III 4

Przewodnictwo elektryczn.

w gazach

Elektrony

Owa twory tylko pewne przestrzeni

Pomińmy dyfuzyę i absorpcyę i nierównomierność rozkładu

Jonizacya przez RR lub β lub γ

$$-\frac{\partial^2 V}{\partial x^2} = \frac{\partial X}{\partial x} = 4\pi\rho = 4\pi e\,(n_p - n_n)$$

$$i = (v_p n_p + v_n n_n)\, X e \qquad \frac{\partial i}{\partial x} = 0$$

gdzie n_p = małe $\frac{q x e}{v_p n_p} = X$

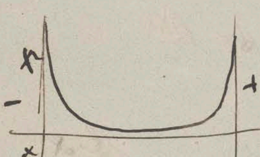
n_n $\frac{c - qx}{v_n n_n} = X$

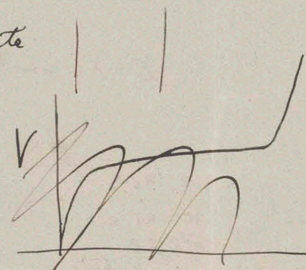
$$q = \alpha n_p n_n + \frac{\partial}{\partial x}(v_p X n_p)$$

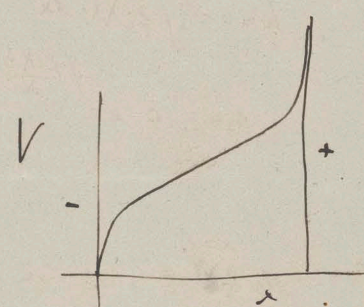
$$q = \alpha n_p n_n - \frac{\partial}{\partial x}(v_n X n_n)$$

$$\left(\frac{1}{v_1} + \frac{1}{v_2}\right)(q - \alpha n_1 n_2) = \frac{\partial}{\partial x}\left[X(n_1 - n_2)\right] = \frac{1}{4\pi e}\frac{\partial}{\partial x}\left(X\frac{\partial X}{\partial x}\right) = \frac{1}{8\pi e}\frac{\partial^2 (X)^2}{\partial x^2}$$

przy ścianach n_1 n_2 małe







Np. $v_2 \gg v_1$ lub $v_1 = 0$ w płomieniach

$$\left.\begin{aligned} i &= X v_2 n_2 e \\ q &= \alpha n_1 n_2 \\ \frac{\partial X}{\partial x} &= 4\pi e(n_1 - n_2) \end{aligned}\right\} \quad \begin{aligned} n_1 &= \frac{q}{\alpha}\frac{v_2 X e}{i} \\ n_2 &= \frac{i}{e X v_2} \end{aligned}$$

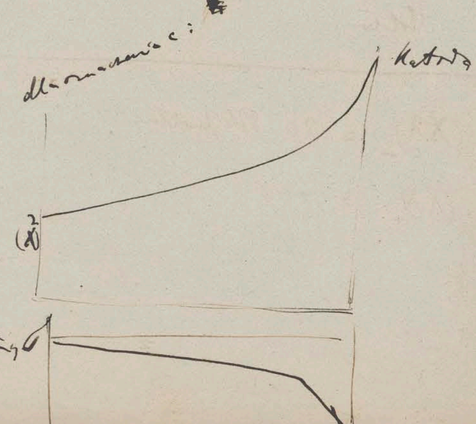
dla wszystkich i: $i = k_2 n_2 X e = \int (q - \alpha n_1 n_2)\, e\, dx$ =

$$X\frac{dX}{dx} = 4\pi e\left(\frac{q e v_2 X^2}{\alpha i} - \frac{i}{e v_2}\right)$$

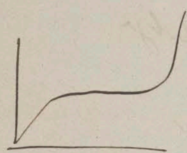
$$X^2 = \frac{\alpha i^2}{q e^2 v_2^2} + \underbrace{c\, e^{8\pi\frac{q e v_2}{\alpha i} x}}_{= c'\, e^{-\frac{8\pi q e v_2}{\alpha i}(l - x)}}$$

dla małych x:

$$X = \frac{i}{e v_2}\sqrt{\frac{\alpha}{q}}$$



Rozbij. samod. mechanizm wył. przez wstęp.
Prąd wyższych pól znowu wzrost przy niesamodzielnym. [Townsend 1901)



UVL

Dopóki mały X przed wyładowaniem proporcjonalny katody wystawiona
$i = k X n \varepsilon$

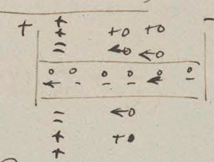
∂

$$\frac{\partial n}{\partial t} + \frac{\partial}{\partial x}(nu) = -N = -2\alpha \overbrace{n\frac{u}{\lambda}}^{\text{liczba spotkań}} \qquad \alpha = f.(X\varepsilon\lambda)$$

$(2\ bo + i -)$

Jeśli tylko ujemne jony jonizują

$$\boxed{i = (k_1 n_1 + k_2 n_2) X \varepsilon}$$



$$\frac{\partial(nu)}{\partial x} = -2\frac{nu}{\lambda} f(X\lambda)$$

$$\lg nu = -\frac{2}{\lambda}\int f(X\lambda)\,dx$$

$$nu = c\, e^{-2\int \frac{f(X\lambda)}{\lambda}dx}$$

jeśli $X = \text{const}$ ~~[illegible]~~

$$nu = n_0 u_0 e^{-\beta x} \qquad \beta = \frac{f(X\lambda)}{\lambda}$$

$$n = n_0 e^{\beta x}$$

~~[illegible]~~

$x = l$

dla ~~[illegible]~~: $\varepsilon n_0 u_0$ = stosunek elektr. (wychodząca z katody) = i_0
(pierwotnie)
~~[illegible]~~ (przy naświetlaniu)

$$\frac{n}{n_0} = e^{\beta(l - x)}$$

$i_0 = \varepsilon n_0 u_0$ ~~$e^{\beta l}$~~

~~[illegible]~~ $i = 0$:

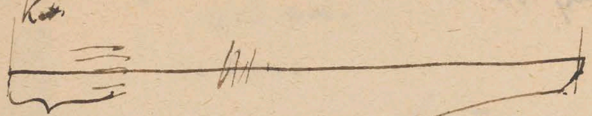
$$n = n_0 e^{\beta l}$$

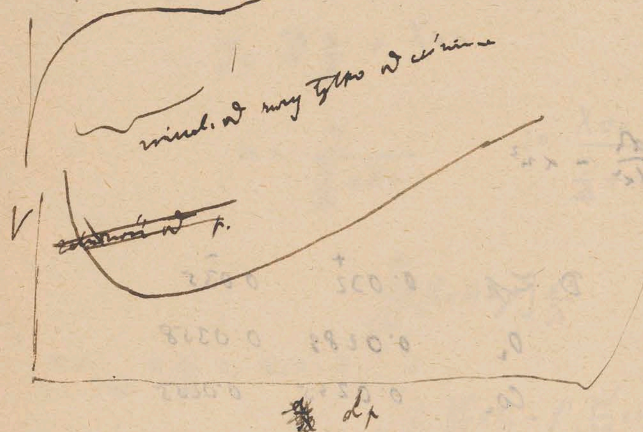
Stoletow

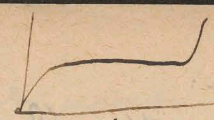
$(X\lambda)_- = 30$ Volt przeciętnie ; $v = 3.3 \cdot 10^8 \frac{cm}{s}$

$(X\lambda)_+ = 440$; $7.7 \cdot 10^6$

Dokładne pomiar. potenc. w kierunku $f = 1$







~~podział~~

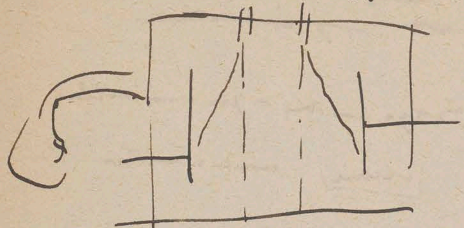
~~podział~~ ~~przy samej katodzie~~

podział przez ... + przy samej katodzie

ciemnie promienie katodowe

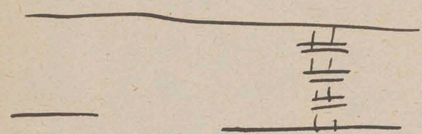
Prądy konwekcyjne:

tokiem CO_2 Zeleny 1898



Dyfuzja.

$$\frac{dn}{dt} = q + D\frac{d^2n}{dx^2} - \alpha n^2$$



D	Luft	$0{\cdot}03\overset{+}{2}$	$0{\cdot}03\bar{5}$
	O_2	$0{\cdot}0288$	$0{\cdot}0358$
	CO_2	$0{\cdot}0245$	$0{\cdot}0255$
	H_2	$0{\cdot}128$	$0{\cdot}142$

$CO - CO_2$	$0{\cdot}131$
Luft - CO_2	$0{\cdot}134$
H_2 CO_2	$0{\cdot}534$
eter - Luft	$0{\cdot}077$
eter - CO_2	$0{\cdot}055$

Jonów absorpcja

~~4~~ W razie jeśli produkcja jonów graniczona do jednej powierzchni

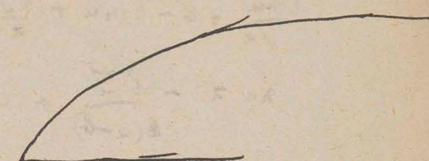
UVL dyfuzja

$$i = Xvne$$

jeśli podłużnie wszystko J

$$J = D\frac{n}{\lambda} + Xvn$$

$$n = \frac{J}{\frac{D}{\lambda} + Xv} \qquad i = \frac{XveJ}{\frac{D}{\lambda} + Xv}$$



$$nu\Big|_{x=d} = \frac{i}{e} = n_0 u_0 e^{\int_0^d \left[f(Xe\lambda) - \mu\right]\frac{dx}{\lambda}}$$

$$= n_0 u_0 e^{\int_0^d \left[f\left(\frac{\partial V}{\partial x}e\lambda\right) - f'\left(\frac{\partial V}{\partial x}e\lambda\right)\right]\frac{dx}{\lambda}} \qquad \frac{x}{\lambda} = \xi$$

$$= n_0 u_0 e^{\int^{\xi = \frac{d}{\lambda}} \left[f\left(e\frac{\partial V}{\partial \xi}\right) - f'\left(e\frac{\partial V}{\partial \xi}\right)\right] d\xi} \qquad \frac{\partial V}{\partial \xi} = \varphi$$

$$\cancel{d\xi = \frac{dV}{X}}$$

$$\frac{n_0 u_0}{kV_0} + n_0 u_0 e = n_0 u_0 e^{\int_0^V \left[\frac{f(e\varphi) - f'(e\varphi)}{\varphi}\right] dV} \qquad d\xi = \frac{dV}{\varphi}$$

$$\varphi = f_1\left(\frac{V}{m}\right) \qquad \frac{V_0}{d}\lambda = f(V_0)$$

$$V_0 = f\left(\frac{\lambda}{d}\right)$$

n = liczba cząsteczek u = prędkość przeciętna (w polu X)   (rekomb. pomin.)

$$\cancel{\frac{\partial n}{\partial t}} + \frac{\partial}{\partial x}(nu) = n\frac{u}{\lambda}\left[f_o(Xe\lambda)\right] + \frac{n'v}{\lambda}\left[f'(Xe\lambda)\right] = anu + bn'v$$

$$J = (nu + n'v)\,\varepsilon = \text{const}$$

$$\frac{d(nu)}{dx} = (a-b)\,nu + \frac{bJ}{\varepsilon}$$

iskra

$$nu = -\frac{bJ}{\varepsilon(a-b)} + c\,e^{(a-b)x}$$

$x=0$ $$\frac{J_0}{\varepsilon} = -\frac{bJ}{\varepsilon(a-b)} + c$$

$x=d$ $$\frac{J}{\varepsilon} = -\frac{bJ}{\varepsilon(a-b)} + c\,e^{(a-b)d}$$

$$\frac{J}{\varepsilon} = \frac{J_0}{\varepsilon}\,\frac{(a-b)\,e^{(a-b)d}}{a - b\,e^{(a-b)d}}$$

w razie jeżeli mianownik $= 0$: $J = \infty$ $\cancel{\frac{a}{b} = \lg}$

$$\frac{a}{e^{ad}} = \frac{b}{e^{bd}}$$

$$\lg a - ad = \lg b - bd$$

$$d = \frac{\lg a - \lg b}{a-b} = \frac{\lg f - \lg f'}{f - f'}\,\lambda$$

$$\frac{d}{\lambda} = f_o\,(X\lambda) = f\left(\frac{V_0\lambda}{d}\right)$$

$$V_0 = f_o\left(\frac{\lambda}{d}\right) = f_o(d\lambda)$$ Paschen

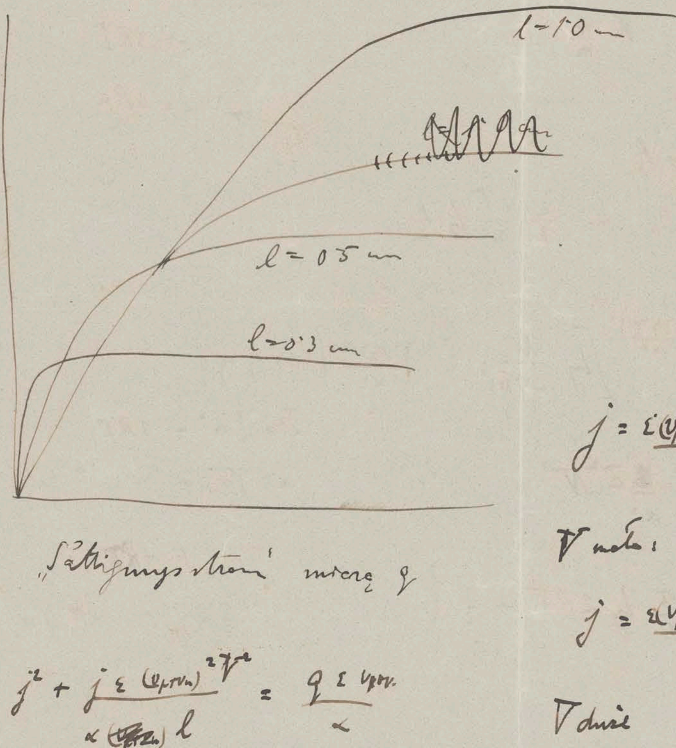
$\cancel{j = \mu \varepsilon l}$

$$\frac{dn}{dt} = q - \alpha n^2 - \frac{j}{l\varepsilon}$$

$$j = n\varepsilon(v_p + v_n)\frac{V}{l} \qquad \mu = \frac{j}{\varepsilon l}$$

$$= q - \alpha \frac{j^2}{\left(\varepsilon(v_p+v_n)\frac{V}{l}\right)^2} - \frac{j}{l\varepsilon}$$

$$j = q\, l\varepsilon \qquad \boxed{j = \varepsilon(v_p+v_n)\frac{V}{l}\sqrt{\frac{q}{\alpha}}}$$



