta}{6}$$

$$=2\Delta-\frac{\Delta^2}{3}$$

$\mu' = 1+\Delta$

$$\sum' T\left(2\Delta-\frac{\Delta^2}{3}\right)$$

coefficient of opacity

$$h = 24\pi^3 n\,\frac{T^2}{\lambda^4}\,\frac{(\mu'^2-1)^2}{(\mu'^2+2)^2} \mp \frac{32\pi^3 n}{3}\,\frac{T^2\Delta^2}{\lambda^4}$$

$$=\frac{32\pi^3(\mu-1)^2}{3n\lambda^4}$$

μ = refr. ind. of composed medium

[Thermodynam. Entropia określona tylko w razie równowagi?]

$\cancel{T\Delta}$

Jeśli Entropia = log[Wahrscheinl. off ^{c}] to należałoby uściślić ogólnie:

$$dW = A\,e^{-\alpha S}\,dv \quad ?$$

$$dW = dW_0\,e^{-\alpha(S-S_0)} \qquad \text{izoterm:}$$

$$\Delta\varphi =$$

$$= dW_0\,e^{-\alpha\frac{\Delta\varphi}{T}} = dW_0\,e^{\underbrace{-\frac{\alpha}{T}\left(\Delta U+\int_{v_0}^{v} p\,dv\right)}}$$

nieprawda, bo to tylko ważne dla normalnych stanów równowagi

Próba: gaz idealny $\Delta U = 0$

$$\int_{v_0}^{v} p\,dv = RT \lg \frac{v}{v_0} = RT \lg(1+\delta) = RT\delta$$

$dw_0\, e^{-\alpha R\delta}$ stimmt nicht!

~~Statt feiner Verteilung ist die wahrscheinlichste insofern jedes Volumelement ∞ f. Mol. enthält, sonst bei Endlichkeit der enthaltenen Mol.-Zahl treten die Ungleichförmigkeiten auf. Hängt das mit der Ersetzung von ... zusammen?~~

Termodynamicznie nie można ΔS określić, bo stan zboczenia przez δ, nie jest stanem równowagi termodyn.

Można tak argumentować:

Gaz zamknięty tłokiem na który ciśnie sprężyna normalnym ciśnieniem p_0 przyjmuje samodzielnie anormalną objętość v; jaka różnica Entropii? $\Delta S = \frac{\delta Q}{T}$

Ten sam stan można było osiągnąć wykonując pracę (dodatkową) (przy normalnych warunkach)

$$\int p\,dv - p_0(v-v_0)$$

Więc: $\delta Q = \Delta U + \int p\,dv - p_0(v-v_0)$

Tożsamy się zgadza z wzorem dla gazów idealnych

ale różnica ogólna ΔU

Zarzuty: 1) rola sprężyny

2). $\Delta S = \frac{\delta Q}{T}$ ważne tylko dla procesów odwracalnych

Izoterma krytyczna $\frac{\cancel{\Delta p}}{\cancel{p_K}} \neq -\frac{3}{2}\left(\frac{\Delta v}{v_K}\right)^3$ według VdW

$$\Delta p = \frac{3}{2} p_K \frac{\Delta v^3}{v_K^3}$$

$$\int_{\Delta v=0}^{\Delta v} \Delta p \, dv = -\frac{3}{8} p_K \frac{\Delta v^4}{v_K^3}$$

$$\frac{3}{8} p_K \frac{\Delta v^4}{v_K^3}$$

4.10^{19}

$$A e^{-\frac{N}{RT} \cdot 9 \cdot \frac{3}{8} p_K \frac{\Delta v^4}{v_K^3}} =$$

$$A e^{-\frac{N}{RT} \frac{3}{8} \frac{V}{v} p_K v_K \left(\frac{\Delta v}{v}\right)^4}$$

$$p_K v_K = RT_K \cdot \frac{3}{8}$$

$$A e^{-\frac{9}{64} \frac{T_K}{T} \delta^4}$$

$$\frac{1}{3.10^{-5}} \sqrt{l} = \lambda \; 3.10^5$$

$0'6.10^{-4}$

$= 1.20$

$l = 400$

$$\delta \neq \sqrt[4]{\frac{64}{9} \frac{T_0}{T_K \, V}}$$

$V = (0'1 \mu)^3 = 5.10^5$ $\delta \neq 10^{-1}$!!!

$n - 1 = \alpha v$

$dn = \alpha \, dv$

$= \alpha v \delta$

$\frac{dn}{n-1} = \delta$

$dn = (n-1)\delta$

$\frac{dn}{n} = \frac{n-1}{n} \cdot \delta$

$$N\pi 6^2 = \frac{1'25}{\sqrt{2}.\lambda}$$

$$(n-1) \frac{2\sqrt{2}}{\sqrt{\pi}} \sqrt{l\sqrt{2}.\lambda}$$

$$= (n-1)\sqrt{\frac{8\sqrt{2}}{\pi} \frac{l\lambda}{1'25}}$$

$$(n-1) l \delta < \frac{\lambda}{2} = (n-1) \frac{l}{\sqrt{2\pi . 4.10^{19} . n^2 \pi l}}$$

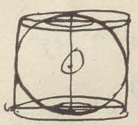
$$3.10^{-5} = 3.10^{-4} \frac{\sqrt{l}}{\pi \sqrt{10^{20}}}$$

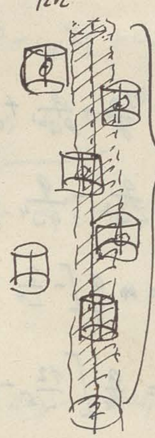
$$\sqrt{l} = 10^{-1} . \pi . 10^{-8} \sqrt{10^{20}} = 10.\pi = 31$$

$l = 900$ cm

Promień przechodzący przez gaz, zastępuje każdą kulę przez walec o równej objętości i równym przekroju:

πr^2

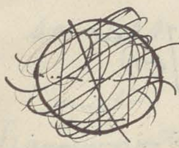




l

Tyle walców zostanie przeciętych ile punktów O się znajduje w obrębie walca o przekroju $\pi \sigma^2$ wykreślonego koło promienia jako osi więc w obrębie objętości $v = \pi r^2 l$

Pierwiastkowe zboczenie od normalnej objętości optycznej nl będzie zatem określone pierwiastkiem zboczenia od gęstości normalnej w obrębie v



$2r = 1 \mu\mu$

$r = 0.5 \cdot 10^{-7}$ cm

| or from optical measurements $0.1 \cdot 10^{-7}$ cm ?

Terpentyna

$v = \frac{\pi}{4} \cdot 10^{-14} l$

$l = 250$ cm

$\nu = n v = \frac{\pi}{4} \cdot 10^{-14} \cdot 250 \cdot 4 \cdot 10^{19}$

$= \pi \cdot 250 \cdot 10^5$

$\delta = \frac{1}{\sqrt{2\nu\pi}} = \frac{1}{\pi\sqrt{5 \cdot 10^7}} = \frac{1}{7 \cdot \pi \cdot 10^3} = \frac{1}{2} \cdot 10^{-4}$

~~[illegible]~~ δ in this case 5 times as much

Krit. Temp. $\delta \neq \frac{1}{\sqrt[4]{\nu}} = 1\% = 3.6\%$ l kürzer

Tosamo w której [illegible] gdzie 1 składnik optycznej objętości

---

Czy interferencyjność zmniejszona?

Droga optyczna $(n)l$

$\Delta = (n)l\,\varepsilon\,\delta < \frac{\lambda}{2}$

$l \neq \frac{\lambda/2}{\sqrt{2\nu\pi} \cdot 10^{-4} \cdot 3} = \frac{\cancel{\lambda}}{3 \cdot 10^{-5} \cdot 3 \cdot 10^{-4} \sqrt{2\pi \frac{\pi}{2^2} \cdot 4 \cdot 10^{19} l}}$

$= \frac{10^{-1}}{\pi\sqrt{10^{20}}\, l} = \frac{10^{-1}}{10^{-8} \cdot 3 \cdot 10^{10} \sqrt{l}} = \frac{10^{-3}}{3\sqrt{l}}$

$\sqrt{\frac{8}{\pi^2 \sigma^2 l N}} = \frac{2\sqrt{2}}{\pi\sigma\sqrt{lN}}$

$\frac{0.00035 \cdot 2\sqrt{2}\,\sqrt{l}}{3.1 \cdot 2.9 \cdot 10^{-8} \sqrt{4 \cdot 10^{19}}}$

$\sqrt{l} = 10^{-10} \frac{\sqrt{10^{20}}}{2\pi} = 10$

$= \frac{3.5 \cdot 2.8 \cdot 10^{-5}}{3.1 \cdot 2.9 \cdot 3.4} = \frac{1}{3} \cdot 10^{5} \sqrt{l}$

$$= f(x,\omega) = f(1,1) + \Delta x\left(\frac{\partial f}{\partial x}\right)_{11} + \Delta\omega\left(\frac{\partial f}{\partial \omega}\right)_{11} + \frac{1}{2}\Delta x^2\left(\frac{\partial^2 f}{\partial x^2}\right)_{11} + \dots$$

$$\vartheta = \frac{\left(x + \frac{3}{\omega^2}\right)(3\omega - 1)}{8} = \frac{x}{8}(3\omega-1) + \frac{9}{8\omega} - \frac{3}{8\omega^2}$$

w bliskim punktu $\omega, x, \vartheta = 1$:

$$\left(\frac{\partial\vartheta}{\partial x}\right) = \frac{3\omega-1}{8}\Big| = \frac{1}{4} \qquad \frac{\partial\vartheta}{\partial\omega} = \frac{3x}{8} - \frac{9}{8\omega^2} + \frac{6}{8\omega^3}\Big| = 0$$

$$\left(\frac{\partial^2\vartheta}{\partial x^2}\right) = 0 \quad \dots \quad \frac{\partial^2\vartheta}{\partial x\partial\omega} = \frac{3}{8} \qquad \frac{\partial^2\vartheta}{\partial\omega^2} = \frac{9}{4\omega^3} - \frac{9}{4\omega^4}\Big| = 0$$

$$\frac{\partial^n\vartheta}{\partial x^n} = 0 \quad \frac{\partial^n\vartheta}{\partial x^m\partial\omega^{n-m}} = 0 \qquad \frac{\partial^3\vartheta}{\partial\omega^3} = \frac{9}{4}\left[-\frac{3}{\omega^4} + \frac{4}{\omega^5}\right]\Big| = \frac{9}{4}$$

$$\frac{\partial^4\vartheta}{\partial\omega^4} = \frac{9}{4}\left[\frac{12}{\omega^5} - \frac{20}{\omega^6}\right]\Big| = -18$$

$$\frac{\partial^n\vartheta}{\partial\omega^n}\Big| = \frac{9}{4}\left[\frac{n!}{2} - \frac{(n+1)!}{2.3}\right](-1)^{n-1}$$

$$= (-1)^{n} \frac{3}{8}\left[3n! - (n+1)!\right]$$

$$= (-1)^{n} \frac{3n!}{8}\left[3 - n - 1\right] = \frac{(-1)^{n-1}3(n-2)n!}{8}$$

$$\Delta\theta = \frac{\Delta x}{4} + \frac{1}{2}\left\{\frac{3}{4}\Delta x\Delta\omega\right\} + \frac{1}{2.3}\left\{\frac{9}{4}\Delta\omega^3\right\} - \frac{18}{2.3.4}\cdot\Delta\omega^4$$

$$= \frac{\Delta x}{4} + \frac{3}{8}\Delta x\Delta\omega + \frac{3}{8}\Delta\omega^3 - \frac{1}{4}\Delta\omega^4 + \dots$$

$$\Delta\theta \neq \frac{\Delta x}{4}\left(1 + \frac{3\Delta\omega}{2}\right) + \frac{3}{8}\Delta\omega^3$$

Istotn. : $\Delta x = \dfrac{4\Delta\theta - \frac{3}{2}\Delta\omega^3}{1 + \frac{3\Delta\omega}{2}} = 4\Delta\theta - 6\Delta\theta\Delta\omega - \frac{3}{2}\Delta\omega^3$

$$\oint \Delta x\, d\omega = 4\Delta\theta(\omega_1 - \omega_2) - 3\Delta\theta(\omega_1^2 - \omega_2^2) - \frac{3}{8}(\omega_1^4 - \omega_2^4)$$

$$= (\omega_1 - \omega_2)\left[4\Delta\theta - 6\Delta\theta\,\omega_1 - \frac{3}{2}\omega_1^3\right] = 0$$

$$4\Delta\theta - 3\Delta\theta(\omega_1+\omega_2) - \frac{3}{8}(\omega_1^3+\omega_1^2\omega_2+\omega_1\omega_2^2+\omega_2^3) - \underbrace{\left[4\Delta\theta \cancel{+} 6\Delta\theta.\omega_1 - \frac{3}{2}\omega_1^3\right]}_{=\left[4\Delta\theta - 3\Delta\theta(\omega_1+\omega_2) - \frac{3}{4}(\omega_1^3+\omega_2^3)\right]} = 0$$

$$2(\omega_1^3+\omega_2^3) = (\omega_1^3+\omega_1^2\omega_2+\omega_1\omega_2^2+\omega_2^3)$$

$$\underbrace{\omega_1^3-\omega_1^2\omega_2-\omega_1\omega_2^2+\omega_2^3}_{\omega_1^2(\omega_1-\omega_2)-\omega_2^2(\omega_1-\omega_2)}=0 \qquad = (\omega_1-\omega_2)^2(\omega_1+\omega_2)=0$$

$$\underbrace{\Delta\omega_1 + \Delta\omega_2 = 0}$$

Nothing!

$$\Delta\omega_1 = -\Delta\omega_2$$

gerade Durchmesser! vertikal

$$\left.\begin{aligned}\Delta\pi &= 4\Delta\theta - 6\Delta\theta\Delta\omega_1 - \tfrac{3}{2}\Delta\omega_1^3\\ \Delta\pi &= 4\Delta\theta + 6\Delta\theta\Delta\omega_1 + \tfrac{3}{2}\Delta\omega_1^3\end{aligned}\right\}$$ unmöglich ausser:

Dampf-Spannungsseite $\left\{ \quad \frac{\Delta\pi}{\Delta\theta} = 4 \qquad \Delta\omega_1^2 = -\cancel{4}\,4\Delta\theta \right.$

$$\underline{\underline{\Delta\pi = 4\Delta\theta = -\Delta\omega^2}}$$

Boltzmann: p 53 (II)

$$c_p = \underbrace{\frac{3\pi(1+\beta)}{2}}_{c_v} + \underbrace{\frac{\pi}{1-\frac{2a(v-b)}{v(a+pv^2)}}} \qquad v = 3b \qquad p = \frac{a}{27b^2}$$

$$\frac{\pi}{1-\frac{2a.2b}{3b\;a(1+\frac{1}{3})}} = \frac{\pi}{1-\frac{4}{4}} = \infty\,!$$

vergl. V d W I p 133

Adiabaten: $v-b \sim \theta^{\frac{1}{k-1}}$

$$\left(\frac{v-b}{2b}\right)^{k-1} = \frac{\theta}{\theta_k} \qquad \vartheta_{red.} = \left(\frac{\omega-\frac{1}{3}}{\frac{2}{3}}\right)^{k-1} = \left(\frac{3\omega-1}{2}\right)^{k-1} = \left[1+\frac{3}{2}\Delta\omega\right]^{k-1}$$

$$\Delta\theta = -(k-1)\frac{3}{2}\Delta\omega \quad \| \quad \Delta\pi = -(k-1)6.\Delta\omega \quad | \text{Kr}$$

$$\left(\frac{\partial \pi}{\partial \theta}\right)_{\Delta\omega = \text{const}} = \frac{4}{1+\frac{3}{2}\Delta\omega} \qquad \left(\frac{\partial \pi}{\partial \omega}\right)_{\Delta\theta = \text{const}} = -\frac{9\,\Delta\omega^2}{8} \qquad \left(\frac{\partial \omega}{\partial \theta}\right)_{\Delta\pi} = +\frac{8}{9\Delta\omega^2}\;\frac{4}{1+\frac{3}{2}\Delta\omega} = \frac{32}{9\Delta\omega^2(1+\frac{3}{2}\Delta\omega)}$$

$$\delta\varphi = c_v\, d\theta + AT.\frac{4}{1+\frac{3}{2}\Delta\omega}\, d\omega$$

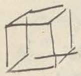
$$C_p = c_v + AT\,\frac{16\;.\,8}{(1+3\Delta\omega)\,9\,\Delta\omega^2}$$

$$= c_v + \frac{128\,.\,AT}{9\,\Delta\omega^2(1+3\Delta\omega)}$$

$$=$$

Andere Art der Abweichung f ein Würfelchen

pro sec Anzahl der die Seitenwände durchfliegenden Molek. wenn n auf $1\,cm^2$



$2(\Delta x\Delta y + \Delta y\Delta z + \Delta x\Delta z)\, n$ theils hinein theils hinaus

$p = \frac{Nmc^2}{3} = \frac{mc}{2}\cdot n$

$\int_0^{\frac{\pi}{2}} \frac{2c\cos\varphi \cdot 2\pi\sin\varphi\, d\varphi}{2\pi}$ = [illegible]

$= 2c\,\frac{\sin^2\varphi}{2} = c$

[illegible]

$n = \frac{2}{3} N c$

Drinnen sind normal enthalten: $N\,\Delta x\Delta y\Delta z$

Während t gesamt Zahl $2nt(\Delta x\Delta y + \ldots)$

Davon wahrscheinliche Abweichung vom Mittelwerth: $\sqrt{2nt\ldots}$

Also kann man berechnen wie lange es durchschnittlich dauert bis die Abweichung verschwindet

Darin aber vernachlässigt dass die links eintretenden meistens rechts wiederaustreten werden

also ist das nur giltig für $\begin{matrix}\Delta x\\ \Delta y\\ \Delta z\end{matrix} \gg \lambda$

Somit wenn die inneren Zusammenstösse unberücksichtigt, geht jedes von auswärts kommende Mol. hindurch

bleibt also durchschnittlich während der ~~Zeit~~ $\frac{\Delta x}{3c}$ drinnen

Während dieses Zeitraumes sind überhaupt $\frac{2\Delta y\Delta z\, n}{2}\frac{\Delta x}{3c}$ Mol ~~[illegible]~~ angekommen, welche alle drinnen sind wenn falls gleichmässig vertheilt

also sind gleichzeitig $= \frac{\Delta x\Delta y\Delta z\cdot n}{3c}$

Entropie = Log (Zustandswahrscheinlichkeit)

Zustandswahrsch. = e^{S}

Wahrsch. eines spez. Vol. $v \ldots v+dv$, wenn das normale sp. Vol. v_0 beträgt $W(v)\,dv$

$$\left.\begin{array}{l} W(v)\,dv = e^{S} \\ W(v_0)\,dv = e^{S_0} \end{array}\right\} \quad W(v) = W(v_0)\,e^{S-S_0}.$$

Jaka różnica entropii w stanach v, v_0?

1). ~~Ga~~ Gaz sam się skurczył ~~bez pracy zewnętrznej~~ tak że sprężyna zewnętrzna napięciowa wisi siłą po wykonaniu pracy $p_0 dv$; i bez ciepła z zewnątrz

$$\delta Q = \frac{\partial U}{\partial T}dT + \frac{\partial U}{\partial v}dv + A p_0\,dv = 0$$

zatem ~~podwyższ.~~ obniżenie temp. $\quad dT = -\dfrac{\frac{\partial U}{\partial v} + A p_0 \ldots}{\frac{\partial U}{\partial T}}dv$

To jest jego stan dla v

2). Przy użyciu stanów równowagi ~~sta~~ sprowadzamy go napowrót do stanu $v_0\,T_0$:

$$\delta Q = \left(\frac{\partial U}{\partial T}\right)dT + \left(\frac{\partial U}{\partial v}\right)dv + Ap\,dv = A(p-p_0)\,dv$$

$$S - S_0 = A\int\frac{(p-p_0)\,dv}{T}$$ ale tutaj ciśnienie p odpowiada normalnemu ciśnieniu gazu przy temp. $T \gtrless T_0$

drogą taką że najpierw zmieniamy p niezmiennego v tylko T aż dojdziemy do izotermy T_0 potem po tej izotermie wracamy do punktu pierwotnego

Wszystkie takie argumentacje nie mają sensu

bo termodynamicznie nie możemy wcale określić sposobu żadnego do obliczenia entropii S w takich stanach które nie odpowiadają normalnej równowadze!

Inna argumentacja:

$$S - S_0 = \int \frac{\frac{\partial U}{\partial T}\,dT + \frac{\partial U}{\partial v}\,dv + Ap\,dv}{T}$$ niezależnie od kształtu drogi

Stan v i v_0 posiadają tę samą temp. zatem $T = T_0$

$$S - S_0 = \frac{1}{T_0}\left(U - U_0 + A\int_{izot.} p\,dv\right)$$

Dla gazu ideal. $S - S_0 = AR\lg\frac{v}{v_0} = AR\lg(1+\delta) = -AR\,\delta$ fałszywie!

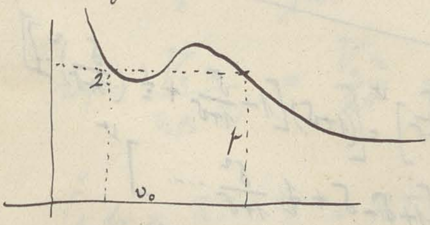
Ale czy nie należy owszem wprowadzić:

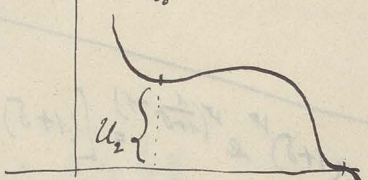
$$W = W_0\, e^{\frac{1}{T_0}\left[\underline{\underline{U - U_0}} + A\int(p-p_0)\,dv\right]}$$?

Przeciwnie, trzeba znaleźć prawdop. W z zasad kinetyki i na tej podstawie można określić co nazwać należy entropią w takich stanach anormalnych.

Wracając do dowodu ·/·/··/·/· /·/·

$$U = \int_{v_0}^{v} p_v\,dv - p_0(v - v_0)$$





Gdy $U = 0$ będziemy mieli całą podziałkę $F_1 = A\sqrt{\frac{\pi}{\alpha}}$ $\alpha = \left(\frac{\partial p}{\partial v}\right)_0 \cdot \frac{N}{RT}\left(\frac{V}{v}\right)$

$F_2 = A\sqrt{\frac{\pi}{\beta}}$ $\beta = \left(\frac{\partial p}{\partial v}\right)_2 \frac{N}{RT}\left(\frac{V}{v}\right)$

więc prawdopod. nie tylko równe, ale gdy się dalej posuniemy (w razie jakby było $F_1 \gtrless F_2$)

$$F_1 = A\sqrt{\frac{\pi}{\alpha}}; \qquad F_2 = e^{-U_2\frac{N}{RT}\frac{V}{v}}\sqrt{\frac{\pi}{\beta}}$$

więc z powodu olbrzymości liczby N już niedużej zmiany może przyrost U_2 więc niedużej może zmiana (p, v) wystarczy do zrównania $F_1 = F_2$, a dalej do $F_1 > F_2$ itd.

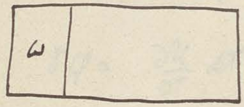
Jaka różnica energii wewnętrznej (potencyalnej) ~~[illegible]~~ powstaje jeżeli ~~gaz~~ gęstości ρ

pierwotnie jednostajnej gęstego tak zmienimy że element ω otrzyma ~~gęstość~~ $\rho(1+\delta)$ obj. ω, v

Ponieważ wyrażamy energię kinetyczną całkowitą niezmienioną więc

$= \delta U$ [różnica energii wewnętrznej termodynamicznej]

$$\delta U = \delta U_1 + \delta U_2 = \frac{\omega}{v}\int_{v_0}^{v_0(1-\delta)}\left[T\left(\frac{\partial p}{\partial T}\right)_v - p\right]dv + \frac{v-\omega}{v}\int_{v_0}^{v_0\left(1-\frac{\omega}{v_0-\omega}\delta\right)}\left[T\left(\frac{\partial p}{\partial T}\right)_v - p\right]dv$$



$$\omega(1+\delta) + (v_0-\omega)x = v_0$$

$$x = \frac{-\omega(1+\delta)+v_0}{v_0-\omega}$$

$$= 1 - \frac{\omega}{v_0-\omega}\delta$$

$$= f(\delta) = f(0) + \delta\left(\frac{\partial f}{\partial \delta}\right)_0 + \frac{\delta^2}{2}\left(\frac{\partial^2 f}{\partial \delta^2}\right)_0$$

$$f(0) = 0$$

$$\frac{\partial f}{\partial \delta} = \frac{\omega}{v}$$

$$\frac{v_0}{v} = 1+\delta$$

$$(1+\delta)^{\nu} e^{\nu\left(\frac{1}{1+\delta}-1\right)} = \left[(1+\delta)e^{-\frac{\delta}{1+\delta}}\right]^{\nu} = \left[(1+\delta)\left[1-\frac{\delta}{1+\delta}+\frac{1}{2}\left(\frac{\delta}{1+\delta}\right)^2 - \dots\right]\right]^{\nu}$$

$$= \left[1+\delta-\delta+\frac{1}{2}\frac{\delta^2}{1+\delta}\dots\right]^{\nu}$$

$$= \left[1+\frac{\delta^2}{2}\right]^{\nu} = e^{\nu\frac{\delta^2}{2}}$$

$$(1+\varepsilon)^{\frac{\alpha}{\varepsilon}} = e^{\alpha}$$

$$(1+x)^y = e^{xy}$$

$$\frac{v}{n} = e^{\lg\frac{v}{n}} = e^{-\lg(1+\delta)} = e^{-\delta+\frac{\delta^2}{2}-\frac{\delta^3}{3}}$$

$$e^{1-\frac{v}{n}} = e^{+\frac{\delta}{1+\delta}} = e^{+\delta-\delta^2 - -}$$

I objstů dána, místem ~~[illegible]~~ ilosů n?
II ilosů n dána, místem řízení objstů

Dle gasu ideálního

Způsobem I: $\log W = n \log\frac{\nu}{n} + (n-\nu) - \frac{1}{2}\log 2\pi n + \log dn$

II: $dW = \underline{A}\, dv \cdot e^{+hq\left[\int_{v_0}^{v} p_v\, dv - p_0(v-v_0)\right]}$ $\quad \rho = \frac{V_0}{v_0}$

$dW = \underline{A}\, e^{-h\frac{m}{2}(\xi^2+\eta^2+\zeta^2)} d\xi\, d\eta\, d\zeta \quad = \frac{1}{n}\left(\frac{mh}{2\pi}\right)^{3/2} e^{-h\frac{m}{2}(\xi^2+\eta^2+\zeta^2)} d\xi \ldots$

$1 = \underline{A}\left(\sqrt{\frac{2\pi}{mh}}\right)^3$

$\int \xi^2 e^{-\frac{hm\xi^2}{2}} d\xi = \frac{1}{hm}\int e^{-h\ldots}$

$p = nm\,\overline{\xi^2}$

$= \frac{nm}{hm} = \frac{\rho}{hm} \qquad \frac{p}{\rho} = RT = \frac{1}{hm}$

$\overline{c^2} = \frac{\int 4\pi c^4\, dc\, e^{-\frac{hmc^2}{2}}}{\int 4\pi c^2\, dc\, e^{-\ldots}} = \frac{3 \cdot 2}{2\, hm} = \frac{3}{hm}$

$p = \frac{nm\overline{c^2}}{3} = \frac{\rho\, 3}{3hm} \qquad \boxed{hm = \frac{1}{RT}} \qquad h = \frac{1}{mRT} = \frac{n}{\rho_0 RT}$

$hq = \frac{V_0 n}{v_0 \rho_0 RT} = \frac{V_0 n}{RT} = \frac{N}{RT}$

$\int_{v_0}^{v} p\, dv = RT\int\frac{dv}{v} = RT\log\frac{v}{v_0} = p_0 v_0 \log\frac{v}{v_0}$

