

5443

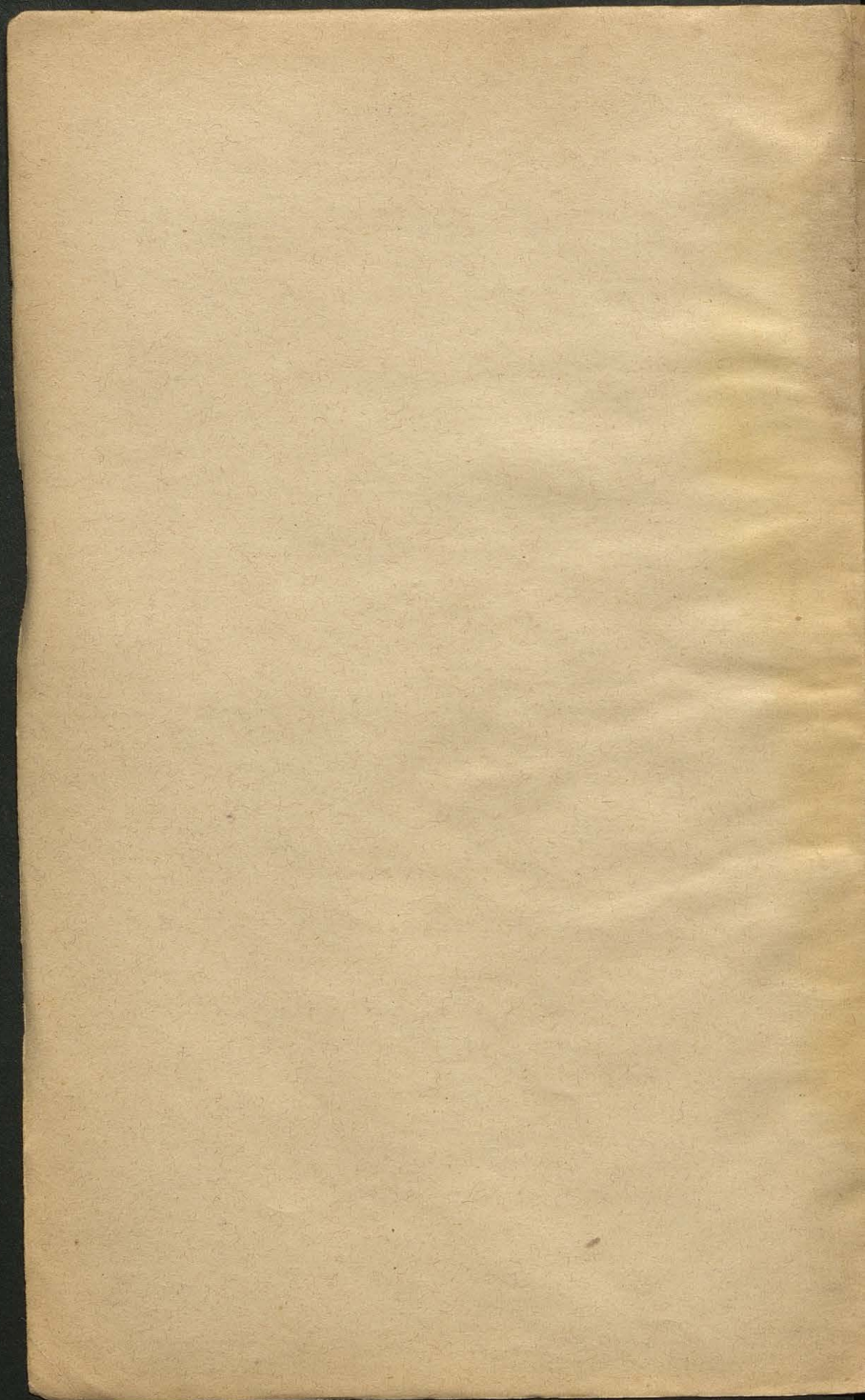
^{1.}
Dr. Josef Stefan II. S. 92.
Akustik.
Rimoluchowski

6873

3448

1873

J. LUZANSKY
WIEN
IV. Wiedener Hauptstr. 29



4/5 ... 1/2 ... = ...

Our Scale C D E F G A H C

e f g a b c d e f g a b c d : Octave ... 1:1/2 Pythagoras

... .. 1:2 Pythagoras

f g a b c d e f g a b c d



2 ...

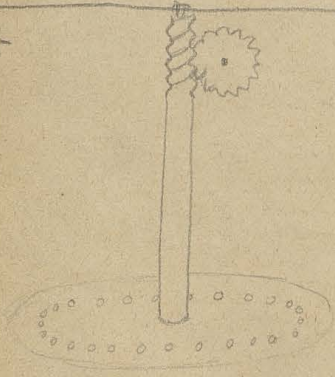
... .. Quint ... $\frac{2}{3} : 1$

$\frac{3}{2} : 1$

rel. ... :

1 $\frac{9}{8}$ $\frac{5}{4}$ $\frac{4}{3}$ $\frac{3}{2}$ $\frac{5}{3}$ $\frac{15}{8}$ 2

4/5



de

... ..

... ..

$$b = a \sin 2n\pi t$$

b = Verdrehung 4

$$b = 0 \quad 2n\pi t = n$$

2n
3n

$$t = \frac{1}{2n}, 2 \frac{1}{2n}, 3 \frac{1}{2n}, \dots$$

$$\frac{db}{dt} = 2n\pi a \cos 2n\pi t$$

a $\frac{db}{dt} +$ etc. Copy \sim δ , ω - etc. \sim δ

$$\frac{db}{dt} = +2n\pi a [-1, +1, -1, +1, \dots]$$

... etc. etc. etc. etc. etc. etc. etc. etc.

$$t_1 = \frac{1}{n} = \text{...} \left[\begin{array}{l} \text{etc. Perioden von } \frac{1}{2n} \\ \text{...} \end{array} \right]$$

... etc.

$$b' = a' \sin 2n'\pi t$$

$$b + b' = \dots \text{ Copy}$$

$$n' > n$$
$$n' = n + x$$

$$b + b' = a \sin 2n\pi t + a' \sin(2n\pi t + 2x\pi t)$$

$$= a \sin 2n\pi t + a' \sin 2n\pi t \cos 2x\pi t +$$

$$+ a' \cos 2n\pi t \sin 2x\pi t$$

$$= (a \cos 2x\pi t) \sin 2n\pi t + a' \sin 2x\pi t \cos 2n\pi t$$

$$a + a' \cos 2\pi n t = A \cos \varphi$$

$$a' \sin 2\pi n t = A \sin \varphi$$

$$b + b' = A \sin(2\pi n t + \varphi)$$

$$a^2 + 2aa' \cos 2\pi n t + a'^2 = A^2$$

(Quadrat)
= 2 per. fe.

u p 2 L ...
p 2. p' ...

$$cA = 0 \quad a^2 + a'^2 + \dots = 0$$

$$a = a' \quad \cos 2\pi n t = -1$$

$$A = 2a$$

$$\text{Nexim. } \cos 2\pi n t = +1$$

$$2\pi n t = 0, 2\pi, 4\pi, \dots$$

$$t = 0, \frac{1}{2}, 2\frac{1}{2}, 3\frac{1}{2}, \dots$$

... x ...

... p ...

5/5 Monochorde ...

vertik. b; v ...

... p ...

H ... M ...

Sauvenc (1701) ...

1/2 lb of ...

- 6 lb of ...

in the ...

at ...

of ...

and ...

in ...

of ...

of ...

at ...

[5 lb] of ...

of ...

at ...

Ground ...

1. ... 2. ... 3. ...

at ...

at ...

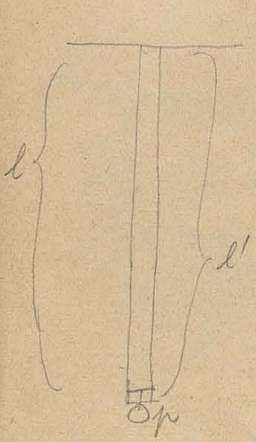
11 lb

in ...

at ...

at ...

6/5 12 gms 1/2 6.
 verjüngte Elastie.