„Sättigungsstrom" mierzy q

$$j^2 + \frac{j\,\varepsilon(v_p+v_n)^2 V^2}{\alpha\,\cancel{(v_p+v_n)}\,l} = \frac{q\,\varepsilon\, v_p v_n}{\alpha}$$

$$j = \frac{\varepsilon(v_p+v_n)V}{l}\left[-\frac{1}{2\alpha}\frac{(v_p+v_n)V}{1} \pm \sqrt{\frac{q}{\alpha}\cancel{(\ldots)} + \cdots}\right]$$

V małe:

$$j = \frac{\varepsilon(v_p+v_n)V}{l}\sqrt{\frac{q}{\alpha}}$$

V duże

$$j = \frac{\varepsilon(v_p+v_n)^2 V^2}{2\alpha l}\left[-1 + \sqrt{1 + \frac{4q\alpha}{(v_p+v_n)^2 V^2}}\right]$$

$$= \frac{q\,l\varepsilon}{2} - $$

$$\frac{\frac{Ne^2X}{2mv}}{\alpha\frac{3}{2}RNmvX} = \frac{e^2}{\alpha m^2v^2R} = \cancel{\frac{e^2}{\ldots}}$$

$$= \frac{3e^2\theta}{\alpha Nm^2v^4}$$

$\frac{e}{m} = 10^7$

$e = 3{\cdot}5 \cdot 10^{-10}$

$R\theta = \frac{Nmc^2}{3}$

$R = \frac{pv}{\theta} = \frac{10^6}{0{\cdot}0013 \cdot 273}$

Kontakt pot.

Thermoelektr.

m

$$E = 0{\cdot}0002\frac{T}{w}\log\frac{p_1}{p_2}$$

$$RT\log\frac{p_1}{p_2} = E\cdot\varphi = \frac{\varepsilon}{m}$$

$\frac{mc^2}{2} = \alpha T$

$c^2 = 3RT$

$\alpha = \frac{3Rm}{2}$

$$E = \frac{m}{\varepsilon}RT\log\frac{p_1}{p_2}$$

$$= \frac{2}{3}\frac{\alpha T}{2}\log\frac{p_1}{p_2}$$

$\int\frac{N}{(\alpha\sqrt{\pi})^3}e^{-\frac{c^2}{\alpha^2}}d\xi$

$\sqrt{\alpha\pi}$

$$\int f\log f\,d\omega = \int\frac{N}{(\alpha\sqrt{\pi})^3}e^{-\frac{\xi^2+\eta^2+\zeta^2}{\alpha^2}}\left[\log\frac{N}{(\alpha\sqrt{\pi})^3} - \frac{\xi^2+\eta^2+\zeta^2}{\alpha^2}\right]d\xi\,d\eta\,d\zeta$$

$\overline{c^2} = \frac{3}{2}\alpha^2 = 3RT$

$\alpha = \sqrt{2RT}$

$$= N\log\frac{N}{(\alpha\sqrt{\pi})^3} - \frac{\overline{c^2}N}{\alpha^2}$$

$\int\frac{c\,dT}{T} + \int\frac{\ldots}{\partial v}$

$c_v\log T + R\log v$

$\frac{3}{2}R\log T + R\log v$

$= \frac{3}{2}R\log\frac{T}{v}$

$$= N\left\{\log N - \frac{3}{2}\log 2RT - \frac{3}{2}\right\}$$

$$= N\log\left(NT^{-3/2}\right)$$

pro Volum

$$= n\log\left(\rho T^{-3/2}\right)$$

$$\frac{16}{3}\pi\frac{\alpha T}{\lambda^4}d\lambda$$

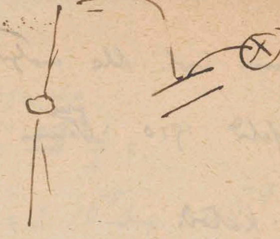
	$T_{1/2}$		
U		α	
U_x	22 d	$\beta\gamma$	
R	1300 y.	α	
Em	3·6 d.	α	gas $-150°$
RA	3 m	α	
B	27 m	β	
C	20 m	$\alpha\beta\gamma$	
D	12 y.	—	
E	6 d.	$\beta\gamma$	
F	143 d.	α	

Th		α
ThX	4 d	α
Em	53 s.	α
Th A	11 h.	β
Th B	55 m.	$\alpha\cancel{\beta}$
Th C		$\alpha\beta\gamma$
Act.		—
Act X	10·2 d.	α
Em	3·9 s.	α
Act A	36 m.	β
B	2·1 m.	α
C		$\alpha\beta\gamma$

Czy wszystkie gazy przewodzą ? powyżej pewnego — — od ...

Coulomb że część przez gaz

Warburg : pyłki

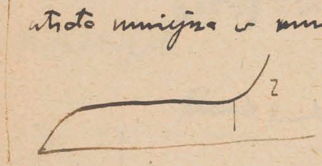


Dokładne pomiary Elster & Geitel, CTR Wilson 1900
1900

naczynie

atoli to mniejsza w mniejszym naczyniu (większa w otwartem)

fundamentalna różnica



wpływ ścian, przydatne itd.

zatem niemożliwa

1) teorya konwekcyjna Coulomb molek. → musiałaby być analog. przewod. cząstek itd:

Warburg pyłki.

2) elektrolit. JJ Thomson

Perrot 1861 El. lyza H_2O pary

JJ Thomson krótkie iskry nadmiar H_2 przy anodzie
<4mm O_2 katodzie

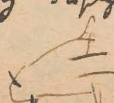
nad— . mniejsze

dłuższe >10mm H_2 kat. O_2 an. } ilość wg praw Farad.

ale nie może to być ogólne bo równie dobrze przewodzą pary pierw. , a nawet Hg, K, Na

coś podobnego : iony , ale które się tworzą tylko w pewnych warunkach i ~~...~~ może większe od atomu

Faraday : wpływ przechod. iskry : władza następuje



Hittorf 1879 jeśli iskra przestała wystarczy bardzo mała napięcie do przewodzenia

Najważniejszy jednak prawdopodobnie Katod.

Vacuum izol. dla małych napięć ale dla wielkich ?

Lilienfeld 1910 : prąd Vacuum między el. wolt... (pozorny rozpl. i por. skrzydła)

Prom. katod.

Plücker 1859 fosf. ściel., Hittorf 1869

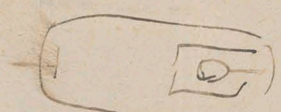
Goldstein 1876 cienie : wiele falowy w eterze

Crookes 1879 strumień cz.

luminesc. Ca Mg Mn Cd Ba Sr Zn Na, Wiedemann Schmidt

ogrz. mechaniczne : tłukące

magn. ~~pole~~ promień „spektrum" Birkeland, ale przy pomocy mocnych wpływów nie ma ...

Perrin 1895 ładunek el. 

Elektrost. odchyl.

Absorpcja Lenard e^{-kh}

	k	ρ	$\frac{k}{\rho}$
H_2 (3 mm h)	0·00149	0·06368	4040
pow. (760)	~~0·476~~ 3·42		~~5610~~ 2780
Collodium	3310		7070
Szkło	7810		3160
Al	7150		2650
Au	55600		2880

$$ne\overline{V} = nm\frac{c^2}{2} = \mathcal{E} = J\,\frac{m}{e}\,\frac{v^2}{2}$$

$$ne = J$$

$$m\frac{d^2x}{dt^2} = e\overline{V} \;\Big\|\; = eHv$$

$$\frac{m}{R}v^2 = eHv$$

$$\frac{1}{R} = \frac{e}{m}\frac{H}{v} \;\Big\|\; \frac{e}{m}\frac{V}{v^2}$$

$$p = 2nmc$$

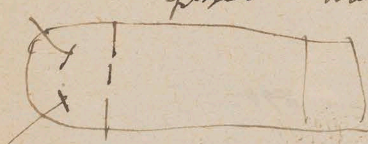
$$= 2J\,\frac{m}{e}\,c$$

$$= 2.10^{-5}\,\text{Amp.} \times 10^{-7}.\ 10^{10}$$

$$= 2.10^{-3}\,\text{dyn}$$

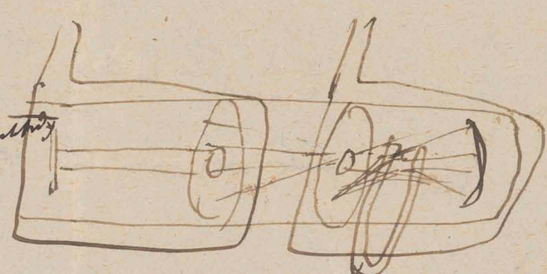
nie dostrzegalne

Kathodenfall



$$\frac{e}{m} = 1{\cdot}8.10^{7}$$ niezależne od gazu i substancji

Des Coudres Wiechert v bezpośrednio



$$22 - 50.000\ \frac{k}{m}$$

$$\overline{V} = \frac{m}{e}\frac{c^2}{2}$$

$$= \frac{\left[5.10^{9}\right]^2}{2}\ \frac{1}{1{\cdot}8.10^{7}} = \frac{25}{3{\cdot}6}.10^{11}$$

$$= 7000\ \text{Volt}$$

200-500 V

Kathodenfall normalny dopóki zależy od gazu i metalu całą katodę nie pokrywa

słabo zależny od ciśnienia i prądu

anormalny zależy od ciśnienia i prądu

wiele podniesienie jeżeli Kathodenfall mniejszy

jeśli przy pokryciu katody CaO

jeśli rozżarzone SrO

BaO

MgO etc.

spada do kilku Voltów

$$v = 0{\cdot}016 - 0{\cdot}11.10^{10}$$

Wehnelt

Kolor natężenie

$2\,cm^2$ Pt CaO nie

$J = 3$ Amp

110 Volt

$p = 0{\cdot}01$ mm Hg

spraw[illegible] druty Richardson H A Wilson

Hertz 1887 Hallwachs 1888 Zn traci – ale nie + UV

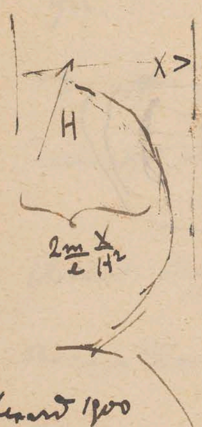
Elster & Geitel	Na	K	Rb
blau	8	30	87
Gelb	8	4	340
Orange	3	2	182
rot	0·7	0·1	21

Topić . Farbstoffy [illegible]
(silnie absorb.
choć nie absorbuje)

zmęczenie

powietrzem w gazach (komplikacje) ale próżnia w próżni

Thomson 1899

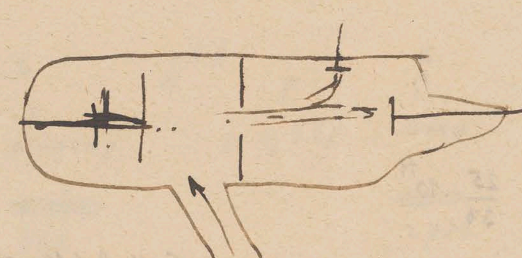


$x = \frac{m}{e}\frac{X}{H^2}\left[1-\cos\left(\frac{e}{m}Ht\right)\right]$

jeżeli odstęp większy niż $\frac{2m}{e}\frac{X}{H^2}$ to żaden elektron nie dosięga płyty –

$\frac{e}{m} = 7\cdot3 . 10^6$

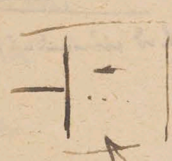
Lenard 1900



skrzywienie w polu magn. i potencjał V

$1\cdot5 . 10^6$ [mniejsze v]

własna prędkość



$\frac{mv^2}{2} = eV$ V ca 8 Volt

Anfangspotential

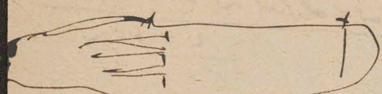
Tak można wytworzyć promienie katodowe o dowolnej prędkości!

Ladenburg: prędkość ~~v~~ (Stern) (Lenard) nie zależy wcale od natężenia światła, tylko od barwy

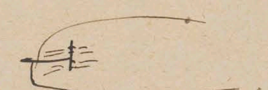
większa dla krótszych fal ale słabo od prop ↗

Dodatnie promienie

I). Kanalstrahlen Goldstein 1886



Także



z otworów katody wolno umieszczonej, ale przykryto katodą [illegible] która może odgiąć [illegible]

żółte w powietrzu

różowe w H_2

cienie; proste, fluoresc. podobnie jak katodowe

1). Goldstein przypuszcza że powstają na katodzie

2). Stark, Wehnelt, Villard etc.... na granicy przestrzeni ciemnej (wyjście pod kat.)

↓

cienie przez ciała w ciemnej przestrzeni

Wien: odchylenie elektr. magn. (3000 V) H_2 $\frac{e}{m}$ ~~7485~~ (wszy H_2) dyspersja spektra
1902 9760

$v = 1{,}5 \cdot 10^8 \frac{cm}{sec}$

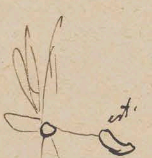
Thomson studiował
1907

Widmo: Goldstein widmo jasne przedłużeniem liniowe

Stark 1905 $\lambda_n - \lambda_p = \lambda_n \frac{v}{c}$ H_2 N_2 Hg Na K

Dopplereffekt

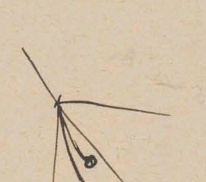
$5 - 6 \cdot 10^7$



$\frac{e}{m} = 1{,}2 \cdot 10^4$

5000

Thomson 1910



H 1

~~He 2~~ He++

C++ 6

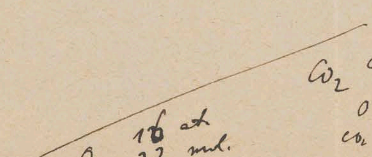
O_2 16 atom
32 mol.