$N = (\nu)$

$dW = \underline{A}\, dv\; e^{+\frac{N}{RT}\left[RT\log\frac{v}{v_0} - RT\left(\frac{v}{v_0}-1\right)\right]}$

$\frac{v_0}{v}(\nu) = (n)$

$= \underline{A}\, dv\; e^{+N\left[\log\frac{v}{v_0} + 1 - \frac{v}{v_0}\right]}$ $\qquad v = \frac{\nu v_0}{n}$

$= A\, dv \cdot \left(\frac{v}{v_0}\right)^N e^{-N\left(1-\frac{v}{v_0}\right)} = \underline{A}\, dv \cdot \left(\frac{\nu}{n}\right)^{\nu} e^{\nu\left(1-\frac{\nu}{n}\right)} = \sqrt{\frac{\nu}{2\pi}}\,\frac{\nu}{n^2}\, dn \cdot \left[\frac{\nu}{n} e^{1-\frac{\nu}{n}}\right]^{\nu}$

I pisane z użyciem symbolów II:

$$\lg W = \frac{v_0}{v} N \lg \frac{v}{v_0} + \frac{v_0}{v} N \left(1 - \frac{v}{v_0}\right) - \frac{1}{2} \lg 2\pi \frac{v_0 N}{v} + \lg v_0 N$$

$$= N \left[\frac{v_0}{v} - 1 + \frac{v_0}{v} \lg \frac{v}{v_0}\right] - \frac{1}{2} \lg 2\pi \frac{v_0 N}{v}$$

$$dW = -\frac{e^{N\left[\frac{v_0}{v} - 1 + \frac{v_0}{v} \lg \frac{v}{v_0}\right]}}{\sqrt{2\pi \frac{v_0 N}{v}}} \cdot \frac{v_0 N}{v^2} dv$$

$$= -\frac{e^{-N\frac{v_0}{v}\left[1 - \frac{v}{v_0} - \lg \frac{v}{v_0}\right]}}{\sqrt{\cdots}} \cdots = -\left(\frac{v}{v_0}\right)^{N\frac{v_0}{v}} \frac{e^{-N\frac{v_0}{v}\left(1 - \frac{v}{v_0}\right)}}{\sqrt{\cdots}} \cdots dv$$

$$\int \left(\frac{\nu}{n}\right)^{\nu} e^{n-\nu}\, dn = e^{-\nu}\cdot\nu^{\nu}\int .$$

$$\frac{N^N}{n^n (N-n)^{N-n}} \left(\frac{v}{V}\right)^n \left(\frac{V-v}{V}\right)^{N-n} \sqrt{\frac{N}{n(N-n)}}$$

$$= \cancel{\frac{\nu^{\nu}}{n^n (\nu-n)^{\nu-n}} (\alpha)^n (1-\alpha)^{\nu-n}} = \left(\frac{\nu}{\nu-n}\right)^{\nu} \left(\frac{\nu-n}{n}\right)^n \alpha^n (1-\alpha)^{\nu-n}$$

$$\nu = \frac{Nv}{V}$$

$$= \left[\frac{N}{N-n}\,\frac{V-v}{V}\right]^N \left[\frac{\cancel{V}}{V-v}\cdot\frac{v}{\cancel{V}}\,\frac{N-n}{n}\right]^n \sqrt{\frac{N}{n(N-n)}}$$

$$\frac{\nu!}{n!\,\nu-n!} = \frac{\nu^{\nu}}{n^n(\nu-n)^{\nu-n}}\,\frac{e^{n-\nu}}{\sqrt{2\pi}}\sqrt{\frac{\nu}{n(\nu-n)}}$$

$$= \frac{\nu^{n+\nu-n}}{n^n(\nu-n)^{\nu-n}} \cdots = \left(\frac{\nu}{n}\right)^n\cdot\left(\frac{\nu}{\nu-n}\right)^{\nu-n} e^{\cdots}$$

$$= \left(\frac{\nu}{n}\right)^n \cancel{(\ldots)}$$

$$= \left[\frac{N-\nu}{N-n}\right]^N \left[\frac{\nu-n\frac{v}{V}}{\left(1-\frac{v}{V}\right)n}\right]^n$$

$$= \left(\frac{1-\frac{\nu}{N}}{1-\frac{n}{N}}\right)^N \left(\frac{\nu}{n}\right)^n \left[\frac{1-\frac{n}{\nu}\frac{v}{V}}{1-\frac{v}{V}}\right]^n$$

stimmt

$$= \frac{e^{-\nu}}{e^{-n}}\left(\frac{\nu}{n}\right)^n \qquad \frac{vN}{v_0} = x \qquad \frac{v}{v_0} = \frac{x}{N} \qquad dv = dx\,\frac{v_0}{N}$$

$$A\, e^N \int_0^{\infty} e^{-\frac{vN}{v_0}} \left(\frac{v}{v_0}\right)^N dv = A\, e^N \int_0^{\infty} e^{-x}\left(\frac{x}{N}\right)^N \frac{v_0}{N}\, dx = \frac{A\, e^N v_0}{N^{N+1}} \int_0^{\infty} e^{-x} x^N\, dx$$

$$= \frac{A\, e^N v_0}{N^{N+1}} \underbrace{\Gamma(N+1)}_{= \sqrt{2N\pi}\left(\frac{N}{e}\right)^N}$$

$$= \frac{A v_0\, e^N \sqrt{2\pi(N+1)}\,(N+1)^{N+1}}{N^{N+1}\, e^{N+1}}$$

$$= \frac{A v_0}{e}\sqrt{2\pi(N+1)}\left(1+\frac{1}{N}\right)^{N+1}$$

$$= \frac{A\, e^N v_0}{N^{N+1}}\,\frac{\sqrt{2N\pi}\, N^N}{e^N} = \frac{A v_0 \sqrt{2N\pi}}{N}$$

$$= A v_0 \sqrt{\frac{2\pi}{N}} = 1$$

$$A = \sqrt{\frac{N}{2\pi}}\,\frac{1}{v_0}$$

$$dW = A\, dW_0\, e^{-h U}$$

dW_0 = prawdop. gdyby nie było energii ~~[illegible]~~ potencyalnej U
więc dla gazu idealnego

U = energia potencyalna danego układu

$$\underbrace{n \text{ w } v,\ N-n \text{ w } V-v}$$

$$U = U_0 + \delta U$$

$$A\, e^{-hU_0} = A'$$

$$\int_{n=0}^{\infty} A'\, dW_0\, e^{-h\delta U} = 1$$

$$\delta U = n m \int_{v_0}^{\frac{v}{n}v_0} + \underbrace{(N-n) m \int_{v_0}^{v_0 \cdot \frac{N-v}{N-n}} \left[T \frac{\partial p}{\partial T} - p\right] dv}_{(N-n) m \int_{\left(1+\frac{v-n}{N}\right)v_0}^{v_0} \cdots\cdots}$$

ale tutaj także implicite istnienie przyjęte? powinni

Nie!

$$\frac{v}{n} v_0 = v$$

$$= -(N-n) m \left[T\left(\frac{\partial p}{\partial T}\right) - p\right]_0 \cdot v_0 \frac{v-n}{N} = -v_0 (v-n) m \left[T \frac{\partial p}{\partial T} - p\right]_0$$

$$\delta U = n m \left[\int_{v_0}^{v} \left[T \frac{\partial p}{\partial T} - p\right] dv + (v_0 - v) \left[T \frac{\partial p}{\partial T} - p\right]_0\right]$$

$$dW = A \left(\frac{v}{n}\right)^n \frac{e^{n-v}}{\sqrt{2 n \pi}}\, dn \cdot e^{-\frac{n}{p_0 v_0}\left[\int_{v_0}^{v} \left(T \frac{\partial p}{\partial T} - p\right) dv + (v_0 - v)\left[T \frac{\partial p}{\partial T} - p\right]_0\right]}$$

$$\int_{v_0}^{v}\left(T\frac{\partial p}{\partial T}-p\right)dv = \mathfrak{F}(v)-\mathfrak{F}(v_0)$$

$$= \mathfrak{F}[v_0+(v-v_0)]-\mathfrak{F}(v_0)$$

$$=(v-v_0)\frac{\partial \mathfrak{F}}{\partial v}\Big/_0+\frac{(v-v_0)^2}{2}\frac{\partial^2\mathfrak{F}}{\partial v^2}\Big/_0+\cdots$$

$$=(v-v_0)\left[T\frac{\partial p}{\partial T}-p\right]_0+\frac{(v-v_0)^2}{2}\frac{\partial}{\partial v}\left[T\frac{\partial p}{\partial T}-p\right]$$

$$p+\frac{a}{v^2}=\frac{RT}{v-b}$$

$$\underbrace{\int_{v_0}^{v}\left(T\frac{\partial p}{\partial T}-p\right)dv=\frac{a}{v_0}-\frac{a}{v}\qquad (v_0-v)\left[T\frac{\partial p}{\partial T}-p\right]_0=(v_0-v)\frac{a}{v_0^2}}$$

$$\frac{a}{v_0}\left[1-\frac{v_0}{v}+\frac{v_0-v}{v_0}\right]=\frac{a}{v_0}\left[2-\frac{v_0}{v}-\frac{v}{v_0}\right]=-\frac{a\,(v_0-v)^2}{v_0^2\,v}$$

To nie wiodłoby zatem żadnych specyalnych zjawisk w punkcie krytycznym!

Dla małych v_0-v : $v_0=v(1+\delta)$:

$$dW=A\,e^{-\frac{\nu\delta^2}{2}+\frac{n}{p_0v_0}\frac{a\,v\,\delta^2}{v_0^2}}\neq A\,e^{-\frac{\nu\delta^2}{2}\left[1-\frac{2a}{p_0v_0^2}\right]}$$

Wykładnik $=0$ jeśli $p=\frac{2a}{v^2}$ t.j.: $\frac{3a}{v^2}=\frac{RT}{v-b}$

podczas gdy w punkcie krytycznym: $\left.\begin{array}{l} v_k=3b \\ p_k=\frac{a}{27b^2}\end{array}\right\}=\frac{a}{3v_k^2}$

Powinno być:

$M \qquad N \quad \left(+\Pi(1-6\delta)\right)$

$$-\underset{(148)}{P} = \frac{4\varphi}{\sqrt{2}} - \frac{1}{2\sqrt{2}\,h}\left(\frac{2\varphi-2\varphi'-\varphi''}{2\varphi+\varphi'}\right) + \delta\left\{\frac{4\varphi'}{\sqrt{2}} - \frac{1}{2\sqrt{2}\,h}\left(\frac{6\varphi'^2+6\varphi\varphi''+2\varphi\varphi''+\varphi''^2}{(2\varphi+\varphi')^2}\right)\right\}$$

$$M = 1 - \frac{3\varphi'+\varphi''}{2\varphi+\varphi'}$$

$\cancel{6\varphi\varphi'' + 3\varphi'\varphi'' - 6\varphi'^2 - 2\varphi'\varphi'' - 3\varphi\varphi''} \quad -\varphi'^2$

$$\frac{\partial M}{\partial l} = -\frac{(2\varphi+\varphi')3\varphi'' - (3\varphi'+\varphi'')(2\varphi'+\varphi'')}{(2\varphi+\varphi')^2}$$

$$\frac{4\varphi}{\sqrt{2}} - \frac{M}{2\sqrt{2}}\,rT = -\Pi\, l^2 \qquad M>0$$

$$\cancel{\frac{4\varphi'}{\sqrt{2}} - \frac{M'}{2\sqrt{2}\,h} = \frac{\partial\Pi}{\partial l}\left(= -6\Pi\right)}$$

$$\cancel{\frac{4\varphi'}{\sqrt{2}} - \frac{Mr}{}} \quad \left(\frac{4\varphi'}{\sqrt{2}} - \frac{rT}{2\sqrt{2}}M'\right)\frac{\partial l}{\partial T} - \frac{Mr}{2\sqrt{2}} = \underbrace{+\,4\Pi\frac{\partial l}{\partial T}\,l}_{= -\frac{\partial \Pi}{\partial T}\cdot l^2 - 2l\Pi\frac{\partial l}{\partial T}}$$

$$\frac{\partial l}{\partial T} = \frac{\frac{Mr}{2\sqrt{2}}}{\frac{4\varphi'}{\sqrt{2}} - \frac{rT}{2\sqrt{2}}M' + 4\Pi} \qquad \Pi = \frac{1}{l^2}\left[-\frac{4\varphi}{\sqrt{2}} + \frac{MrT}{2\sqrt{2}}\right]$$

$$\overline{\mathcal{U}} = n\left\{6\,\overset{12?}{\Phi} + \frac{3}{2}rT\right\} \qquad nm=\rho \qquad n\frac{l^2\cdot l}{\sqrt{2}} = 1 \qquad nl^3 = \sqrt{2}$$

$$c\rho = \frac{\partial \mathcal{U}}{\partial T} = n\left[\overset{12?}{6}\varphi\cdot\frac{\partial l}{\partial T} + \frac{3r}{2}\right] + 3\frac{\Pi}{l}\frac{\partial l}{\partial T} = n\left\{\frac{\partial l}{\partial T}\left[\overset{12?}{6}\varphi + \frac{3}{nl^3}\left(-\frac{4\varphi}{\sqrt{2}} + \frac{MrT}{2\sqrt{2}}\right)\right] + \frac{3r}{2}\right\}$$

$$\cancel{c = \frac{1}{m}\left[6\varphi\frac{\partial l}{\partial T} + \frac{3r}{2}\right] + \frac{3\Pi}{\rho}\frac{\partial l}{\partial T}} = n\left\{\frac{\partial l}{\partial T}\left[\overset{6\varphi?+}{} + \frac{3MrT}{4}\right] + \frac{3r}{2}\right\}$$

$$\frac{3}{2}rT + 6\delta\varphi' + 3\delta^2\varphi''$$

Superponuję ~~Próba~~ $(1+\delta)$

Robię po kolej rozszerzenie w kierunku $X\sqrt{2}$:

$$-P = \frac{4\varphi}{\sqrt{2}} - \frac{1}{2\sqrt{2}h}\,\frac{2\varphi - 2\varphi' l - \varphi' l^2}{2\varphi + \varphi' l}$$

$$+\delta\left\{\frac{4\varphi' l}{\sqrt{2}} + \frac{1}{2\sqrt{2}h}\,\underbrace{\frac{-32\varphi^2 - 32\varphi\varphi' + 64\varphi'^2 - 64\varphi\varphi'' + 8\varphi''^2 + 16\varphi'\varphi''}{8(2\varphi+\varphi')^2}}\right\}$$

$$= \frac{-4\varphi^2 - 4\varphi\varphi' + 8\varphi'^2 - 8\varphi\varphi'' + \varphi'^2 + 2\varphi'\varphi''}{(2\varphi+\varphi')^2}$$

$$(2\varphi+\varphi' l)(2\varphi' - 2\varphi'') - (2\varphi - 2\varphi' - \varphi')(2\varphi' + \varphi'') =$$

Nie zgadza się!

$$= 4\varphi\varphi' + 2\varphi'^2 - 4\varphi\varphi'' - 2\varphi'\varphi''$$
$$-4\varphi\varphi' + 4\varphi'^2 - 2\varphi\varphi'' + 2\varphi'\varphi'' + 2\varphi'\varphi'' + \varphi''^2$$

$$= \frac{6\varphi'^2 - 6\varphi\varphi'' + 2\varphi'\varphi'' + \varphi''^2}{4\varphi^2 + 4\varphi\varphi' + \varphi'^2}$$

$$-l^2 X = \frac{4\varphi}{\sqrt{2}} - \frac{1}{2\sqrt{2}h}\,\frac{2\varphi - 2\varphi' l - \varphi'' l^2}{2\varphi + \varphi' l} + \qquad + \Pi(1-2\delta)$$

$$+\,\delta\left\{\frac{2\varphi}{\sqrt{2}} + \frac{2\varphi' l}{\sqrt{2}} - \frac{1}{2\sqrt{2}h}\,\frac{3\varphi^2 - 30\varphi\varphi' + 27\varphi'^2 - 38\varphi\varphi'' + 3\varphi''^2 + 2\varphi'\varphi''}{8\,(2\varphi + \varphi' l)^2}\right\}$$

$$-l^2(1+\delta) Y = \frac{4\varphi}{\sqrt{2}} - \frac{1}{2\sqrt{2}h}\,\frac{2\varphi - 2\varphi' l - \varphi'' l^2}{2\varphi + \varphi' l} + \qquad + \Pi(1-2\delta)$$

$$+\,\delta\left\{-\frac{\varphi}{\sqrt{2}} + \frac{\varphi' l}{\sqrt{2}} + \frac{1}{2\sqrt{2}h}\,\frac{35\varphi^2 + 2\varphi\varphi' - 37\varphi'^2 + 26\varphi\varphi'' - 5\varphi''^2 - 14\varphi'\varphi''}{16\,(2\varphi + \varphi')^2}\right\}$$

$$\frac{\iint (u^2+v^2+w^2)\, e^{-h(u^2+v^2+w^2)\frac{m}{2}}}{\iint e^{-h\ldots}} = \frac{3}{hm} = \bar{c}^2 \qquad \frac{p}{\rho} = \frac{\bar{c}^2}{3} = \frac{1}{hm} = RT$$

$$\frac{1}{h} = RTm = rT$$

$$\alpha = \frac{1}{l}\frac{\partial l}{\partial T}$$

$$-\frac{4\varphi}{\sqrt{2}} + \frac{rT}{2\sqrt{2}}\,\frac{2\varphi - 2\varphi' l - \varphi'' l^2}{2\varphi + \varphi' l} = +\Pi$$

$M \lessgtr 0$

bo

$$\left[\frac{4\varphi'}{\sqrt{2}} - \frac{rT}{2\sqrt{2}}\,\frac{\partial M}{\partial l} - 6\Pi\right]\alpha l - \frac{r}{2\sqrt{2}} M = 0$$

$$\alpha l = \frac{\frac{r M}{2\sqrt{2}}}{\frac{4(\varphi' + 6\varphi)}{\sqrt{2}} - \frac{rT}{2\sqrt{2}}\left(\frac{\partial M}{\partial l} + 6M\right)}$$

zależy ~~od~~ ... z temperat.

w gruben ... dla $\Pi = 0$

$$\alpha l = \frac{rM}{2(\varphi' + 6\varphi)}$$

$$\Pi = -\frac{4\varphi}{\sqrt{2}} - \frac{rT}{\sqrt{2}}$$

$$0 = -\frac{4\varphi'}{\sqrt{2}}\cdot \alpha l - \frac{r}{\sqrt{2}} \qquad \alpha l = -\frac{4\varphi'}{r}$$

nonsens!

$$\frac{1}{a+b\delta} = \frac{1}{a\left(1-\frac{b}{a}\delta\right)} = \frac{1}{a} + \frac{b\delta}{a^2}$$

$$\int \left[\frac{3(x^2+y^2)\delta}{4\sqrt{2}}(\varphi - \varphi' l) + \frac{2^2\delta}{2\sqrt{2}}\left[3(\varphi-\varphi') + \varphi''\right] \right. e^{-h\left[(x^2+y^2+z^2)\, 2(2\varphi+\varphi') - \right.}$$

$$- x^2\delta\left[3(\varphi-\varphi')-\varphi''\right]$$

$$\left. - \frac{x^2+y^2}{2\sqrt{2}}(\varphi-\varphi'-\varphi'') + \frac{y^2}{\sqrt{2}}\left[3\varphi - 3\varphi' - \varphi''\right] \right.$$

$$\left. - \frac{y^2+z^2}{2}\delta\left[\varphi-\varphi'-\varphi''\right] \right]$$

$$= \frac{\delta}{2h}\left\{ \frac{3(\varphi-\varphi' l)}{4\sqrt{2}} \left\{ \frac{1}{\left[4\varphi+2\varphi' - \delta\left(3\varphi-3\varphi'-\varphi''\right)\right]} + \frac{1}{\left[4\varphi+2\varphi' - \frac{\delta}{2}(\varphi-\varphi'-\varphi'')\right]} \right\} \right.$$

$$\left. + \frac{3\varphi-3\varphi'+\varphi''}{2\sqrt{2}} \frac{1}{4\varphi+2\varphi'-\frac{\delta}{2}(\varphi-\varphi'-\varphi'')} \right\} -$$

$$- \frac{1}{2\sqrt{2}}\frac{1}{2h}\left\{ \frac{(\varphi-\varphi'-\varphi'')}{4\varphi+2\varphi'-\delta(3\varphi-3\varphi'-\varphi'')} + \frac{\varphi-\varphi'-\varphi'' + 6\varphi - 6\varphi' - 2\varphi''}{4\varphi+2\varphi' - \frac{\delta}{2}(\varphi-\varphi'-\varphi'')} \right\}$$

$$= -\frac{1}{4\sqrt{2}\,h} \frac{8\left(\varphi-\varphi'-\frac{\varphi''}{2}\right)}{4\varphi+2\varphi'}$$

$$+ \frac{\delta}{4\sqrt{2}\,h}\left\{ \frac{3(\varphi-\varphi') + 3\varphi - 3\varphi' + \varphi''}{4\varphi+2\varphi'} - \underbrace{\frac{(\varphi-\varphi'-\varphi'')(3\varphi-3\varphi'-\varphi'') + (7\varphi-7\varphi'-3\varphi'')\left(\varphi-\varphi'-\frac{\varphi''}{2}\right)}{(4\varphi+2\varphi')^2}} \right.$$

$$\frac{6\varphi-6\varphi'+\varphi''}{4\varphi+2\varphi'} - \frac{(\varphi-\varphi'-\varphi'')(13\varphi-13\varphi'-5\varphi'')}{2(4\varphi+2\varphi')^2}$$

$$= \left\{ \begin{array}{ll} 48\varphi^2 - 48\varphi\varphi' + 8\varphi''\varphi & \\ \quad + 24\varphi\varphi' & -24\varphi'^2 + 4\varphi'\varphi'' \\ -13\varphi^2 + 13\varphi\varphi' + 5\varphi\varphi'' & \quad -13\varphi'\varphi'' \\ \quad 13\varphi\varphi' + 13\varphi\varphi'' - 13\varphi'^2 & -5\varphi'\varphi'' + 5\varphi''^2 \end{array} \right.$$

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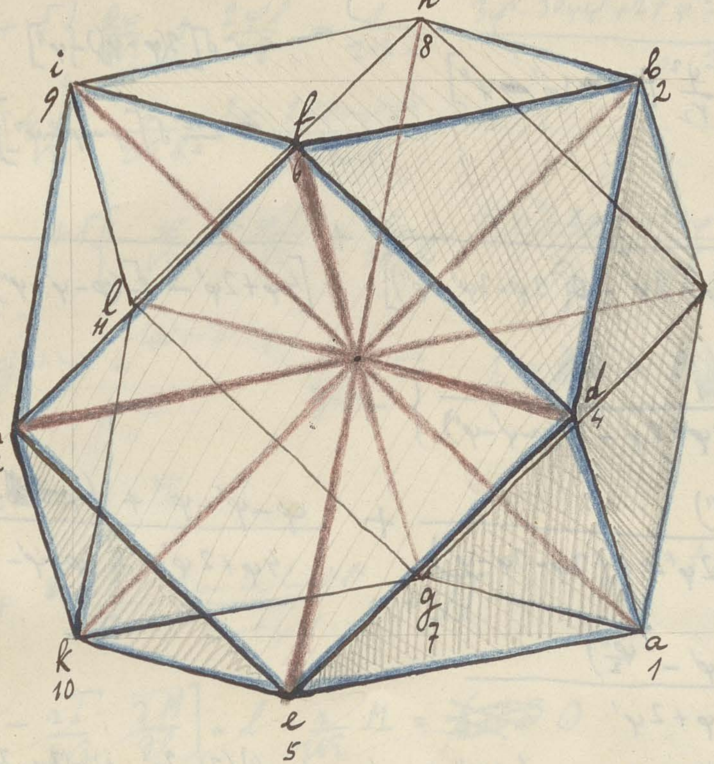
$$+ \frac{\delta}{4\sqrt{2}\,h} \frac{35\varphi^2 + 2\varphi\varphi' + 26\varphi\varphi'' - 37\varphi'^2 - 14\varphi'\varphi'' - 5\varphi''^2}{8(2\varphi+\varphi')^2}$$


h 8
i 9
b 2
f 6
c 3
l 4
d 4
m 12
g 7
k 10
a 1
e 5

Cu : $E = \underline{1200}.098.10^{9}$ (CGS) $\qquad l^2 E \neq (\varphi+\varphi' l)\sqrt{2}$

$\neq 1{,}2.10^{6}$ atm.

$-\frac{\partial U}{\partial r} = -4(2\varphi+\varphi')\frac{r}{l} = +m\frac{d^2r}{dt^2} \qquad \alpha = 2\sqrt{(2\varphi+\varphi')\frac{1}{ml}} \neq \sqrt{\frac{2l^2E}{m\cancel{l}}} = \sqrt{\frac{2\rho E}{l}}$

$r = r_0 \sin\alpha t$ $\qquad$ jeżeli $\varphi' l \gg \varphi$

$\frac{dr}{dt} = \alpha r_0 \cos\alpha t$

$\frac{2}{\pi}\int\left(\frac{dr}{dt}\right)^2 dt = \alpha^2 r_0^2 \frac{2}{\pi}\int_0^{\alpha t=\frac{\pi}{2}} \cos^2\alpha t\, dt = \frac{\alpha^2 r_0^2}{2} = \bar{c}^2$

$r_0 = \sqrt{\frac{2\bar{c}^2\, ml}{4(2\varphi+\varphi' l)}} \neq \sqrt{\frac{2\bar{c}\, m\cancel{l}}{\sqrt{2}.\, l^{2}E}}$

$\bar{c} = 3.10^4$

$\rho = 10$

$E = 1{,}2.10^{12}$

$\cancel{\frac{r_0}{l}}$ $\qquad N l^3 = 1$

$\frac{r_0}{l} \neq \sqrt{\frac{\bar{c}^2 m}{\sqrt{2}\, E\, l^3}}$ $\qquad N m = \rho$

$\frac{l^3}{m} = \frac{1}{\rho}$

$\frac{r_0}{l} = \sqrt{\frac{9.10^8.10}{\sqrt{2}.1{,}2.10^{12}}} \neq \sqrt{\frac{0{,}9.10^{10}}{1{,}8.10^{12}}} = \sqrt{\frac{1}{200}} \qquad \neq \bar{c}\sqrt{\frac{\rho}{\sqrt{2}\, E}} \qquad \sqrt{\frac{\alpha T}{\sqrt{2}\, E}}$ atom. volum.