~ 6,6 3,2

185 6,2 f Newtons ~ 16 ~ 2 Hookes

$$\Delta l = \frac{P}{E} l$$

$$P = \frac{E \Delta l}{l} = E \Delta \epsilon$$

$$\frac{\Delta l}{l} = \text{relat. wjg} \quad \frac{1}{\eta} = E$$

E = Elastizitätskoeffizient

$$\eta = 1 \quad l' - l = \Delta l$$

max 100 % ~ e in P, e

$$P = E \Delta l \quad \Delta l = 3,2 \text{ cm}$$

z.B. $A = 1 \text{ mm}^2 \quad E = 10.000 \text{ kg/cm}^2$

$1 \text{ cm}^2 \quad E = 10.000.000 \text{ kg/cm}^2$

100 gms 1/2 6.

ffo ~ 1/2 N a de ~ e ~ wjg ~ 1/2 m 6

de ~ 1/2 m 6 ~ 1/2 m 6 ~ 1/2 m 6

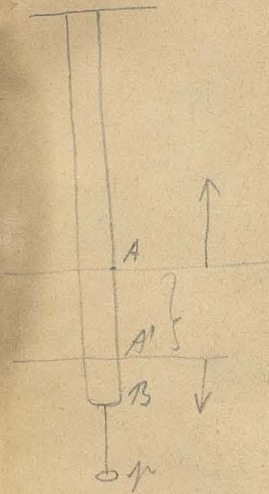
de ~ 1/2 m 6 ~ 1/2 m 6 ~ 1/2 m 6

~ wjg. f ~ 1/2 m 6 ~ 1/2 m 6 ~ 1/2 m 6

~ 1/2 m 6 ~ 1/2 m 6 ~ 1/2 m 6

ffo ~ 1/2 m 6 ~ 1/2 m 6 ~ 1/2 m 6

Handwritten notes at the top left, possibly "Handwritten notes" or similar.



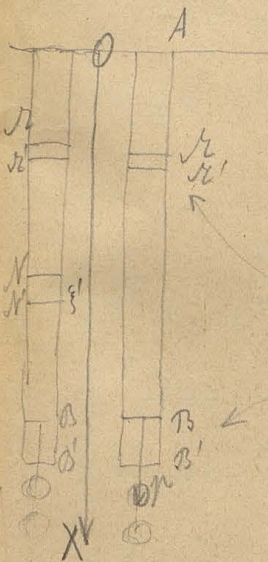
Handwritten notes: $\rho_1' p_0 + \dots$

Handwritten notes: $\rho / \rho_0 m \dots$

Handwritten notes: $\rho_1' p_0 + \dots$

Handwritten notes: $\rho_1' p_0 + \dots$

$1: \rho = \rho_0$



$AB = l$

$AR = x$

$RR' = \xi$

$BB' = \lambda$

$P = \frac{\rho_0 g \lambda}{l}$

$P = \frac{\rho_0 g \xi}{x}$

$\frac{\rho_0 g \lambda}{l} = \frac{\rho_0 g \xi}{x}$

$\frac{\lambda}{l} = \frac{\xi}{x}$

$\frac{\lambda}{l} = \frac{\xi'}{x'}$

$\frac{\lambda}{l} = \frac{\xi}{x} = \frac{\xi'}{x'} = \frac{\xi' - \xi}{x' - x}$

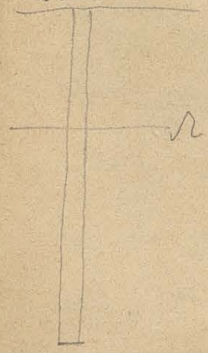
Handwritten notes: $1: \rho = \rho_0$

max $\rho p_{3/2}^2$ des Nenners

$$\xi = f(x)$$

$$\lambda = \frac{q' - q}{x' - x} = \frac{d'q}{dx}$$

$$P = \frac{c q}{205} \frac{d'q}{dx}$$



$s = \text{op. l'}$

f 25 16 l' des / o d' c p p s
1/2 l' + o d' p l' c

~ d' u o l' o r g p f r' c
 $q(l-x) s = 1' o o l' \sqrt{2 s p l' x}$
 $= \frac{c q}{205} \frac{d' q}{dx}$

$$\frac{s}{2} (l-x) = \frac{d'q}{dx}$$

$$\xi = \frac{s}{2} \left[lx - \frac{x^2}{2} \right] + C$$

$$x=0 \quad 0 = 0 + C$$

$$\xi = \frac{s}{2} \left[lx - \frac{x^2}{2} \right]$$

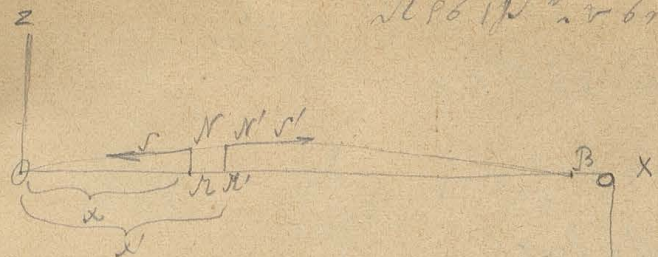
$$\lambda = \frac{s}{2} \frac{dx}{2}$$

$$= \frac{p d}{205}$$

$$sl = p = 1' o o l'$$

o o e s v e t' l' c c v l' o' s e
s' d' o s p u o v l' o' s

206 1/2 2. r 6 70.2 / 8
 6/2



$x' - x = d$

$\xi = f(r, \theta, \dots)$ $\xi' = f(r', \theta', \dots)$
 $\xi = 2 \text{Coord. of } N$ $\xi' = \dots$ N'
 N

$r, \theta \in \xi - f(r, \theta, \dots)$

$\xi = \varphi(x)$ $\xi' = \varphi(x')$
 $= \varphi(x+d) = \varphi(x) + \lambda \varphi'(x)$

$\xi' = \xi + \lambda \frac{d\xi}{dx}$ $\xi = \varphi(x)$ $\xi' = \varphi(x+\lambda)$

$\xi' = \xi + \lambda \frac{d\xi}{dx}$ $\varphi(x) \in \varphi(x+\lambda)$ N, N', \dots, N, N'

$r, \theta \in \xi, N, N', \dots, N, N'$

$\xi, N' - f, \dots$

$X = -S \cos \alpha$ $X' = S' \cos \alpha'$
 $Z = -S \sin \alpha$ $Z' = S' \sin \alpha'$

$\left\{ \begin{aligned} m \frac{d^2 \xi}{dt^2} &= -S \sin \alpha + S' \sin \alpha' \\ m \frac{d^2 (x + \xi)}{dt^2} &= -S \cos \alpha + S' \cos \alpha' = m \frac{d^2 x}{dt^2} \end{aligned} \right. \}$

$$P \approx MM' = NN' = \mu MM' = \mu \lambda$$

PP, P, S, P, X, \alpha \text{ or } f(x) \sim

$$S \sin \alpha = f(x)$$

$$S' \sin \alpha' = f(x + \lambda)$$

$$= f(x) + \lambda f'(x) = S \sin \alpha + \lambda \frac{d(S \sin \alpha)}{dx}$$

$$\mu \lambda \frac{d^2 f}{dx^2} = \lambda \frac{d(S \sin \alpha)}{dx} \quad \text{cosine } \delta \lambda / \alpha$$

$$\mu \frac{d^2 f}{dx^2} = \frac{d(S \sin \alpha)}{dx}$$

$$\mu \frac{d^2 f}{dx^2} = \frac{d(S \sin \alpha)}{dx}$$

$$S = P + \epsilon \rho \frac{(NN' - MM')}{\lambda \lambda'}$$

$$\begin{aligned} NN'^2 &= (x' + \xi' - x - \xi)^2 + (\xi' - \xi)^2 \\ &= (\lambda + \lambda \frac{d\xi}{dx})^2 + \lambda^2 (\frac{d\xi}{dx})^2 \\ &= \lambda^2 \left[1 + 2 \frac{d\xi}{dx} + \left(\frac{d\xi}{dx} \right)^2 \right] + \lambda^2 \left(\frac{d\xi}{dx} \right)^2 \end{aligned}$$

$$\epsilon \lambda^c \epsilon \sigma_y$$

$$S = P + \epsilon \rho \lambda \sqrt{1 + 2 \frac{d\xi}{dx} + \left(\frac{d\xi}{dx} \right)^2 + \left(\frac{d\xi}{dx} \right)^2} - \lambda$$

cos \gamma \text{ or } \cos \beta \text{ (N e P.C.) } \epsilon \text{ or } \xi \text{ or } \xi \text{ or } \sigma \text{ or } \delta

$$\frac{d\xi}{dx} > \frac{d\xi}{dx}$$

$$S = P + Q_2 \frac{\sqrt{1 + 2 \frac{d^2 q}{dx^2}} - 1}{1} = P + Q_2 \frac{d^2 q}{dx^2}$$

$$\sin \alpha = \frac{t x}{\sqrt{1 + t^2 x^2}} = \frac{d^2 q}{dx^2} \sqrt{1 + \left(\frac{d^2 q}{dx^2}\right)^2} + \frac{d^2 q}{dx^2}$$

donc e^{α} sur l'eq.

$$w^2 \cos \alpha = 1$$

$$m \frac{d^2 q}{dt^2} = \frac{d}{dx} [S \sin \alpha] = \frac{d}{dx} \left[P + Q_2 \frac{d^2 q}{dx^2} \right]$$

$$= Q_2 \frac{d^2 q}{dx^2} \quad \text{à corriger la longueur d. corr}$$

à corriger la longueur d. corr

$$m = Q_2 \cdot l$$

$$p \frac{d^2 q}{dt^2} = Q \frac{d^2 q}{dx^2} \quad \text{à corriger la longueur d. corr}$$