(O_3) 48

~~Hg++ 100~~

Hg + 200

$(Hg_4)+$ 800



O 16 at.
32 mol.

CO_2 C 12
O 16
O 32
CO_2 44 ?

He ... 4

C 12
26

Decrease of Velocity

V.l. 1 — 0·64 V_0

al. foil 0 — 12 13 layers

0·00034 cm

Każdy al. foil ~ 0·54 cm powietrza

R = 6·5 cm

α rays $\frac{e}{m} = 6{\cdot}5 \cdot 10^3$

	Range		Bragg
R	3	0·82 V_0	
Em.	88 - 94	87 - 90	
RA			
RC	6·7	1·00 V_0	

α rays 10.000 R = 39 cm

Kathod. - 0·01 cm

elektrost. Pulltrassen

$\frac{1}{R} = \frac{e}{m} \frac{H}{v}$ $\frac{1}{R} = \frac{e}{m} \frac{V}{v^2}$ $v^2 = 2V\frac{e}{m}$ Parabol

	pow.	H_2	CO
ab. $\frac{1}{2}$ intens.	4·3 mm	16	3

Każda cząstka α prod. 100.000 jonów

prod β : α = ~~[illegible]~~ 0·0033 (Ra)

— mit Kohlegehalten

Ra. Em.

= R A A C 15 min. RC

Iskry Entladungspotential niezale od rodzaju metal

(Lane Mansfeld Winkelmann III 1 p. 41) 9

Kula 1cm

0·01 cm	~~3·38~~	410
0·1	~~15·9~~	4800
0·5	~~60~~	17000
1		25800
2		35000
2m		1000000

zależności od promienia kuli, etc.

dla małych iskier łatwiej duży promień (zależności

odstępu odwrotnie

Verzögerung (Warburg)

Chrystal, Baille, Macfarlane, Paschen, Freyberg, dalsze ścisłe

ważne doświadczenia Paschena: zależność od ciśnienia
(1889) o ... promień

Paschen $V = f(p\delta)$ dla ...

Ciśnienie p	powietrze δ	V	wodór	CO_2
750 mm	0·10	16·33	10·44	17·2
300	0·25	16·83	9·58	17·8
150	0·50	16·54	9·22	16·5
100	0·75	16·23	9·50	16·5

$V = a + b\,d$

przybliżenie

z wyjątkiem dla

bardzo krótkich

zachowanie stałej tendencji

Niemagnetyczny Titan α 140° 0·00036 Königsberger & Schilling.

Magnetyt Fe_3O_4 220°

15°	0·00794
124	0·00520
221	434
347	555
485	0·0112

PbS [illegible] <-189°

bardzo dobry przewodnik, ani śladu elektrolizy

O dielektrykach później będziemy mówili

Co do teorii: elektrolityczne przewodzenie zupełnie zrozumiałe

co do metali więc — dopiero z gazów na [illegible] wnioskować możemy o metalach

Gazy: przy zwykłym ciśnieniu: Iskry, szczotki, łuk

ale do tego potrzeba pewnego napięcia

1). [illegible] bardzo mało swobodnych, zależnie od wielkości kolca

różne + i −

2). Łuk elektryczny Davy 1821 kontakt (albo zbliżenie) potem rozsunąć

dla węgla potrzeba 30–60 V.

$$\Delta E = \underset{40V.}{A} + D l J$$ Edlund

elektromotoryczna [illegible] [illegible]

z drugiej strony można to także napisać $JW = A + DlJ$

$$W = \frac{A}{J} + Dl$$

ujemny El.

dodatni węgiel gorętszy 3500°? i [illegible]

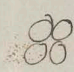
ujemny 2700°

[illegible] przy [illegible]

~~[illegible]~~ A przez impregnowanie solami i [illegible] światła (Bremer) obl. teoret. $v_1 = \frac{X e \lambda}{2 m c}$ przy założ. że jon = drobina:

	He	H_2	O_2	CO_2	SO_2	Cl_2
obl. v =	47·3	26	3·8	2·06	1·25	1·1
obs.	1·4	7·2	1·36	0·78	0·5	1·0
$\frac{\text{obl.}}{\text{obs.}}$ =	34	3·8	2·7	2·7	2·5	1·1

Gdyby zaś jon się składał z n drobin:



$$\lambda = \frac{1}{\sqrt{2}\, \pi N \sigma^2} \qquad R = r \sqrt[3]{n}$$

$$\sigma = \frac{\sigma_0}{2}\left(\sqrt[3]{n} + 1\right)$$

$$\cancel{\lambda = \frac{4\lambda_0}{(1+\sqrt[3]{n})^3}}$$

$$\lambda_1 = \frac{1}{\pi\left[(r_1 + r_2)^2 n_2 \sqrt{\frac{m_1 + m_2}{m_2}} + 4 r_1^2 n_1 \sqrt{2}\right]}$$

$$r_2 = r_1 \sqrt[3]{n}$$

$$\lambda_1 = \frac{1}{\pi r_1^2 N \left(1 + \sqrt[3]{n}\right)^2 \sqrt{1+n}} = \lambda_0 \frac{4\sqrt{2}}{\left(1+\sqrt[3]{n}\right)^2 \sqrt{1+n}}$$

$$w : w_0 = 1 : \sqrt{n}$$

$$v_1 = v_0 \frac{4\sqrt{n}\sqrt{2}}{n\sqrt{1+n}\left(1+\sqrt[3]{n}\right)^2}$$

$n = 2$ $\qquad \frac{4 \cdot 2}{\sqrt{3}\,(2\cdot 26)^2}$ $\qquad$ 0·26

~~1571~~
2385
7082
9065
8467
8564

Thomson $\lambda = \frac{2\lambda_0}{n+1}$ = stosunek przekrojów (ekwatorycznych)

$$v = v_0 \frac{2}{n+1} \frac{\sqrt{n}}{n} = \frac{2 v_0}{n+1\,\sqrt{n}}$$

$n = 2$ $\qquad v_1 = \frac{2}{3.14} \neq \frac{1}{2} v_0$

$n = 4$ $\qquad v = \frac{2}{5.2} \neq \frac{1}{5} v_0$

Hość rekombinacji ~~$\frac{mc^2}{3}$~~ Harms Jahrb. d. R. 3 p. 330

Rutherford $\frac{dn}{dt} = -\alpha n^2$

I). jonizacja

II). pomiar czas t

III). usunięcie pola elektr. wszystkich wyprowadzeń

$$\frac{\alpha}{e} = \frac{i_M^2 - i^2}{i^2} \frac{(u_1+u_2)\cdot V^2}{l} \neq \frac{i_M}{i^2} \frac{(u_1+u_2) V^2}{l}$$

Thomson & Rutherf.

---

według teorji Thomsona α niezal. od ciśnienia

$\frac{\alpha}{e}$ =	powietrze Townsend 3420	
O_2	3380	wszystko prawie niezal. od natury gazu zgodnie z teorją Thomsona
CO_2	3500	
H_2	3020	

Ruchliwość jonów

$v_+ \quad v_- \quad \left[\frac{1\,Volt}{1\,cm} \middle| \frac{cm}{sec} \right]$

760 mm

	v_-	v_+	v_-/v_+
H_2 =	7·18	5·67	1·26
wilg.	5·60	5·30	1·05
~~O_2 =~~	~~1·68~~	1·36	~~1·25~~
pow.	1·71	1·25	1·36
	1·51	1·37	1·10
CO_2	0·84	0·78	1·08
	0·75	0·82	0·91

Langevin: 3 zetknięcia = 3 rodzaje jonów, dwa + jeden −

Air	Vk_-	(800 Volt) pk_2	Vk_+	pk_1
p = 75 mm	~~219~~ 6560	647	4430	437
200	2204	580	1634	430
410	994	530	782	427
760 cm	510	570	480	420
1420	270	505	275	425

$X\frac{e}{m}$ = przysp. $\quad \frac{Xet}{2m}$ = przeciętna prędkość $\quad t = \frac{\lambda}{\omega}$ jeżeli prędkość mała w porówn. z c

$$= \frac{Xe\lambda}{2m\omega}$$

więc jeżeli ion się nie zmienia $\quad v \sim \lambda$

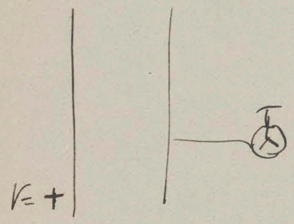
$$= \frac{1}{p}$$

zatem przy rozrzedz. wzrasta, jony się rozdrabniają

Prędkość jonów

Langevin

prędkość w polu X: $v_p X$, $v_n X$



1). gaz ioniz.

2). promieniowanie ust., potem lewej płyty $= +V$ podczas czasu t

3). potem lewej płyty $= -V$ zwykle

2). do elektrometru wyszło $(v_p X)t$ pozostanie $(l - v_p X t)$ +

do płyty $(v_n X)t$ $(l - v_n X t)$ −

I. 3). do elektrom. $(v_p X t) - (l - v_n X)t = [(v_p + v_n)Xt - l] = q$

Jeśli jednak t takie długie że wszystkie ujemne jony wyszły

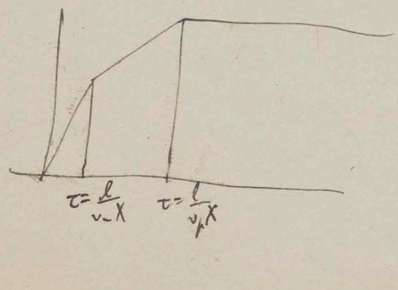
$$l - v_n X t = 0$$

$$\tau = \frac{l}{v_n X}$$

II). do $q = v_p X t$.

Jeśli także wszystkie + jony wyszły τ

III). $q = v_p X \tau$ $\tau = \frac{l}{v_p X}$



Absorpcja Lenard 12

$\frac{1}{\lambda} \propto \sqrt{V}$

Pohl i Pringsheim

Normalny efekt: ~~[illegible]~~ $\frac{1}{\lambda}$ bez optycznej polar.

Selektywny dla E ∥ dla pewnych λ dla Na K

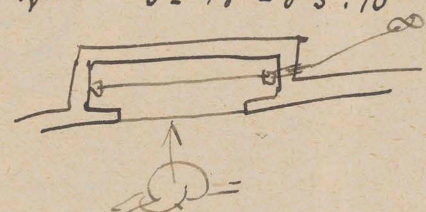
Volt		
4	31'4	Lsp
~~[illegible]~~	28	
100	19	
500	9	
1000	3	
4000	1'2	
20000	0'005	

Tak samo Rö R. ~~[illegible]~~ ale znacznie szybciej

Dorn wg. diss. 1900 $v = 1'8 - 8'5 . 10^9$

Curie & Sagnac

~~Deutty~~



$v_m = 6 - 8 . 10^9$

Innes

Deutty 1910 zależności od rodzaju Rö R

Ladenburg [illegible] Lampe

		Pt	Zn	
λ	260 μμ	4'075	0'685	kiedy λ większe ale
	201	1'86	4'12	

Oprócz tego różne inne siarczki, gorzej przewodzące, węgiel, tlenki metali, sole etc. etc.

Zasadnicza różnica 1), z transportem materii (elektroliza), polaryzacja

2). lub bez

1). Sole ~~[illegible]~~ $NaCl$ etc $BaSO_4$ (Baryt) Königsberger & Schilling Ann. 32, 186

$BaCl_2$ (także SO_4 da się wykazać) [illegible]

2). Fe_3O_4, FeS_2, Si, Ti, Zr, Fe_2O_3, PbS, Adular, Sb_2S_3?, Naphtalin, Anthracen

---

Zachowanie metali się przy zmianach w zależności od temp.:

$\sigma = \sigma_0(1+\alpha t)$

Clausius 1858 opór $\sim \theta$ [zwykle szybka zmienność $\alpha = 0{\cdot}0036 - 0{\cdot}0048$

Clay (Kamerlingh Onnes) 1908 -259° α się zmniejsza tak jakgdyby dążność do niezmiennego oporu

---

Alliaże jeśli mieszaniny kryształów to opór = w. stosunku przez skład.

jeśli roztwory, znacznie ~~większy~~ opór bez porównania większy, a współczynnik temp. mniejszy

Dziwne koleje rozwoju nauki

To co najdawniej znane z elektryczności: rozbrzęczenia w gazach, iskry a wydarzały się najmniej zrozumiałe

W Volty i Ohma przewodzenie w metalach. Wydawało się najprostsze, normalny typ

Opór tego przewodzenia w elektrolitach ogniwa

W tym obrębie ścisłe dokładne badania.

$-259{,}2^\circ: 1\cdot10^{-8}$

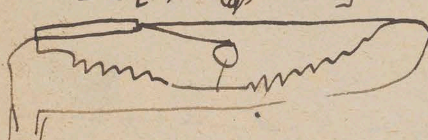
Metale: prawo Ohma, <u>brak transportu materyi</u> Ag $1{,}6\cdot10^{-6}$ Bi $1{,}2\cdot10^{-4}$ // C węgl. $28\cdot10^{-3}$

El. lity: " <u>transport ~~...~~ zmian chemicznych</u>

Ścisłość prawa Ohma?

Maxwell i Chrystal 1876

$$R = a(1+\alpha i^2+\beta i^4+\dots)$$
$$= a\left[1+\alpha\left(\frac{I}{q}\right)^2+\dots\right]$$



1. Czy opór stały

Lecher drut srebrny $q = 0{\cdot}03\,mm^2$ 10 ... Paalzow

dwa druty (z tego samego materyału) jeden cienki długi drugi gruby krótki, o tem samem oporze przy słabym prądzie

Czy taki sam opór przy silnym prądzie?