$= \frac{1}{14} = 7\%$

Przeciętne oddalenie się cząstek z położeń równowagi $r_0 = \frac{l}{14}$

Również tem większe im mniejszy (atom. volum. $\times$ E)

Dla $E = 0$ (punkt topliwości) $r_0 = \infty$ zupełne oddalenie

$T = \frac{2\pi}{\alpha} \qquad n = \frac{\alpha}{2\pi} = \frac{\sqrt{2\rho E}}{2l\pi} = \frac{\sqrt{20.1{,}2.10^{12}}}{2.3{,}1.10^{-8}} = \frac{5}{6{,}3}.10^{14} \neq 10^{14}$

$\lambda = \frac{3.10^{10}}{n} = \frac{3.10^{10}}{10^{14}} = 3.10^{-4} = 3\mu$!!!

ciągnienie

$$X = \frac{4\varphi}{\sqrt{2}}\left(1+\frac{\delta}{2}\right) + \frac{2\varphi' l\delta}{\sqrt{2}} - \frac{1}{2\sqrt{2}h}(M+N\delta) + \Pi(1-2\delta)$$

$$Y = \frac{4\varphi}{\sqrt{2}}\left(1-\frac{5\delta}{4}\right) + \frac{\varphi' l\delta}{\sqrt{2}} - \frac{1}{2\sqrt{2}h}(M+(N'-M)\delta) + \Pi(1-3\delta)$$

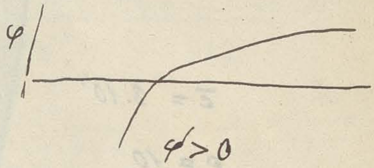
$$0 = \frac{4\varphi}{\sqrt{2}} - \frac{M}{2\sqrt{2}h} + \Pi \qquad \Pi = -\frac{4\varphi}{\sqrt{2}} + \frac{M}{2\sqrt{2}h}$$

$$+c_{11} = \frac{2\varphi}{\sqrt{2}} + \frac{2\varphi' l}{\sqrt{2}} - \frac{N}{2\sqrt{2}h} - 2\Pi$$

$$+c_{11} = \frac{10\varphi}{\sqrt{2}} + \frac{2\varphi' l}{\sqrt{2}} - \frac{(N+2M)}{2\sqrt{2}h}$$

$$+c_{21} = -\frac{5\varphi}{\sqrt{2}} + \frac{\varphi' l}{\sqrt{2}} - \frac{(N'-M)}{2\sqrt{2}h} - 3\Pi$$

$$+c_{21} = \frac{7\varphi}{\sqrt{2}} + \frac{\varphi' l}{\sqrt{2}} - \frac{(N'+2M)}{2\sqrt{2}.h}$$



$\varphi' > 0$

$\varphi'' \le 0$

$$c_{44} = \frac{3\varphi+\varphi'}{\sqrt{2}} - \frac{(\varphi-\varphi'-\varphi'')^2}{4\sqrt{2}(2\varphi+\varphi')} - \frac{4\varphi-4\varphi'-10\varphi''}{16\sqrt{2}(2\varphi+\varphi')h}$$

$$\frac{(1-\mu)}{\mu} = \begin{array}{l} 30\varphi + 6\varphi' l \\ +14 \quad +2 \\ +12 \quad +4 \\ \hline \dfrac{56\varphi + 12\varphi' l}{\sqrt{2}} \end{array} \;\cdots\; - \frac{(\varphi-\varphi'-\varphi'')^2}{\sqrt{2}(2\varphi+\varphi')}$$

$$\begin{array}{l} 10 + 2 \\ 28 \quad 4 \\ -6 \quad -2 \\ \hline \dfrac{32\varphi + 4\varphi' l}{\sqrt{2}} \end{array} + \frac{(\varphi-\varphi'-\varphi'')^2}{2\sqrt{2}(2\varphi+\varphi')}$$

$\cancel{\left(\frac{1-\mu}{\mu}\right) = \frac{7}{8}}$ $\cancel{\mu = \frac{1}{8}}$ $\cancel{\frac{32+4}{42+8}}$

czem większe φ (ujemne) tem mniejsze μ

$$\mu = \frac{1}{1 + \dfrac{56\varphi + 12\varphi' l}{32\varphi + 4\varphi' l}} = \frac{32\varphi + 4\varphi' l}{88\varphi + 16\varphi' l} = \frac{\varphi' l + 8\varphi}{4\varphi' l + 22\varphi}$$

~~$\Phi(l) =$~~

$$\cancel{12\,\Phi(l) + \varphi l\left[4\delta + 4(x^2+y^2+z^2) - 23x^2\delta - \frac{5x^2\delta}{2}\right]}$$

$$12\,\Phi(l) + \varphi l\left\{4\delta + [(x-\xi)^2+(y-\eta)^2+(z-\zeta)^2]\left(4-\frac{\delta}{2}\right) - \frac{5(x-\xi)^2\delta}{2}\right\}$$
$$+\varphi' l^2\left\{[\cdots - \cdots]\left(2+\frac{\delta}{2}\right) + \frac{5(x-\xi)^2\delta}{2}\right\}$$
$$+\varphi'' l^3\left\{[\cdot - \cdot]\frac{\delta}{2} + \frac{(x-\xi)^2\delta}{2}\right\}$$

$$U = 24\,\Phi(l) + 8\varphi l\delta + \left[\varphi l\left(4-\frac{\delta}{2}\right) + \varphi' l^2\left(2+\frac{\delta}{2}\right)\right](x^2+y^2+z^2+\xi^2+\eta^2+\zeta^2)$$
$$+\varphi''\frac{\delta}{2}$$
$$-\left[\frac{5\delta}{2}(\varphi l - \varphi' l^2) + \frac{\delta}{2}\varphi'' l^3\right](x^2+\xi^2)$$

$$-\Phi - \varphi\left\{\frac{\delta}{2} - \frac{x^2}{2}\left(1+\frac{\delta}{2}\right) + \frac{y^2}{2}\left(1-\frac{\delta}{2}\right) + \frac{x^2+y^2+z^2}{2}\left(1-\frac{\delta}{2}\right) - \frac{x^2}{4}\left(1+\frac{\delta}{2}\right) - \frac{y^2}{4}\left(1-3\frac{\delta}{2}\right) + \frac{xy}{2}\left(1-\frac{\delta}{2}\right)\right.$$

$$-\Phi - \varphi\left\{\frac{\delta}{2} + \frac{\xi^2}{2}\left(1+\frac{\delta}{2}\right) - \frac{\eta^2}{2}\left(1+\frac{\delta}{2}\right) + \frac{\xi^2+\eta^2+\zeta^2}{2}\left(1-\frac{\delta}{2}\right) - \frac{\xi^2}{4}\left(1+\frac{\delta}{2}\right) - \frac{\eta^2}{4}\left(1-3\frac{\delta}{2}\right) + \frac{\xi\eta}{2}\left(1-\frac{\delta}{2}\right)\right.$$

$$+\Phi + \varphi\left\{\frac{\delta}{2} - \frac{x+\xi}{2}\left(1+\frac{\delta}{2}\right) + (y-\eta)\cdots + \frac{(x-\xi)^2+\cdots}{2} - \frac{(x-\xi)^2}{4}\left(1+\frac{\delta}{2}\right) - \frac{(y-\eta)^2}{4}\left(1-3\frac{\delta}{2}\right) + \frac{(x-\xi)(y-\eta)}{2}\left(1-\frac{\delta}{2}\right)\right.$$

$$-\Phi - \varphi\left\{\frac{\delta}{2} + (x\xi+y\eta+z\zeta)\left(1-\frac{\delta}{2}\right) + \frac{x\xi}{2}\left(1+\frac{\delta}{2}\right) + \frac{y\eta}{2}\left(1-3\frac{\delta}{2}\right) + \frac{\xi y+x\eta}{2}\left(1-\frac{\delta}{2}\right)\right.$$

[illegible]

$K\left(\frac{dy}{dt}\right)^2 = m v^2$ $\qquad n = \frac{v}{2\pi\pi} = \frac{5.10^4}{6.10^{-8}} = 10^{12}$

$m n^2$

$\left(\frac{2\pi}{\tau}\right)^2 = \left(\frac{v}{r}\right)^2$

$\frac{2\pi}{\tau} = \frac{v}{r}$

$$\Sigma r = 12 + 4\delta + \frac{1+\frac{\delta}{2}}{\sqrt{2}}[X] + \frac{1}{\sqrt{2}}[Y] - \frac{\delta}{2\sqrt{2}}\begin{pmatrix} y_2 + y_9 - y_1 - y_{10} \\ + z_4 + z_{12} - z_3 - z_{11} \end{pmatrix} + \frac{1}{\sqrt{2}}[Z]$$

$$+ 6(x_0^2 + y_0^2 + z_0^2) - (x_0 \Sigma x + y_0 \Sigma y + z_0 \Sigma z) + \frac{\Sigma x^2 + \Sigma y^2 + \Sigma z^2}{2} -$$

$$-\left(1 + \frac{3\delta}{2}\right)\left[2x_0^2 - \frac{1}{2} x_0 \Sigma x + \frac{\Sigma x^2}{4}\right] -$$

$$-\left(1 - \frac{\delta}{2}\right)\left[y_0^2 - \frac{1}{2} y_0 (y_1 + y_2 + \cdot\right.$$

$X = \frac{4\varphi}{\sqrt{2}}$

$$U = A + B(x_0, y_0, z_0) + C(x_0^2, y_0^2, z_0^2)$$

$$\int (A + Bx + Cx^2)\, e^{-h(A+Bx+Cx^2)}\, dx = e^{-hA}\, e^{\frac{h^2B^2}{4C}} \sqrt{\frac{\pi}{C}} \left[A + \frac{B^2 h}{2C} + C\left(\frac{1}{2C} + \frac{h^2B^2}{4C^2}\right)\right]$$

$$= e^{-hA}\, e^{\frac{h^2B^2}{4hC}} \sqrt{\frac{\pi}{hC}} \left[A - \frac{B^2 h}{2hC} + C\left(\frac{1}{2hC} + \frac{h^2B^2}{4h^2C^2}\right)\right]$$

$$= e^{-hA + \frac{B^2 h}{4C}} \sqrt{\frac{\pi}{hC}} \left[A - \frac{B^2}{2C} + \frac{1}{2h} + \frac{B^2}{4C}\right]$$

$$= \cancel{e^{-hA + \frac{B^2 h}{4C}}}$$

$$= e^{-h\left(A - \frac{B^2}{4C}\right)} \sqrt{\frac{\pi}{hC}} \left[A - \frac{B^2}{4C} + \frac{1}{2h}\right]$$

$$\int e^{-h(A+Bx+Cx^2)}\, dx = e^{-h\left(A - \frac{B^2}{4C}\right)} \sqrt{\frac{\pi}{hC}}$$

$$U = A + B_1 x + B_2 y + B_3 z + C_1 x^2 + C_2 y^2 + C_3 z^2$$

$$\int U e^{-hU}\, dx = \sqrt{\frac{\pi}{hC_1}}\, e^{-h\left[A + B_2 y + B_3 z + C_2 y^2 + C_3 z^2 - \frac{B_1^2}{4C_1}\right]} \left[\frac{1}{2h} + A + B_2 y + B_3 z + \quad \right]$$

$$\int \dots\, dy = \sqrt{\frac{\pi^2}{h^2 C_1 C_2}}\, e^{-h\left[A + B_3 z + C_3 z^2 - \frac{B_1^2}{4C_1} - \frac{B_2^2}{4C_2}\right]} \left\{\frac{2}{2h} + A + B_3 z + C_3 z^2 - \frac{B_1^2}{4C_1} - \frac{B_2^2}{4C_2}\right\}$$

$$\int \dots\, dz = \sqrt{\frac{\pi^3}{h^3 C_1 C_2 C_3}}\, e^{-h\left[A - \frac{B_1^2}{4C_1} - \frac{B_2^2}{4C_2} - \frac{B_3^2}{4C_3}\right]} \left[\frac{3}{2h} + A - \frac{B_1^2}{4C_1} - \frac{B_2^2}{4C_2} - \frac{B_3^2}{4C_3}\right]$$

$$\sum r^2 = \overset{12+8\delta}{\cancel{\quad}}) + (1+\delta)\sqrt{2}\left[x_1+x_2-x_9-x_{10}+x_3+x_4-x_{11}-x_{12}\right]+$$

$$+\sqrt{2}\left[y_2+y_9+y_6+y_8-y_1-y_{10}-y_5-y_7\right]+$$

$$+\sqrt{2}\left[z_4+z_5+z_6+z_{12}-z_3-z_7-z_8-z_{11}\right]+$$

$$+12(x_0^2+y_0^2+z_0^2)-2(x_0\Sigma x+y_0\Sigma y+z_0\Sigma z)+\Sigma x^2+\Sigma y^2+\Sigma z^2$$

$$r_1 = 1+\frac{\delta}{2}-\frac{x_0-x_1}{\sqrt{2}}\left(1+\frac{\delta}{2}\right)+\frac{y_0-y_1}{\sqrt{2}}\left(1-\frac{\delta}{2}\right)+\frac{(x_0-x_1)^2+\cdots}{2}\left(1-\frac{\delta}{2}\right)-\frac{(x_0-x_1)^2}{4}\left(1+\frac{\delta}{2}\right)$$

$$-\frac{(y_0-y_1)^2}{4}\left(1-\frac{3\delta}{2}\right)+\frac{(x_0-x_1)(y_0-y_1)}{2}\left(1-\frac{\delta}{2}\right)$$

$$\cancel{\sum r = 12+2\delta+\frac{1+\frac{\delta}{2}}{\sqrt{2}}[X]+\frac{1-\frac{\delta}{2}}{\sqrt{2}}[Y]}$$

$$\sum_{1 \ldots 10} r = 4\left(1+\frac{\delta}{2}\right)+\frac{1+\frac{\delta}{2}}{\sqrt{2}}\sum_{1\,2\,9\,10}X+\frac{1-\frac{\delta}{2}}{\sqrt{2}}\sum_{2\,9\,1\,10}Y+\frac{\Sigma(x_0-x_1)^2\cdots}{2}\left(1-\frac{\delta}{2}\right)$$

$$-\Sigma\frac{(x_0-x_1)^2}{4}\left(1+\frac{3\delta}{2}\right)-\frac{\Sigma(y_0-y_1)^2}{4}\left(1-\frac{\delta}{2}\right)+\left(\frac{1-\frac{\delta}{2}}{2}\right)\Big[(x_0-x_1)(y_0-y_1)-(x_0-x_2)(y_0-y_2)$$

$$-\Sigma(z_0-z_1)^2\cdot\frac{\delta}{4}\qquad +(x_0-x_9)(y_0-y_9)-(x_0-x_{10})(y_0-y_{10})\Big]$$

$$\sum_{3 \ldots 12} r = 4\left(1+\frac{\delta}{2}\right)+\frac{1+\frac{\delta}{2}}{\sqrt{2}}\sum_{3 \ldots}X+\frac{\left(1-\frac{\delta}{2}\right)}{\sqrt{2}}\sum Z+\Sigma\frac{\cdots}{2}\left(1-\frac{\delta}{2}\right)$$

$$-\Sigma\frac{(x_0-x_3)^2}{4}\left(1+\frac{3\delta}{2}\right)-\frac{\Sigma(z_0-z_3)^2}{4}\left(1-\frac{\delta}{2}\right)+\frac{1-\frac{\delta}{2}}{2}\Big[(x_0-x_3)(z_0-z_3)-(x_0-x_4)(z_0-z_4)$$

$$-\Sigma(y_0-y_3)^2\frac{\delta}{4}\qquad +(x_0-x_{12})(z_0-z_{12})-(x_0-x_{11})(z_0-z_{11})\Big]$$

$$\sum_{5 \ldots 8} r = 4+\cancel{\quad}\frac{1}{\sqrt{2}}\Sigma Z+\frac{1}{\sqrt{2}}\Sigma Y+\frac{\Sigma(x_0-x)^2\cdots}{2}$$

$$-\Sigma\frac{(z_0-z_5)^2}{4}-\frac{\Sigma(y_0-y_5)^2}{4}+\frac{1}{2}\Big[(y_0-y_5)(z_0-z_5)-(y_0-y_7)(z_0-z_7)$$

$$+(y_0-y_8)(z_0-z_8)-(y_0-y_6)(z_0-z_6)\Big]$$

$$\cancel{r_{02}^2 = 1+\delta - (1+\delta)(x_0 - x_2)\sqrt{2} - (y_0 - y_2)\sqrt{2} + (x_0-x_2)^2 + (y_0-y_2)^2 + (z_0-z_2)^2}$$

$$r_{01}^2 = 1+\delta - (1+\delta)(x_0 - x_1)\sqrt{2} + (y_0 - y_1)\sqrt{2} + (x_0-x_1)^2 + (y_0-y_1)^2 + (z_0-z_1)^2$$

$$r_{02}^2 = 1+\delta - (1+\delta)(x_0 - x_2)\sqrt{2} - (y_0 - y_2)\sqrt{2} + (x_0-x_2)^2 + (y_0-y_2)^2 + (z_0-z_2)^2$$

$$r_{0,9} \qquad + \qquad -$$

$$r_{0,10} \qquad + \qquad +$$

$$r_{1,2,9,10}^2 = 4(1+\delta) + (1+\delta)(x_1 + x_2 - x_9 - x_{10})\sqrt{2} + (+y_2 + y_9 - y_1 - y_{10})\sqrt{2}$$
$$+4(x_0^2 + y_0^2 + z_0^2) - 2x_0 \Sigma x + \Sigma x^2$$
$$- 2y_0 \Sigma y + \Sigma y^2$$
$$- 2z_0 \Sigma z + \Sigma z^2$$

$$r_3^2 = 1+\delta - (1+\delta)(x_0 - x_3)\sqrt{2} + (z_0 - z_3)\sqrt{2} +$$

$$r_{3,4,11,12}^2 = 4(1+\delta) + (1+\delta)(x_3 + x_4 - x_{11} - x_{12})\sqrt{2} + (z_4 + z_{12} - z_3 - z_{11})\sqrt{2}$$
$$+4(x_0^2 + y_0^2 + z_0^2) - 2x_0 \Sigma x + \Sigma x^2$$
$$- 2y_0 \Sigma y + \Sigma y^2$$
$$- 2z_0 \Sigma z + \Sigma z^2$$

$$r_{5,7,6,8}^2 = 4 + (y_6 + y_8 - y_5 - y_7)\sqrt{2} + (z_5 + z_6 - z_7 - z_8)\sqrt{2}$$
$$+4(x_0^2 + y_0^2 + z_0^2) - 2x_0 \Sigma x$$
$$-2 \ldots \qquad - \ldots$$
$$- \quad -$$

$$\int_{-\infty}^{+\infty} y\, e^{-axy-by^2}\, dy = -\frac{ax}{2b^{3/2}}\, e^{\frac{a^2x^2}{4b}} \sqrt{\pi} \qquad a = -h(\varphi-\varphi'-\varphi'')\beta$$

$$b = 2h(2\varphi+\varphi')$$

$$\int_{-\infty}^{+\infty} e^{-axy-by^2}\, dy = \frac{e^{\frac{a^2x^2}{4b}}}{\sqrt{b}} \sqrt{\pi}$$

$$\frac{\partial}{\partial b}: \quad -\int_{-\infty}^{+\infty} y^2 e^{-axy-by^2}\, dy = e^{\frac{a^2x^2}{4b}} \sqrt{\frac{\pi}{b}} \left[-\frac{a^2x^2}{4b^2} - \frac{1}{2b}\right]$$

$$\overline{X}_y = (3\varphi+\varphi')\frac{\beta}{\sqrt{2}} - \cancel{\frac{\varphi-\varphi'-\varphi''}{\sqrt{2}} \frac{h(\varphi-\varphi'-\varphi'')\beta}{4h(2\varphi+\varphi')}} - \left[\frac{3}{4\sqrt{2}}(\varphi-\varphi'-2\varphi'')\beta + \right.$$

$$\left. \frac{5}{4\sqrt{2}}(\varphi-\varphi')\beta - \frac{1}{\sqrt{2}}(\varphi-\varphi'+\varphi'')\beta\right] \frac{4h(2\varphi+\varphi') \cancel{\frac{(\varphi-\varphi'-\varphi'')^2 h\beta}{\sqrt{2}}}}{h^2\left[16(2\varphi+\varphi')^2 - (\varphi-\varphi'-\varphi'')^2\beta^2\right]}$$

$$= \frac{\beta}{4\sqrt{2}} \frac{(12\varphi+4\varphi')(2\varphi+\varphi') - \cancel{(\varphi-\varphi'-\varphi'')^2}}{2\varphi+\varphi'} - \frac{\beta}{4\sqrt{2}}\left[3\varphi-3\varphi'-6\varphi''+5\varphi-5\varphi'\right.$$

$$\left. -4\varphi+4\varphi-4\varphi''\right] \cancel{\beta} \frac{1}{4h(2\varphi+\varphi')} - \frac{(\varphi-\varphi'-\varphi'')^2\beta}{16h\sqrt{2}(2\varphi+\varphi')^2}$$

$$= \frac{\beta}{4\sqrt{2}} \left\{ 4(3\varphi+\varphi') - \cancel{\frac{(\varphi-\varphi'-\varphi'')^2}{2\varphi+\varphi}} - \cancel{\beta}\frac{(4\varphi-4\varphi-10\varphi'')}{4h(2\varphi+\varphi')} \right\}$$

$$\int e^{-b(y^2 \cdot 2)} dy \, y^4 \underbrace{\int x^2 e^{-axy - bx^2} dx}$$

$$= e^{+\frac{a^2y^2}{4b}} \int x^2 e^{-\left(x\sqrt{b} + \frac{ay}{2\sqrt{b}}\right)^2} dx$$

$$= \frac{e^{+\frac{a^2y^2}{4b}}}{b^{3/2}} \int \left[\left(x\sqrt{b} + \frac{ay}{2\sqrt{b}}\right)^2 \cancel{\;} - ayx \cancel{\;} \frac{a^2y^2}{4b}\right] \cancel{\;} e^{-(x\sqrt{b} + \sim)^2} \, d(x\sqrt{\sim})$$

$$= \frac{e^{+\frac{a^2y^2}{4b}}}{b^{3/2}} \left[\cancel{\;} \int z^2 e^{-z^2} dz - \cancel{\frac{a}{b} \int y x e^{-x^2 b - ayx} dx} - \cancel{\frac{a^2y^2}{4b} \int e^{-x^2 b - ayx} dx} \right.$$

$$\left. - \frac{ay}{\sqrt{b}} \left[\cancel{\int z e^{-z^2} dz} - \underbrace{\frac{ay}{2\sqrt{b}} \int e^{-z^2} dz + \frac{ay}{4\sqrt{b}} \int e^{-z^2} dz}_{-\frac{ay}{4\sqrt{b}} \sqrt{\pi}} \right] \right]$$

$$= \frac{e^{+\frac{a^2y^2}{4b}}}{b^{3/2}} \left[\frac{\sqrt{\pi}}{2} + \frac{a^2y^2}{4b} \sqrt{\pi} \right] \text{stimmt}$$

$$= \frac{\int e^{-bz^2} dz \cdot \frac{1}{2b} \sqrt{\frac{\pi}{b}} \int e^{-by^2} e^{\frac{a^2y^2}{4b}} \left(1 + \frac{a^2y^2}{2b}\right) dy}{\int e^{-bz^2} dz \sqrt{\frac{\pi}{b}} \int e^{-\left(b - \frac{a^2}{4b}\right) y^2} dy} =$$

$$= \frac{1}{2b} \left\{ 1 + \frac{a^2}{2b} \frac{\int y^2 e^{-\left(b - \frac{a^2}{4b}\right) y^2} dy}{\int e^{-\left(b - \frac{a^2}{4b}\right) y^2} dy} \right\} = \frac{1}{2b} \left\{ 1 + \frac{a^2}{2b} \frac{1}{2\left(b - \frac{a^2}{4b}\right)} \right\}$$

$$= \frac{1}{2b} \frac{4b^2}{4b^2 - a^2} = \frac{2b}{4b^2 - a^2}$$

$$X_y = 2\varphi\left\{\frac{3\beta}{2\sqrt{2}} + \frac{\beta y}{4} - \frac{3x}{2} - \frac{yx}{2\sqrt{2}} - \frac{3x^2\beta}{8\sqrt{2}} - \frac{5y^2\beta}{8\sqrt{2}} + \frac{z^2\beta}{2\sqrt{2}}\right\}$$

$$-2\varphi'\left\{-\frac{\beta}{2\sqrt{2}} + \frac{\beta y}{4} + \frac{x}{2} - \frac{xy}{2\sqrt{2}} - \frac{3x^2\beta}{8\sqrt{2}} - \frac{5y^2\beta}{8\sqrt{2}} + \frac{z^2\beta}{2\sqrt{2}}\right\}$$

$$-\frac{\varphi''}{2}\left\{\beta y - \frac{2xy}{\sqrt{2}} - \frac{3x^2\beta}{\sqrt{2}} - \frac{2z^2\beta}{\sqrt{2}}\right\}$$

$$U = \Phi(b) + \varphi\left[-xy\beta + 4(x^2y^2+z^2y)\right]$$

$$+\varphi'\left[xy\beta + 2(x^2y^2+z^2y)\right]$$

$$+\varphi''\, xy\beta$$

$$\int e^{-axy - b(x^2+y^2+z^2)}\,dx\,dy\,dz = \int e^{-b(y^2+z^2)}\,dy\,dz \int e^{-axy - bx^2}\,dx$$

$$= \int e^{-b(y^2+z^2)}\,dy\,dz \int e^{-\left(x\sqrt{b} + \frac{ay}{2\sqrt{b}}\right)^2 + \frac{a^2y^2}{4b}}\,dx$$

$$= \cancel{e^{\frac{a^2}{4b}} \int e^{-by^2+z^2}\,dy\,dz \sqrt{\frac{\pi}{b}}} = \sqrt{\frac{\pi}{b}} \int e^{-bz^2}\,dz \int e^{-\left(b - \frac{a^2}{4b}\right)y^2}\,dy$$

$$\frac{\int xy\, e^{\cdots}}{\int e^{\cdots}} = -\frac{a\sqrt{\pi}}{2b^{3/2}} \cdot \sqrt{\frac{b}{\pi}} \cdot \frac{1}{2\left(b - \frac{a^2}{4b}\right)} = -\frac{a}{4b^2 - a^2}$$

$$\overline{X}_y = (3\varphi + \varphi')\frac{\beta}{\sqrt{2}}$$

$$+ \frac{\int y\, e^{-b[xy + 4(x^2 + \dots)]} \, dx\,dy\,dz}{\int \dots\dots} + \int yx\, \frac{e^{-xy + \dots}}{\text{---}}$$

$$\int e^{-b\,4(x^2 + z^2)} \, dx\,dz \underbrace{\int_{-\infty}^{+\infty} y\, e^{-bxy - 4by^2} \, dy}$$

$$\frac{-2\beta}{-2\beta} \int_{-\infty}^{+\infty} y\, e^{-\alpha y - \beta y^2} \, dy = \int \frac{-\alpha - 2\beta y + \alpha}{-2\beta} e^{-\alpha y - \beta y^2} \, dy$$

$$= -\frac{1}{2\beta}\left\{ \cancel{e^{-\alpha y - \beta y^2}\Big|} + \alpha \int_{-\infty}^{+\infty} e^{-\alpha y - \beta y^2} \, dy \right\} = -\frac{\alpha}{2\beta} \int_{-\infty}^{+\infty} e^{-\left(y\sqrt{\beta} + \frac{\alpha}{2\sqrt{\beta}}\right)^2 + \frac{\alpha^2}{4\beta}} \, dy$$

$$= -\frac{\alpha}{2\beta^{3/2}} e^{\frac{\alpha^2}{4\beta}} \sqrt{\pi}$$

$$\frac{\int y\, e^{-axy - b(x^2 + y^2 + z^2)} \, dx\,dy\,dz}{\int \dots} = -\int e^{-b(x^2 + z^2)} \frac{ax}{2b^{3/2}} e^{\frac{a^2x^2}{4b}} \sqrt{\pi} \, dx\,dz = ()$$

$$\frac{\int xy\, e^{-axy - b(x^2 + y^2 + z^2)} \, dx\,dy\,dz}{\int \dots} = -\frac{a\sqrt{\pi}}{2b^{3/2}} \int e^{-bz^2} dz \int e^{-\left(b - \frac{a^2}{4b}\right)x^2} \cdot x^2 \, dx$$

$$r = r_0 - \frac{x x_k + y y_k + z z_k}{r_0} + \frac{x^2+y^2+z^2}{2r_0} - \frac{x^2 x_k^2}{2r_0^3} - \frac{y^2 y_k^2}{2r_0^3} - \frac{z^2 z_k^2}{2r_0^3} - xy\frac{x_k y_k}{r_0^3} - \dots$$

$$\left[1 \pm \frac{\beta}{2} \mp \frac{x}{\sqrt{2}}\left(1 \pm \frac{\beta}{2}\right) - \frac{y}{\sqrt{2}}\left(1 \mp \frac{\beta}{2}\right) + \frac{x^2+y^2+z^2}{2}\left(1 \mp \frac{\beta}{2}\right) - \frac{x^2}{4}\left(1 \pm \frac{\beta}{2}\right) - \frac{y^2}{4}\left(1 \mp \frac{3\beta}{2}\right) \mp \frac{xy}{2}\left(1 \mp \frac{\beta}{2}\right)\right] x$$

$$-\left(\frac{1+\beta}{\sqrt{2}}\right)\left[1 + \frac{\beta}{2} - \frac{x}{\sqrt{2}}\left(1+\frac{\beta}{2}\right) - \frac{y}{\sqrt{2}}\left(1-\frac{\beta}{2}\right) + \frac{x^2+y^2+z^2}{2}\left(1-\frac{\beta}{2}\right) - \frac{x^2}{4}\left(1+\frac{\beta}{2}\right) - \frac{y^2}{4}\left(1-\frac{3\beta}{2}\right) - \frac{xy}{2}\right.$$

$$\left.+ \frac{1-\beta}{\sqrt{2}}\left[1 - \frac{\beta}{2} + \frac{x}{\sqrt{2}}\left(1-\frac{\beta}{2}\right) - \frac{y}{\sqrt{2}}\left(1+\frac{\beta}{2}\right) + \dots \left(1+\frac{\beta}{2}\right) - \frac{x^2}{4}\left(1-\frac{\beta}{2}\right) - \frac{y^2}{4}\left(1+\frac{3\beta}{2}\right) + \frac{xy}{2}\right.$$

$$\cancel{2x\left[\frac{\beta}{2} - \frac{x}{\sqrt{2}} + \frac{y\beta}{2\sqrt{2}} + \frac{x^2+y^2+z^2}{2} - \frac{x^2}{4} - \frac{y^2}{4}\right.}$$

$$\cancel{2x\left[\frac{\beta}{2} - \frac{x}{\sqrt{2}} + \frac{y\beta}{2\sqrt{2}}\right] - (x^2+y^2+z^2)\frac{\beta}{4} - \frac{x^2\beta}{8} + y^2\frac{3\beta}{8} - \frac{xy}{2}}$$

$$\left\{ -\frac{3\beta}{\sqrt{2}} + \frac{x}{\sqrt{2}} + \frac{y\beta}{2} - \frac{x^2+y^2+z^2}{2}\frac{\beta}{\sqrt{2}} + \frac{x^2}{4}\frac{3\beta}{\sqrt{2}} - \frac{y^2\beta}{4\sqrt{2}} + \frac{xy}{\sqrt{2}} \right.$$

$$\left. +2x\left[1 - \frac{x\beta}{2\sqrt{2}} - \frac{y}{\sqrt{2}}\right] \right.$$