à corriger la longueur d. corr

$$m \frac{d^2 q}{dt^2} = \frac{d}{dx} \left[\left(P + Q_2 \frac{d^2 q}{dx^2} \right) \frac{d^2 q}{dx^2} \right]$$

$$= \frac{d}{dx} \left[P \frac{d^2 q}{dx^2} + Q_2 \frac{d^2 q}{dx^2} \frac{d^2 q}{dx^2} \right]$$

qui est révisé

$$m \frac{d^2 q}{dt^2} = P \frac{d}{dx} \left(\frac{d^2 q}{dx^2} \right)$$

$$m \frac{d^2 q}{dt^2} = P \frac{d^2 q}{dx^2}$$

à corriger la longueur d. corr

à corriger la longueur d. corr

12/5

$$m \frac{d^2 y}{dt^2} = \frac{P}{\mu} \frac{d^2 y}{dx^2} \quad \frac{P}{\mu} = a^2$$

$$\frac{d^2 y}{dt^2} = a^2 \frac{d^2 y}{dx^2} \quad \text{w/o } P \text{ and } \mu$$

$y = f(x, t) = X \cdot T$... Prod. of 2 variables $f(x, t)$

$$\text{Subst. } X \frac{d^2 T}{dt^2} = a^2 T \frac{d^2 X}{dx^2}$$

$$\frac{d^2 T}{dt^2} = a^2 \frac{d^2 X}{dx^2} = c$$

$$a^2 \frac{d^2 T}{dt^2} = c T \quad \frac{d^2 X}{dx^2} = \frac{c}{a^2} X$$

$$T = A e^{\sqrt{c} t} + B e^{-\sqrt{c} t}$$

$a e^{+ \dots} + B e^{- \dots}$... per δ^2 ; $a \omega < \infty$

e^{\dots} - per ω ; $\omega < \infty$

$$\frac{d^2 T}{dt^2} = -a^2 T$$

$$T = A \cos a t + B \sin a t$$

$$\frac{d^2 X}{dx^2} = -\frac{a^2}{a^2} X$$

$$X = C \cos \frac{a x}{a} + D \sin \frac{a x}{a}$$

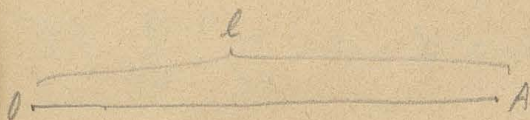
$$\xi = \left[C \cos \frac{\alpha x}{a} + D \sin \frac{\alpha x}{a} \right] \left[A \cos \alpha t + B \sin \alpha t \right] \quad 9$$

$\omega = \alpha \sqrt{l}$ - per. ω ; ρ Amplitude; ρ of ξ of η .

$\omega \rho \sin \alpha t \cos \alpha x$

$$\xi \Big|_{x=0} = 0$$

$$\xi \Big|_{x=l} = 0$$



$$\xi = 0 = C \cdot T \quad C = 0$$

$$= 0 \Rightarrow$$

$$\xi = D \sin \frac{\alpha x}{a} \cdot T$$

$x=l$

$$\xi = 0 = D \sin \frac{\alpha l}{a} \cdot T \quad D \sin \frac{\alpha l}{a} = 0$$

$f^c = 0 \Rightarrow D = 0$ \leftarrow or $\sin \frac{\alpha l}{a} = 0$

αl is a const. α :

$$\frac{\alpha l}{a} = 0, \pi, 2\pi, 3\pi, \dots$$

$$\frac{\alpha l}{a} = \pi \quad \alpha = \frac{a\pi}{l}$$

$$\xi = D \sin \frac{\pi x}{l} \left[A \cos \frac{\pi a t}{l} + B \sin \frac{\pi a t}{l} \right]$$

$$DA = F \quad DB = G$$

$$\xi = \sin \frac{\pi x}{l} \left[F \cos \frac{\pi a t}{l} + G \sin \frac{\pi a t}{l} \right]$$

$$\frac{n \pi a t}{l} = 2n \quad a \approx \text{lead}$$

$$\tau = \frac{2l}{a} = \rho \mu c$$

$$f \rho \mu c \approx \mu c = \frac{1}{\tau} = n = \frac{a}{2l} = \frac{1}{2l} \sqrt{\frac{P}{\mu}}$$

and so on:

$$\frac{\alpha l}{a} = 2n, 3n, \dots, kn$$

$$\alpha = \frac{akn}{l}$$

$$y' = \sin \frac{k\pi x}{l} \left[F \cos \frac{k\pi a t}{l} + G \sin \frac{k\pi a t}{l} \right]$$

$$\frac{k\pi a t'}{l} = 2\pi$$

$$\tau' = \frac{2l}{ka}$$

$$n' = \frac{1}{\tau'} = \frac{k}{2l} a = \frac{k}{2l} \sqrt{\frac{P}{\mu}}$$

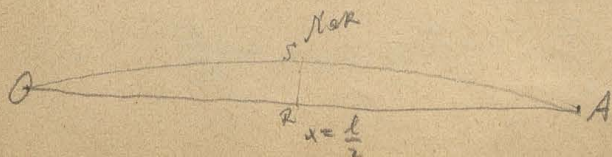
and so on

$n = \dots$ $n = \dots$ $n = \dots$ $n = \dots$ $n = \dots$

$2, 3, 4, \dots$

$k=1$

$e^{\dots} \sin \frac{\pi x}{l}$ \dots



$$RS = aF \cos \frac{n\pi x}{l} + G \sin \frac{n\pi x}{l}$$

Curve + B - sin - lin.

$$h=2$$

$$x = \frac{l}{2} \quad \xi = 0$$



$f(x) = \sin \frac{\pi x}{l}$

$$h=3$$

$$\left. \begin{aligned} x = \frac{l}{3} \\ x = \frac{2l}{3} \end{aligned} \right\} \xi = 0$$



$$h=4$$

etc.



Curve + B - sin - lin. ~ 2nd order of ξ or η
or $\eta = \delta \frac{1}{4} \sim \eta_0 \cos \sim 2\text{th. der. of } \eta \text{ or } \eta \text{ or } \xi$

$$\begin{aligned} \xi = & \sin \frac{\pi x}{l} \left[F_1 \cos \frac{n\pi x}{l} + G_1 \sin \frac{n\pi x}{l} \right] + \\ & + \sin \frac{2\pi x}{l} \left[F_2 \cos \frac{2n\pi x}{l} + G_2 \sin \frac{2n\pi x}{l} \right] + \\ & + \sin \frac{3\pi x}{l} \left[F_3 \cos \frac{3n\pi x}{l} + G_3 \sin \frac{3n\pi x}{l} \right] + \\ & + \dots \end{aligned}$$

PFs ξ or η ~ all ξ or η

2. 10 P. Cond. F S 9 & 2

20 el 10 / n a - 616 625 617 602.

216 / t=0.

$$\frac{12}{5} \xi = \sin \frac{\pi x}{l} \left[F_1 \cos \frac{\pi a t}{l} + G_1 \sin \frac{\pi a t}{l} \right] +$$

$$+ \sin \frac{2\pi x}{l} \left[F_2 \cos \frac{2\pi a t}{l} + G_2 \sin \frac{2\pi a t}{l} \right] +$$

+ ...

$$\frac{d\xi}{dt} = -\frac{\pi a}{l} \sin \frac{\pi x}{l} \left[F_1 \sin \frac{\pi a t}{l} - G_1 \cos \frac{\pi a t}{l} \right] +$$

$$- \frac{2\pi a}{l} \sin \frac{2\pi x}{l} \left[F_2 \sin \frac{2\pi a t}{l} - G_2 \cos \frac{2\pi a t}{l} \right] +$$

$$\overline{t=0} \quad \xi = \varphi(x) \quad \frac{d\xi}{dt} = \psi(x)$$

$$\varphi(x) = F_1 \sin \frac{\pi x}{l} + F_2 \sin \frac{2\pi x}{l} + F_3 \dots$$

$$\psi(x) = \frac{\pi a}{l} G_1 \sin \frac{\pi x}{l} + \frac{2\pi a}{l} G_2 \sin \frac{2\pi x}{l} + \dots$$

20. 10 P. Cond. F S 9 & 2
 225 6 / n a & l.

20. 10 P. Cond. F S 9 & 2
 225 6 / n a & l.