Różnica $< 10^{-12}$ pierwotnej wartości dla silnych i słabych prądów

trudność ogrzania, doraźnie ominięta

2. Czy opór stały dla prądów zmiennych?

Z pewnością znaczny ale dopiero przy prądach przemiennych o szybkości zmian jak dla drgań świetlnych (Rubens & Hagen)

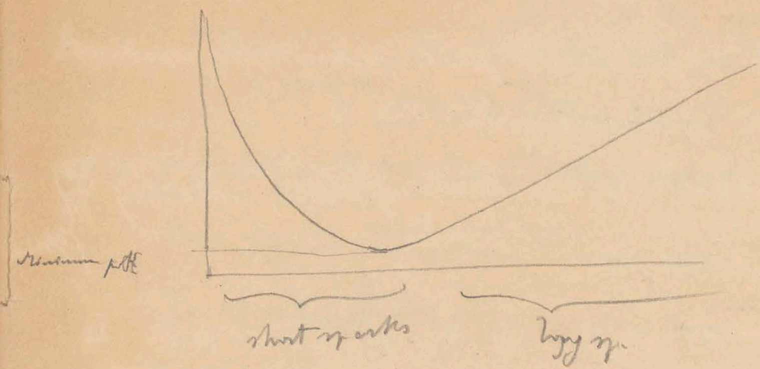
Elektrolity — Trudności: polaryzacya, zmiany koncentracyi, ogrzanie

zarazem tutaj tylko prądy przemienne dokładny zbadane

Fitzgerald i Trouton 1887

$e = iw(1-ci^2)$ $c<3\cdot10^{-6}$ jak jest w elektrolitach?

V increasing for l diminishing! (Short spark region)



exactly such curves have been found

cf.

in other words spark length = fn (p.l)

(putting c = l we get the minimum value)

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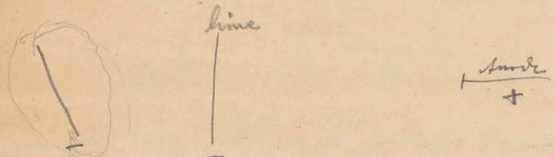
cathode: piece of hot lime (gives out large number of corpusc.)



sending disch. between the 2 electr.

applying pot diff. to the other plate so as to make it an independent cathode

dark space is formed round it

for greater pt. diff. dark space small, for decreasing pot diff it spreads out

if = 0 through the whole tube

Thus dark space depends not only on pressure of gas, but also on intensity of current.

The disch between lime and anode provides the carriers of the current

Dark space behaves like surface of contact potential (certain diff betw c. and a.)

∴ in order to produce luminosity the corp. have to acquire certain amount of energy

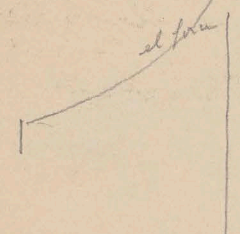
they don't make the gas luminous until they get this amount of energy

this amount may depend also on strength density of current itself (if storage of energy)

Very abrupt change, dark space can be determined to a fraction of a mm

Striation of discharge (in + column) Wehnelt tube { if we get an independent supply of corp. (hot lime) then is no cathode fall (because not necessary for production of − corp.)

If small pot. diff. current runs through, but not luminous; current carried by those corpuscles emitted by hot lime



force increasing from c. towards anode

When force at anode ~~so large~~ sufficiently large,

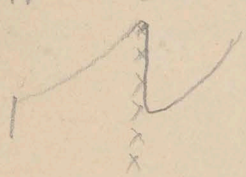
luminosity appears at the anode

With increasing pot. diff. this glow gets thicker, the glowing part is a good conductor nearly as if anode were pushed forward by the same distance; the field gets stronger on the left

Then sometimes there appears a luminous sheet (in the middle) between anode and cathode

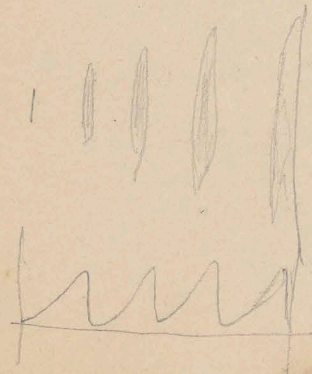
Suppose with great current density luminosity may appear easier for smaller corp. than with small

Here current decreases while force increases with distance of cathode, so there may happen a stage where ionization produced



here the force must fall, because good conductor

then it may be too small to prod. luminosity, therefore there will adjoin a dark space



In the normal case when glow on anode only: as long as ~~...~~ glow confined to neighbourhood of anode, corpuscules produced afresh have not any occasion to produce any more corpuscules increase of ions will not be very large.

But if field increasing, until strong enough to prod. fresh ions close to cathode itself, then very big increase of ionization, glow suddenly starts at cathode enormous increase of current

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Anode fall of pot. dep. on pressure of the gas (circa 20-30 V.)

It takes place in a very small space (one cannot come close enough to the anode as to get anything less than 20 V.)

much more abruptly than cathode fall // distance seems to be << mean free path like double layer of el.

In Wehnelt tubes we get rid of cathode fall (300 V) but not of anode fall

this gives a lower limit for the pot diff required to get luminosity (20 V.)

No explanation. Depends on nature of metall (those which have big c.f. have small a.f.)

In order to avoid anode fall it seems to be necessary to emitt ions from anode itself otherwise accumul. of − el. which cause going up of dV/dx

Is anode fall not there in order to provide for emission of ions from anode?

If metall plate bombarded by − ary., will it give out + ions?

Secondary radiations does it contain + ions?

insulation loss

zero 313 glow lamp screened
x 318 with dark paper
325
331
336 glow lamp put out
341
344 all light out
348
351 glow lamp screened
354
zero 321
x 328 ||
332
550 218
731 181 } 621 : 3 = 207
953 222 put out
988
986
985
zero 329
zero 304
x 306 organd lamp on
376 70
453 77 687 : 9 = 75.7
532 79
610 78
687 77
762 75 zero 320 | 2^h15
834 72
913 79 987 74

4^h15
zero 296 Argand lamp the whole time shining
363 67
439 76 ?
515 66 705 : 10 =
582 67
653 71 70.5
725 72
795 70
864 69
932 68
1002 69

4^h33 glow lamp
zero 296
470 174
657 187
845 188

4^h49 fresh surface
zero 295 Argand
363 68
435 72 695 : 10 = 69.5
507 72
577 70
642 65
712 70
781 69
852 71
921 69
990 69

$$r^2 = [x + (t_0 - t)u]^2 + y^2 + z^2$$

$$t = t_0 - \frac{r_0}{c}$$

$$r^2 = \underbrace{x^2 + y^2 + z^2}_{r_0^2} + \frac{r_0^2 u^2}{c^2} + 2xu\frac{r_0}{c}$$

Crookes [illegible] !

jeśli się przytrzymuje [illegible] o [illegible] póki to prędzej obraca się

fałszywe tłomaczenie

$\frac{e}{m} = 10^7$ $i = 0.001$

$m = 10^{-10}$

$v = 10^{10}$

$mv = 1 \frac{Dyn}{cm^2}$

D. Katodowe

U. β

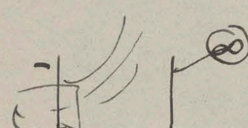
Katodowe [illegible]

Kathodenfall		
Pt – H_2	298 V.
N_2	232
H_2O	469
NH_3	482
O_2	369

3). Fotoelektr.

Hertz 1887 Hallwachs Lenard 1900

Thomson



4). Wtórne prędkości mniejsza

5). Rozżarzone metale Richardson

J J Thomson $\frac{e}{m} \neq 10^7$

6). Rozgrzane ~~[illegible]~~ tlenki ~~[illegible]~~ Wehnelt

CaO SrO BaO

Kathodenfall spada aż do 5 Voltów

$\frac{e}{m} = 1.4 \cdot 10^7$

różnice:

$1.6 \cdot 10^8 < v < 10.7 \cdot 10^8$

Absorpcja:

Lenard $c \ldots \frac{c}{100}$ $\alpha \ldots 10^6$

				$\frac{\alpha}{\rho}$
H_2	$p = 3.3$ m	0.00149	$\rho = 0.000000368$	4040
	760	0.476	0.000085	5610
pow.	760	3.42	0.00123	2780
papier	2690	1.30		2070
szkło		7810	2.47	3160
Al		7150	2.70	2650
Ag		32200	10.50	3070
Au		55.600	19.30	2880

Biot Savart

$$\vartheta = \frac{e}{r^2}$$

$$\mathfrak{H} = \frac{1}{c} V \sin\vartheta = \frac{e u \sin\theta}{c r^2}$$

$$H = \frac{n e \sin\theta}{r^2} ds \quad \dots \quad ds = \frac{u}{n} \quad = \frac{u e \sin\theta}{r^2}$$

$$\int_0^\pi \sin^3\theta \, d\theta = \int \sin\theta \, d\theta - \int \cos^2\theta \sin\theta \, d\theta = -\cos\theta + \frac{\cos^3\theta}{3}\Big|_0^\pi = \frac{4}{3}$$

$$\frac{1}{8\pi}\iiint H^2 dv = \frac{e^2u^2}{8\pi}\int_a \frac{\sin^2\theta}{r^4} r^2 \, 2\pi \sin\theta \, d\theta \, dr$$

$$= \frac{e^2u^2}{4} \cdot \frac{4}{3} \cdot \frac{1}{r}\Big|_a^\infty$$

$$= \frac{e^2u^2}{3a} = \frac{mu^2}{2}$$

$$m = \frac{2e^2}{3a}$$

$$a = \frac{2}{3}\frac{e}{m} \cdot e =$$

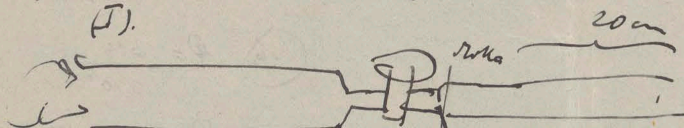
$$e = 3{\cdot}4 \cdot 10^{-10} \cdot \frac{\cancel{}}{3.10^{10}} = 10^{-20} \text{ (em)}$$

$$\frac{e}{m} = 2.10^{7} \text{ (em)} =$$

$$a = \frac{4}{3} \cdot 10^{7} \cdot 10^{-20} = 1{\cdot}3 . 10^{-13} \text{ cm}$$

α cząstki Rutherford a Geiger

(I).



(II) Scyntylacje

$N = 3{\cdot}4 . 10^{10}$ dla Ra samego

4 razy tyle dla R. z RaC ...

Ład

$$\frac{E = 9{\cdot}3 . 10^{-10}}{e = 4{\cdot}65 . 10^{-10}}$$

Thomson 3·4

Wilson 3·1

Millikan 4·06

Planck 4·69

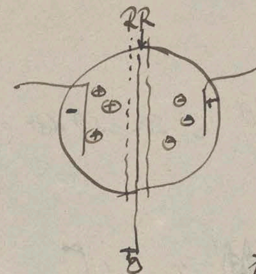
(Rutherford p. 50)

C T R Wilson $1{\cdot}25 < \frac{v_2}{v_1} < 1{\cdot}38$

$$u = \frac{2}{9} \frac{\rho g a^2}{\mu}$$

Thomson $\frac{v_2}{v_1} = 1{\cdot}31$ różnica w $\pm$ rozm.?

C T R Wilson 1899:



$$e\,V = \frac{4\pi}{3} \rho g a^3$$

ale $\frac{v_2}{v_1} = 1{\cdot}31$ tylko dla —

jeśli dość duża expansya to równe ilości kropelek i równe szybkos opadają

$$a = \sqrt{\frac{9}{2} \frac{\mu}{\rho g} u}$$

$$e = \frac{\frac{4\pi}{3} \rho g}{V} \left(\cdot\ - \right)^3$$

e JJ Thomson $= 3{\cdot}4 . 10^{-10}$ (est)

H A Wilson $= 3{\cdot}1 . 10^{-10}$

równe e w O_2, H_2

większe różnice w $\frac{m}{e}$ pochodzą od m

$$H = \frac{\overset{i\,ds}{e u \sin\theta}}{r^2}$$

$\overset{e}{o} \rightarrow$

$$T = \frac{1}{8\pi} \int_{r=a}^{\infty} H^2 dv$$

$$= \frac{e^2 u^2}{3a}$$

Rowland:

$$4\pi\sigma = \frac{\partial V}{\partial n} = 12.300\,V. = \frac{3600\,V.}{1\,cm}$$

$$\sigma = 1\,(est) = \frac{1}{3.10^{10}}\,(em)$$

(10cm) $\varphi = \frac{314}{3.10^{10}}$

$$i = n\varphi = \frac{100.\,100}{10^{10}} = 10^{-6}\,em$$

$$= 10^{-5}\,Amp.$$

$$H = \frac{2\pi i}{a} \neq 10^{-5}$$

$$tg\,\varphi = \frac{10^{-5}}{0{\cdot}2} = \frac{1}{2}.10^{-4}$$

$$\delta = 1\,m\ tg\varphi$$

$$= \frac{10^{-1}}{2} = \frac{1}{20}\,mm$$

18

$$A = \sin\delta + \ldots = \frac{-(1-\cos m\delta)\sin\delta + (1-\cos\delta)\sin m\delta}{1-\cos\delta} = \sin m\delta + \sin\delta\,\frac{1-\cos m\delta}{1-\cos\delta}$$

$$B = 1+\cos\delta + \ldots = \frac{(1-\cos\delta)(1-\cos m\delta) - \sin\delta\sin m\delta}{1-\cos\delta} = (1-\cos m\delta) - \sin\delta\,\frac{\sin m\delta}{1-\cos\delta}$$

$$A^2+B^2 = \sin^2 m\delta + \cos^2 m\delta - 2\cos m\delta + 1 + \frac{\sin^2\delta}{(1-\cos\delta)^2}(1-2\cos m\delta+1) +$$

$$+\,2\,\frac{\sin\delta}{1-\cos\delta}\left[\cancel{\sin m\delta\,(1-\cos m\delta) - \sin\delta\,(\ldots)}\right]$$

$$= 2\left\{(1-\cos m\delta) + \frac{\sin^2\delta\,(1-\cos m\delta)}{(1-\cos\delta)^2}\right\} = 4\,\frac{(1-\cos m\delta)}{(1-\cos\delta)}$$

$$= 4\,\frac{\sin^2\frac{m\delta}{2}}{\sin^2\frac{\delta}{2}}$$

$$I = I_{1\cdot\frac{\delta a}{2}}\;\frac{\sin^2\left(\pi\,\frac{\delta\sin\beta}{2\lambda}\right)}{\pi\,\frac{\delta\sin\beta}{2\lambda}}\;\;\frac{\sin^2\left(m\pi\,\frac{b\sin\beta}{2\lambda}\right)}{\sin^2\left(\pi b\,\frac{\sin\beta}{2\lambda}\right)}$$

$$\frac{\sin^2 m b\mu}{\sin^2 b\mu}$$

Maximum:

$b\mu = k\pi \qquad \| \; \mu = \pi\,\frac{\sin\beta}{2\lambda}$

$\frac{b\pi\beta}{2\lambda} = k\pi$

$b\beta = 2k\lambda$

następne min: $mb(\mu+d\mu) = mk\pi+\pi$

$$d\mu = \frac{\pi}{mb} = \pi\,\frac{\cos\beta\,d\beta}{2\lambda}$$

$$d\beta = \frac{2\lambda}{mb\cos\beta} = \text{szerokość prążka}$$

dla $\lambda+d\lambda$:

$$d\beta = \frac{2k\,d\lambda}{b}$$

$$\frac{2k\,d\lambda}{b} > \frac{2\lambda}{mb}$$

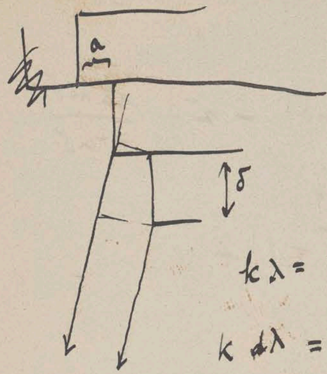
$$\frac{d\lambda}{\lambda} > \frac{1}{mk}$$

Na D: $\frac{d\lambda}{\lambda} = 0.001$

$m\,k_{II} = 500$

(trzeba 500 rysy …)

Promienie równoległe



$$k\lambda = n\delta + a\sin\beta - \delta\cos\beta \neq \delta(n-1)$$

$$k\,d\lambda = \delta\,dn + (a\cos\beta + \delta\sin\beta)\,d\beta$$

$$\neq \delta\,dn + a\,d\beta$$

$$d\beta = \frac{k\,d\lambda - \delta\,dn}{a} = \frac{\delta}{a}\left[\left(\frac{n-1}{\lambda}\right)d\lambda - dn\right]$$

zdolność rozszczepiania bardzo wielka

N.p. $\delta = 18$ mm, $a = 1$ mm, $N = 20$

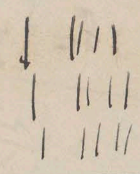
$\frac{1}{500}$ odstępu

5 m przypada

$$(k+1)\lambda = n\delta + a\sin(\beta+d\beta) - \delta\cos(\beta+d\beta)$$

$$\lambda = (a\cos\beta + \delta\sin\beta)\,d\beta \neq a\,d\beta$$

$$d\beta = \frac{\lambda}{a}$$



$$U(x, y, z, r)$$

$$\frac{dU}{dx} = \frac{\partial U}{\partial x} + \frac{\partial U}{\partial r}\cos(rx)$$

$$\frac{d}{dx}\left(\frac{1}{r}\frac{\partial U}{\partial x}\right) = \frac{1}{r}\frac{\partial^2 U}{\partial x^2} - \frac{1}{r^2}\frac{\partial U}{\partial x}\cos(rx) + \frac{1}{r}\frac{\partial^2 U}{\partial r\,\partial x}\cos rx$$

$$\frac{d}{dy}$$

$$\frac{d}{dz}$$

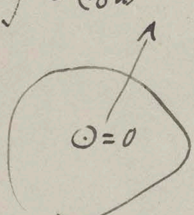
$$\frac{d}{dx}(\ \ ) + \frac{d}{dy}\ \ \cdots = \frac{1}{r}\nabla^2 U - \frac{1}{r^2}\underbrace{\left[\frac{\partial U}{\partial x}\cos rx + \cdots\right]}_{= \frac{dU}{dr} - \frac{\partial U}{\partial r}} + \frac{1}{r}\underbrace{\left[\frac{\partial^2 U}{\partial r\,\partial x}\cos rx + \cdots\right]}_{= \frac{d}{dr}\left(\frac{\partial U}{\partial r}\right) - \frac{\partial^2 U}{\partial r^2}}$$

$$\frac{d}{dr} = \frac{\partial}{\partial r} + \frac{\partial}{\partial x}\cos rx + \frac{\partial}{\partial y}\cos ry + \frac{\partial}{\partial z}\cos rz$$

$$= \frac{1}{r}\nabla^2 U - \frac{1}{r}\frac{\partial^2 U}{\partial r^2} + \underbrace{\left[\frac{1}{r}\frac{d}{dr}\frac{\partial U}{\partial r} - \frac{1}{r^2}\left(\frac{dU}{dr} - \frac{\partial U}{\partial r}\right)\right]}_{\frac{1}{r^2}\frac{d}{dr}\left(r\frac{\partial U}{\partial r} - U\right)}.$$

$$\cancel{\frac{1}{r^2}\frac{\partial U}{\partial r} + \frac{1}{r}\frac{d}{dr}\frac{\partial U}{\partial r} - \frac{1}{r^2}\frac{dU}{dr}}$$

$$-\int \frac{1}{r}\left(\frac{\partial U}{\partial n}\right) dS = \iiint \frac{\nabla^2 U - \frac{1}{r}\frac{\partial^2 U}{\partial r^2}}{r}\, d\tau + \iiint \frac{1}{r^2}\frac{d}{dr}\left(r\frac{\partial U}{\partial r} - U\right) d\tau$$

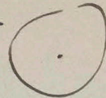


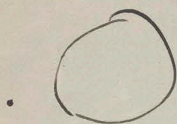
$$\int \frac{1}{r^2}\frac{d}{dr}\left(r\frac{\partial U}{\partial r} - U\right) d\tau = \int d\varphi \int^{r} dr\, \frac{d}{dr}\left(r\frac{\partial U}{\partial r} - U\right) =$$

$$\int d\varphi \left[\left(r\frac{\partial U}{\partial r} - U\right)_{r=r} - \underbrace{(\ \ \ )_{r=0}}_{4\pi U_0}\right]$$

$$d\varphi = \frac{dS\cos(n r)}{r^2}$$

$$-\int\left[\frac{1}{r}\frac{\partial U}{\partial n} - \cos(n r)\frac{\partial}{\partial r}\left(\frac{U}{r}\right)\right] dS = \int \frac{1}{r}\left(\nabla^2 U - \frac{\partial^2 U}{\partial r^2}\right) d\tau + 4\pi U_0 .$$

Źródło w kole 



$$-\int\left[\frac{1}{r}\frac{\partial u}{\partial n}-\cos(n r)\,\frac{\partial}{\partial r}\left(\frac{\partial u}{\partial n}\right)\right]dS' = \int\frac{1}{r}\left(\nabla^2 u-\frac{\partial^2 u}{\partial t^2}\right)d\tau$$

$$u = s\left(t-\frac{r}{v}\right)$$

$$r=0 \qquad u_0 = s_0$$

$$\frac{\partial^2 u}{\partial t^2} = v^2\,\nabla^2 u$$

$$= v^2\frac{\partial^2 u}{\partial r^2}$$

$$4\pi s_0 = \int\left\{\frac{\partial\left[\dfrac{s\left(t-\frac{r}{v}\right)}{r}\right]}{\partial r}\cos n r-\frac{1}{r}\,\frac{\partial\left[s\left(t-\frac{r}{v}\right)\right]}{\partial n}\right\}dS'$$

porównaj z Huyghens $\int s\left(t-\frac{r}{v}\right)\cos n\,dS$

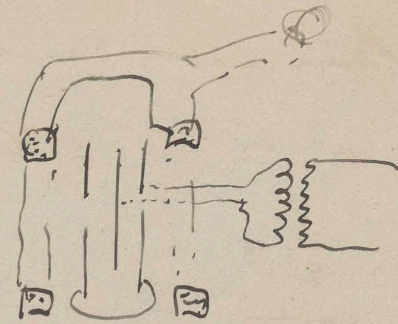
Rowland 1876, 1889

Crémieu 1900

Pender 1901

Crémieu & Pender 1903

Vasilesco Karpen



Lenard: zewnątrz w powietrzu tylko kilka cm, w rozrzedzonym H_2 do 1 m

obrob. zależna tylko od masy

robią gazy przewodnikiem, ozonizują, kondensują parę

V). Prąd. ~~Ele~~ Elektr. zilość prom. $J = Ne$

Ciepło $\quad \mathcal{Q} = \frac{Nmv^2}{2}$

(Thomson)

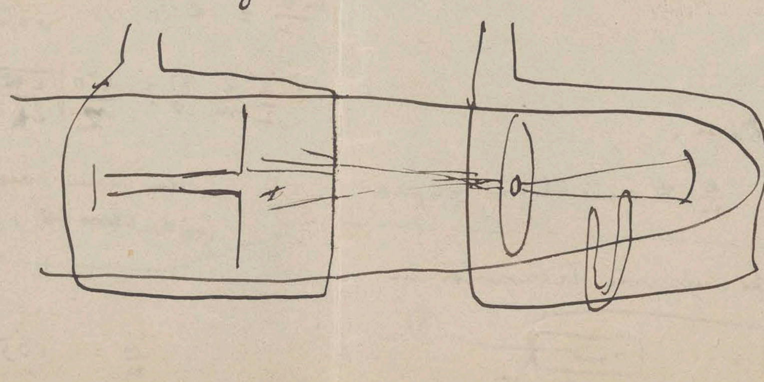
$$\frac{\mathcal{Q}}{J} = \frac{1}{2}\frac{mv^2}{e}$$

$$R = \frac{m}{e}\frac{v}{H}$$

$$\frac{\mathcal{Q}}{JR} = \frac{v}{2H} \quad \text{itd.}$$

Des Coudres Wiechert $\frac{r}{8}$

ziemia 29'5 km



Stała Faradaya $= 96513$ Kulomb $=$ ilość wyz. 1 równoważnik gramowy jonu ($= 1{,}01$ gr H_2

8 gr O_2

1 Amper na minutę $6{,}96$ norm cm^3 H_2

$n = \frac{8 \cdot 10^{19}}{0{,}000089}$

w wyst. bardz. en $\left(\frac{e}{m}\right)_H = 9651{,}3 : 1{,}01 = 9556$

561
563
583

$e = \frac{96513 \cdot 8 \cdot 10^{-5}}{8 \cdot 10^{19}} = 10^{-19}$ Coul. $= 10^{-20}$ (em)

$\frac{7 \cdot 0{,}0013}{16 \cdot 60 / 1000} = 1 \cdot 10^{-5}$ $= m$, cm 1 Amp.

$\rho = 0{,}000089873$

$e = 4{,}69 \cdot 10^{-10}$ (eA)

---

magn. Ablenkung

$\frac{dx}{dt} = v \qquad t = \frac{x}{v}$

(I)

JJ Thomson

$$m \frac{d^2y}{dt^2} = eHv = m \frac{d^2y}{dx^2}\left(\frac{dx}{dt}\right)^2 = m \frac{d^2y}{dx^2} v^2$$

$$\frac{d^2y}{dx^2} = \frac{e}{m}\frac{H}{v}$$

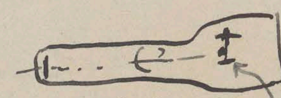
$$R = \frac{\left(1+\frac{dy}{dx}\right)^{3/2}}{\frac{d^2y}{dx^2}} \neq \frac{m}{e}\frac{v}{H}$$

---

elektr. Abl.

$$\frac{dx}{dt} = v$$

$$m \frac{d^2y}{dt^2} = eE = m \frac{d^2y}{dx^2} v^2$$



cień się

zwęża gdy dodatnio

III).

$$\frac{d^2y}{dx^2} = \frac{e}{m}\frac{E}{v^2}$$

$$y = \frac{e}{2m}\frac{E}{v^2} x^2$$

$2 \cdot 1{,}86 \cdot 10^7 \cdot 10^8$

~~$2 \cdot 10^{16}$~~

$\cancel{4} 6 \cdot 10^7 \sqrt{\text{Volt}}$

---

z drugiej strony Kaufmann:

$$\frac{mv^2}{2} = eV_0$$ = Entladungspotential

$$v = \sqrt{\frac{2eV_0}{m}}$$

(II).

$$y = \frac{x^2}{2}\frac{e}{m}\frac{H}{v} = \frac{x^2 H}{\cancel{2}}\sqrt{\frac{e}{2V_0 m}}$$

przesunięcie obrazu na ekranie fluor.

$$y \sim \frac{1}{\sqrt{V_0}}$$

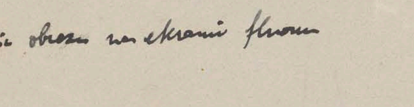
y niezależne od gazu!

tj. że $\frac{e}{m}$ to samo dla wszystkich gazów

~~im~~ większe rozrzedzenie tem większe V_0 (~~do~~ do ~~5000~~ Volt) i tem mniejsze R

Perrin Pedrunkt ujemne transportowane

IV).



$$\frac{e}{m} = 1{,}865 \cdot 10^7$$

Becquerel 1896 Ur

Tho : Curie, Schmidt

Radium } 1898 Curie ← Demarçay . . . . Pechblende Tonne : 0·2–3 gr. Radium
Polonium } spekt. nyttt.

Aktinium Debierne Ba åtskil 225 at 1°

Rad. akt. före

α Ga ... Goldstein Canal.

β Kath.

γ Rntg.

β: $\mu H \rho = \frac{mv}{e}$ magn Abl.

$\frac{h L^2}{2} y = \frac{mv^2}{e}$ elektr. Abl. / y =

$L = V$

$h = \text{fri}$

Kaufmann u. fr.:	$\frac{e}{m}$	v		Elektrolyse $\frac{e}{m} = 9650$
	$1{\cdot}865 . 10^7$	$0{\cdot}7 . 10^{10}$	Kath.	u e b $m = \frac{1}{2000}$ Hatom
	$1{\cdot}31$	$2{\cdot}36$	R.	
10^8 Condukt rornov. premary	$0{\cdot}97$	$2{\cdot}59$		
	$0{\cdot}63$	$2{\cdot}83$		

α: ~~H~~ 5 er ... β ~~Abl~~ Alum $\delta \angle \frac{1}{100}$ mm 56 ds. ; ~~...~~ / 1 cm ... as 10 cm

< 5 ... R. gr (Towns)

Rutherford }
Des Coudres } Vacuum ablegten

$v = 1{\cdot}65 . 10^9$ $\frac{e}{m} = 6400$ ... = Hatom !