$\frac{3}{4} - 1$

$$= \left\{ -\frac{3\beta}{\sqrt{2}} + 3x + \frac{y\beta}{2} - \frac{x^2+y^2+z^2}{2}\frac{\beta}{\sqrt{2}} + \frac{x^2}{4}\frac{\beta}{\sqrt{2}} - \frac{y^2\beta}{4\sqrt{2}} - \frac{xy}{\sqrt{2}} \right\}$$

$$\left[1 - \frac{\beta x}{\sqrt{2}} - \frac{y \mp z}{\sqrt{2}} + \frac{x^2+y^2+z^2}{2} - \frac{(y\mp z)^2}{4} - \frac{x(y\mp z)\beta}{2}\right]\left(x - \frac{\beta}{\sqrt{2}}\right)$$

$$+2\left\{x - \frac{\beta x^2}{\sqrt{2}} - \frac{yx}{\sqrt{2}} - \frac{\beta}{\sqrt{2}} + \frac{\beta y}{2} - \frac{x^2+y^2+z^2}{2}\frac{\beta}{\sqrt{2}} + \frac{(y^2+z^2)\beta}{4\sqrt{2}}\right\}$$

$$= \left\{ -\frac{5\beta}{\sqrt{2}} + 5x - \frac{(x^2+y^2+z^2)3}{2}\frac{\beta}{\sqrt{2}} - \frac{x^2 9\beta}{4\sqrt{2}} - \frac{3yx}{\sqrt{2}} + \frac{z^2\beta}{2\sqrt{2}} \right\}$$

$+3y\frac{\beta}{2}$

$+\frac{y^2\beta}{4\sqrt{2}}$

$$\sum r(x - x_k) = x \sum\left(r_0 - \frac{x x_k + y y_k + z z_k}{r_0}\right)$$

$$- \sum\Big[\underbrace{x_k r_0}_{} - \frac{x x_k^2}{r_0} + \frac{x^2+y^2+z^2}{2}\frac{x_k}{r_0} - \frac{x^2}{2}\frac{x_k^3}{r_0^3} - \frac{y^2}{2}\frac{x_k y_k^2}{r_0^3} - \frac{z^2}{2}\frac{x_k z_k^2}{r_0^3}\Big]$$

$$\frac{5\beta}{\sqrt{2}} \quad - y\sum\frac{x_k y_k}{r_0} - xy\frac{\sum x_k^2 y_k}{r_0^3} - xz\cancel{\frac{\sum x_k y_k z_k}{r_0^3} - yz}$$

$$= -\sum x_k r_0 + x\underbrace{\sum\left(r_0 + \frac{x_k^2}{r_0}\right)}_{=5} - x^2\sum\left(\frac{x_k}{r_0} + \frac{1}{2}\frac{x_k^3}{r_0^3}\right) + \frac{y^2}{2}\underbrace{\sum\frac{x_k y_k^2}{r_0^3}}_{\frac{\beta}{2\sqrt{2}}} + \frac{z^2}{2}\underbrace{\cancel{\sum\frac{x_k z_k^2}{r_0^3}}}_{=\frac{\beta}{\sqrt{2}}}$$

$$- \frac{x^2+y^2+z^2}{2}\sum\frac{x_k}{r_0} - xy\left(\sum\frac{y_k}{r_0} - \frac{x_k^2 y_k}{r_0^3}\right) - \cancel{xz\sum\frac{z_k}{r_0}} + y\left[\beta + \frac{1+\beta}{2}\left(1-\frac{\beta}{2}\right) + \frac{1-\beta}{2}\left(1+\frac{\beta}{2}\right)\right]$$

$$= -\frac{5\beta}{\sqrt{2}} + 5x - x^2\frac{9\beta}{4\sqrt{2}} - \frac{x^2+y^2+z^2}{2}\frac{3\beta}{\sqrt{2}} - xy\frac{4}{\sqrt{2}} + y\cdot\frac{3\beta}{2}$$

$$+ y^2\frac{\beta}{4\sqrt{2}} + \frac{xy}{\sqrt{2}}\underbrace{\left(\frac{1+2\beta}{2}\left(1-\frac{3\beta}{2}\right) + \frac{1-2\beta}{2}\left(1+\frac{3\beta}{2}\right)\right)}_{=\frac{1}{\sqrt{2}}}$$

$$\frac{3\beta}{\sqrt{2}} - \frac{3\beta}{4\sqrt{2}}$$

$$= -\frac{5\beta}{\sqrt{2}} + 5x - x^2\cdot\frac{15\beta}{4\sqrt{2}} + \frac{3y\beta}{2} - y^2\frac{5\beta}{4\sqrt{2}} - z^2\frac{\beta}{\sqrt{2}} - xy\frac{3}{\sqrt{2}}$$

stimmt

$$+\left\{-\frac{3\beta}{\sqrt{2}} + 3x - \frac{y\beta}{2} + \frac{3x^2\beta}{4\sqrt{2}} + \frac{5y^2\beta}{4\sqrt{2}} - \frac{z^2\beta}{\sqrt{2}} + \frac{xy}{\sqrt{2}}\right\}$$

$$-4\left\{-\beta\sqrt{2} + 2x\right\}$$

$$= y\beta - \frac{3x^2\beta}{\sqrt{2}} \quad \frac{2z^2\beta}{\sqrt{2}} \quad - xy\frac{2}{\sqrt{2}}$$

$$\frac{\partial}{\partial x}\sum(r-1)^3 = 3\left\{\beta\left[y - \frac{2z^2}{\sqrt{2}} - \frac{3x^2}{4\sqrt{2}}\right] - 2\frac{xy}{\sqrt{2}}\right\}$$

$$= -\frac{5\rho}{\sqrt2} + 3x - \frac{3y\rho}{2} + \frac{3x^2\rho}{4\sqrt2} + \frac{xy}{\sqrt2} + \frac{x^2+y^2+z^2}{2}\frac{3\rho}{\sqrt2} - \frac{3y^2\rho}{4\sqrt2}$$

$$= -\frac{5\rho}{\sqrt2} + 3x - \frac{3y\rho}{2} + \frac{9\rho}{4\sqrt2}x^2 + y^2\frac{3\rho}{4\sqrt2} + \frac{z^2\, 3\rho}{2\sqrt2} + \frac{xy}{\sqrt2}$$

$$(r_2-1)^3 = 2\left\{-\frac{y^3}{2\sqrt2} - \frac{3x^2y}{2\sqrt2} + \frac{3\rho}{2}\left[xy + \cancel{\frac{xy^2}{2\sqrt2}} - \frac{xz^2}{\sqrt2} - \frac{x^3}{2\sqrt2}\right]\right\}$$

$\cancel{\rho=0 \quad x \cdots z \quad y \cdots (x+y)}$

$\rho=0$, $x \cdots \rho x$, $y \cdots y+2$

$$(r_6-1)^3 = 2\left\{-\frac{(y\pm2)^3}{2\sqrt2} \ldots\right\}$$

$$-\frac{3}{2\sqrt2}\rho x(y\pm2)^2 = 2\left\{-\frac{y^3+3yz^2}{2\sqrt2} + \frac{3\rho}{2}\left[\frac{y^2+z^2}{2} - \frac{y(x^2\ldots)}{2\sqrt2} + \frac{3(y^3+3yz^2)}{4\sqrt2}\right]\right\}$$

$$\sum_y (z-1)^3 = 2\left\{-\frac{y^3}{\sqrt2} - \frac{3x^2y}{2\sqrt2} - \frac{3yz^2}{2\sqrt2} + \frac{3\rho}{2}\left[xy + \frac{xy^2}{\sqrt2} - \frac{xz^2}{\sqrt2} - \frac{x^3}{2\sqrt2}\right]\right\}$$

$$\frac{\partial}{\partial x} = 3\rho\left[y - \frac{z^2}{\sqrt2} - \frac{3x^2}{2\sqrt2}\right] - \frac{3xy}{\sqrt2} - \frac{xy^2+z^2}{\sqrt2}$$

$$\frac{\partial}{\partial x}\sum(z-1)^3 = 3\sum r^2\frac{\partial r}{\partial x} - 3\frac{\partial}{\partial x}\sum r^2 + 3\frac{\partial}{\partial x}\sum r$$

$4\sqrt2 - \frac{5}{\sqrt2} - \frac{3}{\sqrt2}$

$$= 3\sum r^2\frac{\partial r}{\partial x} - 3\frac{\partial}{\partial x}\sum(r-1)^2 - 3\frac{\partial}{\partial x}\sum r$$

$$-\frac{5\rho}{\sqrt2} + 3x - \frac{3y\rho}{2} + \frac{9\rho}{4\sqrt2}x^2 + \frac{3\rho}{4\sqrt2}y^2 + \frac{3\rho}{2\sqrt2}z^2 + \frac{xy}{\sqrt2} - 4\left\{-\rho\sqrt2 + 2x\right\}$$

$$+2\left\{-\frac{3\rho}{2\sqrt2} + \frac{3x}{2} - \frac{y\rho}{4} + \frac{3x^2\rho}{8\sqrt2} + \frac{5y^2\rho}{8\sqrt2} - \frac{z^2\rho}{2\sqrt2} + \frac{xy}{2\sqrt2}\right\}$$

$$\frac{\partial}{\partial x}\sum(r-1)^2 = \frac{\partial}{\partial x}\sum r^2 - 2\,\frac{\partial \sum r}{\partial x}$$

$$\sum r^2 = 4\left[1-\rho x\sqrt{2} - y\sqrt{2} + x^2+y^2+z^2\right]$$

$$\cancel{+\left[1-\rho x\sqrt{2} - y\sqrt{2} + \dots\right]}$$

$$\frac{\partial}{\partial x}\sum(r-1)^2 = 4\left\{-\rho\sqrt{2}+2x\right\} - 4\left\{-\frac{3\rho}{2\sqrt{2}} - \frac{\rho y}{4} + \frac{3x}{2} + \frac{xy}{2\sqrt{2}} + \frac{3x^2\rho}{8\sqrt{2}} + \frac{5y^2\rho}{8\sqrt{2}} - \frac{z^2\rho}{2\sqrt{2}}\right\}$$

$$= 4\left\{-\frac{\rho}{2\sqrt{2}} + \frac{x}{2} + \frac{\rho y}{4} - \frac{xy}{2\sqrt{2}} - \frac{3x^2\rho}{8\sqrt{2}} - \frac{5y^2\rho}{8\sqrt{2}} + \frac{z^2\rho}{2\sqrt{2}}\right\}$$ stimmt

$$\sum(x-x_k)r_k = -\sum x_k r_0 + x\sum\left(r_0 - \frac{x_k^2}{r_0}\right) - y\sum\frac{x_k y_k}{r_0} + x^2\sum\left(\frac{x_k}{r_0} - \frac{3}{2}\frac{x_k^3}{r_0^3}\right) +$$

$$\cancel{y^2\sum\left(\frac{x_k}{r_0} - \frac{3}{2}\frac{y_k^3}{r_0^3}\right)} - + xy\sum\left(\frac{y_k}{r_0} - 3\frac{x_k^2 y_k}{r_0^3}\right) + \frac{x^2+y^2+z^2}{2}\sum\frac{x_k}{r_0} -$$

$$-\frac{3}{2}y^2\sum\frac{x_k y_k^2}{r_0^3} - \cancel{\frac{3}{2}z^2\sum\frac{x_k z_k^2}{r_0^3}}$$

$$= -\left[\frac{2\rho}{\sqrt{2}} + \frac{1+\rho}{\sqrt{2}}\left(1+\frac{\rho}{2}\right) - \frac{1-\rho}{\sqrt{2}}\left(1-\frac{\rho}{2}\right)\right] + x\left[4 - \frac{1+2\rho}{2}\left(1-\frac{\rho}{2}\right) + \frac{1-2\rho}{2}\left(1+\frac{\rho}{2}\right)\right]$$

$$-y\left[\rho + \frac{1}{\sqrt{2}}\left(\frac{1+\rho}{\sqrt{2}}\left(1-\frac{\rho}{2}\right) - \frac{1-\rho}{\sqrt{2}}\left(1+\frac{\rho}{2}\right)\right)\right] + x^2\left[\frac{2\rho}{\sqrt{2}} + \frac{1+\rho}{\sqrt{2}}\left(1-\frac{\rho}{2}\right) - \frac{1-\rho}{\sqrt{2}}\left(1+\frac{\rho}{2}\right)\right] -$$

$$-\frac{3}{2}\left(\frac{1+3\rho}{2\sqrt{2}}\left(1-\frac{3\rho}{2}\right) - \left(\frac{1-3\rho}{2\sqrt{2}}\left(1+\frac{3\rho}{2}\right)\right)\right] + xy\left[\frac{4}{\sqrt{2}} - \frac{3}{\sqrt{2}}\left(\frac{1+2\rho}{2}\left(1-\frac{2\rho}{2}\right) + \frac{1-2\rho}{2}\left(1+\frac{2\rho}{2}\right)\right)\right]$$

$$+\frac{x^2+y^2+z^2}{2}\left[\frac{2\rho}{\sqrt{2}} + \frac{1+\rho}{\sqrt{2}}\left(1-\frac{\rho}{2}\right) - \frac{1-\rho}{\sqrt{2}}\left(1+\frac{\rho}{2}\right)\right] - \frac{3y^2}{2}\,\frac{1}{2}\left[\frac{2\rho}{\sqrt{2}} + \frac{1+\rho}{\sqrt{2}}\left(1-\frac{3\rho}{2}\right) - \frac{1-\rho}{\sqrt{2}}\left(1+\frac{3\rho}{2}\right)\right]$$

$$-\cancel{\dots} = -\frac{5\rho}{\sqrt{2}} + 3x - \frac{3y\rho}{2} + \frac{x^2\left[3\rho - \frac{9\rho}{4}\right]}{\sqrt{2}} + \frac{xy}{\sqrt{2}} +$$

$$+\frac{x^2+y^2+z^2}{2}\,\frac{3\rho}{\sqrt{2}} - \frac{3y^2}{4}\,\frac{\rho}{\sqrt{2}}$$

$$\sum \frac{x-x_k}{r} = \frac{\ }{r_0} - x = \sum (x-x_k)\Big[\frac{1}{r_0} + \frac{x x_k + y y_k + z z_k}{r_0^2} - \frac{x^2+y^2+z^2}{2 r_0^3} + \frac{3}{2}\,\frac{x^2 x_k^2 + y^2 y_k^2 + z^2 z_k^2}{r_0^5}$$

$$+ \frac{3}{r_0^5}\left(xy\, x_k y_k + xz\, x_k z_k + yz\, y_k z_k\right)\Big]$$

$$= x\sum\frac{1}{r_0} + x^2\sum\frac{x_k}{r_0^3} + xy\sum\frac{y_k}{r_0^3} - \cancel{\frac{x(x^2+y^2+z^2)}{2}\sum\frac{1}{r_0^3}} + \frac{3}{2}\cancel{\Big[x^3\sum\frac{x_k^2}{r_0^5} + xy^2\sum\frac{y_k^2}{r_0^5} + xz^2\sum\frac{z_k^2}{r_0^5}\Big]}$$

$$\cancel{+3\left(x^2y\sum x_k y_k + x^2 z\sum x_k z_k + xyz\sum y_k z_k\right)} -$$

$$-\Big\{\sum\frac{x_k}{r_0} + x\sum\frac{x_k^2}{r_0^3} + y\sum\frac{x_k y_k}{r_0^3} + z\sum\cancel{\frac{x_k z_k}{r_0^3}} - \frac{x^2+y^2+z^2}{2}\sum\frac{x_k}{r_0^3} + \frac{3}{2}x^2\sum\frac{x_k^3}{r_0^5}$$

$$+\frac{3}{2}y^2\sum\frac{x_k y_k^2}{r_0^5} + \frac{3}{2}z^2\sum\frac{x_k z_k^2}{r_0^5} + 3xy\sum\frac{x_k^2 y_k}{r_0^5} + 3xz\sum\cancel{\frac{x_k^2 z_k}{r_0^5}} + 3yz\sum\cancel{\frac{x_k y_k z_k}{r_0^5}}\Big\}$$

$$= \cdot 4x + x^2\Big[\frac{2\beta}{\sqrt2} + \cancel{\frac{1+\beta}{\sqrt2}\Big(1-\frac{3\beta}{2}\Big)} - \cancel{\frac{1-\beta}{\sqrt2}\Big(1+\frac{3\beta}{2}\Big)}\Big] + xy\,\frac{4}{\sqrt2} - \frac{x(x^2+y^2+z^2)}{2}\cdot 4 +$$

$$+\frac{3x^3}{2}\underbrace{\Big[\cancel{\frac{1+2\beta}{2}\Big(1-\frac{5\beta}{2}\Big)} + \frac{1-2\beta}{2}\Big(1+\frac{5\beta}{2}\Big)\Big]}_{=1} + \frac{3xy^2}{2}\cdots + \frac{3}{2}xz^2\cdots + 3x^2y\,[\beta+\beta]$$

$$-\Big[\frac{3\beta\sqrt2}{2} + \cancel{\frac{1+\beta}{\sqrt2}\Big(1-\frac{\beta}{2}\Big)} + \cancel{\frac{1-\beta}{\sqrt2}\Big(1+\frac{\beta}{2}\Big)}\Big] - x\underbrace{\Big[\frac{1+2\beta}{2}\Big(1-\frac{5\beta}{2}\Big) + \frac{1-2\beta}{2}\Big(1+\frac{5\beta}{2}\Big)\Big]}_{=1}$$

$$-y\Big[\beta + \cancel{\frac{1+\beta}{2}\Big(1-\frac{3\beta}{2}\Big)} - \cancel{\frac{1-\beta}{2}\Big(1+\frac{3\beta}{2}\Big)}\Big] + \frac{x^2+y^2+z^2}{2}\Big[\frac{\beta\sqrt2}{2} + \cancel{\frac{1+\beta}{\sqrt2}\Big(1-\frac{3\beta}{2}\Big)} - \cancel{\frac{1-\beta}{\sqrt2}\Big(1+\frac{3\beta}{2}\Big)}\Big]$$

$$-\frac{3x^2}{2}\Big[\cancel{\frac{1+3\beta}{2\sqrt2}\Big(1-\frac{5\beta}{2}\Big)} - \frac{1-3\beta}{2\sqrt2}\Big(1+\frac{5\beta}{2}\Big)\Big] - \frac{3}{2}y^2\Big[\frac{\beta}{\sqrt2} + \cancel{\frac{1+\beta}{2\sqrt2} - \frac{1-\beta}{2\sqrt2}}\Big] = \frac{\beta}{2\sqrt2}$$

$$1 - \frac{3\beta}{2} - \Big(1+\frac{3\beta}{2}\Big) = -\frac{3\beta}{2\sqrt2}$$

$$-\frac{3z^2}{2}\Big[\frac{2\beta}{2\sqrt2}\Big] + 3xy\Big[\cancel{\frac{1+2\beta}{2\sqrt2}\Big(1-\frac{5\beta}{2}\Big)} - \cancel{\frac{1-2\beta}{2\sqrt2}\Big(1+\frac{5\beta}{2}\Big)}\Big]$$

$$= -\frac{3\beta}{\sqrt2} + 3x - y\frac{\beta}{\sqrt2} + x^2\underbrace{\Big[\frac{\beta}{\sqrt2} - \frac{3\beta}{4\sqrt2}\Big]}_{=\frac{1}{\sqrt2}} + \frac{3y^2\beta}{4\sqrt2} + xy\,\frac{1}{\sqrt2} + \frac{(x^2+y^2+z^2)\beta}{2\sqrt2} - \frac{3z^2\beta}{2\sqrt2}$$

$$\cancel{2x(x^2+y^2+z^2) + \frac{3x^3}{2} + 3xy^2 + 6x^2y\beta + \frac{3}{2}xz^2}$$

$$\frac{4-3+2}{4\sqrt2} = \frac{3}{4\sqrt2} \qquad 3+$$

$(r_6-1)^3 = \cancel{\ } - \frac{(y+z)^3}{2\sqrt{2}} = -\frac{y^3+3z^2y}{2\sqrt{2}}$

8

$\frac{6}{7} = \quad + $

$\Sigma = 0$

$\Sigma_{1\ldots 12} = 6\beta . xy$

$\Sigma (r-1)^3 = 12 + 24\,(x^2 y^2 z^2) + 9 . xy . \beta - 36\,(1 + x^2 y^2 z^2) + 36 - 3xy\beta$

$+ 12\,(x^2 y^2 z^2) - 12$

$= 6 xy\beta$ stimmt

$$\Sigma \frac{x - x_k}{r} = -\Sigma \frac{x_k}{r_0} + x\,\Sigma\left(\frac{1}{r_0} - \frac{x_k^2}{r_0^3}\right) - y\,\Sigma \frac{x_k y_k}{r_0^3} + x^2 \underbrace{\Sigma\left(\frac{x_k}{r_0^3} - \frac{3}{2}\frac{x_k^3}{r_0^5}\right)}^{\beta/4\sqrt{2}} + xy\,\underbrace{\Sigma\left(\frac{y_k}{r_0^3} - 3\frac{x_k^2 y_k}{r_0^5}\right)}_{\frac{1}{\sqrt{2}}}$$

$$+ \frac{x^2 y^2 z^2}{2} \underbrace{\Sigma \frac{x_k}{r_0^3}}_{\beta/\sqrt{2}} - \frac{3}{2} y^2\, \Sigma \frac{x_k y_k^2}{r_0^5} - \frac{3}{2} z^2\, \Sigma \frac{x_k z_k^2}{r_0^5} = \beta/\sqrt{2}$$

$$\sum (z-1)^2 = \sum z^2 - 2\sum z + 12 =$$

$$= 12(1+x^2y^2z^2) - 2\left[12 - xy\beta + 4(x^2y^2z^2)\right] + 12 =$$

$$= \cancel{\ } + 4(x^2y^2z^2) + 2xy\beta$$

stimmt

$$4\left\{\frac{x^2y^2}{2} + \frac{y^2z^2}{2} + \frac{x^2z^2}{2} + xy\frac{\beta}{2}\right\} = \uparrow$$

$$\left(\frac{z}{2}-1\right)^3 = -\frac{x^3}{2\sqrt2}\left(1+\frac{3\beta}{2}\right) - \frac{y^3}{2\sqrt2}\left(1-\frac{3\beta}{2}\right) + \frac{3\beta}{2}\left[\frac{x^2}{2} + \frac{y^2}{2} + xy - \frac{x(x^2y^2z^2)}{4\sqrt2} + \frac{x^3(\cancel{\ })}{2\sqrt2}\right.$$

$$\cancel{-\frac{x(x^2y^2z^2)}{2\sqrt2} - \frac{y(x^2y^2z^2)\beta-\beta}{2\sqrt2} + \frac{x^3(1+\beta)}{}} + \frac{xy^2}{2\sqrt2}(\cancel{\ }) + \frac{x^2y}{4\sqrt2}$$

$$-\frac{x^3}{2\sqrt2}\cdot\frac{3\beta}{2} \quad -\frac{y(x^2y^2z^2)}{4\sqrt2} + \frac{yx^2}{2\sqrt2} + \frac{y^3}{2\sqrt2} + \left.\frac{xy^2}{4\sqrt2}\right] - \frac{3x^2y}{2\sqrt2}\left(1+\frac{\beta}{2}\right) - \frac{3xy^2}{2\sqrt2}\left(1-\frac{\beta}{2}\right)$$

9 $$(z_2-1)^3 = \pm\frac{3\beta}{2}\left[\cancel{\ } \frac{x^2y^2z^2}{2} \pm xy \mp \frac{x^3}{4\sqrt2} + \frac{yx^2}{4\sqrt2} \mp \frac{xy^2}{2\sqrt2} \pm \frac{y^3}{4\sqrt2} - \frac{xz^2}{2\sqrt2} - \frac{yz^2}{2\sqrt2}\right]$$

$$\mp\frac{x^3}{2\sqrt2} - \frac{y^3}{2\sqrt2} \;-\;-\;—$$

$$= 2\left\{-\frac{y^3}{2\sqrt2} + \frac{3\beta}{2}\left[xy + \frac{xy^2}{2\sqrt2} - \frac{xz^2}{4\sqrt2}\right] - \frac{x^3}{2\sqrt2}\Bigg] - \frac{3x^2y}{2\sqrt2} + \frac{3xy^2}{4\sqrt2}\beta\right\}$$