Lagrange's 10 P. Cond. F S 9 & 2
 225 6 / n a & l.

for $h \neq k$... Fourier ... M
 $\int_0^l \sin \frac{hnx}{l} \sin \frac{knx}{l} dx = 0$ if $h \neq k$

$\int_0^l \sin \frac{hnx}{l} \cos \frac{knx}{l} dx = 0$ if $h \neq k$

$$\int_0^l \sin \frac{hnx}{l} \sin \frac{knx}{l} dx = 0 \quad h, k \text{ w/o}$$

$$= \frac{1}{2} \int_0^l dx \left[\cos \frac{(h-k)nx}{l} - \cos \frac{(h+k)nx}{l} \right]$$

$$= \frac{1}{2} \left. \frac{l}{(h-k)n} \sin \frac{(h-k)nx}{l} - \frac{1}{2} \frac{l}{(h+k)n} \sin \frac{(h+k)nx}{l} \right|_0^l = 0$$

\neq for $h=k$... $\int_0^l \sin^2 \frac{hnx}{l} dx$

$$\int_0^l \sin^2 \frac{hnx}{l} dx = \frac{1}{2} \int_0^l 1 - \cos \frac{2knx}{l} dx$$

$$= \frac{l}{2}$$

... $F_1 \frac{l}{2}$

$$\int_0^l \sin \frac{nx}{l} \cos \frac{nx}{l} dx = \int_0^l \sin \frac{nx}{l} \frac{dx}{2} + \int_0^l \cos \frac{nx}{l} \frac{dx}{2}$$

$$= F_1 \frac{l}{2}$$

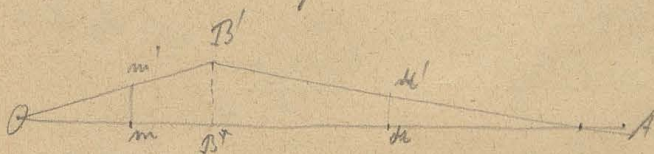
$$\int_0^l \sin \frac{2nx}{l} \cos \frac{2nx}{l} dx = F_2 \frac{l}{2}$$

etc. ...

$$F_h = \frac{2}{l} \int_0^l \varphi(x) \sin \frac{h\pi x}{l} dx$$

$$g_h = \frac{2}{2\pi a} \int_0^l \varphi(x) \sin \frac{h\pi x}{l} dx$$

203. $-\delta u / \delta \rho_1 \delta^2$



$$P \eta'' = 0 \quad \varphi(x) = 0$$

$$u'' = 0 \quad g_h = 0$$

$$O m = x \quad m m' = c \quad B B' = c \quad O B = x$$

$$f: c = x: l$$

$$f = \frac{cx}{l} \quad \left(\frac{c}{l} \right) \int_0^l f(x) dx$$

$$\varphi(x) = \frac{cx}{l} \quad \text{for } x=0 \text{ to } x=l$$

$$n n' = c \quad f: c = A n: A B \\ = l-x: l-l$$

$$\varphi(x) = f = \frac{c(l-x)}{l-l} \quad \text{for } x=l \text{ to } x=l$$

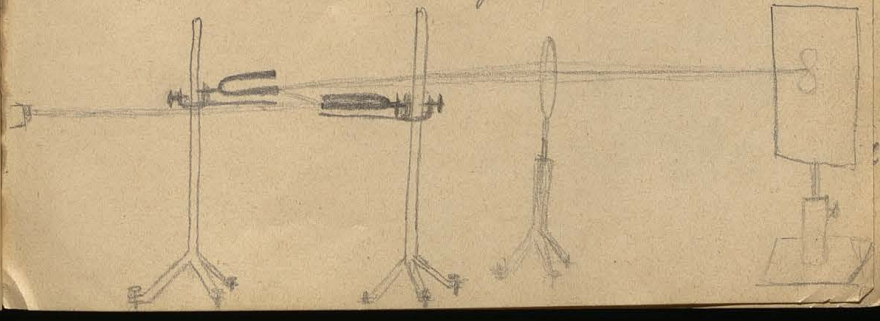
$$F_h = \frac{2}{l} \int_0^x \frac{cx}{l} \sin \frac{h\pi x}{l} dx + \frac{2}{l} \int_x^l \frac{c(l-x)}{l-l} \sin \frac{h\pi x}{l} dx$$

$$\begin{aligned}
 \frac{dF_h}{dc} &= \frac{1}{\lambda} \left[-\frac{x l}{\hbar \pi} \cos \frac{\hbar x}{l} + \int_0^x \frac{d}{\hbar \pi} \int_0^x \cos \frac{\hbar x}{l} dx \right] \\
 &+ \frac{1}{\hbar \lambda} \left[\frac{(b-x) l}{\hbar \pi} \cos \frac{\hbar x}{l} + \frac{d}{\hbar \pi} \int_x^b \cos \frac{\hbar x}{l} dx \right] \\
 &= \frac{1}{\lambda} \left[-\frac{\lambda l}{\hbar \pi} \cos \frac{\hbar \lambda}{l} + \frac{d}{\hbar \pi} \sin \frac{\hbar \lambda}{l} \right] \\
 &+ \frac{1}{\hbar \lambda} \left[+\frac{(b-\lambda) l}{\hbar \pi} \cos \frac{\hbar \lambda}{l} + \frac{d}{\hbar \pi} \sin \frac{\hbar \lambda}{l} \right] \\
 &= \frac{d}{\hbar \pi} \left(\frac{1}{\lambda} + \frac{1}{\hbar \lambda} \right) \sin \frac{\hbar \lambda}{l} \\
 F_h &= \frac{2c l^2}{\hbar \pi \lambda (\lambda + \hbar)} \sin \frac{\hbar \lambda}{l}
 \end{aligned}$$

$a \quad \lambda = l \quad \text{oder} \quad F_h = 0 \quad \text{wenn} \quad \lambda = 2l \quad \text{oder} \quad F_2 = 0$
 wenn $\lambda = 2l$ oder $\lambda = 3l$ oder $\lambda = 4l$
 oder $\lambda = 5l$, ... oder $\lambda = n l$.

$$\lambda = \frac{l}{3} \quad \text{oder} \quad \lambda = \frac{l}{2} \quad \text{oder} \quad \lambda = \frac{l}{4} \quad \text{oder} \quad \lambda = \frac{l}{5} \quad \text{oder} \quad \lambda = \frac{l}{6}$$

13/5 Experimente Lissajousche Figuren.



u, v, w, x, y, z

$$x = a \sin at \quad y = b \sin at$$

$$\frac{x}{a} = \frac{y}{b} = -\text{per cent in the } 4^{\text{th}} - \frac{1}{2}$$

$$x = a \cos at \quad y = b \cos at$$

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \quad \text{ellipse}$$

2 - ref. line

$$x = \frac{a \cos at}{\cos} \quad y = \frac{b \sin at}{\sin}$$

1/2 the velocity of

speed of the particle is $\frac{a}{\cos} \omega \sin at$ & $\frac{b}{\sin} \omega \cos at$

$$t=0 \quad x=0 \quad z=0$$

at $t=\pi$ $z \neq \pi$ & the direction of

is $\frac{a}{\cos} \omega \sin at$ - all the x & all the y

of the - ω dt

about - 6 degrees of ω π \cos

the \sin \cos [ref. line]

the \sin \cos ω dt π \cos

$$\alpha t - \alpha' t = 2\pi$$

13

$$\alpha = 2\pi n$$

$$\alpha' = 2\pi(n + \frac{1}{2})$$

$$2\pi n t = 2\pi$$

$$n = \frac{1}{t}$$

$$t = \frac{1}{n} = \frac{1}{2\pi n} = \frac{1}{2\pi} \cdot \frac{1}{n}$$

f. d.

W. p. u. n. 1. 2. 3. 4. 5. 6. 7. 8. 9. 10. 11. 12. 13. 14. 15. 16. 17. 18. 19. 20.

f. f. l. 1. 2. 3. 4. 5. 6. 7. 8. 9. 10. 11. 12. 13. 14. 15. 16. 17. 18. 19. 20.