Spinthariskop Zn SO_4 phosph. $\frac{1}{2}$ mm ...

Wärmeentwicklung stündlich 80 cal pro 1 gr. | 73 cm³ H_2 ... H_2

Heydweiller Adolf 1902: zatopione w rurce szklanej

5g (zmniejszenie działanie 0·02

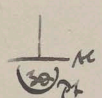
Becquerel oblicza ilość energii

Dorn Ernst 3/12 1902 29·9 mg przeczenie

$\frac{e}{m}$ takie znalezione poruszają w [illegible] gorące identyczne, później równy do [illegible] (Abraham)

W. Wien elektryzacja dodatnia

jeśli Al



zrazu nie [illegible] ± w równej ilości wysyłane

Induzierte Radioaktivität Phys. Ztschr. 2 p. 500, 513

$$\int E^2 dv = \int X^2 dv + \int R^2 dv$$

$$x = x_0\sqrt{s}$$

$$\frac{\partial\varphi}{\partial x} = \frac{\partial\varphi}{\partial x_0}\frac{1}{\sqrt{s}}$$

$$= \sqrt{s}\int X^2 dv_0 = \frac{1}{\sqrt{s}}\int X_0^2 dv_0 + \sqrt{s}\underbrace{\int R_0^2 dv_0}$$

$$= \frac{2}{3}\int X_0^2 dv_0$$

$$\frac{1}{2}\frac{e^2}{r^2}\ \ \int r^2 dr \cdot \frac{e^2}{r^4}\ \ \frac{e^2}{2a}$$

$$= \frac{1}{\sqrt{s}}\left(\frac{1+2s}{3}\right)\underbrace{\int E_0^2 dv_0}$$

$$= \frac{1}{\sqrt{s}}\left(1+\frac{2}{3}\frac{u^2}{v^2}\right)\frac{e^2}{2a}$$

$$\sin^2\psi : \sin^2\varphi = \sqrt{x^2+y^2} : \sqrt{sx^2+y^2}$$

$$H^2 = E^2\left(\frac{u^2}{c^2}\sin^2\psi\right)$$

$$\sin^2\psi = \sin^2\varphi\cdot\frac{1}{\sin^2\varphi + s\sin^2\varphi}$$

$$= \frac{\sin^2\varphi}{1-\frac{u^2}{c^2}\sin^2\varphi}$$

$$D_r = \frac{A\sqrt{s}}{r^2\sqrt{1-\frac{u^2}{c^2}\sin^2\psi}^{\,3}}$$

$$\frac{\partial}{\partial s}\int\frac{dx}{\sqrt{s+x^2}} = \lg\left(x+\sqrt{s+x^2}\right)$$

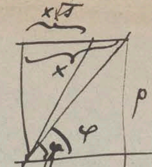
$$A\sqrt{s}\int\frac{2\pi r^2 \sin\psi\, d\psi}{r^2\sqrt{1-\frac{u^2}{c^2}\sin^2\psi}^{\,3}} = \varrho\cdot 4\pi$$

$$\int\frac{dx}{\sqrt{1+x^2}^{\,3}} = \frac{1}{\left(x+\sqrt{1+x^2}\right)\sqrt{1+x^2}}$$

$$\int\frac{\sin\psi\, d\psi}{\sqrt{s+\frac{u^2}{c^2}\cos\psi}^{\,3}} = \frac{2\varrho}{A\sqrt{s}} = \frac{c}{u}\int\frac{dx}{\sqrt{s+x^2}^{\,3}} = \frac{2c}{u}\left(\frac{1}{\frac{u^2}{c^2}+1} + \frac{1}{1-\frac{u^2}{c^2}}\right)$$

$$\cancel{2\left.\frac{1}{\sqrt{s+\frac{u^2}{c^2}\cos\psi}}\right|_{\cos\psi=1}^{0} = \frac{2\varrho u}{cA\sqrt{s}} = 2\left(\frac{1}{\sqrt{s}} - 1\right)} = \frac{4c}{us}$$

$$A = \frac{\varrho\sqrt{s}}{2}\frac{u}{c}$$



$$r^2 : r_0^2 = x^2 s + p^2 : x^2 + p^2$$

$$= 1 : \frac{\sin^2\psi}{s} + \cos^2\psi \qquad = s\cos^2\varphi + \sin^2\varphi : 1$$

$$= 1 - \frac{u^2}{c^2}\cos^2\varphi : 1$$

$$= s : \sin^2\psi + s\cos^2\psi = s : 1 - \frac{u^2}{c^2}\sin^2\psi$$

$$r_0^2 s = r^2\left(1 - \frac{u^2}{c^2}\sin^2\psi\right) \qquad 1 - \frac{u^2}{c^2}\sin^2\psi = \frac{s}{1 - \frac{u^2}{c^2}\cos^2\varphi}$$

$$E_0^2 = \frac{e^2}{r^4}$$

$$E^2 = \frac{e^2 s^2}{r^4\left(1 - \frac{u^2}{c^2}\sin^2\psi\right)^3} = \frac{e^2 s^2}{r_0^4 s^2\left(1 - \frac{u^2}{c^2}\sin^2\psi\right)} = \frac{e^2}{r_0^4 s}\left(1 - \frac{u^2}{c^2}\cos^2\varphi\right)$$

$$H^2 = E^2 \cdot \frac{u^2}{c^2}\sin^2\psi = E^2\left\{1 - \frac{s}{1 - \frac{u^2}{c^2}\cos^2\varphi}\right\} = E^2 - \frac{\cancel{s}\, e^2}{\cancel{s}\, r_0^4}$$

$$\frac{1}{8\pi}\int E^2 dv = \frac{\sqrt{s}}{8\pi}\int E^2 dv_0 = \frac{\sqrt{s}}{4}\int \frac{e^2}{r_0^4 s}\left(1 - \frac{u^2}{c^2}\cos^2\varphi\right) r^2 dr \sin\varphi\, d\varphi$$

$$T_e = \frac{e^2}{4\sqrt{s}\, a}\int 2 - \frac{u^2}{c^2}\frac{2}{3} = \frac{e^2}{2a\sqrt{s}}\left(1 - \frac{u^2}{3c^2}\right)$$

$$T_h = T_e - \sqrt{s}\cdot\frac{e^2}{2a}$$

$$T_e + T_h = \frac{e^2}{2a\sqrt{s}}\left(2 - \frac{2u^2}{3c^2} - s\right) = \frac{e^2}{2a\sqrt{s}}\left(1 + \frac{u^2}{3c^2}\right) \qquad \text{(Lorentz!)}$$

$$\frac{dT}{du} = \frac{\cancel{2}\, e^2}{\cancel{2}\, a\sqrt{s}}\left\{\frac{u}{3c^2} + \left(1 + \frac{u^2}{3c^2}\right)\frac{\frac{u}{c^2}}{1 - \frac{u^2}{c^2}}\right\} = \frac{e^2}{a\sqrt{s}}\,\frac{\frac{u}{3c^2} - \cancel{\frac{u^3}{3c^4}} + \frac{u}{c^2} + \cancel{\frac{u^3}{3c^3}}}{1 - \frac{u^2}{c^2}} = \frac{e^2}{a\sqrt{s^3}}\,\frac{4u}{3c^2}$$

$$m_l = \frac{4e^2}{3a\sqrt{s}^3 c^2}$$

Starre Flächenladung:

$$\cancel{m_s = \frac{1}{2}\frac{e^2}{ac^2}\left[\frac{1}{\beta^2}\right.}$$

$$m_s = \frac{2}{3}\frac{e^2}{ac^2}\left\{1 + \frac{6}{5}\beta^2 + \frac{9}{7}\beta^4 + \frac{12}{9}\beta^6 + \cdots\right.$$

$$m_r = \quad \left\{1 + \frac{6}{3.5}\beta^2 + \frac{9}{5.7}\beta^4 + \cdots\right.$$

Starre Volumladung:

$$m_s = \frac{4}{5}\frac{e^2}{ac^2}\left\{1 + \frac{6}{5}\beta^2 + \frac{9}{7}\beta^4 \cdots \right\}$$

$$m_r = \quad \left\{1 + \frac{6}{3.5}\beta^2 + \frac{9}{5.7}\beta^4 + \cdots \right\}$$

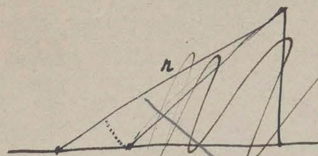
Heaviside Ellipsoid (Lorentz) Bucherer Ellipsoid

$$m_s = \frac{m_0}{\left(1 - \frac{u^2}{c^2}\right)^{3/2}} =$$

$$m_r = \frac{m_0}{\left(1 - \frac{u^2}{c^2}\right)^{1/2}} =$$

$$m_1 = \frac{2T}{u^2}$$

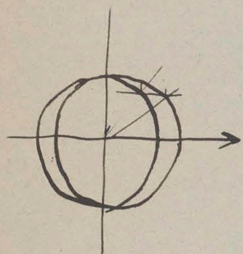
$$F_e \, \Delta s = \Delta T \qquad \therefore \quad m_e = \frac{1}{\frac{du}{dt}}\frac{dT}{ds} = \frac{1}{u}\frac{dT}{du}$$



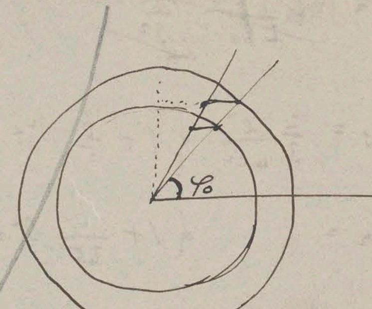
$$r = \sqrt{y^2+x^2}$$

$$\frac{dr}{dt} = \frac{x}{\sqrt{y^2+x^2}}\,\frac{dx}{dt} = u\cos\varphi$$

$$\frac{1}{8\pi}\int E^2\,dv + \frac{1}{8\pi}\int H^2\,dv$$



$$dv : dv_0 = \sqrt{s} : 1$$



$$E^2 : E_0^2 = \Delta r_0^2 : \Delta r^2$$

$$= \frac{1}{\sin^2\varphi_0} : \frac{1}{\sin^2\varphi}$$

$$= \sin^2\varphi : \sin^2\varphi_0$$

$$= \frac{y^2}{(y^2+sx^2)} : \frac{y^2}{(y^2+x^2)} = y^2+x^2 : y^2+sx^2$$

$$= 1 : \sin^2\varphi_0 + s\cos^2\varphi_0$$

$$= 1 : s + \sin^2\varphi_0\,(1-s)$$

$$= 1 : s + \frac{u^2}{c^2}\sin^2\varphi_0$$

$$= 1 : 1 - \frac{u^2}{c^2}\cos^2\varphi_0$$

$$E_0^2\,dv_0 = E^2\,dv\cdot\left(s + \frac{u^2}{c^2}\sin^2\varphi_0\right)\sqrt{s}$$

$$E^2\,dv = \frac{E_0^2\,dv_0}{\sqrt{s}\left(s+\frac{u^2}{c^2}\sin^2\varphi_0\right)}$$

$$\frac{1}{8\pi}\int\cdots = \frac{1}{4}\int \frac{\frac{e^2}{r^4}\,r^2\,dr\,\sin\varphi\,d\varphi}{\sqrt{s}\left(s+\frac{u^2}{c^2}\sin^2\varphi\right)} = \frac{1}{4a\sqrt{s}}\int\frac{\sin\varphi\,d\varphi}{s+\frac{u^2}{c^2}\sin^2\varphi}$$

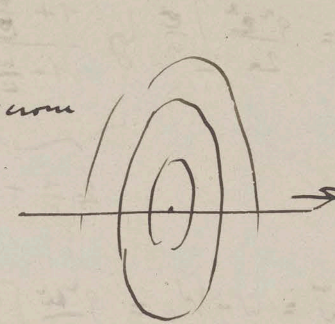
$$= \frac{1}{4a\sqrt{s}}\int\frac{\sin\varphi\,d\varphi}{1-\frac{u^2}{c^2}\cos^2\varphi} = \frac{c}{2au\sqrt{s}}\int_1^0\frac{dy}{1-y^2} = \frac{c}{2au\sqrt{s}}\,\frac{1}{2}\lg\frac{1+y}{1-y}\Big|_1^0$$

$$= \frac{c}{4au\sqrt{s}}$$

Potencjał: $U = \frac{A}{\sqrt{(\frac{x}{s})^2+y^2+z^2}}$ $\mathfrak{f} = -\nabla U$

Pow. ekwipot. $(\frac{x}{s})^2 + y^2 + z^2 = \text{const}$ elipsoidy spłaszczone

Heavisidea Elipsoidy

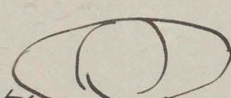


Jeśli kulisty elektron się porusza nie może być taki rozkład pot., bo $\mathfrak{f}$ = siła na masę równocześnie się poruszającą z prędkością u musiałaby być ⊥ powierzchni, wskutek zmniejszenia eliptyczności.

Więc zadanie znaleźć takie rozwiązanie

$s \frac{\partial^2 U}{\partial x^2} + \frac{\partial^2 U}{\partial y^2} + \frac{\partial^2 U}{\partial z^2} = 0$ które jest stałe na kuli a i spełnia warunki ...