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10 $$(z_1-1)^3 = \mp\frac{x^3}{2\sqrt2} + \frac{y^3}{2\sqrt2} \mp \frac{3\beta}{2}\left[\frac{x^2y^2}{2} \mp xy \mp \frac{3x^3}{4\sqrt2} - \frac{yx^2}{4\sqrt2} \pm \frac{xy^2}{4\sqrt2} - \frac{y^3}{4\sqrt2} \mp \frac{xz^2}{4\sqrt2} + \frac{yz^2}{2\sqrt2}\right]$$

$$= 2\left\{\frac{y^3}{2\sqrt2} + \frac{3\beta}{2}\left(xy + \frac{3x^3}{4\sqrt2} - \frac{xy^2}{4\sqrt2} + \frac{xz^2}{4\sqrt2}\right) + \frac{3x^2y}{2\sqrt2} - \frac{3xy^2\beta}{4\sqrt2}\right\}$$

$$\sum_{12\,9\,10} = 4\left\{\frac{3\beta\,xy}{2}\right\} = 6\beta\cdot xy$$

$$\sum\left(r_{\substack{2\\9}}-1\right)^2 = 2\left\{-\frac{\beta x}{\sqrt2}+\frac{x^2+y^2}{2}-xy\frac{\beta}{2}+\frac{3x^3\beta}{4\sqrt2}-\frac{y^3}{2\sqrt2}+\frac{x^2y}{2\sqrt2}-\frac{xy^2\,7\beta}{4\sqrt2}+\frac{xz^2\beta}{2\sqrt2}+\frac{yz^2}{\sqrt2}\right.$$

$$(r_1-1)^2 = 1+\frac{\beta x}{\sqrt2}\mp\frac{\beta y}{\sqrt2}+\frac{x^2}{2}\left(1\mp\frac{5\beta}{4}\right)+\frac{y^2}{2}\left(1\pm\frac{3\beta}{4}\right)\mp xy\left(1\pm\frac{\beta}{2}\right)\mp z^2\frac{\beta}{2}\mp\frac{x^3}{2\sqrt2}\left(1\pm\frac{3\beta}{2}\right)$$

$$+\frac{y^3}{2\sqrt2}\left(1\pm\frac{\beta}{2}\right)-\frac{x^2y}{2\sqrt2}\left(1\mp\frac{3\beta}{2}\right)\pm\frac{xy^2}{2\sqrt2}\left(1\pm\frac{7\beta}{2}\right)\mp\frac{xz^2}{\sqrt2}\left(1\pm\frac{\beta}{2}\right)+\frac{yz^2}{\sqrt2}\left(1\pm\frac{3\beta}{2}\right)$$

$$\sum_{10} = 2\left\{\frac{\beta x}{\sqrt2}+\frac{x^2+y^2}{2}-yx\frac{\beta}{2}-\frac{3x^3\beta}{4\sqrt2}+\frac{y^3}{2\sqrt2}-\frac{x^2y}{2\sqrt2}+\frac{7xy^2\beta}{4\sqrt2}-\frac{xz^2\beta}{4\sqrt2}+\frac{yz^2}{\sqrt2}\right.$$

$$\sum_{\substack{1\\2\\9\\10}} = 4\left\{\frac{x^2+y^2}{2}-xy\frac{\beta}{2}\right\}$$

$$\left(r_{\substack{6\\8}}-1\right)^2 = \frac{(y\pm z)^2}{2}\left(1+\cancel{\beta x\sqrt2}\right)+\beta x(y\pm z)\left(\cancel{\ldots}\right)-\frac{\beta x}{\sqrt2}(x^2+y^2+z^2)+\frac{\beta x}{\sqrt2}\frac{(y\pm z)^2}{2}$$

$$-\frac{(y\pm z)}{\sqrt2}\left(1+\cancel{2\beta\frac{x}{\sqrt2}}\right)(x^2+y^2+z^2)+\frac{(y\pm z)^3}{2\sqrt2}\left(\cancel{1+2\beta x\sqrt2}\right)$$

$$=2\left\{\frac{y^2+z^2}{2}\left(1+\beta x\sqrt2\right)+\beta xy\left(\cancel{\ldots}\right)-\frac{\beta x^3}{\sqrt2}-\frac{\beta x(y^2+z^2)}{2\sqrt2}\right.$$

$$\left.-\frac{y(x^2+y^2+z^2)}{\sqrt2}+\frac{y^3+3yz^2}{2\sqrt2}\right\}$$

$$=2\left\{\frac{y^2+z^2}{2}+xy\beta-\frac{\beta x^3}{\sqrt2}+\frac{\beta x(y^2+z^2)}{2\sqrt2}-\frac{y(x^2+y^2+z^2)}{\sqrt2}+\frac{y^3+3z^2y}{2\sqrt2}\right\}$$

$\substack{5\\7}$ $$=2\left\{\frac{y^2+z^2}{2}+xy\beta+\frac{\beta x^3}{\sqrt2}-\frac{\beta x(y^2+z^2)}{2\sqrt2}+y(\;--\;)-\ldots\right\}$$

$$=4\left\{\frac{y^2+z^2}{2}+xy\beta\right\}$$

$$\sum_{34\,11\,12} = 4\left\{\frac{x^2+z^2}{2}\right\}$$

$$r_2 = 1+\frac{\beta}{2} - \frac{x}{\sqrt2}\left(1+\frac{\beta}{2}\right) - \frac{y}{\sqrt2}\left(1-\frac{\beta}{2}\right) + \frac{x^2+y^2-2z^2}{2}\left(1-\frac{\beta}{2}\right) - \frac{x^2}{4}\left(1+\frac{\beta}{2}\right) - \frac{y^2}{4}\left(1-\frac{3\beta}{2}\right) - \frac{xy}{2}\left(1-\frac{\beta}{2}\right)$$

$$+\frac{x(x^2+y^2-2z^2)}{2\sqrt2}\left(1-\frac{\beta}{2}\right) + \frac{y(x^2+y^2-2z^2)}{2\sqrt2}\left(1-\frac{3\beta}{2}\right) - \frac{x^3}{4\sqrt2}\left(1+\frac{\beta}{2}\right) - \frac{y^3}{4\sqrt2}\left(1-\frac{5\beta}{2}\right) - \frac{3x^2y}{4\sqrt2}\left(1-\frac{\beta}{2}\right)$$

$$-\frac{3xy^2}{4\sqrt2}\left(1-\frac{3\beta}{2}\right)$$

$$(r_2-1)^2 = \frac{x^2}{2}(1+\beta) + \frac{y^2}{2}(1-\beta) - \frac{\beta x}{\sqrt2} - \frac{\beta y}{\sqrt2} + \frac{x^2+y^2-2z^2}{2}\beta - \frac{x^2\beta}{4} - \frac{y^2\beta}{4} - \frac{xy\beta}{2}$$

$$+\frac{x(x^2+y^2-2z^2)\beta}{2\sqrt2} + \frac{y(x^2+y^2-2z^2)\beta}{2\sqrt2} - \frac{x^3\beta}{4\sqrt2} - \frac{y^3\beta}{4\sqrt2} - \frac{3x^2y\beta}{4\sqrt2} - \frac{3xy^2\beta}{4\sqrt2}$$

$$+xy - \frac{x(x^2+y^2-2z^2)}{\sqrt2} + \frac{x^3}{2\sqrt2}(1+\beta) + \frac{xy^2}{2\sqrt2}(1-\beta) + \frac{x^2y}{\sqrt2} \qquad 1-\frac{3}{4}+1+\frac12 \quad \frac74$$

$$-\frac{y(x^2+y^2-2z^2)}{\sqrt2}(1-\beta) + \frac{x^2y}{2\sqrt2} + \frac{y^3}{2\sqrt2}(1-2\beta) + \frac{xy^2}{\sqrt2}(1-\beta)$$

$$= -\frac{\beta x}{\sqrt2} - \frac{\beta y}{\sqrt2} + \frac{x^2}{2}\underbrace{\left(1+\beta+\frac{\beta}{2}-\frac{\beta}{4}\right)}_{1+\frac{5\beta}{4}} + \frac{y^2}{2}\underbrace{\left(1-\beta+\frac{\beta}{2}-\frac{\beta}{4}\right)}_{1-\frac{3\beta}{4}} + xy\left(1-\frac{\beta}{2}\right) + \frac{z^2\beta}{2}$$

$$-\frac{x(x^2+y^2-2z^2)}{\sqrt2}\left(1-\frac{\beta}{2}\right) - \frac{y(x^2+y^2-2z^2)}{\sqrt2}\left(1-\frac{3\beta}{2}\right) + \frac{x^3}{2\sqrt2}\left(1+\frac{\beta}{2}\right) + \frac{y^3}{2\sqrt2}\left(1-\frac{5\beta}{2}\right)$$

$$+\frac{x^2y}{\sqrt2}\left(\frac32-\frac{3\beta}{4}\right) + \frac{xy^2}{\sqrt2}\left(\frac32-\frac{3\beta}{2}-\frac{3\beta}{4}\right)$$

$$\frac{x^3}{\sqrt2}\left(-1+\frac{\beta}{2}+\frac12+\frac{\beta}{4}\right) + \frac{y^3}{2\sqrt2}\left(1-\frac{5\beta}{2}-2+3\beta\right) + \frac{x^2y}{\sqrt2}\left(\frac32-\frac{3\beta}{4}-1+\frac{3\beta}{2}\right)$$

$$-\frac12+\frac{3\beta}{4} \qquad \frac12+\frac{3\beta}{4}$$

$$+\frac{xy^2}{\sqrt2}\left(\frac32-\frac{9\beta}{4}-1+\frac{\beta}{2}\right) - \frac{xz^2}{\sqrt2}\left(1-\frac{\beta}{2}\right) - \frac{yz^2}{\sqrt2}\left(1-\frac{3\beta}{2}\right)$$

$$\underset{9}{r_2} = -\frac{\beta x}{\sqrt2} \mp \frac{\beta y}{\sqrt2} + \frac{x^2}{2}\left(1\pm\frac{5\beta}{4}\right) + \frac{y^2}{2}\left(1\mp\frac{3\beta}{4}\right) \pm xy\left(1\mp\frac{\beta}{2}\right) \pm \frac{z^2\beta}{2} \mp \frac{x^3}{2\sqrt2}\left(1\mp\frac{3\beta}{2}\right) - \frac{y^3}{2\sqrt2}\left(1\mp\frac{\beta}{2}\right)$$

$$+\frac{x^2y}{2\sqrt2}\left(1\pm\frac{3\beta}{2}\right) \pm \frac{xy^2}{2\sqrt2}\left(1\mp\frac{7\beta}{2}\right) \mp \frac{xz^2}{\sqrt2}\left(1\mp\frac{\beta}{2}\right) - \frac{yz^2}{\sqrt2}\left(1\mp\frac{3\beta}{2}\right)$$

$$\sum r^2 = 12\,(1 + x^2+y^2+z^2)$$

$$r^3 = r_0^3 + \frac{3(x^2+y^2+z^2)}{2} \sum r_0 + \frac{3x^2}{2} \sum \frac{x_k^2}{r_0} + \frac{3y^2}{2} \sum \frac{y_k^2}{r_0} + \frac{\Sigma z^2}{2} \sum \frac{z_k^2}{r_0}$$

$$+ 3xy \sum \frac{x_k y_k}{r_0} + \cdots$$

$$8\left[\frac{1-2\beta}{4}\left(1+\frac{\beta}{2}\right) + \frac{1+2\beta}{4}\left(1-\frac{\beta}{2}\right)\right] + \frac{4}{2} = \sum \frac{x_k^2}{r_0} = 4$$

$$1 - \cancel{\frac{3\beta}{2}} + 1 + \cancel{\frac{3\beta}{2}}$$

$$\sum \frac{y_k^2}{r_0} = 4$$

$$\sum \frac{z_k^2}{r_0} = 4$$

$$\sum \frac{x_k y_k}{r_0} = \cancel{4}\left\{\cancel{4}(1-\beta)\left(1+\frac{\beta}{2}\right) + \cancel{4}(1+\beta)\left(1-\frac{\beta}{2}\right)\right\} + \frac{4}{2}\beta = 3\beta$$

$$-1+\frac{\beta}{2} \quad +1+\frac{\beta}{2}$$

$$\sum \frac{x_k z_k}{r_0} = 0$$

$$\sum y_k z_k = 0$$

$$\sum r^3 = 12\left[1 + \frac{3(x^2+y^2+z^2)}{2}\right] + 6(x^2+y^2+z^2) + 9xy\beta$$

$$= 12 + 24(x^2+y^2+z^2) + 9xy\beta$$

$$\frac{\partial}{\partial x} = 2\left\{-\frac{3\rho}{2\sqrt2} - \frac{\rho y}{4} + \ldots + \frac{3x}{2} + \frac{yx}{2\sqrt2}\right.$$

$$+\frac{3x\rho}{8\sqrt2} + \frac{5y\rho}{8\sqrt2} - \frac{z^2\rho}{2\sqrt2}$$

stimmt

$$\sum_{5678} r = 4\left\{1 - \rho\frac{xy}{2} + \frac{x^2+y^2+z^2}{2} - \frac{x^2y^2z^2}{4}\right\}$$

$$\sum_{34} r = 4\left\{1 - \ldots + \ldots + \frac{x^2+y^2+z^2}{2} - \frac{x^2}{4} - \ldots\right\} + \frac{x\ldots}{2} + \frac{x(x^2+y^2+z^2)}{2\sqrt2} - \frac{2(x^2+y^2+z^2)}{2\sqrt2}$$

$$-\frac{x^3}{4\sqrt2} + \frac{z^3}{4\sqrt2} + \frac{3x^2z}{4\sqrt2} - \frac{3xz^2}{4\sqrt2}$$

$$\sum_{1-12} r = 12\left\{1 - \frac{xy\rho}{12} + \frac{x^2+y^2+z^2}{2} - \frac{x^2+y^2+z^2}{6}\right\}$$

$$= 12\left\{1 - \frac{xy\rho}{12} + \frac{x^2+y^2+z^2}{3}\right\} = 12 - xy\rho + (x^2+y^2+z^2)4$$

(stimmt)

$$\sum r = \sum\left\{r_0 - \frac{x\,x_k + y\,y_k + z\,z_k}{r_0} + \frac{x^2+y^2+z^2}{2r_0} - \frac{x^2x_k^2}{2r_0^3} - \frac{y^2y_k^2}{2r_0^3} - \frac{z^2z_k^2}{2r_0^3}\right.$$

$$\left. - \frac{xy\,x_ky_k}{r_0^3} - \frac{xz\,x_kz_k}{r_0^3} - \frac{yz\,y_kz_k}{r_0^3}\right\}$$

$$\sum r = 12 + \frac{x^2+y^2+z^2}{2} - \frac{x^2}{2}\left[\frac{1-2\rho}{2}\left(1+\frac{3\rho}{2}\right) + \frac{1+2\rho}{2}\left(1-\frac{3\rho}{2}\right) + 2\right] - \frac{y^2}{2}\,\frac{8}{2} - \frac{z^2}{2}\,\frac{8}{2} -$$

$$-\frac{xy}{2}\left[-(1-\rho)2\left(1+\frac{3\rho}{2}\right) + (1+\rho)2\left(1-\frac{3\rho}{2}\right) + \ldots\right] \quad \begin{matrix}\rho - 1 & -\frac{3\rho}{2}\\ \rho + 1 & -\frac{3\rho}{2}\end{matrix} \quad \begin{matrix}-\left(1+\frac{\rho}{2}\right)\\ +\left(1-\frac{\rho}{2}\right)\end{matrix}$$

$$= 12\left(1 + \frac{x^2+y^2+z^2}{2}\right) - 2x^2\left[\ldots\right] - 2y^2 - 2z^2 + xy\rho$$

$$= 12 + 6(x^2+y^2+z^2) - 2x^2 - 2y^2 - 2z^2 - xy\rho$$

$$= 12 + 4(x^2+y^2+z^2) - xy\rho$$

$$\sum \frac{x_k}{r_0} =$$

$$r_6 = \sqrt{1 - \beta x\sqrt{2} - (y+2)\sqrt{2} + x^2+y^2+z^2}$$

$\delta \cdots -\beta x\sqrt{2}$ $\quad \delta = 0$

$x \cdots$ $\quad x \cdot -\beta x$

$y \quad -(y+2)$ $\quad y \quad -(y+2)$

$$\frac{r_6}{8} = 1 - \frac{\beta x}{\sqrt{2}} - \frac{(y\pm 2)}{\sqrt{2}}\left(\cancel{\ldots}\right) + \cancel{\ldots}\left(\cancel{\ldots}\right) - \frac{(y\pm2)^2}{4}\left(1+\frac{3\beta x\sqrt{2}}{2}\right) \pm \frac{\beta x\sqrt{2}(y+2)}{2}$$

$$+\frac{(y\pm2)(x^2+y^2+z^2)}{2\sqrt{2}}\left(\cancel{1+\frac{3\beta x\sqrt{2}}{2}}\right) - \frac{(y\pm2)^3}{4\sqrt{2}}\left(\cancel{1+\frac{5\beta x\sqrt{2}}{2}}\right)$$

$$-\frac{\beta x}{\sqrt{2}} - \frac{y}{\sqrt{2}} + \frac{x^2+y^2+z^2}{2} - \frac{x^2}{4} - \frac{2\beta xy+y^2}{4} + \frac{(\beta x+y)}{2\sqrt{2}}(x^2+y^2+z^2) - \frac{(\beta x+y)^3}{4\sqrt{2}} - \frac{3z^2(\beta x+y)}{4\sqrt{2}}$$

$$= 2\left\{1 - \frac{\beta x}{\sqrt{2}} - \frac{y}{\sqrt{2}}\left(1+\frac{\beta x}{\sqrt{2}}\right) + \frac{x^2+y^2+z^2}{2}\left(1+\frac{\beta x}{\sqrt{2}}\right) - \frac{y^2+z^2}{4}\left(1+\frac{3\beta x}{\sqrt{2}}\right)\right.$$

$$\left. + \frac{y(x^2+y^2+z^2)}{2\sqrt{2}}\left(1+\cancel{\frac{3\beta x}{\sqrt{2}}}\right) - \frac{y^3+3zy^2}{4\sqrt{2}}\left(1+\cancel{\frac{5\beta x}{\sqrt{2}}}\right)\right\}$$

$$1 - \frac{\beta x}{2\sqrt{2}} - \frac{y}{\sqrt{2}} + \frac{x^2+y^2+z^2}{2}\left(1-\frac{\beta x}{2\sqrt{2}}\right) - \frac{x^2+y^2}{4} + \frac{xy\beta}{4} + y\frac{(x^2+y^2+z^2)}{2\sqrt{2}}$$

$$-\frac{x^3\beta}{8\sqrt{2}} - \frac{y^3}{4\sqrt{2}} - \frac{3x^2y}{4\sqrt{2}} + \frac{9xy^2\beta}{8\sqrt{2}}$$

$$\sum r_2 \rho\, 6\, 8 = 2\left\{2 - \frac{3\beta x}{2\sqrt{2}} - \frac{2y}{\sqrt{2}} - \frac{\beta xy}{4} + x^2+y^2+z^2 - \frac{x^2+2y^2+z^2}{4} + y\frac{(x^2+y^2+z^2)}{\sqrt{2}}\right.$$

$$+ \frac{x(x^2+y^2+z^2)\beta}{4\sqrt{2}} - \cancel{\frac{3\beta xy^2}{2\sqrt{2}}} - \frac{3\beta xz^2}{4\sqrt{2}} - \frac{y^3}{2\sqrt{2}} - \frac{3yz^2}{4\sqrt{2}} - \frac{x^3\beta}{8\sqrt{2}}$$

$$\left. - \frac{3x^2y}{4\sqrt{2}} + \frac{3xy^2\beta}{8\sqrt{2}}\right\}$$

$$\frac{9}{8} - \frac{3}{3} = \frac{9-6}{8} \qquad = 2\left\{2 - \frac{3\beta x}{2\sqrt{2}} - \frac{2y}{\sqrt{2}} - \frac{\beta xy}{4} + \frac{3x^2+2y^2+3z^2}{4} + \frac{yx^2}{4\sqrt{2}} + \frac{y^3}{2\sqrt{2}} + \frac{yz^2}{4\sqrt{2}}\right.$$

$$\left. + \frac{3x^3\beta}{8\sqrt{2}} + \frac{xy^2\beta}{8\sqrt{2}} - \frac{xz^2\beta}{8\sqrt{2}}\right\}$$

$\delta \quad \ldots -\beta$

$$r_1 = 1 - \frac{\beta}{2} - \frac{x}{\sqrt2}\left(1-\frac{\beta}{2}\right) + \frac{y}{\sqrt2}\left(1+\frac{\beta}{2}\right) + \frac{x^2+y^2-r^2}{2}\left(1+\frac{\beta}{2}\right) - \frac{x^2}{4}\left(1-\frac{\beta}{2}\right) - \frac{y^2}{4}\left(1+\frac{3\beta}{2}\right) + \frac{xy}{2}\left(1+\frac{\beta}{2}\right)$$

$$+ \frac{x(x^2+y^2-r^2)}{2\sqrt2}\left(1+\frac{\beta}{2}\right) - \frac{y(x^2+y^2-r^2)}{2\sqrt2}\left(1+\frac{3\beta}{2}\right) - \frac{x^3}{4\sqrt2}\left(1-\frac{\beta}{2}\right) + \frac{y^3}{4\sqrt2}\left(1+\frac{5\beta}{2}\right) + \frac{3x^2y}{4\sqrt2}\left(1+\frac{\beta}{2}\right) -$$

$$- \frac{3xy^2}{4\sqrt2}\left(1+\frac{3\beta}{2}\right)$$

$$r_2 = 1 + \frac{\beta}{2} - \frac{x}{\sqrt2}\left(1+\frac{\beta}{2}\right) - \frac{y}{\sqrt2}\left(1-\frac{\beta}{2}\right) + \frac{x^2+y^2-r^2}{2}\left(1-\frac{\beta}{2}\right) - \frac{x^2}{4}\left(1+\frac{\beta}{2}\right) - \frac{y^2}{4}\left(1-\frac{3\beta}{2}\right) - \frac{xy}{2}\left(1-\frac{\beta}{2}\right)$$

$$+ \frac{x(x^2+y^2-r^2)}{2\sqrt2}\left(1-\frac{\beta}{2}\right) + \frac{y(x^2+y^2-r^2)}{2\sqrt2}\left(1-\frac{3\beta}{2}\right) - \frac{x^3}{4\sqrt2}\left(1+\frac{\beta}{2}\right) - \frac{y^3}{4\sqrt2}\left(1-\frac{5\beta}{2}\right) - \frac{3x^2y}{4\sqrt2}\left(1-\frac{\beta}{2}\right)$$

$$- \frac{3xy^2}{4\sqrt2}\left(1-\frac{3\beta}{2}\right)$$

$$X_y = -\frac{\partial}{\partial x}\left\{\sum_{2,9,6,8}\Phi + \varphi\,\Sigma(r-l) + \frac{\varphi'}{2}\,\Sigma(r-l)^2 + \frac{\varphi''}{2.3}\,\Sigma(r-l)^3\right\}$$

$$r_9 = 1 - \frac{\beta}{2} + \frac{x}{\sqrt2}\left(1-\frac{\beta}{2}\right) - \frac{y}{\sqrt2}\left(1+\frac{\beta}{2}\right) + \frac{x^2+y^2-r^2}{2}\left(1+\frac{\beta}{2}\right) - \frac{x^2}{4}\left(1-\frac{\beta}{2}\right) - \frac{y^2}{4}\left(1+\frac{3\beta}{2}\right) + \frac{xy}{2}\left(1+\frac{\beta}{2}\right)$$

$$- \frac{x(x^2+y^2-r^2)}{2\sqrt2}\left(1+\frac{\beta}{2}\right) + \frac{y(x^2+y^2-r^2)}{2\sqrt2}\left(1+\frac{3\beta}{2}\right) + \frac{x^3}{4\sqrt2}\left(1-\frac{\beta}{2}\right) - \frac{y^3}{4\sqrt2}\left(1+\frac{5\beta}{2}\right) - \frac{3x^2y}{4\sqrt2}\left(1+\frac{\beta}{2}\right) +$$

$$+ \frac{3xy^2}{4\sqrt2}\left(1+\frac{3\beta}{2}\right)$$

$$r_{29} = 2\left\{1 - \frac{x\beta}{2\sqrt2} - \frac{y}{\sqrt2} + \frac{x^2+y^2-r^2}{2} - \frac{x^2}{4} - \frac{y^2}{4} + \frac{xy\beta}{4} - \frac{x(x^2+y^2-r^2)\beta}{4\sqrt2} + \right.$$

$$\left. + \frac{y(x^2+y^2-r^2)}{2\sqrt2} - \frac{x^3\beta}{8\sqrt2} - \frac{y^3}{4\sqrt2} - \frac{3x^2y}{4\sqrt2} + \frac{9xy^2\beta}{8\sqrt2}\right\}$$

$$\sum_{\substack{1\\2\\9\\10}} r = 4\left\{1 + \frac{xy\beta}{4} + \frac{x^2+y^2-r^2}{2} - \frac{x^2+y^2}{4}\right\} = \cancel{2\left\{1 + \frac{xy}{2}\left(1+\frac{\beta}{2}\right) + \frac{3x^2+y^2+2r^2}{4}\right\}}$$

Sumdzeie $\quad \operatorname{tg}\varphi = \beta \quad x = x_0 + y\beta$

1). $x = \frac{1}{\sqrt{2}}(1-\beta) =$, $y = -\frac{1}{\sqrt{2}}$, $z = 0$, $\boxed{r_0 = 1 - \frac{\beta}{\sqrt{2}}}$

9). $x = -\frac{1-\beta}{\sqrt{2}}$, $y = \frac{1}{\sqrt{2}}$, $z = 0$, $\boxed{r_0 = 1 - \frac{\beta}{\sqrt{2}}}$

2). $x = \frac{1+\beta}{\sqrt{2}}$, $y = \frac{1}{\sqrt{2}}$, $z = 0$, $\boxed{r_0 = 1 + \frac{\beta}{\sqrt{2}}}$

10). $x = -\frac{1+\beta}{\sqrt{2}}$, $y = -\frac{1}{\sqrt{2}}$, $z = 0$, $\boxed{r_0 = 1 + \frac{\beta}{\sqrt{2}}}$

6). $x = \frac{\beta}{\sqrt{2}}$, $y = \frac{1}{\sqrt{2}}$, $z = \frac{1}{\sqrt{2}}$

8). $x = \frac{\beta}{\sqrt{2}}$, $y = \frac{1}{\sqrt{2}}$, $z = -\frac{1}{\sqrt{2}}$

5). $x = -\frac{\beta}{\sqrt{2}}$, $y = -\frac{1}{\sqrt{2}}$, $z = \frac{1}{\sqrt{2}}$

7). $x = -\frac{\beta}{\sqrt{2}}$, $y = -\frac{1}{\sqrt{2}}$, $z = -\frac{1}{\sqrt{2}}$

$\sqrt{\left(-\frac{\beta}{\sqrt{2}}-x\right)^2 + \left(-\frac{1}{\sqrt{2}}-y\right)^2 + \left(\frac{1}{\sqrt{2}}-z\right)^2}$

3). $x = \frac{1}{\sqrt{2}}$, $y = 0$, $z = -\frac{1}{\sqrt{2}}$

4). $x = \frac{1}{\sqrt{2}}$, $y = 0$, $z = \frac{1}{\sqrt{2}}$

11). $x = -\frac{1}{\sqrt{2}}$, $y = 0$, $z = -\frac{1}{\sqrt{2}}$

12). $x = -\frac{1}{\sqrt{2}}$, $y = 0$, $z = \frac{1}{\sqrt{2}}$

$r_1^2 = (1-\beta) - (1-\beta)x\sqrt{2} + y\sqrt{2} + x^2y^2z^2$

$r_{10}^2 = (1+\beta) + (1+\beta)x\sqrt{2} + y\sqrt{2} +$

$r_9^2 = 1-\beta + (1-\beta)x\sqrt{2} - y\sqrt{2} +$ $\left\| \begin{matrix} \beta \cdots & -\beta \\ x & -x \\ y & -y \end{matrix} \right.$

$r_2^2 = 1+\beta - (1+\beta)x\sqrt{2} - y\sqrt{2} +$

$r_3^2 = 1 - x\sqrt{2} + \cancel{z\sqrt{2}} + x^2y^2z^2$

$r_4^2 = 1 - x\sqrt{2} - z\sqrt{2} + \cdots$

$+$

$+$

$r_5^2 = 1 + \beta x\sqrt{2} + y\sqrt{2} - 2\sqrt{2} + x^2y^2z^2$

$r_7^2 = 1 + \beta x\sqrt{2} + y\sqrt{2} + 2\sqrt{2} + \cdots$

$r_6^2 = 1 - \beta x\sqrt{2} - y\sqrt{2} - 2\sqrt{2} + \cdots \quad \cdots 2$ $\quad \begin{matrix} \beta = 0 & \\ x & \beta x \\ y & y+2 \end{matrix}$