L. O. t. e. r. e.

882)

$$x = a \cos 2at$$

$$y = b \sin at$$

$$x = a - 2a \sin^2 at$$

$$x = a - 2a \sin^2 at$$

$$x = a - \frac{2a}{b^2} y^2$$

$$(a-x) \frac{dx}{2a} = y^2 \quad \text{Parabel}$$

$$x = a \sin 2at$$

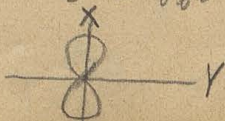
$$y = b \sin at$$

$$x = 2a \sin at \cos at$$

$$x^2 = 4a^2 \sin^2 at [1 - \sin^2 at]$$

$$x^2 = 4a^2 \frac{y^2}{b^2} [1 - \frac{y^2}{b^2}] \quad \text{C. 4. t. 2. 6}$$

W. x. 2. 4. y. 4. y. 4. y. 4. y.



$$\text{Print } x = \sin \omega t$$

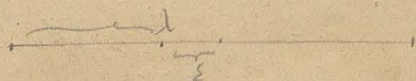
$$y = \sin (\omega t + \theta)$$

17/5 by up & down of 600. v. s. z

$\omega < 1$ C. v. l. $\sqrt{b^2} = 0^2 < 1$ $\omega^2 \geq 0$ v. s. k. l. v. s.

f' t=0 v. s. r. v. s. r. e. b. - 1/2 e. l. e. v. s. r.

v. s. r. b.



$$f = 0 \quad \frac{df}{dt} = c \quad \text{if } x = \lambda \text{ s. } x = \lambda + \varepsilon$$

$$f = \sin \frac{n\pi x}{l} \left[F_1 \cos \frac{n\pi t}{l} + G_1 \sin \frac{n\pi t}{l} \right] + \dots$$

$$\frac{df}{dt} = -\frac{n\pi}{l} \sin \frac{n\pi x}{l} \left[F_1 \sin \frac{n\pi t}{l} + G_1 \cos \frac{n\pi t}{l} \right] + \dots$$

$$0 = F_1 \sin \frac{n\pi x}{l} + F_2 \sin \frac{2n\pi x}{l} + \dots$$

$$F_1 = F_2 = \dots = 0$$

$$\int_x^{x+\varepsilon} f(x) dx \sin \frac{n\pi x}{l} = \frac{n\pi a}{l} \int_n \frac{l}{2}$$

$$-\frac{2c}{n\pi} \cos \frac{n\pi x}{l} \Big|_x^{x+\varepsilon} = \dots$$

$$G_n = \frac{2lc}{n^2 \pi^2 a} \left[\cos \frac{n\pi x}{l} - \cos \frac{n\pi (x+\varepsilon)}{l} \right]$$

$$I_{n0} = \frac{4lc}{n^2 n^2 a} \sin \frac{n\pi(\lambda + \frac{\epsilon}{2})}{l} \sin \frac{n\pi \frac{\epsilon}{2}}{l}$$

$$= \frac{4c\epsilon}{n^2 a} \sin(n\pi \frac{\lambda + \frac{\epsilon}{2}}{l})$$

cu ~ 2π / λ

$$n\pi \frac{\lambda + \frac{\epsilon}{2}}{l} = n \quad a^c e \beta = 0$$

$$\lambda + \frac{\epsilon}{2} = \frac{l}{n} \quad \text{cu } \beta = n \text{ te } l \text{ cu } \lambda = 0, \text{ cu } \epsilon = 0$$

2 1/2 pt / octave

3 l / cu v ~ 1/3 ~ 2/3 pt.

cu ~ 1/6 ~ 1/6 pt ~ m ~ 1/6 pt.

longitudinale: ρ

$$\frac{d^2 \xi}{dt^2} = \frac{c}{\rho} \frac{d^2 \xi}{dx^2} \quad \text{for } \rho = \rho(x) \text{ and } c = c(x)$$

cu 1 pt. ρ & c cu x .

cu ρ & c long. ρ & c cu x cu ρ & c cu x .



$$\begin{aligned} \xi &= -2g \frac{d\xi}{dx} \\ &= -f(x) \end{aligned}$$

$$f' = f(x+\varepsilon)$$

$$\rho g \varepsilon \frac{d^2 \eta}{dt^2} = f' - f = f(x+\varepsilon) - f(x)$$

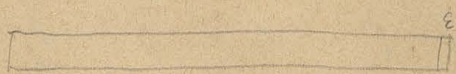
$$\rho g \frac{d^2 \eta}{dt^2} = \frac{f(x+\varepsilon) - f(x)}{\varepsilon} = \frac{df(x)}{dx}$$

$$\rho g \frac{d^2 \eta}{dt^2} = \frac{d}{dx} \left(\rho g \frac{d\eta}{dx} \right) \quad \text{if } \rho, g \text{ are const.}$$

$$= \rho g \frac{d^2 \eta}{dx^2} \quad \text{const.}$$

$$\frac{d^2 \eta}{dt^2} = \frac{c^2}{\rho} \frac{d^2 \eta}{dx^2}$$

$\rho = \rho_0 \sqrt{1 - v^2/c^2}$; ρ_0 is rest mass density, v is velocity.
 ρ_0 is constant; ρ is variable. ρ_0 is constant, v is variable.
 ρ is variable, v is variable. ρ is variable, v is variable.
 ρ is variable, v is variable. ρ is variable, v is variable.



$$\rho g \varepsilon \frac{d^2 \eta}{dt^2} = -g \frac{d\eta}{dx}$$

reverse

$$\frac{d^2 \eta}{dt^2} = -\frac{g}{\rho \varepsilon} \frac{d\eta}{dx} \quad \text{if } \rho \times \varepsilon \frac{d^2 \eta}{dt^2} = \text{const.}$$

$$\text{if } \rho \times \varepsilon \frac{d^2 \eta}{dt^2} = 0 \sim ; \rho \times \varepsilon = 0.$$

$$\frac{g}{\rho} = a^2$$

$$\xi = \sin \alpha t [A \cos \beta x + B \sin \beta x]$$

$$\alpha^2 = a^2 \beta^2$$

$$+ \cos \alpha t [C \cos \beta x + D \sin \beta x] \quad \text{part. f}$$

$$L + \text{part. f}$$

$$x=0 \quad x=l \quad \frac{d\xi}{dx} = 0$$

$$\frac{d\xi}{dx} = \sin \alpha t [-A\beta \sin \beta x + B\beta \cos \beta x] + \cos \alpha t [-C\beta \sin \beta x + D\beta \cos \beta x]$$

$$x=0$$

$$\frac{d\xi}{dx} = \sin \alpha t B\beta + \cos \alpha t D\beta = 0$$

$$B=0 \quad D=0 \quad \text{L, part. f}$$

$$\xi = \cos \beta x [A \sin \alpha t + C \cos \alpha t]$$

$$\frac{d\xi}{dx} = -\beta \sin \beta x [A \sin \alpha t + C \cos \alpha t]$$

$$x=0 \quad x=l$$

$$\sin \beta l = 0$$

$$\beta l = n, 2n, 3n \dots$$

$$\begin{aligned} \beta l &= x = a\beta \\ &= \frac{2n}{l} = 2n\pi \end{aligned}$$

$$2n\pi = a\beta$$

$$= a \left[\frac{n\pi}{l}, \frac{2n\pi}{l} \dots \right]$$

$$n = \frac{a}{2l} [1, 2, 3 \dots]$$

$$n = \sqrt{\frac{g}{p}} \frac{1}{2l} [1, 2, \dots]$$

$\therefore \xi \in \mathbb{C}$, Octave etc.

$$\xi = \cos \frac{n\pi x}{l} [A_1 \sin \omega t + C_1 \cos \omega t]$$

$$\beta = \frac{2n\pi}{l}$$

$$\xi = \cos \frac{2n\pi x}{l} [A_2 \sin \dots]$$

$\therefore \text{D.P.E.}$

$$\xi = \dots$$

... ..

... ..

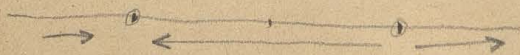
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Octave $x = \frac{l}{4}, \frac{3l}{4}, \frac{5l}{4}$

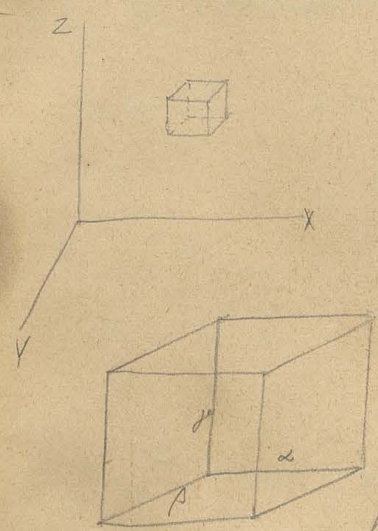
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Octave:



... ..

verwendet \sim 2 d. h. p. f. d.



$\rho_0 = \rho_0 \alpha \beta \gamma$

ρ_0 ist in allen Punkten gleich

I. s. v. / v. s. g.

z. P. H. ρ_0 ist / $\rho_0 \alpha \beta \gamma$ u.