Będzie to pot. elips. rozciągniętej



bo się skurczy w kierunku osi tak że się stanie kulą

$$W = \frac{q^2c^2}{2a}\Bigg[\frac{c}{u}\underbrace{\log\frac{1+\frac{u}{c}}{1-\frac{u}{c}}}_{2\left(\frac{u}{c}+\frac{u^3}{3c^3}+\frac{u^5}{5c^5}\right)} - 1\Bigg] \qquad \boxed{-\frac{q^2c^2}{2a}}$$

$$T = W - W_0 = \frac{q^2u^2}{a}\left[\frac{1}{3}+\frac{1}{5}\left(\frac{u}{c}\right)^2+\cdots\right]$$

$$= m\frac{u^2}{2} \qquad // \qquad m = \frac{u^2}{a}\left[\frac{2}{3}+\cdots\right.$$

$$m_e\,du$$

$$\frac{dT}{dt} = F\frac{dx}{dt} = M\frac{du}{dt}\cancel{\frac{dx}{dt}}\ u$$

$$\frac{dT}{du} = \frac{M}{u} \qquad M = u\frac{dT}{du} \qquad m_e = \frac{2}{3}\frac{q}{a}\left[1+\frac{6}{5}\frac{u^2}{c^2}+\frac{9}{7}\left(\frac{u}{c}\right)^4+\cdots\right]$$

$$\int \mathfrak{E}\rho\,d\tau = 4\pi c^2\int \vartheta(\operatorname{div}\vartheta)\,d\tau + \int[[\rho u]\mathfrak{H}]\,dv$$

$$\rho u = \frac{1}{4}\operatorname{curl}\mathfrak{H} - \frac{d\vartheta}{dt}$$

$$\int \vartheta\operatorname{div}\vartheta\,dv \qquad \int \vartheta(\vartheta N)\,dS - \underbrace{\int(\vartheta\nabla)\vartheta\,dv}_{=[\vartheta\operatorname{rot}]+\frac{1}{2}\nabla\vartheta^2}$$

$$\ldots\int dS \qquad -\int\nabla\ \frac{d}{dt}\int\nabla\mathfrak{H}\vartheta\,dv$$

$$\int \frac{\pi^2 s^2}{r^4}\, 2\pi r\, dr \quad \int \frac{1+\varepsilon^2 \sin^2 y}{(1-\varepsilon^2 \sin^2 y)^3} \sin y\, dy = \int \frac{2 \sin y\, dy}{(1-\varepsilon^2 \sin^2 y)^3} - \frac{\sin y\, dy}{(1-\varepsilon^2 \sin^2 y)^2}$$

$$\cancel{\int \frac{1+\varepsilon^2 - \varepsilon^2 \cos^2 y}{[1-\varepsilon^2+\varepsilon^2\cos^2 y]^3} \sin y\, dy} =$$

$$= 2\int_0^{\pi} \frac{\sin y\, dy}{[1-\varepsilon^2+\varepsilon^2\cos^2 y]^3} = 4\int_0^{+1} \frac{dx}{(1-\varepsilon^2+\varepsilon^2 x^2)^3} = \cancel{\frac{2}{\varepsilon}\int \frac{dx}{(1-\varepsilon^2+x^2)^3}}$$

$$= 4\int_0^1 \frac{dx}{[s^2+(1-s^2)x^2]^3} = \frac{2}{(1-s^2)^3}\int_0^1 \frac{dx}{\left[\frac{s^2}{1-s^2}+x^2\right]^3} = \frac{2}{\varepsilon^{4/3}}\int_0^{\pi} \ldots$$

$$\text{Cap. } \sqrt{c^2-a^2}\ \underbrace{\log\left[\frac{a}{c+\sqrt{c^2-a^2}}\right]}_{\log\left(\frac{s}{1+\sqrt{1-s^2}}\right)} = \frac{Q}{U} \qquad c = \frac{a}{s}$$

$$= a \log \frac{1-\frac{u}{v}}{1+\frac{u}{v}}$$

$$\frac{\sqrt{1-s^2}}{s} = \frac{\frac{u}{v}}{\sqrt{1-\frac{u^2}{v^2}}}$$

$$\cancel{\frac{\partial X}{\partial t} = -u\frac{\partial X}{\partial x} - u}$$

$$U = \frac{A}{\sqrt{\left(\frac{x}{s}\right)^2 + y^2 + z^2}}$$

$$\frac{\partial X}{\partial t} + 4\pi\rho u = c\left(\frac{\partial N}{\partial y} - \frac{\partial M}{\partial z}\right) = 4\pi\rho u - u\frac{\partial X}{\partial x}$$

$$\frac{\partial Y}{\partial t} = c\left(\frac{\partial L}{\partial z} - \quad\right) = 0 - u\frac{\partial Y}{\partial x}$$

$$\frac{\partial Z}{\partial t} = c\left(\quad\right) = 0 - u\frac{\partial Z}{\partial x}$$

$$\mathfrak{D}_1 = \frac{\partial U}{\partial x} = \frac{Ax}{\sqrt{x^2 + s^2(y^2+z^2)}^3}$$

$$\mathfrak{D}_2 = -\frac{\partial U}{\partial y} = \frac{Ay}{\sqrt{\quad}^3}$$

$$\mathfrak{D}_3 \qquad \frac{\partial U}{\partial z} = \frac{Az}{\sqrt{\quad}^3}$$

$$D = A$$

$$-\cancel{\ }(1-\tfrac{u^2}{v^2})$$

$$D = \frac{Az}{s^2\left(\frac{x^2}{s^2} + r^2 - x^2\right)^{3/2}} = \frac{As}{\cancel{4\pi} r^2\left(1 - \frac{u^2}{v^2}\sin^2\gamma\right)^{3/2}}$$

$$\frac{Ars}{r^2 s^2 + x^2(1-s^2)}$$

$$\int_0^\pi \frac{2\pi r^2 \sin\gamma\, d\gamma}{\cancel{4\pi r^2}\left(1 - \frac{u^2}{v^2}\sin^2\gamma\right)^{3/2}} = \frac{2\pi}{\cancel{4\pi}}\int_\pi^0 \frac{d(\cos\gamma)}{\left(1 - \frac{u^2}{v^2} + \frac{u^2}{v^2}\cos^2\gamma\right)^{3/2}} =$$

$$= \frac{1}{2v^2}\ \frac{1}{s^3}\int \frac{d\xi}{\left[1 + \frac{1-s^2}{s^2}\xi^2\right]^{3/2}} = \frac{1}{2v^2 s^3}\ \frac{\sqrt{\frac{1-s^2}{s^2}}}{\sqrt{\frac{1}{s^2}}} = \frac{\frac{u^2}{v^2}}{2v^2 s^3}$$

$$\int \frac{dx}{(a^2+x^2)^{3/2}} = \frac{x}{a^2\sqrt{a^2+x^2}}$$

$$\frac{d}{dx}\left(\frac{x}{\sqrt{1+x^2}}\right) = \frac{1}{\sqrt{\ }} - \frac{x^2}{\sqrt{\ }^3}$$

$$\vartheta = \frac{(v_1)\ \varrho s}{4\pi r^2\left(1 - \frac{u^2}{v^2}\sin^2\gamma\right)^{3/2}}$$

$$\mathfrak{f} = \tfrac{1}{c}\left[\mathfrak{v}\,\mathfrak{v}\right]\vartheta = \frac{\varrho u s \sin\gamma}{4\pi r^2(1 - \cdots)^{3/2}}$$

$$4\pi J = \frac{\partial \mathfrak{D}}{\partial t} + 4\pi\rho\bar{u} = c \operatorname{rot} \mathfrak{H}$$

$$\frac{\partial \mathfrak{H}}{\partial t} = -c \operatorname{rot} \mathfrak{E}$$

$$\mathfrak{E} = \mathfrak{D} + \frac{V\bar{u}\mathfrak{H}}{c}$$

$$\begin{array}{ccc} i & j & k \\ \hline \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ \begin{vmatrix} v & w \\ Y & Z \end{vmatrix} & \begin{vmatrix} w & u \\ Z & X \end{vmatrix} & \begin{vmatrix} u & v \\ X & Y \end{vmatrix} \end{array}$$

Ruch jednostajny

$$\frac{\partial}{\partial t} X = -u \frac{\partial X}{\partial x} + v\frac{\partial Y}{\partial y} - w\frac{\partial X}{\partial z} = -u\left(\frac{\partial X}{\partial x} + \frac{\partial Y}{\partial y} + \frac{\partial Z}{\partial z}\right) + u\left(\frac{\partial Y}{\partial y} + \frac{\partial Z}{\partial z}\right) - v\frac{\partial X}{\partial y} - w\frac{\partial X}{\partial z}$$

$$\frac{\partial}{\partial y}(uY - vX) - \frac{\partial}{\partial z}(wX - uZ)$$

$$-v\frac{\partial X}{\partial y} - w\frac{\partial X}{\partial z}$$

$$\frac{\partial}{\partial t} Y = -v \frac{\partial Y}{\partial y}$$

$$\frac{\partial}{\partial t} Z = -$$

$$\frac{\partial \mathfrak{D}}{\partial t} = -4\pi\rho\bar{u} + \operatorname{rot} V\bar{u}\mathfrak{D}$$

$$\operatorname{rot} V\bar{u}\mathfrak{D} = c \operatorname{rot} \mathfrak{H}$$

$$\boxed{\mathfrak{H} = \frac{1}{c} V\bar{u}\mathfrak{D}} + \mathfrak{N}$$

$$\frac{\partial \mathfrak{H}}{\partial t} = -\cancel{4\pi\rho\bar{u}} + \operatorname{rot} V\bar{u}\mathfrak{H} = -c \operatorname{rot} \mathfrak{D}$$

$$\mathfrak{D} = -\frac{1}{c} V\bar{u}\mathfrak{H} + \mathfrak{M}$$

$$\mathfrak{M} = \mathfrak{E}$$

nic nowego tylko to

że $\operatorname{rot} \mathfrak{E} = 0$

zatem $\mathfrak{E} = -\nabla U$

$$\cancel{\frac{\partial \mathfrak{H}}{\partial t} = \frac{1}{c} V\bar{u} \frac{\partial \mathfrak{D}}{\partial t} + \frac{\partial \mathfrak{N}}{\partial t} = -c \operatorname{rot} \mathfrak{D}}$$

$$[i(kD_2 - jD_3)] = -(jD_2 + kD_3)\frac{u^2}{c^2}$$

$$\mathfrak{E} = -\nabla U = \mathfrak{D} + \frac{1}{c^2}\underbrace{V\bar{u}V\bar{u}\mathfrak{D}}_{\bar{u}(\bar{u}\mathfrak{D}) - \bar{u}^2\mathfrak{D}}$$

$$\bar{u} = \bar{u} i$$

$$-\nabla U = \mathfrak{D} + \frac{1}{c^2}(i u^2 D_1 - u^2 \mathfrak{D})$$

$$= \mathfrak{D}\left(1 - \frac{u^2}{c^2}\right) + i\frac{u^2}{c^2} D_1$$

$$s = \sqrt{1 - \frac{u^2}{c^2}}$$

$$-\frac{\partial U}{\partial x} = D_1$$

$$-\frac{\partial U}{\partial y} = D_2\left(1 - \frac{u^2}{c^2}\right)$$

$$-\frac{\partial U}{\partial z} = D_3\left(1 - \frac{u^2}{c^2}\right)$$

$$\underline{\operatorname{div} \mathfrak{D} = 0}$$

$$s^2 \frac{\partial^2 U}{\partial x^2} + \frac{\partial^2 U}{\partial y^2} + \frac{\partial^2 U}{\partial z^2} = 0$$

$$\frac{\partial^2 U}{\partial \left(\frac{x}{s}\right)^2} + \frac{\partial^2 U}{\partial y^2} \cdots\cdots = 0$$

$$U = \frac{A}{\sqrt{\left(\frac{x}{s}\right)^2 + y^2 + z^2}}$$

$4\pi u = \ldots$

$\rho = \operatorname{div} \vartheta$

$\cancel{4\pi}\frac{\partial \vartheta}{\partial t} + \cancel{4\pi}\rho \bar{u} = c \operatorname{curl} \mathfrak{H}$ $\qquad \nabla\left(\rho \bar{u} + \frac{\partial \vartheta}{\partial t}\right) = 0$

$\operatorname{div} \mathfrak{H} = 0$

Ogólne

$\frac{\partial \mathfrak{H}}{\partial t} = - \cancel{} c^2 \operatorname{curl} \vartheta$

$$\mathfrak{E} = \cancel{} \vartheta + \frac{V \bar{u}}{c} \mathfrak{H}$$

Ruch jednostajny:

$$-\frac{\partial \vartheta}{\partial t} = +(\bar{u}\nabla)\vartheta$$

$$= \bar{u} \operatorname{div} \vartheta - \vartheta(\overset{\operatorname{div}}{\nabla \bar{u}}) - \operatorname{curl} V \bar{u} \vartheta + (\vartheta\nabla)\bar{u}$$

$$= \bar{u}\rho - \operatorname{curl} V \bar{u} \vartheta$$

$\therefore \operatorname{curl} \mathfrak{H} = \operatorname{curl} V \bar{u} \vartheta$

$$\boxed{\mathfrak{H} = \tfrac{1}{c} V \bar{u} \vartheta + \mathfrak{N}}$$

$i\left\{u \frac{\partial E_1}{\partial x} + u \frac{\partial E_2}{\partial x} + \ldots\right\}$

$= iu\left(\frac{\partial E_1}{\partial x} + \frac{\partial E_2}{\partial y} + \frac{\partial E_3}{\partial z}\right)$

$- \ldots i \frac{\partial}{\partial y}(\ldots E_3 - \ldots) \ldots$

$\cancel{4\pi} V \bar{u} \frac{\partial \vartheta}{\partial t} = - \cancel{} c^2 \operatorname{curl} \vartheta$

$$\operatorname{curl} \mathfrak{E} = \cancel{} \operatorname{curl} \vartheta + \cancel{\bar{u}(\overset{\operatorname{div} \mathfrak{H}}{\nabla \mathfrak{H}})} - \cancel{\mathfrak{H}(\nabla \bar{u})} + \cancel{(\mathfrak{H}\nabla)\bar{u}} \underbrace{-(\bar{u}\nabla)\mathfrak{H}}_{= \frac{1}{c}\frac{\partial \mathfrak{H}}{\partial t}} = 0$$

$$\mathfrak{E} = -\nabla \mathfrak{U} = \cancel{} \vartheta + \underbrace{\frac{1}{c^2} V \bar{u} V \bar{u} \vartheta}_{= \frac{1}{c^2}\left[\bar{u}(\bar{u}\vartheta) - \bar{u}^2 \vartheta\right]}$$