$r_8^2 = 1 - \beta x\sqrt{2} - y\sqrt{2} + 2\sqrt{2} + \cdots$

$$U = A + B(x^2+y^2+z^2)$$

$$\frac{\int \alpha x^2 e^{-hax^2} dx}{\int e^{-hax^2} dx} = \frac{1}{2h}$$

$$\bar{U} = A + \cancel{B} \frac{\iiint B(x^2+y^2+z^2)\, e^{-B(x^2+y^2+z^2)}\, dx}{\iiint e^{-B(x^2+y^2+z^2)}\, dx} =$$

$$e^{-\frac{h}{l^2}\left\{x^2\left[\varphi(4-3\delta)+\varphi l(2+3\delta)+\varphi'' l^2 \delta\right]\right.}$$

$$\cancel{B = x^2\left[\left(\frac{4\varphi}{l} + 2\varphi'\right) - 3\right]} \quad e^{-\frac{h(y^2+z^2)}{l^2}\left[\varphi\left(4-\frac{\delta}{2}\right)+\varphi l\left(2+\frac{\delta}{2}\right)+\varphi'' l^2 \frac{\delta}{2}\right]}$$

$$\bar{U} = \cancel{12}\,\Phi(l) + 4\varphi l\delta + \underbrace{\frac{3}{2h}}_{=}$$

Przy jednolitym rozszerzeniu w wszystkich kierunkach o stosunek $1+\delta$:

$$\bar{U} = n\left\{\cancel{12}\, 6\left[\Phi(l) + l\varphi(l)\cdot\frac{\delta}{\cancel{2}}\right] + \frac{3}{2h}\right\} + \Pi(1+3\delta)$$

Jeśli ..., to φ zmienne

$$p.\,d\Pi.\,c = \frac{3n}{h} + \left[\cancel{12}\, 6 n l \varphi + 3\Pi\right]\delta$$

$$+\Pi = \left(\frac{4\varphi}{\sqrt{2}} - \frac{M}{2\sqrt{2}\,h}\right)\frac{1}{l^2}$$

$$n = \frac{1}{4\sqrt{2}\,l^3}$$

$$\rho c = 3n + \left(\cancel{12}\, 6 n l \varphi \cancel{+} \frac{12\varphi}{\sqrt{2}}\,\frac{1}{l^2} + \frac{3M}{2\sqrt{2}\, l^2}T\right)\frac{\delta}{dT}$$

$$= 3n - \left[\left(\frac{12}{\sqrt{2}} - \cancel{6}\right) - \frac{3MT}{2\sqrt{2}}\right]\frac{1}{l^2}\,\frac{\delta}{dT}$$

~~4 Punkte auf einen Cubus von $(2\sqrt{2})^3 = 16\sqrt{2}$~~

~~1 Punkt auf : $4\sqrt{2}\, l^3$~~

~~$n = \frac{1}{4\sqrt{2}\, l^3}$~~

Schmelzpunkt = Punkt wo Druckfestigkeit = 0

$$U = n\left\{6\left[\Phi(l) + \delta l \varphi + \frac{\delta^2}{2} l^2 \varphi'\right] + \frac{3}{2h}\right.$$

$$c = \frac{3n}{h} + \left[6 n l \varphi + 6\delta\, l^2 \varphi' n\right]$$

$$\frac{y^3}{2\sqrt2}\left(1-\frac{3\delta}{2}\right)+3\frac{\delta x^2}{4}+3\frac{y^2\delta}{4}-\frac{3x^2y\,\delta}{2\sqrt2}+\frac{3y(x^2+y^2+z^2)\delta}{2\sqrt2}-\frac{3x^2y\,\delta}{4\sqrt2}-\frac{3y^3\delta}{4\sqrt2}$$

$$\cancel{\frac{y^3}{2\sqrt2}}\left(\cancel{1-\tfrac{3}{2}\delta}\right)+\frac{3(x^2+z^2)\delta}{4}+\frac{3y^2\delta}{2}+\frac{3y(z^2+x^2)\delta}{4\sqrt2}\cancel{-\frac{3x^2y\,\delta}{4\sqrt2}}$$

$$\frac{3y^2}{\sqrt2}+3y\delta+\frac{3(x^2+z^2)y\,\delta}{4\sqrt2}$$

$$+\overset{2}{V}=2\varphi\left\{-\sqrt2+\frac{\delta}{2\sqrt2}+y\left(1+\frac{\delta}{4}\right)+\frac{x^2+z^2}{4\sqrt2}+\frac{3y^2}{2\sqrt2}-\frac{3(x^2+z^2)y\,\delta}{8\sqrt2}+\frac{3z^2\delta}{4\sqrt2}\right\}$$

$$+\varphi'\left\{-\frac{\delta}{\sqrt2}+y\left(2-\frac{\delta}{2}\right)-\frac{x^2+z^2}{2\sqrt2}+\frac{3y^2}{\sqrt2}+\frac{3(x^2+z^2)y\,\delta}{4\sqrt2}+\frac{3z^2\delta}{2\sqrt2}\right\}$$

$$+\varphi''\left\{\frac{y\,\delta}{2}-\frac{x^2+z^2}{2\sqrt2}-\frac{y^2}{\sqrt2}-\frac{z^2\delta}{2\sqrt2}\right\}$$

2
9
6
8

$$-l^2(1+\delta)\overline{V}=\frac{4\varphi}{\sqrt2}\left(1-\frac{\delta}{4}\right)+\frac{\varphi' l\,\delta}{\sqrt2}-\frac{1}{2\sqrt2\,h}\left(M+N'\delta\right)$$

$$\therefore\ -l^2\overline{V}=\frac{4\varphi}{\sqrt2}\left(1-\frac{5\delta}{4}\right)+\frac{\varphi' l\,\delta}{\sqrt2}-\frac{1}{2\sqrt2\,h}\left(M+(N'-M)\delta\right)$$

Dla równomiernego we wszystkich kierunkach rozszerzenia w stosunku $1+\delta$:

$$+l^2P=-\left\{\frac{4\varphi}{\sqrt2}-\frac{M}{2\sqrt2\,h}+\frac{4\varphi}{\sqrt2}\left(\frac{\delta}{2}-\frac{5\delta}{2}\right)+\frac{4\varphi' l\,\delta}{\sqrt2}-\frac{\delta}{2\sqrt2\,h}\left(N+2N'-2M\right)\right\}$$

$$=\left\{\frac{4\varphi}{\sqrt2}-\frac{M}{2\sqrt2\,h}+\delta\left\{-\frac{8\varphi}{\sqrt2}+\frac{4\varphi' l}{\sqrt2}-\frac{N+2N'-2M}{2\sqrt2\,h}\right\}\right\}(1-6\delta)\,\Pi\, l^2$$

$$P=-\frac{1}{(1+\delta)^2 l^2}\left\{\frac{4\varphi}{\sqrt2}-\frac{M}{2\sqrt2\,h}+\delta\left[\frac{4\varphi' l}{\sqrt2}-\frac{N'}{2\sqrt2\,h}\right]\right\}-(1-4\delta)\,\Pi$$

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$$y\left[\sum r_0 - \frac{y y_k}{r_0} \cdot\right\} - \sum\left\{y_k r_0 - \frac{y y_k^2}{r_0} + \frac{x^2+y^2+z^2}{2}\frac{y_k}{r_0} - \frac{y^2}{2}\frac{y_k^3}{r_0^3} - \frac{z^2}{2}\frac{y_k z_k^2}{r_0^3} - \frac{x^2}{2}\frac{y_k x_k^2}{r_0^3}\right\}$$

$$y(4+\delta) + y^2\frac{4}{\sqrt2}\left(1-\frac{\delta}{4}\right) + \frac{4+\delta}{\sqrt2} + y\,2\left(1-\frac{\delta}{4}\right) + \frac{x^2+y^2+z^2}{2\sqrt2}4\left(1-\frac{\delta}{4}\right) - \frac{y^2}{2}\frac{4}{2\sqrt2}\left(1-\frac{3\delta}{4}\right)$$

$$-\frac{z^2}{2}\frac{2}{2\sqrt2} - \frac{x^2}{2}\frac{2(1+2\delta)}{\sqrt2\cdot 2}\underbrace{\left(1-\frac{3\delta}{2}\right)}_{\left(1+\frac{\delta}{2}\right)}$$

$$= \frac{4+\delta}{\sqrt2} + y\left(6+\frac{\delta}{2}\right) + \frac{x^2}{2\sqrt2}\left(4-\delta-1-\frac{\delta}{2}\right) + \frac{y^2}{2\sqrt2}\left(8-2\delta+4-\delta-2+\frac{3\delta}{2}\right)$$

$$+\frac{z^2}{2\sqrt2}(4-\delta-1)$$

$$= \frac{4+\delta}{\sqrt2} + y\left(6+\frac{\delta}{2}\right) + \frac{x^2}{2\sqrt2}\left(3-\frac{3\delta}{2}\right) + \frac{y^2}{2\sqrt2}\left(10-\frac{3\delta}{2}\right) + \frac{z^2}{2\sqrt2}(3-\delta)$$

$$\frac{\partial}{\partial y}\sum(r_k-l)^2 = \frac{\partial}{\partial y}\left(\sum r_k^2 - 2l\frac{\partial}{\partial y}\sum r_k\right)$$

$$\frac{\partial}{\partial y}4\left(1+\delta+y\sqrt2+x^2+y^2+z^2\right) = 4\left(\sqrt2+2y\right).$$

$$-\frac{x^3}{2\sqrt2}\left(1+\frac{3\delta}{2}\right) + \frac{3x^2\delta}{4} + \frac{3y^2\delta}{4} - \cancel{\frac{3xy\delta}{2}} + \frac{3\delta}{4}\left\{-\frac{x(x^2+y^2+z^2)}{2\sqrt2} + \frac{x^3}{4\sqrt2} + \right.$$

$$\left. + \frac{xy^2}{4\sqrt2} - \cancel{\frac{x^2y}{2\sqrt2}} + \frac{y(x^2+y^2+z^2)}{2\sqrt2}\right\}$$

$$= -\frac{x^3}{2\sqrt2}\left(1+\frac{3\delta}{2}\right) + \frac{3x^2\delta}{4} + \frac{3y^2\delta}{4} - \frac{3\delta x(x^2+y^2+z^2)}{2\sqrt2} + \frac{3\delta x^3}{4\sqrt2} + \cancel{\frac{3\delta xy^2}{4\sqrt2}}$$

$$= -\frac{x^3}{2\sqrt2}\left[1+\cancel{\frac{3\delta}{2}} - 3\delta + \cancel{\frac{3\delta}{2}}\right] + \frac{3x^2\delta}{4} - \frac{3y^2\delta}{4} - \frac{3\delta xy^2}{4\sqrt2} - \frac{3\delta x z^2}{2\sqrt2}$$

$$-\frac{z^3}{2\sqrt2} \quad \cancel{+\frac{3z^2\delta}{4} - \frac{3y^2\delta}{4} - \frac{3\delta zy^2}{4\sqrt2} - 3\delta} \quad -\frac{3y\delta}{2} - \frac{3xy\delta}{2\sqrt2}$$

$$-\cancel{\frac{x^3}{2\sqrt2}\left(1+\frac{3\delta}{2}\right)} + \frac{y^3}{2\sqrt2}\left(1-\frac{3\delta}{2}\right) + \frac{3\delta}{2}\left\{\frac{x^2}{2}\cancel{(1+\delta)} + \frac{y^2}{2}\cancel{(1-\delta)} - \cancel{xy}\right.$$

$$-\cancel{\frac{x(x^2+y^2+z^2)}{\sqrt2}} + \cancel{\frac{x^3}{2\sqrt2}(1+\delta)} + \cancel{\frac{xy^2}{2\sqrt2}(1-\delta)} - \frac{x^2y}{\sqrt2} + y\frac{(x^2+y^2+z^2)}{\sqrt2}\cancel{(1-\delta)}$$

$$\left. -\frac{x^2y}{2\sqrt2} - \frac{y^3}{2\sqrt2}\cancel{(1-2\delta)} + \cancel{\frac{xy^2}{\sqrt2}(1-\delta)}\right\}$$

$$2\left(1-\frac{3\delta}{2}\right)+2 = 4$$

$$\frac{3y^2}{2\sqrt2}\left(1-\frac{3\delta}{2}\right) + \frac{3\delta}{2}\left(y - \frac{x^2}{\sqrt2} + \frac{x^2+3y^2+z^2}{\sqrt2} - \frac{x^2}{2\sqrt2} - \frac{3y^2}{2\sqrt2}\right) + \frac{3x^2}{2\sqrt2}\left(1+\frac{\delta}{2}\right)$$

$$\left\{\begin{array}{l} \frac{3y^2}{2\sqrt2} - \cancel{\frac{9y^2\delta}{4\sqrt2}} + \frac{3\delta}{2}\left(y - \frac{x^2}{2\sqrt2} + \cancel{\frac{3y^2}{2\sqrt2}} + \frac{z^2}{\sqrt2}\right) \\ \frac{3y^2}{2\sqrt2} \qquad + 3\frac{\delta}{2}\left(y - \frac{z^2}{2\sqrt2} + \frac{x^2}{\sqrt2}\right) \end{array}\right.$$

$$-\frac{1}{4}\ \frac{r}{2}$$

$$+\frac{3(x^2+z^2)}{2\sqrt2}$$

$$2\left|\frac{3y^2}{\sqrt2} + 3y\delta + \frac{3}{2\sqrt2}(x^2 + z^2)\delta\right.$$

$$4y + \frac{4}{\sqrt2}$$

$$\cancel{\frac{x}{2\sqrt2}} + \cancel{y} + \frac{y\delta}{4}$$

$$+\frac{3}{2} + \frac{\delta}{2\sqrt2} + \cancel{y} + \frac{y\delta}{4}$$

$$-\cancel{y} - \frac{4}{2}$$

$$\underbrace{\left(\varphi - \varphi' + \frac{\varphi''}{2}\right)}_{B}\frac{\partial z}{\partial x} + \underbrace{(\varphi' - \varphi'')}_{2C} r\frac{\partial z}{\partial x} + \underbrace{\frac{\varphi''}{2}}_{3D} r^2\frac{\partial z}{\partial x}$$

$$\varphi\frac{\partial z}{\partial x} - \varphi'\left(\frac{\partial z}{\partial x} - r\frac{\partial z}{\partial x}\right) + \varphi''\left(\frac{1}{2}\frac{\partial z}{\partial x} - r\frac{\partial z}{\partial x} + \frac{1}{2}r^2\frac{\partial z}{\partial x}\right)$$

$$\frac{4-\delta}{2\sqrt2} + \frac{4+\delta}{2\sqrt2} - \frac{4}{\sqrt2} \qquad y + 3y - 4y \qquad \frac{y\delta}{2} \qquad -\frac{x^2}{4\sqrt2} + \frac{3x^2}{4\sqrt2} \qquad -\frac{3y^2}{2\sqrt2} + \frac{5y^2}{2\sqrt2}$$

$$\frac{x^2}{2\sqrt2} \qquad +\frac{2y^2}{2\sqrt2}$$

$$\frac{3x^2\delta}{8\sqrt2} - \frac{9x^2\delta}{8\sqrt2} \qquad \frac{3y^2\delta}{8\sqrt2} - \frac{3y^2\delta}{8\sqrt2} \qquad \frac{3}{4} - \frac{1}{4} \qquad \frac{x^2\delta}{2\sqrt2}$$

$$\frac{3}{2\sqrt2}\ \frac{1}{4} + \frac{9}{8\sqrt2} - \cancel{\frac{3x^2\delta}{8\sqrt2}}$$

$$E\frac{\partial u}{\partial x} = X_x - \mu(Y_y + Z_z)$$

$$E\frac{\partial v}{\partial y} = Y_y - \mu(X_x + Z_z)$$

$$E\frac{\partial w}{\partial z} = Z_z - \mu(X_x + Y_y)$$

$$X_x = \frac{E\begin{vmatrix} \frac{\partial u}{\partial x} & -\mu & -\mu \\ \frac{\partial v}{\partial y} & 1 & -\mu \\ \frac{\partial w}{\partial z} & -\mu & 1 \end{vmatrix}}{\begin{vmatrix} 1 & -\mu & -\mu \\ -\mu & 1 & -\mu \\ -\mu & -\mu & 1 \end{vmatrix}} = E\,\frac{\frac{\partial u}{\partial x}(1-\mu^2) - \frac{\partial v}{\partial y}(-\mu-\mu^2) + \frac{\partial w}{\partial z}(\mu+\mu^2)}{1-\mu^2 + \mu(-\mu-\mu^2) \mp \mu(\mu+\mu^2)}$$

$$= E\,\frac{\frac{\partial u}{\partial x}(1-\mu) + \mu\left(\frac{\partial v}{\partial y} + \frac{\partial w}{\partial z}\right)}{1-\mu-2\mu^2} \qquad = (1+\mu)(1-2\mu)$$

$= 2:1$

$$\frac{\mu}{1+\mu}\;\frac{2(1-\mu)}{1-2\mu} : \frac{\mu}{(1+\mu)(1-2\mu)} = 2(1-\mu) : 1$$

$$= E\left[\frac{\partial u}{\partial x}\left(\frac{1-\mu}{1-3\mu}\right) + \frac{\mu}{1-3\mu}\left(\frac{\partial v}{\partial y} + \frac{\partial w}{\partial z}\right)\right]$$

$-\frac{3y^2}{V_2} + \frac{3y^2}{2V_2}$

$\frac{9}{4}$ $\frac{15}{8}$ $\frac{3}{4} - \frac{3}{8}$

$$= E\left[\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z}\right)\frac{\mu}{1-3\mu} + \frac{\partial u}{\partial x}\left(\frac{1-2\mu}{1-3\mu}\right)\right]$$

$$\frac{X_x}{E} = \frac{\partial u}{\partial x}\,\frac{1-\mu}{(1-2\mu)(1+\mu)} + \left(\frac{\partial v}{\partial y} + \frac{\partial w}{\partial z}\right)\frac{\mu}{(1-2\mu)(1+\mu)}$$

$$\begin{vmatrix} 1+\mu & 0 & -\mu \\ -(\mu+1) & 1+\mu & -\mu \\ 0 & -(\mu+1) & 1 \end{vmatrix} \qquad \begin{vmatrix} 0 & 1+\mu & -2\mu \\ -(\mu+1) & 1+\mu & -\mu \\ 0 & -(\mu+1) & 1 \end{vmatrix}$$

$-(1+\mu)\left[1+\mu+2\mu+2\mu^2\right]$

$1-\mu-2\mu^2$

$$= \frac{E\mu}{(1+\mu)(1-2\mu)}\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z}\right) + \frac{E\mu}{1+\mu}\,\frac{\partial u}{\partial x}$$

$$= \frac{E\mu(1-\mu)2}{(1+\mu)(1-2\mu)}\,\frac{\partial u}{\partial x} + \frac{E\mu}{(1+\mu)(1-2\mu)}\left(\frac{\partial v}{\partial y} + \frac{\partial w}{\partial z}\right)$$

$\underset{15740}{\leq}$ $$Y = \sum\left(B\frac{\partial r}{\partial y} + 2Cr\frac{\partial r}{\partial y} + 3Dr^2\frac{\partial r}{\partial y}\right)$$

$$= B\sum(y-y_0)\left[\frac{1}{r_0} + \dots\right.$$

$$= B\Big\{y\sum\frac{1}{r_0} + y^2\sum\frac{y_\kappa}{r_0^3} + yz\cancel{\sum\frac{z_\kappa}{r_0^3}} + yx\cancel{\sum\frac{x_\kappa}{r_0^2}} - \sum\frac{y_\kappa}{r_0} - y\sum\frac{y_\kappa^2}{r_0^3} - z\cancel{\sum\frac{y_\kappa z_\kappa}{r_0^2}}$$

$$-x\cancel{\sum\frac{y_\kappa x_\kappa}{r_0^2}} + \frac{x^2+y^2+z^2}{2}\sum\frac{y_\kappa}{r_0^3} - \frac{y^2}{2}\sum\frac{3y_\kappa^3}{r_0^5} - \frac{z^2}{2}\sum\frac{3y_\kappa z_\kappa^2}{r_0^5} - \frac{x^2}{2}\sum\frac{3y_\kappa x_\kappa^2}{r_0^5}\Big\}$$

$$+ 8Cy - 2C\sum y_\kappa + D\dots\dots$$

$$-l^2\overline{X} = \frac{4\varphi}{\sqrt{2}}\left(1+\frac{\delta}{2}\right) + \frac{2\varphi' l\delta}{\sqrt{2}} - \frac{1}{2\sqrt{2}\,\cancel{h}}\left(M + N\delta\right)$$

$$-l^2\overline{X}_0 = \frac{4\varphi}{\sqrt{2}} - \frac{M}{2\sqrt{2}\,\cancel{h}}$$

w tem l oznacza m.p. odstępy dla $T=0$, $\mu=0$

w takim razie tutaj ostrzeżenie przy temp. T w takich odstępach utrzymane

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Dwie drogi: albo ograniczamy się na tylko (3) odcinkach siły: φ, φ', φ''.

~~W takim razie wszystek~~

Wtedy siła $X_0 = \alpha T \dfrac{E}{1-2\mu}$ (α = lin. Ausd. Coeff.)

$\cancel{X = \frac{E}{3(1-2\mu)}\left(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}\right)}$

$\cancel{X = E\left[\frac{\partial u}{\partial x} - \mu\left(\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}\right)\right]}$

$$\Sigma \frac{1}{r_0} = 2\left(1-\frac{\delta}{2}\right)+2 = 4-\delta$$

$$\Sigma \frac{1}{r_0^3} = 2\left(1-\frac{3\delta}{2}\right)+2 = 4-3\delta$$

$$= B\Big\{ y\left(4-\cancel{\delta}\right) + y^2\frac{(4-3\delta)}{\sqrt{2}} + \frac{(4-\delta)}{\sqrt{2}} - y.2\left(1-\frac{3\delta}{4}\right) + \frac{x^2+y^2+z^2}{2}\frac{(4-3\delta)}{\sqrt{2}}$$

$$\cancel{\tfrac{3y}{2}} + \frac{3y^2}{2}\frac{(4-5\delta)}{2\sqrt{2}} + \frac{3z^2}{2}\frac{2(\ldots)}{2\sqrt{2}} + \frac{3x^2(1+2\delta)}{2\sqrt{2}.2}\left(1-\frac{5\delta}{2}\right)$$

$$1+\frac{3\delta}{2}$$

$$+\,8Cy + 2C\frac{4}{\sqrt{2}} + D\ldots$$

$$-6+\frac{3\delta}{2}$$

$$\cancel{-10+\frac{21\delta}{2}}\quad \cancel{\left(2-\frac{9}{2}\delta\right)}$$

$$\cancel{\frac{12}{2}+\frac{15}{4}\delta\,2}$$

$$= B\left\{ \frac{4-\delta}{\sqrt{2}} + y\left(2+\frac{\delta}{2}\right) + \frac{y^2}{2\sqrt{2}}\left(-4+3\delta - 8+6\delta+6+\frac{15\delta}{2}\right) + \frac{x^2}{\sqrt{2}}\left(-4+3\delta+\frac{3}{2}-\frac{3}{2}\delta\right)\right.$$

$$\cancel{-\frac{5}{2}+\frac{9\delta}{4}}$$

$$+\frac{z^2}{2\sqrt{2}}\left(-4+3\delta+3\ldots\right)$$

$$-1+\frac{3}{2}\delta$$

$$\cancel{(1-3\delta)}\quad \cancel{\left(1-\frac{12\delta}{4}\right)}\quad (-1+3\delta)$$

$$\overline{X} = -\frac{1}{2\sqrt{2}\,h\,l}\left\{\frac{3\varphi\left(1-\frac{3\delta}{2}\right)-3\varphi' l\left(1-\frac{3\delta}{2}\right)-\varphi'' l^2(1+3\delta)}{\varphi(4-3\delta)+\varphi' l(2+3\delta)+\varphi'' l^2\delta}\right. +$$

$$\left. + \frac{\varphi\left(1+\frac{5\delta}{2}\right)-\varphi' l\left(1+\frac{5\delta}{2}\right)-\varphi'' l^2}{\varphi\left(4-\frac{\delta}{2}\right)+\varphi' l\left(2+\frac{\delta}{2}\right)+\varphi'' l^2\frac{\delta}{2}}\right\} + 4\varphi\left\{\frac{1}{\sqrt{2}}+\frac{\delta}{2\sqrt{2}}\right\} + \frac{2\varphi' l\delta}{\sqrt{2}}$$

$$\{\ \} = \frac{3\varphi - 3\varphi' l - \varphi'' l^2 \cancel{+} \frac{9\varphi\delta}{\cancel{\varphi}2} + 9\frac{\varphi' l\delta}{2} - 3\varphi'' l^2\delta}{(4\varphi+2l\varphi')\left(1+\frac{-3\varphi+3\varphi' l+\varphi'' l^2}{4\varphi+2l\varphi'}\delta\right)} + \cdots$$

$$\frac{A+\alpha}{B+\beta} = \frac{A\left(1+\frac{\alpha}{A}\right)}{D\left(1+\frac{\beta}{D}\right)} = \frac{A}{B}\left(1+\frac{\alpha}{A}\right)\left(1-\frac{\beta}{B}\right) = \frac{A}{D}\left(1+\frac{\alpha}{A}-\frac{\beta}{B}\right)$$

$$= \frac{A}{B} + \cancel{\frac{\alpha}{B} - \frac{\beta A}{}} \frac{\alpha B - \beta A}{B^2}$$

$$\{\ \} = \frac{3\varphi - 3\varphi' l - \varphi' l^2 + \varphi - \varphi' l - \varphi'' l^2}{2(2\varphi + l\varphi')} + \delta(4\varphi+2l\varphi')\left[-\frac{9\varphi}{2}+\frac{9\varphi' l}{2} - 3\varphi'' l^2 + \frac{5\varphi}{2} - \frac{5\varphi' l}{2}\ \cancel{\ldots}\right]$$

$$\frac{-(-3\varphi+3\varphi' l+\varphi'' l^2)\cancel{(\ldots)}(3\varphi-3\varphi' l-\varphi'' l^2) - \left(-\frac{\varphi}{2}+\frac{\varphi' l}{2}+\frac{\varphi'' l^2}{2}\right)(\varphi-\varphi' l-\varphi'' l^2)}{(4\varphi+2l\varphi')^2}$$

$$= \frac{4\varphi - 4\varphi' l - 2\varphi'' l^2}{4\varphi + 2l\varphi'} + \delta\frac{(4\varphi+2l\varphi')(-2\varphi+2\varphi' l-3\varphi'' l^2) + (3\varphi-3\varphi' l-\varphi'' l^2)^2 + \frac{1}{2}(\varphi-\varphi' l-\varphi'' l^2)^2}{(4\varphi+2l\varphi')^2}$$

$$= \underbrace{\frac{2\varphi - 2\varphi' l - \varphi'' l^2}{2\varphi + \varphi' l}}_{M} + \delta\,\underbrace{\frac{3\varphi^2 - 30\varphi\varphi' + 27\varphi'^2 - 38\varphi\varphi'' + 3\varphi''^2 + 2\varphi'\varphi''}{8(2\varphi+\varphi' l)^2}}_{N}$$

$$6x^2\delta + 3(y^2+z^2)\delta + 12 - 36\left(1-\frac{\delta}{3}\right) - 3x^2(4-3\delta) - 3(y^2+z^2)\left(4-\frac{\delta}{2}\right)$$

$$+ 36\left(1+\frac{2\delta}{3}\right) + 36(x^2+y^2+z^2)$$

$$= 12(1+\delta) + x^2(24+15\delta) + (y^2+z^2)\left(24+\frac{9\delta}{2}\right)$$

$$-2 + 6\left(1+\frac{\delta}{3}\right) + x^2\left(2-\frac{3\delta}{2}\right) + (y^2+z^2)\left(2-\frac{\delta}{4}\right)$$

$$-6\left(1+\frac{2\delta}{3}\right) - 6x^2 - 6(y^2+z^2)$$

$$+2(1+\delta) + 3(x^2+y^2+z^2)\left(1+\frac{\delta}{3}\right) + x^2\left(1+\frac{3\delta}{2}\right) + (y^2+z^2)\left(1-\frac{\delta}{4}\right)$$

$$x^2\left(\cancel{2-6}+3+1-\cancel{\frac{3\delta}{2}}+\delta+\cancel{\frac{3\delta}{2}}\right) + (y^2+z^2)\left(\cancel{2}-\frac{\delta}{4}-\cancel{6}+\cancel{3}+\delta+\cancel{1}-\frac{\delta}{4}\right)$$

$$\underline{+X} = 4\varphi\left[\frac{1}{\sqrt{2}}+\frac{\delta}{2\sqrt{2}}\right] + \frac{2\varphi l\delta}{\sqrt{2}} - \iiint$$

$$\int_{-\infty}^{+\infty} e^{-\alpha x^2}\,dx = \sqrt{\frac{\pi}{\alpha}}$$

$$\int x^2 e^{-\alpha x^2}\,dx = \frac{1}{2}\sqrt{\frac{\pi}{\alpha^3}}$$

$$\frac{\int x^2 e^{-\alpha x^2}\,dx}{\int e^{-\alpha x^2}\,dx} = \frac{1}{2\alpha} \qquad \frac{\int (y^2+z^2)\, e^{-\alpha(y^2+z^2)}\,dy\,dz}{\int e^{-\alpha(y^2+z^2)}\,dy\,dz} = \frac{1}{\alpha}$$