ρ_0 ist in allen Punkten gleich

$\rho_0 \alpha \beta \gamma X = \rho_0 \alpha \beta \gamma$

folgt: $\rho_0 \alpha \beta \gamma = \rho_0 \alpha \beta \gamma$

p / s. v. d. w.

z. P. H. ρ_0 ist in allen Punkten gleich. $\rho_0 \alpha \beta \gamma = \rho_0 \alpha \beta \gamma$

z. P. H. ρ_0 ist in allen Punkten gleich. $\rho_0 \alpha \beta \gamma = \rho_0 \alpha \beta \gamma$

$\rho_0 \alpha \beta \gamma = \rho_0$

$\rho_0 \alpha \beta \gamma = \rho_0 \alpha \beta \gamma$

$-\rho_0 \alpha \beta \gamma = \rho_0 \alpha \beta \gamma$

$\rho_0 \alpha \beta \gamma X + \rho_0 \alpha \beta \gamma - \rho_0 \alpha \beta \gamma = 0$ u. s. v. d. w. / $X = 0$

$p = f(x, y, z)$
 $p = f(x + \alpha, y, z)$

$\rho X + \frac{\rho - \rho'}{\alpha} = 0$

$\rho X - \frac{\partial p}{\partial x} = 0$ u. s. v. d. w. / $X = \frac{\partial p}{\partial x}$

o / u r C. 1/3

$$\rho X - \frac{\partial p}{\partial x} = 0$$

$$\rho Y - \frac{\partial p}{\partial y} = 0$$

$$\rho Z - \frac{\partial p}{\partial z} = 0$$

Mass Hydrostatik

x, y, z / y r o n s g r.

a f 9/3 0 redne. o ^c - y g r n ^c

$$\rho \alpha \beta \gamma X - \alpha \beta \gamma \frac{dp}{dx} = \rho \alpha \beta \gamma F_x$$

$$F_x = \gamma g \quad || \quad X=0$$

f' r / v o r s $\frac{d^2}{dt^2}$ s o a e x " " w i t ; o p k e p

o m v $\frac{dy}{dx}$ o p o g t.

es Euler'sche Ue: 1 e Par. u s o ' N s o u r g n

a r r e 9 p u r g f o e w i d a g e o m. Par. v e l.

e f o e u d o p i g e l s p l t a r Par.

1 p o - r 2 Par. p i g e l u, v, w r p t r / v

$$\frac{u' - u}{\tau} = \gamma g = F_x$$

$$u = \varphi(x, y, z, t)$$

$$u' = \varphi(x + u\tau, y + v\tau, z + w\tau, t + \tau) \quad \text{Taylor}$$

$$= u + \frac{du}{dx} u\tau + \frac{dv}{dy} v\tau + \frac{dw}{dz} w\tau + \frac{du}{dt} \tau$$

$$\frac{u'-u}{\tau} = F_x = \frac{du}{dt} + u \frac{du}{dx} + v \frac{du}{dy} + w \frac{du}{dz}$$

$$\left. \begin{aligned} -\rho X - \frac{dp}{dx} &= \rho \left[\frac{du}{dt} + u \frac{du}{dx} + v \frac{du}{dy} + w \frac{du}{dz} \right] \\ \rho Y &= \dots \\ \rho Z &= \dots \end{aligned} \right\} \begin{array}{l} \text{Terse} \\ \text{Hydrodyn.} \end{array}$$

$\rho x y z$ y^2 z^2 \dots ρ ρ^2 ρ^3 \dots ρ^n ρ^{n+1} \dots

f 3 25 \times 1 f 2 3 5 \dots u v w \dots ρ ρ^2 ρ^3 \dots

ρ ρ^2 ρ^3 \dots ρ^n ρ^{n+1} \dots ρ^m ρ^{m+1} \dots

ρ ρ^2 ρ^3 \dots ρ^n ρ^{n+1} \dots ρ^m ρ^{m+1} \dots

ρ ρ^2 ρ^3 \dots ρ^n ρ^{n+1} \dots ρ^m ρ^{m+1} \dots

$$\rho u = \psi(x, y, z, t)$$

$$(\rho u)' = \psi + \frac{\partial \psi}{\partial x} \alpha = \rho u + \frac{d(\rho u)}{dx} \alpha$$

$$\beta \gamma \rho u \tau - \beta \gamma (\rho u)' \tau = \beta \gamma \tau (\rho u - (\rho u)' + \frac{d(\rho u)}{dx} \alpha)$$

$$= -\alpha \rho y z \frac{d(\rho u)}{dx}$$

$$\text{rot } \vec{a} = \rho \vec{v} \times \vec{\omega} \parallel X: -\alpha \rho y z \frac{d(\rho u)}{dx}$$

$$\text{" " " Y: } -\alpha \rho y z \frac{d(\rho v)}{dy}$$

$$\text{" " " Z: } -\alpha \rho y z \frac{d(\rho w)}{dz}$$

$$\text{resultieren die rot } \vec{a}: -\alpha \rho y z \left[\frac{d(\rho u)}{dx} + \frac{d(\rho v)}{dy} + \frac{d(\rho w)}{dz} \right]$$

$$f = v \cdot \text{rot } \vec{a} = \alpha \rho y z \frac{d\rho}{dt} \tau$$

$$\left. \begin{aligned} \frac{d\rho}{dt} + \frac{d(\rho u)}{dx} + \frac{d(\rho v)}{dy} + \frac{d(\rho w)}{dz} = 0 \end{aligned} \right\} \text{V.N.} \\ \text{= Continuitätsgl.}$$

$$- \alpha \rho y z \frac{d\rho}{dt} \tau; \quad \rho = \rho_0 \left(1 - \frac{\alpha y z}{l} \right)$$

$$u, v, w = 0$$

$$\text{spez. } \rho = \rho_0 \left(1 - \frac{\alpha y z}{l} \right)$$

$$X, Y, Z = 0$$

$$\left\{ \begin{aligned} -\frac{dn}{dt} &= \rho \frac{dn}{dt} + \rho u \frac{dn}{dx} \quad \parallel \quad -\frac{dn}{dy} = -\frac{dn}{dz} = 0 \\ \frac{d\rho}{dt} + \frac{d(\rho u)}{dx} &= 0 \\ \rho &= A \cdot \rho^k \end{aligned} \right\} \text{d. } \rho \text{ } \text{spez. } \rho = \rho_0 \left(1 - \frac{\alpha y z}{l} \right)$$

$$n, \rho, u = ?$$

$$-\frac{dn}{dx} = p \frac{dn}{dt} + pn \frac{dn}{dx}$$

Eigenschaften von ρ & n

ρ & n sind v.

$$\frac{dp}{dt} + \frac{d(pn)}{dx} = 0$$

$$p = p_0 \cdot (1 + \delta) \quad \delta = nk \quad \rho = \rho_0 \cdot (1 - \delta)$$

$$-\frac{dn}{dx} = p_0 \frac{dn}{dt} \left[p_0 \delta \frac{dn}{dt} + p_0 (1 + \delta) n \frac{dn}{dx} \right]$$

us δ von n & ρ sind v. n & ρ sind v.

$$-\frac{dn}{dx} = p_0 \frac{dn}{dt} \quad \text{I).} \quad \frac{d}{dx} \left[\dots \right] \quad 7$$

$$\frac{dp}{dt} = p_0 \frac{d\delta}{dt} = - \frac{d}{dx} [p_0 n + p_0 n \delta]$$

$$= - p_0 \frac{dn}{dx}$$

$$\frac{d\delta}{dt} = - \frac{dn}{dx} \quad \text{II).} \quad p_0 \frac{d}{dx} [p_0 \dots]$$

$$n = A e^k \quad A \text{ const}$$

$$p_0 = A p_0^k$$

$$\frac{n}{p_0} = \left(\frac{p}{p_0} \right)^k$$

$$\frac{n}{p_0} = (1 + \delta)^k = 1 + k\delta + \dots \quad \text{III)}$$

$$n = p_0 [1 + k\delta] \quad \text{III)}$$

$$\frac{d(\Pi)}{dx} + \frac{d(\Pi_0)}{dx}$$

$$- \frac{d^2 \psi}{dx^2} + \rho_0 \frac{d^2 \psi}{dx^2} = 0$$

$$- \rho_0 k \frac{d^2 \psi}{dx^2} + \rho_0 \frac{d^2 \psi}{dx^2} = 0$$

$$\frac{d^2 \psi}{dx^2} = \frac{\rho_0 k}{\rho_0} \frac{d^2 \psi}{dx^2} \quad \text{in E.H. 16 / 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100}$$

for wave N.S. $\psi = \psi_0 \sin kx$ $\therefore k=1$

$$\rho_0 = 76 \cdot 1359 = 103504 \text{ g/cm}^3$$

$$\times 980.6 = 101470 \text{ g/cm}^3 \text{ [corrected value]}$$

$$\rho_0 = 0.00129$$

$$\sqrt{\frac{\rho_0}{\rho_0}} = 27000 \text{ cm} = \text{frequency of } \rho_0 \text{ [corrected]}$$

for Newton's law.

the wave $\lambda = 2\pi / k = 2\pi / 1 = 6.28 \text{ m}$; phase

of the wave is Laplace wave N.S. $\psi = \psi_0 \sin kx$

the wave $\lambda = 2\pi / k = 2\pi / 1 = 6.28 \text{ m}$; phase

$$\rho_0 = \rho_0 \left(\frac{\rho_0}{\rho_0} \right)^k \quad k = 1.70 \quad 14$$

$k = \text{const. } \rho \text{ const. } h : \text{const. Vol.}$

$$\frac{d^2 \delta}{dt^2} = a^2 \frac{d^2 \delta}{dx^2} \quad a^2 = \frac{\rho_0 k}{\rho}$$

$$\delta = \sin \alpha t [A \cos \beta x + B \sin \beta x] + \cos \alpha t [C \cos \beta x + D \sin \beta x]$$