$\bar{u} = ui$

$$4\pi c^2 \vartheta + 4\pi i u^2 D_1 - 4\pi u^2 \vartheta = -\nabla \mathfrak{U}$$

$$4\pi\left(1 - \frac{u^2}{v^2}\right)\vartheta + 4\pi \frac{u^2}{v^2} i D_1 = -\frac{1}{v^2} \nabla \mathfrak{U} \quad \therefore$$

$$D_1 = -\frac{1}{4\pi v^2}\frac{\partial V}{\partial x}$$

$$D_2 = -\frac{1}{4\pi v^2 s}\frac{\partial V}{\partial y}$$

$$D_3 = -\frac{1}{4\pi v^2 s}\frac{\partial V}{\partial z}$$

V = Convectionspotential

$\underline{\operatorname{div} \mathfrak{D} = 0}$

$$s\frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2} = 0$$

$$D_1 = \frac{Ax}{s\sqrt{}^3} = \frac{Ax\sqrt{s}}{\sqrt{x^2 + s(y^2+z^2)}^3}$$

$$D_2 = \frac{Ay}{s\sqrt{-}^3} = \frac{A\sqrt{s}\cos\gamma}{r^2\sqrt{\cos^2\gamma + s\sin^2\gamma}^3}$$

$$D_3 = \text{--} = \frac{A\sqrt{s}\cos\gamma}{r^2\sqrt{1-\frac{u^2}{c^2}\sin^2\gamma}^3}$$

$$V = \frac{A}{\sqrt{\frac{x^2}{s}+y^2+z^2}} = \frac{A}{r\sqrt{1+\frac{x^2}{r^2}\left(\frac{1}{s}-1\right)}} = \frac{A}{r\sqrt{1+\frac{x^2}{r^2}\frac{1-s}{s}}} = \frac{A\sqrt{s}}{r\sqrt{1+\frac{u^2}{c^2 s}\cos^2\gamma}}$$

$$= \frac{A\sqrt{s}}{r\sqrt{1-\frac{u^2}{c^2}+\frac{u^2}{c^2}\cos^2\gamma}}$$

$$= \frac{A\sqrt{s}}{r\sqrt{1-\frac{u^2}{c^2}\sin^2\gamma}}$$

$$s = 1-\frac{u^2}{c^2}$$

$$\mathfrak{D}^2 = \left(\frac{q\,s}{\cancel{4\pi}\, r^2\left(1-\frac{u^2}{c^2}\sin^2\gamma\right)^{3/2}}\right)^2 = \frac{q^2 s^2}{\cancel{(4\pi)}\, r^4\left(1-\frac{u^2}{c^2}\sin^2\gamma\right)^3}$$

$$H^2 = \frac{q^2 u^2 s^2 \sin^2\gamma}{c^2 r^4\left(1-\frac{u^2}{c^2}\sin^2\gamma\right)^3}$$

$$A = qc^2\sqrt{s} \quad = \frac{qc^2 s}{r\sqrt{1-}}$$

$$= \frac{qc^2 s}{r\left(1-\frac{ur}{c}\right)}$$

$$D^2 + H^2 = \frac{q^2 s^2}{r^4\left(1-\frac{u^2}{c^2}\sin^2\gamma\right)^3}\left[1+\frac{u^2}{c^2}\sin^2\gamma\right]$$

$$\frac{1}{8\pi}\int D^2\,dv = \frac{q^2 s^2}{4}\int\frac{r^2\sin\gamma\, d\gamma\, dr}{r^4\left(1-\frac{u^2}{c^2}\sin^2\gamma\right)^3} = \frac{q^2 s^2}{4a}\int\frac{\sin\gamma\, d\gamma}{\left(1-\frac{u^2}{c^2}+\frac{u^2}{c^2}\cos^2\gamma\right)^3}$$

$$= \frac{q^2 s^2}{4a}\int_{-1}^{1}\frac{dx}{\left(s+\frac{u^2x^2}{c^2}\right)^3} = \frac{q^2 \cdot c}{4a\, u}\frac{1}{\sqrt{s}}\int_{-\frac{u}{c\sqrt{s}}}^{+\frac{u}{c\sqrt{s}}}\frac{dy}{(1+y^2)^3}$$

$$\int\frac{dy}{a^2+y^2} = \frac{1}{a}\operatorname{arctg}\left(\frac{y}{a}\right)$$

$$-2\int\frac{dy}{(a^2+y^2)^2} = -\frac{1}{a^3}\operatorname{arctg}\frac{y}{a} - \frac{y}{a^2}\frac{1}{a^2+y^2}$$

$$+8\int\frac{a\,dy}{(\ )^3} = \frac{3}{a^4}\operatorname{arctg}\frac{y}{a} + \frac{y}{a^5}\operatorname{arctg}\frac{y}{a} + \frac{2y}{a^3(a^2+y^2)} + \frac{2y}{a(a^2+y^2)^2}$$

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$$\int \mathfrak{h}\rho\, dv = \int \vartheta \rho\, dv + \frac{1}{c}\int \rho\,[u\, \mathfrak{H}]\, dv$$

$$= \frac{1}{4\pi}\int \vartheta \operatorname{div} \vartheta\, dv + \frac{1}{4\pi c}\int \cancel{\operatorname{div} \vartheta [u \mathfrak{H}]}\, dv$$

$$\int [\rho u, \mathfrak{H}]\, dv$$

$$\downarrow$$

$$\frac{1}{4\pi}\left(c \operatorname{rot} \mathfrak{H} - \frac{\partial \vartheta}{\partial t}\right)$$

$$\cancel{\int \vartheta \operatorname{div} \vartheta\, dv = \int \vartheta(\vartheta n)\, dS - \int (\vartheta \nabla)\vartheta\, dv}$$

$$D_1 \frac{\partial D_1}{\partial x} + D_2 \frac{\partial D_2}{\partial x} + D_3 \frac{\partial D_3}{\partial x}$$
$$- D_2 \frac{\partial D_1}{\partial y} - D_3 \frac{\partial D_1}{\partial z}$$

$$\int \vartheta \operatorname{div} \vartheta\, dv = \int \vartheta(\vartheta n)\, dS - \int \underbrace{\frac{1}{2}\nabla(\vartheta^2)\, dv}_{\frac{1}{2}\int \vartheta^2 n\, dS} + \int [\vartheta \operatorname{rot} \vartheta]\, dv$$

$$\operatorname{rot}\vartheta = -\frac{1}{c}\frac{\partial \mathfrak{H}}{\partial t}$$

$$\int [\mathfrak{H} \operatorname{rot} \mathfrak{H}]\, dv = \int \cancel{\mathfrak{H}(\operatorname{div}\mathfrak{H})}_{=0}\, dv - \int \mathfrak{H}(\mathfrak{H} n)\, dS + \frac{1}{2}\int \nabla(\mathfrak{H}^2)\, dv$$

$$(\mathfrak{H}^2 n)\, dS$$

$$\int \mathfrak{h}\rho\, dv = -\int \quad dS + \frac{d}{dt}\int [\mathfrak{H}\vartheta]\, dv$$

Zde v našich rovnicích

síla $\displaystyle \frac{d}{dt}\int \underbrace{[\vartheta \mathfrak{H}]}_{\mathfrak{M}}\, dv$

$\mathfrak{M}$ hybnost

$$\int D_1\left(\frac{\partial D_1}{\partial x} + \frac{\partial D_2}{\partial y} + \frac{\partial D_3}{\partial z}\right) dx\,dy\,dz =$$

$$\int D_1 (D_1 \cos nx + D_2 \cos ny + D_3 \cos nz)\, dS$$

$$- \int \underbrace{\left(\frac{\partial D_1}{\partial x} D_1 + \frac{\partial D_1}{\partial y} D_2 + \frac{\partial D_1}{\partial z} D_3\right)}\, dv$$

$$\frac{1}{2}\frac{\partial[(D_1)^2 + (D_2)^2 + (D_3)^2]}{\partial x}$$

$$- \begin{vmatrix} D_2 & D_3 \\ \begin{vmatrix} \frac{\partial}{\partial z} & \frac{\partial}{\partial x} \\ D_3 & D_1 \end{vmatrix} & \begin{vmatrix} \frac{\partial}{\partial x} & \frac{\partial}{\partial y} \\ D_1 & D_2 \end{vmatrix} \end{vmatrix}$$

$$V' = \varrho s v^2 \underbrace{\int_h^\infty \frac{dh}{h^2-l^2}}_{\frac{1}{2}\lg\left(\frac{h+l}{h-l}\right)} \qquad l^2 = a^2(1+s^2)$$

$$\frac{x^2}{h^2} + \frac{\rho^2}{h^2-l^2} = 1$$

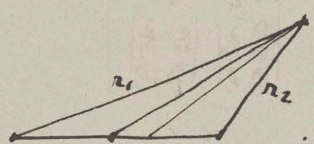
$$\cancel{x^2(h^2-l^2)+\rho^2 h^2 = h^4 - h^2 l^2}$$

$$\cancel{h^4 - h^2(x^2+l^2+\rho^2) = -x^2 l^2}$$

$$\cancel{h^2 = \frac{x^2+l^2+\rho^2}{2} \pm \sqrt{(\quad)^2 - x^2 l^2}}$$

$$x = h\cos\alpha$$

$$\rho = \frac{\sqrt{h^2-l^2}}{\sqrt{s}}\sin\alpha$$



$$\int \frac{d\xi}{\sqrt{(x-\xi)^2+y^2}} = -\lg\left[x-\xi+\sqrt{(x-\xi)^2+y^2}\right]_{-a}^{a}$$

$$= \lg\frac{x+a+\sqrt{(x+a)^2+y^2}}{x-a+\sqrt{(x-a)^2+y^2}} = \lg\frac{x+a+r_1}{x-a+r_2}$$

$$x+a+r_1 = c\,(x-a+r_2)$$

$$x(1-c)+a(1+c)+r_1 = c\,r_2$$

$$= \cancel{\lg\frac{(x^2-a^2)}{(x-a)^2 + (x-a)^2+y^2}} = \lg\frac{r_1(1+\cos\theta_1)}{r_2(1+\cos\theta_2)}$$

$$\cancel{r_1 : r_2 = \sin\theta_2 : \sin\theta_1}$$

$$\cancel{x+a+\frac{a\,y}{\sin\theta_1}}$$

$$\cancel{x^2(1-c)^2+a^2(1+c^2)+(x+a)^2+y^2+2ax(1-c^2)+r_1}$$

$$m_l = \frac{dM}{du} = \frac{1}{u}\frac{dW}{du}$$

$$M = \int \frac{1}{u}\frac{dW}{du}\,du$$

$$m_t = \frac{1}{u}\int\frac{1}{u}\frac{dW}{du}\,du$$

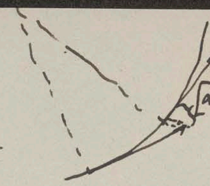
$$W = \frac{q^2}{a}u^2\left(\frac{1}{3}+\frac{1}{5}\frac{u^2}{v^2}\right) \qquad 2/3 + \frac{4}{5}$$

$$m_l = \frac{1}{u}\frac{dW}{du} = \frac{2q^2}{3a}\left(1+\frac{6}{5}\frac{u^2}{v^2}\right) \qquad \frac{12}{10}$$

$$\frac{2q^2}{3a}\left(u+\frac{2u^3}{5v^2}\right)$$

$$m_t = \frac{2q^2}{3a}\left(1+\frac{2}{5}\frac{u^2}{v^2}\right)$$

$$\mathfrak{G} = m\frac{d^2 v}{dt^2} = m\frac{dv}{dt} = \frac{d}{dt}\overbrace{(mv)}^{\mathfrak{A}}$$



$[d\mathfrak{A}_0] : ds = ds : R$

$$\mathfrak{A} = A\,\mathfrak{A}_0$$

$$\frac{d\mathfrak{A}_0}{dt} = \frac{d\mathfrak{A}_0}{ds}\frac{ds}{dt} = \frac{\mathfrak{N}_0}{R}v$$

$$\mathfrak{G} = \underbrace{\frac{dA}{dt}\mathfrak{A}_0}_{\mathfrak{G}_e} + \underbrace{A\frac{d\mathfrak{A}_0}{dt}}_{A\frac{\mathfrak{N}_0 v}{R}}$$

$$\underbrace{A\frac{\mathfrak{N}_0 v}{R}}_{\mathfrak{G}_t}$$

w ruchu $\mathfrak{A} = mv$ $\qquad \frac{mv^2}{R}$

$$F_e = \frac{dA}{dt} = m_e\frac{dv}{dt} \qquad\qquad F_t = m_t\frac{v^2}{R} = \frac{A}{R}v$$

$$m_e = \frac{dA}{dv} \qquad\qquad m_t = \frac{A}{v}$$

$$dW = F_e\,ds$$

$$F_e = \frac{dW}{ds} = m_e\frac{dv}{dt}$$

$$m_e = \frac{dW}{dv}\frac{1}{\frac{ds}{dt}} = \frac{1}{v}\frac{dW}{dv} = \frac{dA}{dv}$$

$$A = \int\frac{1}{v}\frac{dW}{dv}\,dv$$

$$m_t = \frac{1}{v}\int\frac{1}{v}\frac{dW}{dv}\,dv$$

więc znając energię można już obliczyć osobno m_t, m_l!

$$W = \frac{\varrho^2}{a}u^2\left[\frac{1}{3} + \frac{1}{5}\frac{u^2}{v^2} + \frac{1}{7}\frac{u^4}{v^4}\cdots\right]$$

$$m_l = \frac{2}{3}\frac{\varrho^2}{a}\left(1 + \frac{6}{5}\frac{u^2}{v^2} + \frac{9}{7}\frac{u^4}{v^4}\cdots\right)$$

$$m_t = \frac{2}{3}\frac{\varrho^2}{a}\left(1 + \frac{2}{5}\frac{u^2}{v^2} + \frac{9}{5\cdot 7}\frac{u^4}{v^4} + \cdots\right)$$

dwa sposoby mierzenia masy

siła potrzebna do wywoł. przysp. kątowego

siła dośrodkowa

we wszystkich metodach tych mierzymy:

siłę wywołaną przez napięcie

sprężyny i wartość tego napięcia

mierzymy wielkość masy

zupełnie niezależny sposób:

$$\mathfrak{A} = \int[\dot{v}\,\mathfrak{G}]\,dv$$