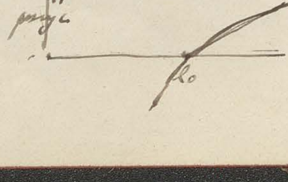
$$-8\varphi^2 - 4\varphi\varphi + 8\varphi\varphi' + 4\varphi'^2 - 12\varphi\varphi'' - 6\varphi'\varphi''$$

$$+9\varphi^2 - 9\varphi\varphi' - 9\varphi\varphi' + 9\varphi'^2 - 3\varphi\varphi'' + 3\varphi'\varphi'' - 3\varphi\varphi'' + 3\varphi\varphi'' + \varphi''^2$$

$$2(\varphi^2 - 14\varphi\varphi' + 13\varphi'^2 - 18\varphi\varphi'' + \varphi''^2)$$

$$+\varphi^2 - 2\varphi\varphi' + \varphi'^2 - 2\varphi\varphi'' + \varphi''^2 + 2\varphi'\varphi''$$

$\varphi' > 0 \quad \varphi'' < 0$



$$4\varphi'\left\{-\frac{1+\frac{\delta}{2}}{\sqrt2}+\frac{x}{2}\left(1-\frac{3\delta}{2}\right)+\frac{5x^2}{8\sqrt2}\left(1-\frac{23\delta}{10}\right)+\frac{y^2+z^2}{8\sqrt2}\left(1+\frac{5\delta}{2}\right)+\frac{1+\delta}{\sqrt2}\cancel{+x}\right\}$$

$$=-2\varphi\left\{+\frac{\delta}{4\sqrt2}-\frac{x}{4}\left(1+\frac{3\delta}{2}\right)+\frac{5x^2}{4\sqrt2}\left(1-\frac{23\delta}{10}\right)+\frac{y^2+z^2}{4\sqrt2}-\right\}$$

$$2\varphi''\left\{-\frac{1+\frac{\delta}{2}}{\sqrt2}+\frac{x}{2}\left(1-\frac{3\delta}{2}\right)+\frac{5x^2}{8\sqrt2}\left(1-\frac{23\delta}{10}\right)+\frac{x^2+y^2+z^2}{8\sqrt2}\left(1+\frac{5\delta}{2}\right)-2x+\frac{2(1+\delta)}{\sqrt2}\right.$$

$$\left.+\frac{1+\frac{3\delta}{2}}{\sqrt2}+\frac{3x}{2}\left(1+\frac{5\delta}{6}\right)+\frac{x^2}{8\sqrt2}\left(7-\frac{\delta}{2}\right)-\frac{y^2+z^2}{8\sqrt2}\left(3+\frac{5\delta}{2}\right)\right\}$$

$$\cancel{\frac{1+\frac{3\delta}{2}}{\sqrt2}\ \frac{1}{\sqrt2}\left(-1-\frac{\delta}{2}+1+\delta\cdots\right.}$$

$$x\left(\frac{1}{2}-\frac{3\delta}{4}-2+\frac{3}{2}+\frac{5\delta}{4}\right)$$

$$\cancel{\frac{1}{\sqrt2}\left(-1-\frac{\delta}{2}+2+2\delta-1-\frac{3\delta}{2}\right.}\qquad\cancel{x(3-2\delta)}\qquad\frac{x\delta}{2}$$

$$\frac{x^2}{8\sqrt2}\left(5+7-\frac{23}{2}\delta-\frac{\delta}{2}+1+\frac{5\delta}{2}-3-\frac{5\delta}{2}\right)$$

$$\frac{y^2+z^2}{8\sqrt2}\left(1+\frac{5\delta}{2}-3-\frac{5\delta}{2}\right)=\frac{y^2+z^2}{8\sqrt2}(-2)$$

$$4(x^2+y^2+z^2)+\delta(6x^2+y^2+z^2)-12+24\left(1+\frac{\delta}{3}\right)+2x^2(4-3\delta)+4(y^2+z^2)\left(8-\frac{\delta}{4}\right)$$

$$=12+8\delta+\cancel{4}(12(x^2+y^2+z^2)$$

$$6x^2\delta+3(y^2+z^2)\delta+12-12\delta+\cancel{12(x^2+y^2+z^2)}+9x^2\delta+\frac{3}{2}(y^2+z^2)\delta$$

$$+4(x^2+y^2+z^2)+6x^2\delta+4(y^2+z^2)\delta$$

$$=\cancel{12(1-\delta)+33x^2\delta}$$

$$=12(1-\delta)-8(x^2+y^2+z^2)+21x^2\delta+\frac{11}{2}(y^2+z^2)\delta$$

$$\frac{\partial r}{\partial x} = 2\left(1+\frac{\delta}{2}\right)\Big[-\frac{1}{\sqrt{2}} + \frac{x}{1+\delta} - \frac{x}{2} + \frac{y}{2(1+\delta)} + \frac{3x^2}{2\sqrt{2}(1+\delta)} + \frac{y^2+z^2}{2\sqrt{2}(1+\delta)}$$

$$-\frac{2xy}{2\sqrt{2}(1+\delta)^2} + \frac{2\sqrt{2}}{16}\left(-3x^2 + \frac{6xy}{1+\delta} - \frac{3y^2}{(1+\delta)^2}\right)$$

$$\sum_{1234} \frac{\partial r}{\partial x} = 4\left(1+\frac{\delta}{2}\right)\Big[-\frac{1}{\sqrt{2}} + \frac{x}{1+\delta} - \frac{x}{2} + \frac{3x^2}{2\sqrt{2}(1+\delta)} - \frac{3x^2}{4\sqrt{2}} + \frac{y^2+z^2}{2\sqrt{2}(1+\delta)} - \frac{3}{8\sqrt{2}}\frac{(y^2+z^2)}{(1+\delta)^2}\Big]$$

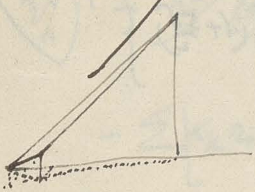
$$=$$

$$-\frac{1}{\sqrt{2}} + \frac{x(2-1-\delta)}{2(1+\delta)} + \frac{3x^2\,(2-(1+\delta))}{4\sqrt{2}(1+\delta)} + \frac{y^2+z^2}{8\sqrt{2}}\left(\frac{4}{1+\delta} - \frac{3}{(1+\delta)^2}\right)\Big]$$

$$-\frac{1}{\sqrt{2}} + \frac{x}{2}(1-2\delta) + \frac{3x^2(1-2\delta)}{4\sqrt{2}} + \frac{y^2+z^2}{8\sqrt{2}}(4-4\delta-3+6\delta)$$

$$1+2\delta$$

$$\sum \frac{\partial r}{\partial x} = 4\left\{-\frac{1+\frac{\delta}{2}}{\sqrt{2}} + \frac{x}{2}\left(1-\frac{3\delta}{2}\right) + \frac{3x^2}{4\sqrt{2}}\left(1-\frac{3\delta}{2}\right) + \frac{y^2+z^2}{8\sqrt{2}}\left(1+\frac{5\delta}{2}\right)\right\}$$



$$\frac{\partial r}{\partial x} = \frac{\frac{1+\delta}{\sqrt{2}} - x}{\sqrt{\ldots}}$$

$$\sum X = \mathcal{B} \sum \frac{\partial r}{\partial x} + C \sum 2r\frac{\partial r}{\partial x} + D \sum 3r^2 \frac{\partial r}{\partial x}$$

$$\frac{\varphi''}{2} \qquad = -\frac{\varphi''}{2} \qquad \frac{\varphi''}{2\cdot 3}$$

$$= \varphi''\left\{\frac{\sum \frac{\partial r}{\partial x}}{2} - \frac{\sum 2r\frac{\partial r}{\partial x}}{2} + \frac{\sum 3r^2\frac{\partial r}{\partial x}}{2\cdot 3}\right. \quad -$$

$$2_B \quad - \quad 4_C \quad + \quad 2_D$$

$-1-\frac{1}{2}+\frac{1}{4}$

$\cancel{\sum x_k = 4\frac{(1+\delta)}{\sqrt{2}}}$

$$12D\Big\{-\frac{(1+\frac{3\delta}{2})}{\sqrt2} + x\Big[\frac{(1+\frac{3\delta}{2})}{2}+1+\frac{\delta}{2}\Big] - x^2\frac{(1+\frac{\delta}{2})}{\sqrt2} - \frac{x^2+y^2+z^2}{2\sqrt2}(1+\tfrac{\delta}{2}) +$$
$$+\frac{x^2}{4\sqrt2}(1+\tfrac{3\delta}{2}) + \frac{(y^2+z^2)}{8\sqrt2}(1-\tfrac{\delta}{2})\Big\}$$

$$=12D\Big\{-\frac{1+\frac{3\delta}{2}}{\sqrt2} + \Big(\frac{3}{2}+\frac{5\delta}{4}\Big)x + (x^2+y^2+z^2)\Big[\frac{1}{8\sqrt2}(1-\tfrac{\delta}{2}) - \frac{1}{2\sqrt2}(1+\tfrac{\delta}{2})\Big] +$$
$$+\frac{x^2}{8\sqrt2}\underbrace{\Big[2(1+3\tfrac{\delta}{2}) - (1-\tfrac{\delta}{2}) - 8(1+\tfrac{\delta}{2})\Big]}_{(-7-\frac{\delta}{2})}$$

$\frac{1}{8\sqrt2}(1-\frac{\delta}{2}-4-2\delta)$

$(-3-\frac{5\delta}{2})$

$$=12D\Big\{-\frac{1+\frac{3\delta}{2}}{\sqrt2} + \frac{3x}{2}\Big(1+\frac{5\delta}{6}\Big) + \frac{x^2}{8\sqrt2}\Big(-7-\frac{\delta}{2}\Big) - \frac{x^2+y^2+z^2}{8\sqrt2}\Big(3+\frac{5\delta}{2}\Big)\Big\}$$

$$+8C\Big\{x-\frac{1+\delta}{\sqrt2}\Big\}$$

$$+4B\Big\{-\frac{1+\frac{\delta}{2}}{\sqrt2} + \frac{x}{2}\Big(1-\frac{3}{2}\delta\Big) + \frac{5x^2}{8\sqrt2}\Big(1-\frac{23\delta}{10}\Big) + \frac{x^2+y^2+z^2}{8\sqrt2}\Big(1+\frac{5\delta}{2}\Big)\Big\}$$

stimmt

$$-\frac{1}{\sqrt2}(1+\tfrac{\delta}{2}) + x\Big(1-\frac{\delta}{2}-\frac{1}{2}-\frac{\delta}{4}\Big) + \frac{3x^2}{2\sqrt2}(1-\tfrac{\delta}{2}) - \frac{3x^2}{4\sqrt2}(1+\tfrac{\delta}{2})$$

$\frac{1}{2}-\frac{3\delta}{4}$

$-\frac{3\delta x^2}{4\sqrt2}-\frac{3\delta x^2}{8\sqrt2}$

$-\frac{25}{2}+\frac{5}{2}$

$=\frac{30}{2}$

$$\frac{1}{2\sqrt2}(1-\tfrac{\delta}{2}) - \frac{3}{8\sqrt2}(1-\tfrac{3\delta}{2}) = \frac{1}{8\sqrt2}\Big(4-2\delta-3+9\frac{\delta}{2}\Big)$$

$1+\frac{5\delta}{2}$

$\frac{15}{8\sqrt2}$

$$4\frac{1}{\sqrt2}(1-\tfrac{7\delta}{2}) + \frac{3}{8\sqrt2}(1-\tfrac{3\delta}{2}) = \cancel{(1-\tfrac{2\delta}{2})}\cancel{(\tfrac{3}{8}+\tfrac{1}{4})}$$

$-\frac{23}{10\,2}\cancel{5}+\frac{5}{2}$

$=$

$$=\frac{1}{8\sqrt2}\Big(2-7\delta+3\overset{8}{-}9\frac{\delta}{2}\Big) = 5-\frac{23}{2}$$

$$B\left\{4x\left(1-\frac{\delta}{2}\right)+4x^2\left(1-\frac{3}{2}\delta\right)\frac{(1+\delta)}{\sqrt2}-\frac{4(1+\delta)}{\sqrt2}\left(1-\frac{\delta}{2}\right)-x\,\frac{4\overset{1+\frac{\delta}{2}}{(1+2\delta)}}{2}\left(1-\frac{3\delta}{2}\right)\right.$$

$$+\frac{x^2+y^2+z^2}{2}\,\frac{4(1+\delta)}{\sqrt2}\left(1-\frac{3\delta}{2}\right)-\frac{3x^2}{2}\,\frac{4(1+3\delta)}{2\sqrt2}\left(1-\frac{5\delta}{2}\right)$$

$$\left.-\frac{3(y^2+z^2)}{2}\frac{1+\delta}{\sqrt2}\left(1-\frac{5\delta}{2}\right)\right\}+8C\left\{x-\frac{(1+\delta)}{\sqrt2}\right\}$$

$$\cancel{=4B\left\{x\left(1-\frac{\delta}{2}-\frac{1}{2}-\frac{\delta}{4}\right)-\frac{\left(1+\frac{\delta}{2}\right)}{\sqrt2}+\frac{x^2}{\sqrt2}\left(1-\frac{3}{4}\left(1+\frac{\delta}{2}\right)\right)\right.}$$

$$=4B\left\{-\frac{\left(1+\frac{\delta}{2}\right)}{\sqrt2}+x\,\frac{\left(1-\frac{3}{2}\delta\right)}{2}+\frac{x^2}{\sqrt2}\left(\frac{1}{4}-\frac{7\delta}{8}\right)-\frac{3(y^2+z^2)}{8\sqrt2}\left(1-\frac{3\delta}{2}\right)+\frac{x^2+y^2+z^2}{2\sqrt2}\left(1-\frac{\delta}{2}\right)\right\}+8C\left\{x-\frac{1+\delta}{\sqrt2}\right\}$$

$\frac{1}{2}-\frac{3\delta}{4}$

$$3\,D\Big/\sum r(x-x_k)=x\sum\left\{r_0-\frac{x x_k+y y_k+z z_k}{r_0}+\frac{x^2+y^2+z^2}{2}\frac{1}{r_0}\right.$$

$$\left.\cancel{-\frac{x^2}{2}\frac{x_k^2}{r_0^3}-\frac{y^2}{2}\frac{y_k^2}{r_0^3}-\frac{z^2}{2}\frac{z_k^2}{r_0^3}-\frac{xy\,x_ky_k+xz\,x_kz_k+yz\,y_kz_k}{r_0^3}}\right\}$$

$$-\sum\left\{x_k r_0-\frac{x x_k^2}{r_0}+\frac{x^2+y^2+z^2}{2}\frac{x_k}{r_0}-\frac{x^2}{2}\frac{x_k^3}{r_0^3}-\frac{y^2}{2}\frac{x_k y_k^2}{r_0^3}-\frac{z^2}{2}\frac{x_k z_k^2}{r_0^3}\right\}$$

$$\cancel{-xy\,x_k^2 y_k}$$

$$=x\left\{4\left(1+\frac{\delta}{2}\right)-x\left(1-\frac{\delta}{2}\right)\frac{4(1+\delta)}{\sqrt2}\right\}-\frac{4\left(1+\frac{\delta}{2}\right)(1+\delta)}{\sqrt2}+x\,\frac{4(1+2\delta)}{2}\left(1-\frac{\delta}{2}\right)$$

$$-\frac{x^2+y^2+z^2}{2}\,\frac{4(1+\delta)}{\sqrt2}\left(1-\frac{\delta}{2}\right)+\frac{x^2}{2}\,\frac{4(1+3\delta)}{2\sqrt2}\left(1-\frac{3\delta}{2}\right)+\frac{(y^2+z^2)}{2}\frac{1+\delta}{\sqrt2}\left(1-\frac{3\delta}{2}\right)$$

$$\sum \frac{x_k^2}{r_0^5} = 4(1+2\delta)\left(1-\frac{5\delta}{2}\right) = 4\left(1-\frac{\delta}{2}\right)$$

$$\sum \frac{y_k^2}{r_0^5} = 2\left(1-\frac{5\delta}{2}\right)+2 \quad = 4\left(1-\frac{5}{4}\delta\right)$$

$$r^2 = x_k^2+y_k^2+z_k^2 + x^2+y^2+z^2 - 2(x x_k + y y_k + z z_k)$$

$$r\frac{\partial r}{\partial x} = 2(x - x_k)$$

$$\sum r\frac{\partial r}{\partial x} = -24x$$

$$\sum r^2\frac{\partial r}{\partial x} = \sum r^2\frac{x-x_k}{r} = \sum r(x-x_k) = x\sum r_k - \sum r_k x_k$$

$$\sum r_k x_k = -x\sum\frac{x_k^2}{r_0}$$

$$\sum r^2\frac{\partial r}{\partial x} = 12x\left(1+\frac{\delta}{3}\right) + \cancel{x^3(4-3\delta)} + \cancel{x(y^2+z^2)(4-\delta)} - x.4\underbrace{(1+2\delta)\left(1-\frac{\delta}{2}\right)}_{1+\frac{3\delta}{2}}$$

$$= x\left[12+4\delta-4-6\delta\right] = x\left[8-2\delta\right]$$

$$\sum X = x\left\{B(1-\delta)8. + 2C.24 + 3D(8-2\delta)\right\}$$

$$\sum_{1234} X = B\sum(x-x_k)\left[\frac{1}{r_0} + \frac{x x_k + y y_k + z z_k}{r_0^2} + \cdots\right.$$

$$= B\left\{x\sum\frac{1}{r_0} + x^2\sum\frac{x_k}{r_0^3} + xy\cancel{\sum\frac{y_k}{r_0^2}} + xz\cancel{\sum\frac{z_k}{r_0^2}} - \sum\frac{x_k}{r_0} - x\sum\frac{x_k^2}{r_0^3} - y\cancel{\sum\frac{x_k y_k}{r_0^2}} - z\cancel{\sum\frac{x_k z_k}{r_0^2}}\right.$$

$$-\frac{x^2}{2}\sum\left(-\frac{x_k}{r_0^3}+\frac{3x_k^3}{r_0^5}\right) - \frac{y^2}{2}\sum\left(-\frac{x_k}{r_0^3}+\frac{3x_k y_k^2}{r_0^5}\right) - \frac{z^2}{2}\sum\left(-\frac{x_k}{r_0^3}+\frac{3x_k z_k^2}{r_0^5}\right)$$

$$\left.-3xy\cancel{\sum\frac{x_k^2 y_k}{r_0^5}} - 3yz\cancel{\sum\frac{x_k y_k z_k}{r_0^5}} - 3xz\cancel{\sum\frac{x_k^2 z_k}{r_0^5}}\right\} + 8Cx - 2C\sum x_k$$

$\frac{\varphi''}{2}$ $\quad - \varphi''$ $\quad + \frac{\varphi''}{2}$

$X = \cancel{\varphi' \frac{\partial r}{\partial x}} \cancel{+ 2\varphi' (r-l) \frac{\partial r}{\partial x}}$

$B \frac{\partial r}{\partial x} + 2 C r \frac{\partial r}{\partial x} + 3 D r^2 \frac{\partial r}{\partial x}$

$\cancel{\Sigma X = \Sigma \{ B}$

$\frac{\partial (\frac{1}{r})}{\partial x} = \frac{x_k - x}{r^3}$

$\frac{\partial r}{\partial x} = - \frac{x_k - x}{r}$

$\frac{\delta (\frac{1}{r})}{\partial x^2} = - \frac{1}{r^3} + \frac{3}{r^5}(x_k - x)^2 \Bigg\| \frac{\delta^2}{\partial x \partial y} = \frac{3(x_k - x)(y_k - y)}{r^5}$

$\frac{1}{r} = \frac{1}{r_0} + x \left(\frac{-\frac{\partial r}{\partial x}}{r^2} \right)_0 \cdots + \frac{x^2}{2} \left(- \frac{1}{r^3} + \frac{3 (x_k - x)^2}{r^5} \right)_0 + \sim$

$+ xy \left(\frac{3 (x_k - x)(y_k - y)}{r^5} \right)_0 + \cdots$

$= \frac{1}{r_0} + \frac{x x_k + y y_k + z z_k}{r_0^2} + \frac{x^2}{2} \left(- \frac{1}{r_0^3} + \frac{3 x_k^2}{r_0^5} \right) + \frac{y^2}{2} \left(- \frac{1}{r_0^3} + \frac{3 y_k^2}{r_0^5} \right) + \sim$

$+ 3 \left(xy \frac{x_k y_k}{r_0^5} + \cdots \sim \right)$

$\Sigma B \left(\frac{x - x_k}{r} \right) = x \Sigma \frac{1}{r_0} - \frac{x (x^2 + y^2 + z^2)}{2} \Sigma \frac{1}{r_0^3} + \frac{3}{2} \left(x^3 \Sigma \frac{x_k^2}{r_0^5} + x y^2 \Sigma \frac{y_k^2}{r_0^5} + x z^2 \Sigma \right)$

$- \left\{ x \Sigma \frac{x_k^2}{r_0^2} + y \cancel{\Sigma \frac{x_k y_k}{r_0^2}} \right.$

$\Sigma \frac{\partial r}{\partial x} = x (12 - 4\delta) - x \cdot 4 (1 + \delta) - \frac{x (x^2 + y^2 + z^2)}{2} 12 (1 - \delta) +$

$+ \frac{3}{2} 4 \left[x^3 \left(1 - \frac{\delta}{2}\right) + x y^2 \left(1 - \frac{5\delta}{4}\right) + x z^2 \left(1 - \frac{5\delta}{4}\right) \right]$

$= \cancel{[x \delta} \; 8x (1 - \delta)$

$\cancel{\Sigma r^2 =}$ $\qquad 8(1+\delta)+4 = 12\left(1+\frac{2\delta}{3}\right)$

$$\Sigma r^2 = 12\left[\left(1+\frac{2\delta}{3}\right) + (x^2+y^2+z^2)\right]$$

$$r^3 = r_0^3 + \cancel{3r_0^2 \frac{x x_k}{r_0}} + \frac{x^2}{2}\left[3r_0 + \frac{3x_k^2}{r_0}\right] + \frac{y^2}{2}\left[\quad\right] \frac{z^2}{2}\left[\quad\right]$$

$$+ 3\,\frac{xy\; x_k y_k}{r} + \dots$$

$$\Sigma \frac{x_k^2}{r_0} = 4(1+2\delta)\left(1-\frac{\delta}{2}\right) = 4\left(1+\frac{3\delta}{2}\right)$$

$$\Sigma \frac{y_k^2}{r_0} = 2\left(1-\frac{\delta}{2}\right)+2 = 4\left(1-\frac{\delta}{4}\right)$$

$$\Sigma \frac{z_k^2}{r_0} = 4\left(1-\frac{\delta}{4}\right)$$

$$\Sigma r^3 = 12(1+\delta) + \frac{3}{2}(x^2+y^2+z^2)\,12\left(1+\frac{\delta}{3}\right) + \frac{3}{2}\,4\left(1+\frac{3\delta}{2}\right)x^2 +$$

$$+ \frac{3}{2}\,4\left(1-\frac{\delta}{4}\right)(y^2+z^2)$$

$$= 12(1+\delta) + 18(x^2+y^2+z^2)\left(1+\frac{\delta}{3}\right) + 6x^2\left(1+\frac{3\delta}{2}\right) + 6(y^2+z^2)\left(1-\frac{\delta}{4}\right)$$

$$U = A + B\,\Sigma r + C\,\Sigma r^2 + D\,\Sigma r^3 \qquad = \alpha + \gamma\cdot[x^2+y^2+z^2]$$

$$\frac{\partial U}{\partial x} = B.8\bar{x} + 2Cx + 18D.2x\left(1+\frac{\delta}{3}\right) + 12Dx\left(1+\frac{3\delta}{2}\right)$$

$$= x\left[8B + 2C + 48.D + 30D\delta\right]$$

$$\sum x_k^2 = 8\frac{(1+\delta)^2}{2} = 4(1+2\delta)$$

$$\sum \frac{1}{r_0} = 8 - 4\delta + 4 = 12 - 4\delta$$

$$\sum \frac{x_k^2}{r_0^3} = 4(1+2\delta)\left(1-\frac{3\delta}{2}\right) = 4\left(1+\frac{\delta}{2}\right)$$

$$\sum \frac{y_k^2}{r_0^3} = 2\left(1-\frac{3\delta}{2}\right)+2 = 4\left(1-\frac{3\delta}{4}\right)$$

$$\sum r_0 = 8\left(1+\frac{\delta}{2}\right)+4 = 12\left(1+\frac{\delta}{3}\right)$$

$$\sum \frac{z_k^2}{r_0^3} = 2\left(1-\frac{3\delta}{2}\right)+2 = 4\left(1-\frac{3\delta}{4}\right)$$

$$\sum r_0^3 = 8\left(1+\frac{3\delta}{2}\right)+4 = 12(1+\delta)$$

$$\sum \frac{x_k y_k}{r_0^3} = \cancel{\left(1-\frac{\delta}{2}\right)} = 0$$

$$\sum \frac{1}{r_0^3} = 8\left(1-\frac{3\delta}{2}\right)+4 = 12(1-\delta)$$

$$\sum r = 12\left(1+\frac{\delta}{3}\right) + \frac{x^2+y^2+z^2}{2}\,12\left(1-\frac{\delta}{3}\right) - \frac{4}{2}\left\{x^2\left(1+\frac{\delta}{2}\right)+y^2\left(1-\frac{3\delta}{4}\right)+z^2\left(1-\frac{3\delta}{4}\right)\right\}$$

$$= 12\left(1+\frac{\delta}{3}\right) + x^2(6-2\delta-2-\delta) + y^2\left(6-2\delta-2+\frac{3\delta}{2}\right) + z^2\left(6-2\delta-2+\frac{3\delta}{2}\right)$$

$$\sum r = 12\left(1+\frac{\delta}{3}\right) + x^2(4-3\delta) + \cancel{\ldots} + (y^2+z^2)\left(4-\frac{\delta}{2}\right)$$ stimmt

$$\frac{\partial r^3}{\partial x} = 3r^2\frac{\partial r}{\partial x}$$

$$\frac{\partial^2}{\partial x^2} = 6r\left(\frac{\partial r}{\partial x}\right)^2 + 3r^2\frac{\partial^2 r}{\partial x^2} = 6r\left(\frac{x_k-x}{r}\right)^2 + 3r^2\left[\frac{1}{r}-\frac{(x_k-x)^2}{r^3}\right]$$

$$= 3r + 3\frac{(x_k-x)^2}{r}$$

$$\frac{\partial^2}{\partial x\partial y} = 6r\frac{\partial r}{\partial x}\frac{\partial r}{\partial y} + 3r^2\frac{\partial^2 r}{\partial x\partial y} = 6r\frac{(x_k-x)(y_k-y)}{r^2} - 3r^2\left[\frac{(x_k-x)(y_k-y)}{r^3}\right]$$

$$= \frac{3(x_k-x)(y_k-y)}{r}$$

$$r-l = r_0 - l + \frac{x^2+y^2+z^2}{r_0} + \frac{x^2}{2}\frac{\partial^2 r}{\partial x^2} + \frac{y^2}{2}\frac{\partial^2 r}{\partial y^2} + \frac{z^2}{2}\frac{\partial^2 r}{\partial z^2} + xy\frac{\partial^2 r}{\partial x \partial y} + \cdots$$

$$\frac{\partial r}{\partial x} = \frac{x_k - x}{r}$$

$$\frac{\partial^2 r}{\partial x^2} = +\frac{1}{r} - \frac{(x_k-x)^2}{r^3} \quad \Big\| \quad \frac{\partial^2 r}{\partial x \partial y} = -\frac{(x_k-x)(y_k-y)}{r^3}$$

$$r-l = r_0 - l + \frac{3}{2}\frac{x^2+y^2+z^2}{r_0} - \frac{x^4+y^4+z^4+2x^2y^2+2x^2z^2+2z^2y^2}{2r_0^3}$$

$$r-l = r_0 - l \cancel{\frac{x(x-x_k)+y(y-y_k)+z(z-z_k)}{r}}$$

$$- x\cdot\left(\frac{x_k-x}{r}\right)_0 + \cdots + \frac{x^2}{2}\left\{\frac{1}{r} - \frac{(x_k-x)^2}{r^3}\right\}_0 + \cdots$$

$$r = r_0 - \frac{x x_k + y y_k + z z_k}{r_0} + \frac{x^2}{2}\left\{\frac{1}{r_0} - \frac{x_k^2}{r_0^3}\right\} + \frac{y^2}{2}\left\{\frac{1}{r_0} - \frac{y_k^2}{r_0^3}\right\} + \frac{z^2}{2}\left\{\frac{1}{r_0} - \frac{z_k^2}{r_0^3}\right\}$$

$$- \frac{xy\, x_k y_k}{r_0^3} - \cdots$$

$$\cancel{r^2 = r_0^2 - 2(x x_k + y y_k + z z_k) + \frac{(x x_k + y y_k + z z_k)^2}{r_0^2} + x^2\left(1 - \frac{x_k^2}{r_0^2}\right)} + \cdots$$