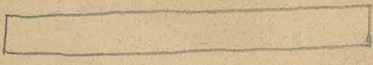
$$\alpha^2 = a^2 \beta^2$$

2^o u_1 : $\sim e^{\alpha t} + e^{-\alpha t} - e^{\alpha t} - e^{-\alpha t}$

u_2 $e^{\alpha t} - e^{-\alpha t}$

$$\begin{matrix} x=0 & u=0 \\ x=l & u=0 \end{matrix} \quad \text{I} \frac{du}{dt} = 0 \quad \text{II} \frac{du}{dx} = 0$$

$$\text{I} \frac{dv}{dx} = 0 \quad \frac{d\delta}{dx} = 0$$



$$\begin{aligned} \frac{d\delta}{dx} = \sin \alpha t [-\beta A \sin \beta x + \beta B \cos \beta x] + \\ + \cos \alpha t [-\beta C \sin \beta x + \beta D \cos \beta x] \end{aligned}$$

$$B_1 = D = 0$$

$$\frac{d\delta}{dx} = -\beta \sin \beta x [A \sin \alpha t + C \cos \alpha t]$$

$$e^{\beta l} \sin \beta l = 0$$

$$\beta l = \pi, 2\pi, 3\pi$$

$$\beta = \frac{\pi}{l}, \frac{2\pi}{l} \quad \alpha = \frac{a\pi}{l}$$

... ..

... ..

$$\underline{\underline{\delta = \cos \beta x [A \sin \alpha t + C \cos \alpha t]}}$$

$$p = p_0 (1 + k \delta)$$

$$\frac{dp}{dx} = p_0 k \frac{d\delta}{dx}$$

$$- p_0 k \frac{d\delta}{dx} = p_0 \frac{dn}{dt}$$

$$- a^2 \frac{d\delta}{dx} = \frac{dn}{dt}$$

$$\frac{dn}{dt} = - a^2 \beta \sin \beta x [A \sin \alpha t + C \cos \alpha t]$$

$$n = - a^2 \beta \sin \beta x \frac{B \sin \alpha t - A \cos \alpha t}{\alpha}$$

$$\underline{\underline{u = a \sin \beta x [C \sin \alpha t - A \cos \alpha t]}}$$

$$\delta \sim \delta \text{ at } x=0$$

$$\beta x = \frac{\pi}{2} \quad \delta = 0 \quad \text{at } x = \frac{l}{2}$$

$$x = \frac{l}{2} \quad \delta = 0 \quad \text{at } x = \frac{l}{2}$$

$$\delta \sim \delta \text{ at } x=0 = 0$$

$$\text{... .. } x = \frac{l}{2}$$

Octave:

$$v = v_0 \cos \frac{2\pi x}{l}$$

$$u = a \sin \frac{2\pi x}{l}$$

$$v=0 \quad \frac{2\pi x}{l} = \frac{\pi}{2} \quad x = \frac{l}{4}$$

$$* \quad \neq 0 \quad x = \frac{3l}{4}$$

$$< 0$$

$$> 0 \quad \text{or}$$

$$x = l$$

as cond. of G

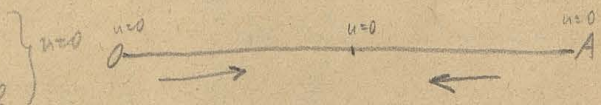
pts:



pts:

$$x=0$$

$$\frac{2\pi x}{l} = n \quad x = \frac{l}{2}$$



as v y p del v

no cond. in the middle; $\frac{2\pi x}{l} = n$ in $0, l$

20/5 Experimente: Longitud. p, l, $\frac{2\pi l}{\lambda}$ $\frac{2\pi l}{\lambda} = n$

$$y'' + p y' + q y = e^{px}$$

$$\frac{dy}{dx} = \frac{dy}{dt} \quad \text{Let } p = \dots$$

$$IV: \quad y = e^{-\alpha x} B = 0 \quad \text{put } B = \dots$$

$$B = \dots$$

$$B = 0$$

$$x = 0 \quad x = l$$

$$y = \dots$$

$$y = \sin \frac{\pi x}{l} \left[A_1 \cos \frac{\pi a t}{l} + B_1 \sin \frac{\pi a t}{l} \right] +$$

$$+ \sin \frac{2\pi x}{l} \left[A_2 \cos \frac{2\pi a t}{l} + B_2 \sin \frac{2\pi a t}{l} \right] + \dots$$

\dots re C , Octave etc.

$$- \frac{dy}{dx} = p_0 \frac{dy}{dt} \quad p = p_0 (1 + k \delta)$$

$$- p_0 k \frac{dy}{dx} = p_0 \frac{dy}{dt}$$

$$- a \frac{dy}{dx} = \frac{dy}{dt}$$

$$\frac{dy}{dt} = 0 \quad \text{and} \quad \frac{dy}{dx} = 0$$

$$\text{re } C \quad y = \sin \frac{\pi x}{l} [\dots]$$

$$\frac{dy}{dx} = \frac{\pi}{l} \cos \frac{\pi x}{l} [\dots]$$

$$\frac{\pi x}{l} = \frac{\pi}{2} \quad x = \frac{l}{2} \quad \text{etc.}$$

$$p a = 0$$

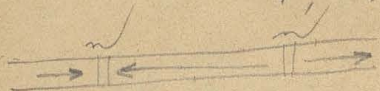


$$\text{Octave } b = \sin \frac{2\pi x}{l} [\text{---}]$$

$$\frac{db}{dx} = \frac{2\pi}{l} \cos \frac{2\pi x}{l} [\text{---}]$$

$$\frac{db}{dx} = 0 : \frac{2\pi x}{l} = \frac{\pi}{2}, \frac{3\pi}{2}$$

$$x = \frac{l}{4}, \frac{3l}{4}$$



$$\text{Quint Octave } b = \sin \frac{3\pi x}{l} [\text{---}]$$

$$\frac{db}{dx} = \frac{3\pi}{l} \cos \frac{3\pi x}{l} [\text{---}]$$

$$\frac{db}{dx} = 0 : \frac{3\pi x}{l} = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}$$

$$x = \frac{l}{6}, \frac{3l}{6}, \frac{5l}{6}$$



underepiter b o p d' = 0 ~ ~ ~

9 ~ 106 - p o e n ; ~

10. I ~ ~ ; ~ ~ ~ ~ ~

1 Octave ~ ~ ~ ~ ~ ~ ~ ~ ~

~ ~ ~ ~ ~ ~ ~ ~ ~ ~ ~ ~ ~ ~ ~

~ ~ ~ ~ ~ ~ ~ ~ ~ ~ ~ ~ ~ ~ ~

Job 1st 7.

$$b = \sin \alpha t [A \cos \beta x + B \sin \beta x]$$

$x=0$ $y=0$ $x=l$ $y=0$

$$x=0 \quad \frac{db}{dt} = 0 = \alpha^2 \frac{db}{dx} = 0$$

$$x=l \quad b=0$$

$$\frac{db}{dx} = \sin \alpha t [-\beta A \sin \beta x + \beta B \cos \beta x]$$

$$B=0$$

$$b = \sin \alpha t A \cos \beta x$$

$$0 = \sin \alpha t A \cos \beta l$$

$$\beta l = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2} \dots$$

$$\beta = \frac{\pi}{2l}, \frac{3\pi}{2l}, \dots$$

$7\pi/2, 5\pi/2, \dots$

for the Bernoulli Eqn

for y : $b=l$, $b=0$ & $x \neq 0$ & $b=0$

the b is 0 if l is 0 or $2l$ or $4l$ or $6l$ or

$2l$ or $4l$ or $6l$ or $8l$ or

$10l$ or $12l$ or $14l$ or $16l$ or $18l$ or $20l$ or

$\rho = \rho_0 (1 + k\phi)$ $\rightarrow \rho = \rho_0 (1 + k(u + v + w))$
 for $\rho = \rho_0 (1 + k(u + v + w))$ $\rightarrow \rho = \rho_0 (1 + k(u + v + w))$
 $\frac{d\rho}{dt} = \rho_0 k \left(\frac{du}{dt} + \frac{dv}{dt} + \frac{dw}{dt} \right)$

$$-\frac{d\rho}{dt} = \rho \left[\frac{du}{dt} + u \frac{du}{dx} + v \frac{du}{dy} + w \frac{du}{dz} \right]$$

$$-\frac{d\rho}{dt} = \rho \frac{du}{dt} \neq \rho_0 \frac{du}{dt} \quad \parallel X$$

$$-\frac{d\rho}{dt} = \rho_0 \frac{dv}{dt} \quad \parallel Y$$

$$-\frac{d\rho}{dt} = \rho_0 \frac{dw}{dt} \quad \parallel Z$$

$$\frac{d\rho}{dt} = - \frac{d(\rho u)}{dx} - \frac{d(\rho v)}{dy} - \frac{d(\rho w)}{dz}$$