$(x_k - x)^2 + \cdots \qquad -2(x_k - x)$

$$-\frac{2xy\, x_k y_k}{r_0^2}$$

$$(r^2) = (r_0^2) - x \cdot 2(x_k \cdots \qquad \cdots + \cancel{\frac{x^2}{2}} + y^2 + z^2$$

$$(r^2) = (r_0^2) - 2(x x_k + y y_k + z z_k) + x^2 + y^2 + z^2$$

$$r_0\,[1,2,3,4,9,10,11,12] = \sqrt{\frac{(1+\delta)^2}{2} + \frac{1}{2}} = \sqrt{1+\delta+\frac{\delta^2}{2}} = 1 + \frac{\delta}{2}$$

$$+\frac{3}{2} - 2$$

$$\frac{1}{r_0} = 1 - \frac{\delta}{2}$$

$$\sum x_k = \sum y_k = \sum z_k = 0$$

$$r = 1 + \frac{\delta}{2} - \left(1-\frac{\delta}{2}\right)\left(\frac{x(1+\delta)}{\sqrt{2}} - \frac{y_k}{\sqrt{2}}\right) + \frac{x^2+y^2+z^2}{2}\left(1-\frac{\delta}{2}\right) - \left(1 - \frac{3\delta}{2}\right)\left[\frac{x^2}{4}(1+\delta)^2 + y^2\right]$$

$$x \cdot \frac{1+2\delta}{2 \cdot 2}\left(1 - \frac{3\delta}{2}\right)$$

1

$$U = \Phi_{(r)} = \Phi(l) + (r-l)\varphi + \frac{(r-l)^2}{2}\varphi' + \frac{(r-l)^3}{2.3}\varphi''$$

$R = -\frac{\partial U}{\partial r} = -\varphi$

dla sił odpych. ~~...~~

φ ujemne

Rozwijając w kierunku x:

$$1\begin{cases} x = \frac{1+\delta}{\sqrt2}l \\ y = -\frac{1}{\sqrt2}l \\ z = 0 \end{cases} \quad 9\begin{cases} x = -\frac{1+\delta}{\sqrt2} \\ y = \frac{1}{\sqrt2} \\ z = 0 \end{cases} \quad 2\begin{cases} x = \frac{1+\delta}{\sqrt2} \\ y = \frac{1}{\sqrt2} \\ z = 0 \end{cases} \quad 10\begin{cases} x = -\frac{1+\delta}{\sqrt2} \\ y = +\frac{1}{\sqrt2} \\ z = 0 \end{cases}$$

$$6\begin{cases} x = 0 \\ y = \frac{1}{\sqrt2} \\ z = \frac{1}{\sqrt2} \end{cases} \quad 8\begin{cases} x = 0 \\ y = \frac{1}{\sqrt2} \\ z = -\frac{1}{\sqrt2} \end{cases} \quad 5\begin{cases} x = 0 \\ y = -\frac{1}{\sqrt2} \\ z = +\frac{1}{\sqrt2} \end{cases} \quad 7\begin{cases} x = 0 \\ y = -\frac{1}{\sqrt2} \\ z = -\frac{1}{\sqrt2} \end{cases}$$

$$3\begin{cases} x = +\frac{1+\delta}{\sqrt2} \\ y = 0 \\ z = -\frac{1}{\sqrt2} \end{cases} \quad 4\begin{cases} x = +\frac{1+\delta}{\sqrt2} \\ y = 0 \\ z = \frac{1}{\sqrt2} \end{cases} \quad 11\begin{cases} x = -\frac{1+\delta}{\sqrt2} \\ y = 0 \\ z = -\frac{1}{\sqrt2} \end{cases} \quad 12\begin{cases} x = \frac{1+\delta}{\sqrt2} \\ y = 0 \\ z = \frac{1}{\sqrt2} \end{cases}$$

$$U = \sum\left\{\Phi(l) + \left[\sqrt{(x_k-x)^2+(y_k-y)^2+(z_k-z)^2} - l\right]\varphi + (\sqrt{} - l)^2\frac{\varphi'}{2} + (\sqrt{} - l)^3\frac{\varphi''}{2.3}\right\}$$

$$= \sum\left\{\Phi - \varphi l + \frac{\varphi' l^2}{2} - \frac{\varphi'' l^3}{2.3} + \overbrace{\left(\varphi - l\varphi' + \frac{l^2\varphi''}{2}\right)}^{\Xi}\sqrt{} + (\sqrt{} - l)^4\frac{\varphi'''}{4!}\right.$$

$$\left. + \left(\frac{\varphi'}{2} - \frac{\varphi''}{2}l\right)\sqrt{}^{\,2} + \frac{\varphi''}{2.3}\sqrt{}^{\,3}\right\}$$

$$\left(\frac{1+\delta}{\sqrt2} - x\right)^2 + \left(-\frac{1}{\sqrt2} - y\right)^2 + z^2 = 1 + \delta \mp (1+\delta)x\sqrt2 + y\sqrt2 + x^2 y^2 z^2$$

$$(0-x)^2 + \left(-\frac{1}{\sqrt2} - y\right)^2 + \left(\frac{1}{\sqrt2} - z\right)^2 = 1 + y\sqrt2 \mp z\sqrt2 + x^2 y^2 z^2$$

$16 \frac{\partial^4 \psi}{\partial\alpha^2 \partial\beta^2} = \left(u \frac{\partial}{\partial x} + v \frac{\partial}{\partial y}\right) \zeta$ $\qquad f(\alpha) = \sqrt{\alpha}$ $\qquad g(\alpha) = \frac{1}{2}\sqrt{\alpha} - \sqrt{\alpha} = -\frac{\sqrt{\alpha}}{2}$

$g(\alpha) = -\frac{\sqrt{\alpha^3}}{3}$

$\frac{\alpha\sqrt{\beta} - \beta\sqrt{\alpha}}{i} - \frac{1}{3}\frac{\sqrt{\alpha^3} - \sqrt{\beta^3}}{i}$

$\cancel{= r^{3/2}\left\{\sin\frac{\theta}{2} - \frac{1}{3}\sin\frac{3\theta}{2}\right\}}$ $\qquad 2\sin\frac{\theta}{2} \cancel{- 2\sin\theta \cos\frac{\theta}{2}}$

$= 2\sin\frac{\theta}{2}\cos^2\frac{\theta}{2}$

$f'(\alpha) = \frac{1}{2\sqrt{\alpha}}$

$f''(\alpha) = -\frac{1}{4\sqrt{\alpha^3}}$

$\frac{1}{4}\left\{\frac{1}{\beta} - \frac{1}{\alpha}\right\} - \cancel{\frac{1}{8}\left\{\frac{1}{\sqrt{\beta\alpha}}\right\}} + \frac{1}{8}\left\{\sqrt{\frac{\beta}{\alpha^3}} - \sqrt{\frac{\alpha}{\beta^3}}\right\}$

$\int \log\beta \, d\beta = \beta(\log\beta - 1)$ $\qquad \sqrt{\alpha\beta^5} \quad \frac{5}{2}\sqrt{\alpha\beta^3} \quad \frac{15}{4}\sqrt{\alpha\beta}$

$\frac{15}{8}\sqrt{\frac{\beta}{\alpha}}$

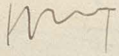
$-\frac{15}{16}\sqrt{\frac{\beta}{\alpha^3}}$

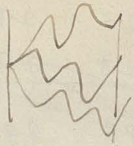
$\psi = \frac{\alpha^2}{2}\{\beta\log\beta\} - \frac{\beta^2}{2}[\alpha\log\alpha] - \frac{8}{15}\left[\sqrt{\alpha\beta^5} - \sqrt{\beta\alpha^5}\right]$

$u = \frac{1}{i}\left(\frac{\partial\psi}{\partial\alpha} - \frac{\partial\psi}{\partial\beta}\right) \qquad v = \frac{\partial\psi}{\partial\alpha} + \frac{\partial\psi}{\partial\beta}$

$u = \frac{1}{i}\left\{\alpha\beta(\log\beta - \log\alpha) - \frac{1}{2}\left[\beta^2(\log\alpha + 1) - \alpha^2(\log\beta + 1)\right]\right.$

+	−	
~~4~~	6	−2
~~7~~	8	−1
~~7~~	5	+2
6	7	−1
4	8	−4
~~28~~	34	





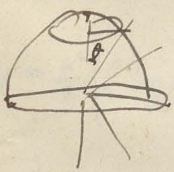
0p34	8921	7634	6232	4150
5051	~~6628~~	~~7709~~	~~0086~~	~~3541~~
5883	2293	9925	6146	0609
	4472	4150	1139	p532
	4449	3484	50p3	4608

$2\sqrt{\frac{64}{27}} = \frac{2.8}{5} = 3$

$\tau = \frac{M}{S} = \frac{M}{6\pi\mu R} = \frac{\frac{4}{3}\pi R^3 \rho}{6\pi\mu R} =$

$\tau = \frac{2}{9}\rho\frac{R^2}{\mu}$

$\frac{2}{9}\cdot(2{\cdot}5.10^{-6})^2$



$$\frac{2\pi\int_0^{\frac{\pi}{2}}\cos\theta\sin\theta\,d\theta}{2\pi} = \frac{1}{2}$$

Svedberg misst somit nur $\frac{1}{2}$ der wirkl. Bewegungs[illegible].

~~dazu kommt~~ $\cancel{\frac{1}{2}}$ ~~wegen Ersetzung der ganzen anstatt Viertel[illegible]~~

$\cancel{\Delta = \frac{8}{3\sqrt{\pi}}\frac{1}{8{\cdot}3.10^7}\sqrt{}}$ $\quad \sqrt{\pi} \quad c\sqrt{\pi} = 48000\sqrt{\frac{0{\cdot}0013}{4.10^{19}}}$

$$\Delta = \frac{8}{\rho\sqrt{\pi}}\frac{c\sqrt{m}}{\sqrt{2{\cdot}3.10^{-3}.\,2{\cdot}5.10^{-6}}}\sqrt{\frac{0{\cdot}032}{4}}\cdot\frac{1}{2}$$

$$= \frac{2}{9}\sqrt{\frac{\cancel{0{\cdot}0013}.\,\cancel{0{\cdot}032}}{10^{10}.\,2{\cdot}3.\,2{\cdot}5.\,\cancel{10^{-9}}}}\cdot\frac{48\cancel{000}}{\sqrt{\pi}}$$

$$= \frac{2}{\cancel{9}\,3}\sqrt{\frac{13.\cancel{32}}{\cancel{2{\cdot}3.\,2{\cdot}5.\,3{\cdot}1}}}\;10^{-5}.\cancel{48}\,16$$

$5{\cdot}7 \quad = \frac{32}{3}\sqrt{2{\cdot}2}\,.10^{-5} = 1{\cdot}6.\,10^{-4}$

The Svedberg	$4A$	τ	η	$\frac{A}{\sqrt{\frac{\tau}{\eta}}}$	$\frac{A}{\tau}$
Aceton	12·4	0·032	3·2	1·24	38
Äthylalkohol	7·8	0·028	4·6	1·00	28
Amylalkohol	5·8	0·026	5·9	0·875	22
Wasser	4·2	0·013	10·2	1·18	32
n-Propylalk.	2·6	0·009	22·6	1·30	29

4472
6628
1·7844
0·9921
0·8922

4150
7709
6441
7634
8220
9414

1139
0086
1053
6233
55265
0706

0·9542
2·3541
0·6001 −1
4150
0·3001
1150

$2·6 \cdot \sqrt{\frac{22·6}{9}}$ $\sqrt{25}$

$12·4 \sqrt{\frac{32}{3·2}}$

0·032

$$4A = \sqrt{\frac{\tau}{\eta}} \sqrt{\frac{RT}{N} \frac{1}{3\pi P}} = \sqrt{\frac{\tau}{\eta}} \sqrt{\frac{\cancel{8·3}.10^7.\cancel{292}}{4.10^{23}} \frac{100}{\cancel{25}.10^{-6}}}$$

$$= \sqrt{\frac{\tau}{\eta}} \; \frac{1}{2} \sqrt{\frac{10^9}{10^{17}}} = \frac{1}{2} \sqrt{\frac{\tau}{\eta}} \sqrt{10^{-8}} =$$

$$A = \frac{1}{8} \sqrt{\frac{\tau}{\eta}} . 10^{-4} =$$

$14 : 18 = \frac{7}{9}$
$= 0·8$

$\frac{0·032}{0·0032}$

1038
580
6038

|| 403

$\frac{0·032}{0·0023}$

5·051
− 3617
1·1434
5·717
1038
6768

474

$\frac{0·009}{0·0226}$

9542
3541
1·6001
0·8000
1038
0·9038

801

$\frac{9}{13} \cdot \frac{10·2}{22·6} =$

$\frac{1}{1·44 . 2·2}$

$\frac{2·9}{\sqrt{3·2}} = 1·79$

Voigt Rapports I p. 320

$$-X_x = c_{11} x_x + c_{12} y_y + \dots \quad c_{16} x_y$$

$$-Y_z = c_{41} x_x + c_{42} y_y + \dots c_{44} y_z\ c_{45} z_x\ c_{46} x_y$$

Corps quasi isotrope :

~~Corps isotropes:~~

$$-X_x = c\, x_x + c_1 y_y + c_1 z_z$$

$$-Y_z = \frac{c_2\, y_z}{2}$$

$(c_2 = c - c_1$

$c = \frac{1}{5}(3A + 2B + 4C)$ $\quad c_1 = \frac{1}{5}(A + 4B - 2C)$ $\quad c_2 = \frac{1}{5}(2A - 2B + 6C)$

où l'on a posé pour abréger

$A = \frac{1}{3}(c_{11} + c_{22} + c_{33})$ $\quad B = \frac{1}{3}(c_{23} + c_{31} + c_{12})$ $\quad C = \frac{1}{3}(c_{44} + c_{55} + c_{66})$

$$-X_x = c_{11} x_x = c_{11} \frac{\partial \xi}{\partial x}$$

$$-Y_y = c_{21} x_x = c_{21} \frac{\partial \xi}{\partial x}$$

$$-Y_z = c_{44} y_z = c_{44}\left(\frac{\partial \eta}{\partial z} + \frac{\partial \zeta}{\partial y}\right)$$

$$c = \frac{(1-\mu)}{(1+\mu)(1-2\mu)} E \; ; \; c_1 = \frac{\mu}{(1+\mu)(1-2\mu)} E \; ;$$

$$\frac{c}{c_1} = \frac{(1-\mu)}{\mu} = \frac{3A + 2B + 4C}{A + 4B - 2C}$$

$$\frac{c}{c_1} + 1 = \frac{1}{\mu}$$

$$\mu = \frac{1}{1 + \frac{c}{c_1}} = \frac{A + B - 2C}{4A + 6B + 2C}$$

$$-X_x = c\, x_x + c_1(y_y + z_z) = c\, x_x + (c - c_2)(y_y + z_z) = c\,(x_x + y_y + z_z) - c_2\,(y_y + z_z)$$
$$= c_2 x_x + (c - c_2)(x_x \dots)$$

$$-Y_z = \frac{c_2}{2} y_z$$

$r = 1 + \delta r$

$$\left[1 + \delta r + \underbrace{\frac{3}{(1+\delta\omega)^2}}_{3(1-2\delta\omega)}\right](3 + 3\delta\omega - 1) = 8(1+\delta\vartheta)$$

$$(4 + \delta r - 6\delta\omega)(2 + 3\delta\omega) = 8(1+\delta\vartheta)$$

$$8 + 2\delta r - \cancel{12\delta\omega} + \cancel{12\delta\omega} = 8(1+\delta\vartheta)$$

$$\boxed{\delta r = 4\delta\vartheta}$$

$\frac{T}{T_k} =$

$$\vartheta = \frac{1}{8}\left(r + \frac{3}{\omega^2}\right)(3\omega - 1) = \frac{\left[(3\omega-1)r + \frac{9}{\omega} - \frac{3}{\omega^2}\right]}{8}$$

$$\vartheta = f(r=1, \omega=1) + \left(\frac{\partial f}{\partial r}\right)_{11} \cdot dr + \frac{\partial f}{\partial}$$

$$\left(\frac{\partial\vartheta}{\partial r}\right) = \left(\frac{3\omega-1}{8}\right) = \frac{1}{4} \qquad \left(\frac{\partial^2\vartheta}{\partial r^2}\right) = 0 \; - - - -$$

$$\frac{\partial^2\vartheta}{\partial r\,\partial\omega} = \frac{3}{8}$$

$$\frac{\partial^3\vartheta}{\partial r\,\partial\omega^2} = 0 \; - -$$

$$\frac{\partial\vartheta}{\partial\omega} = \frac{1}{8}\left[3r - \frac{9}{\omega^2} + \frac{6}{\omega^3}\right]_0 = 0$$

$$\frac{\partial^2\vartheta}{\partial\omega^2} = \frac{1}{8}\left[\frac{18}{\omega^3} - \frac{18}{\omega^4}\right] = \frac{9}{4}\left(\frac{1}{\omega^3} - \frac{1}{\omega^4}\right)_0 = 0$$

$$\frac{\partial^3\vartheta}{\partial\omega^3} = \frac{9}{4}\left(-\frac{3}{\omega^4} + \frac{4}{\omega^5}\right) = \frac{9}{4}$$

$$\Delta\theta = \frac{\Delta r}{4} + \frac{3}{8}\Delta r \cdot \Delta\omega + \frac{3}{8}\Delta\omega^3$$

Izotermy $\Delta r = \dfrac{-3\Delta\omega^3}{2 + 3\Delta\omega}$

$$\neq -\frac{3}{2}\Delta\omega^3$$

$$\cancel{\frac{\Delta r}{\Delta\theta}} = 4\Delta\theta = -\frac{3}{2}\Delta\omega^3$$

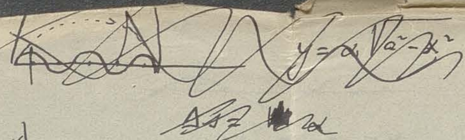
$$\Delta\omega = \sqrt[3]{\frac{8}{3}\Delta\theta}$$

Ogólnie: $\Delta r = \dfrac{8\Delta\theta - 3\Delta\omega^3}{2 + 3\Delta\omega} \neq 4\Delta\theta\cancel{\left(1 - \frac{3}{2}\Delta\omega\right)} - \frac{3}{2}\Delta\omega^3$

$$\int \Delta r\,\Delta\omega = 4\Delta\theta\,\Delta\omega - 3\Delta\theta\,\Delta\omega^2 - \frac{3}{8}\Delta\omega^4 = 0$$

$$4\Delta\theta - 3\Delta\theta\,\Delta\omega - \frac{3}{8}\Delta\omega^3 = 0$$

$\Delta s = \alpha^2 \beta \frac{\pi}{8}$



biorę za długość fal: λ

$\beta\lambda = 2\pi$

$\beta = \frac{2\pi}{\lambda}$ $\qquad \Delta s = \frac{\alpha^2\pi^2}{\lambda}$

nachylenie przeciętne: $\frac{\int \frac{dy}{dx}\,dx}{\int dx} = \frac{\alpha}{\lambda} = \cancel{\alpha}$

$\Delta s = \cancel{\alpha}^2 \lambda \pi^2$

$\Delta U = a\,\cancel{\alpha}^2 \lambda \pi^2$

$$\overline{x^2} = \frac{\int x^2 e^{-a\lambda\pi^2 h x^2}\,dx}{\int - \; -} = \frac{1}{2a\lambda\pi^2 h}$$

$$\bar{x} = \frac{1}{\sqrt{2a\lambda\pi^2 h}}$$

$$h = \frac{N}{RT} = \frac{4\cdot 10^{23}}{8\cdot 10^7 \cdot 273} = \frac{10^{16}}{500}$$
$$= 2\cdot 10^{13}$$

$a = 80$

$\lambda = 10^{-4}$ (mikron)

$$\bar{x} = \frac{1}{\sqrt{160\cdot 10\cdot 10^{-4}\cdot 2\cdot 10^{13}}} = \frac{1}{\sqrt{3\cdot 10^{12}}}$$
$$= \frac{1}{\sqrt{3}}\cdot 10^{-6}$$

$$h = \frac{1}{\pi f}\,\frac{a^2(\rho_1-\rho_2)^2}{H} \qquad \frac{H}{\rho_1-\rho_2} = c\left(1-\frac{T}{T_k}\right)$$

$$= \frac{a^2}{\pi f}\cdot\frac{\rho_1-\rho_2}{c\left(1-\frac{T}{T_k}\right)}$$

h Dolezalek $20\,7^{54}$ für Äther 110°

$= 8^{\cdot 5}$ μμ

$$h_1 : h_2 = \frac{(\rho_1-\rho_2)_0}{\cancel{T}_k - T} : \frac{\Delta\rho}{\Delta T}$$

$$h_1 \frac{\Delta\rho}{\Delta T} = h_2 \frac{\rho_1-\rho_2}{T_k - T}$$

$$\Delta T = \frac{h_1}{h_2}\,\frac{\Delta\rho}{\rho_1-\rho_2}\left(\frac{T_k - T}{\cancel{\Delta T}}\right)$$

1970°
1080°
———
890

$\frac{h_1}{h_2} = \frac{1}{100}$

890

197
273
———
470

$\overline{W}\, d\alpha \sim e^{-hU}\, d\alpha$

$\int e^{-hz^2} dz = \sqrt{\frac{\pi}{h}}$

$\int z^2 e^{-hz^2} dz = \frac{1}{2}\sqrt{\frac{\pi}{h^3}}$

$y = \alpha \sin\beta x$

$\frac{dy}{dx} = \alpha\beta \cos\beta x$

$s = \sqrt{1 + \alpha^2\beta^2\cos^2\beta x}$

$$\Delta s = \int_0^{\beta x = \frac{\pi}{2}} \left[\left(1 + \frac{\alpha^2\beta^2}{2}\cos^2\beta x\right) dx - dx\right] = \frac{1}{\beta}\int_0^{\varphi = \frac{\pi}{2}} \frac{\alpha^2\beta^2}{2}\cos^2\varphi\, d\varphi = \frac{\alpha^2\beta}{2}\,\frac{\pi}{4}$$

$$\frac{\Delta s}{\Delta x = \frac{\pi}{2\beta}} = \frac{\alpha^2\beta^2}{4}$$

$\parallel \frac{\mathrm{tg}\,\varphi^2}{4}$

$$\mathrm{tg}\, y\big|_{x=0} = \alpha\beta$$

$$\Delta U = \frac{\alpha\, \mathrm{tg}^2 y}{4}$$

$\mathrm{tg}\, y = \alpha$

$$\overline{x^2} = \frac{\int x^2 e^{-hU} dx}{\int e^{-hU} dx} = \int \frac{x^2 e^{-\frac{h\alpha x^2}{4}} dx}{\int \cdots} = \frac{1}{2\frac{h\alpha}{4}} = \frac{2}{h\alpha}$$

$\frac{n\, mc^2}{3} = RT = \frac{p}{\rho}$

$$\overline{\mathrm{tg}\, y} = \sqrt{\frac{2}{h\alpha}}$$

$$h = \frac{2}{mc^2} = \frac{2N}{3RT} = \frac{2N\cdot\rho}{3p}$$

$$= \sqrt{\frac{2}{3\alpha}}\; 10^{-5}\cdot\sqrt{\frac{2}{3\alpha}}$$

$$h = \frac{2}{3}\,\frac{4\cdot 10^{19}\cdot 0{\cdot}0013}{10^6} = 3\cdot 10^{10}$$

Etwa $\alpha = 20$

$\sqrt{30} =$

$$\overline{\mathrm{tg}\, y} = 0{\cdot}2\cdot 10^{-5} = (0{\cdot}2\cdot(60)^3\cdot 10^{-5})'' = 0{\cdot}4''\quad (!)$$

7.

Wasilkowski Władysław: Prądy przemienne w zestawieniu z prądem o stałym kierunku. Sprawozdanie Dyrekcyi c.k. Gimnazyum VI. we Lwowie za rok szkolny 1903/4. Lwów 1904. Str. 32 i jedna tablica.

Czytam we wstępie: „.... wiem z doświadczenia, że brak czasu nie pozwala abituryentów dokładnie z tą rzeczą zaznajomić. Niech więc ta rozprawka będzie dla nich uzupełnieniem tego, co słyszeli w gimnazyum z nauki o indukcyi elektrycznej. A może i który z nauczycieli będzie mi wdzięczny za tę pracę."

Ładne zadanie i dobre zamiary! Niestety jednak, przeczytałem rozprawkę i zamiast wdzięczności uczułem żal do autora. Trudno bowiem o rozprawkę, napisaną gorzej, niż książeczka p. Wasilkowskiego. Nie chciałbym być satyrycznym, ale pomimo to mogę powiedzieć, iż jest to zbiór usterek i błędów.

I tak zaraz na początku (str. 3.) można ze zdumieniem czytać, że maszyna dynamo wytwarza prądy, zmieniające co chwila swój kierunek. I cóż powie na to biedny abituryent, którego nauczono w gimnazyum, że dynamo dostarcza prądu o kierunku stałym? — Lub dalej (str. 6.) dowiadujemy się, że „sama obecność prądu galwanicznego w jednym łączniku wzbudza w drugim drucie, nie mającym żadnej łączności z pierwszym, prąd elektryczny." Tymczasem sześć wierszy przedtem jest mowa o tem, że obecność prądu, byle on stale płynął, wcale nie może wywołać prądu indukowanego. — Jednorodnem polem magnetycznem nazywa prof. Wasilkowski to pole (str. 14.), „w którem linie siłowe (?) są do siebie równoległe," nie mówiąc nic o tem, jak gęsto w rozmaitych miejscach uważanej przestrzeni linie owe są rozsiane. — Rozróżnienie dzielności od pracy jest tak nieścisłe, że dzielność wyraża autor (str. 23.) kilogramometrami lub siłą (sic!) koni.

I takie fatalne omyłki spotykamy co krok. Nie chcę mnożyć liczby podobnych przykładów; może już te, które podałem, wystarczą do tego, aby czytelnik wyrobił

II 3

Budowa

Atomu

z rysunkami