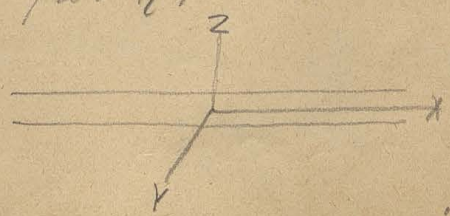
$$\rho_0 \frac{d\rho}{dt} = - \rho_0 \left(\frac{du}{dx} + \frac{dv}{dy} + \frac{dw}{dz} \right)$$

$$-\left[\frac{d\rho}{dx} + \frac{d\rho}{dy} + \frac{d\rho}{dz} \right] = \rho_0 \frac{d\rho}{dt} \left[\frac{du}{dx} + \frac{dv}{dy} + \frac{dw}{dz} \right]$$

$$\rho = \rho_0 (1 + k\phi) \quad \Rightarrow \rho_0 k \frac{d\phi}{dt}$$

$$\rho_0 k \left[\frac{d\phi}{dx} + \frac{d\phi}{dy} + \frac{d\phi}{dz} \right] = \rho_0 \frac{d\phi}{dt}$$

for $\rho = \rho_0 (1 + k\phi)$



note in the case $\parallel X$
 $\rho = \rho_0 (1 + k\phi)$
 $\rho_0 k \frac{d\phi}{dt} = \rho_0 \frac{d\phi}{dt}$
 $\rho_0 k = \rho_0$

$$\pm \sigma \rho = 0^2$$

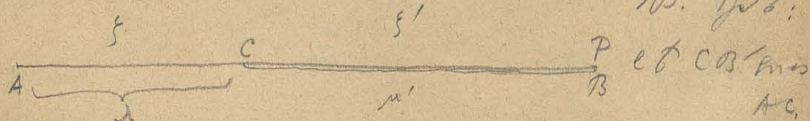
2. $\cos \rho \approx 1$: $\rho \approx \sqrt{\sigma \rho}$ / $\rho \approx \sqrt{\sigma \rho}$ in V , $\rho \approx \sqrt{\sigma \rho}$

$\rho \approx \sqrt{\sigma \rho}$ / $\rho \approx \sqrt{\sigma \rho}$ in V ; $\rho \approx \sqrt{\sigma \rho}$

Helmholtz $\Delta \psi = 0$; $\rho \approx \sqrt{\sigma \rho}$ in V

$\rho \approx \sqrt{\sigma \rho}$ in V & $\rho \approx \sqrt{\sigma \rho}$ in V

24/5 $\cos \rho \approx 1$ $\rho \approx \sqrt{\sigma \rho}$ in V & $\rho \approx \sqrt{\sigma \rho}$ in V



$$\mu \frac{d^2 \xi}{dt^2} = P \frac{d^2 \xi}{dx^2}$$

$$\mu' \frac{d^2 \xi'}{dt^2} = P \frac{d^2 \xi'}{dx'^2}$$

$$\frac{P}{\mu} = a^2 \quad \frac{P}{\mu'} = a'^2$$

$$\xi = [A \sin at + B \cos at] [C \sin \beta x + D \cos \beta x]$$

$$\left. \begin{array}{l} x=0 \\ \xi=0 \end{array} \right\} D=0$$

$$\xi = \sin \beta x [F \sin at + G \cos at]$$

$$a^2 = a'^2 \beta^2$$

$$\xi' = [A' \sin \alpha' t + B' \cos \alpha' t] [C' \sin \beta' x + D' \cos \beta' x]$$

$$\left. \begin{array}{l} \text{if } \mu \neq \mu' \text{ or } \rho \neq \rho' \\ \text{or } \rho \neq \rho' \end{array} \right\} \begin{array}{l} x=l \\ \xi'=0 \end{array}$$

$$C' \sin \beta' l + D' \cos \beta' l = 0$$

$$\xi' = [A' \sin \alpha' t + B' \cos \alpha' t] \left[-\frac{D' \sin \beta' x \cos \beta' l + D' \cos \beta' x}{\sin \beta' l} \right]$$

$$= [\quad] \left[\frac{D'}{\sin \beta' l} \sin \beta' (l-x) \right]$$

$$\xi' = \sin \beta' (l-x) [F' \sin \alpha' t + G' \cos \alpha' t]$$

$$\alpha' = \alpha / \beta'$$

17 C:

$$x = a \quad \xi = \xi'$$

$$\sin \beta' x [F' \sin \alpha' t + G' \cos \alpha' t] = \sin \beta' x' [F' \sin \alpha' t + G' \cos \alpha' t]$$

$$l - x = x'$$

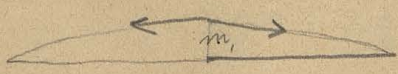
$f^2 / \mu^2 = \omega^2$; $\alpha = \alpha'$; $\omega_1 + \omega_2$ per h.

α and β of $f = C$ is prob α and β .

$$F \sin \beta' x = F' \sin \beta' x'$$

$$G \sin \beta' x = G' \sin \beta' x'$$

of ω_1 or ω_2 of ω_1 or ω_2 in, ω_1 and ω_2 are constant. ω_1 and ω_2 are prob ω_1 and ω_2 .



$$m_1 \frac{d^2 \xi}{dt^2} = P \sin \alpha' - P \sin \alpha$$

$$= P \frac{d\xi'}{dx} - P \frac{d\xi}{dx}$$

$\omega = 9.10 \text{ m/s}$
 mass of ξ and ξ'
 no cons. of prob.

ρ/σ a const. v_0 a const.

$$\text{and: } P \frac{d\rho'}{dt} = P \frac{d\rho}{dt}$$

$$\frac{d\rho'}{dt} = \frac{d\rho}{dt} \quad \text{or } \rho' = \rho \text{ for } \text{const. } P \text{ and } v_0$$

$$\beta \cos \beta \lambda [F \sin \alpha t + G \cos \alpha t] = -\beta' \cos \beta' \lambda' [F' \sin \alpha' t + G' \cos \alpha' t]$$

$$\left\{ \begin{aligned} \beta \cos \beta \lambda \cdot F &= -\beta' \cos \beta' \lambda' F' \\ \beta \cos \beta \lambda \cdot G &= -\beta' \cos \beta' \lambda' G' \end{aligned} \right.$$

$$\beta \cos \beta \lambda \cdot G = -\beta' \cos \beta' \lambda' G'$$

$$\sin \beta \lambda F = -\sin \beta' \lambda' F'$$

$$\sin \beta \lambda G = -\sin \beta' \lambda' G'$$

$$\frac{\sin \beta \lambda}{\beta \cos \beta \lambda} = -\frac{\sin \beta' \lambda'}{\beta' \cos \beta' \lambda'}$$

$$\frac{\tan \beta \lambda}{\beta} = -\frac{\tan \beta' \lambda'}{\beta'} \quad \text{— transverse velocity}$$

$$\beta \lambda = u$$

$$\beta' \lambda' = u'$$

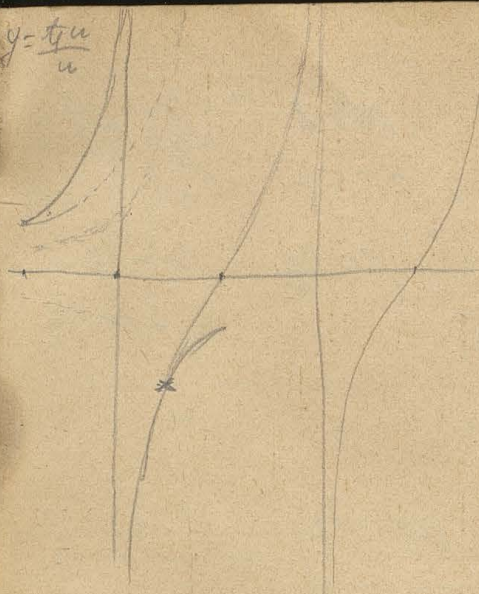
$$\beta' = \beta \frac{a}{a'}$$

$$\beta' \lambda' = \beta \frac{a}{a'} \lambda' = u \frac{a \lambda'}{a' \lambda}$$

$$\frac{\lambda \tan u}{u} = -\frac{\tan(u \frac{a \lambda'}{a' \lambda})}{u \frac{a \lambda'}{a' \lambda}}$$

$$\frac{\tan u}{u} \lambda = -\frac{\tan(u \frac{a \lambda'}{a' \lambda})}{u \frac{a \lambda'}{a' \lambda}} \lambda'$$

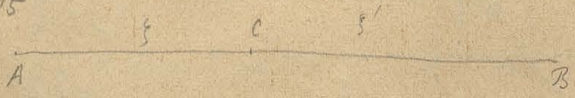
$$y = \frac{tu}{u}$$



$v = \frac{1}{\beta} \ln \frac{u}{u_0}$
 s. const. $\frac{1}{\beta} \ln \frac{u}{u_0}$ $\frac{1}{\beta} \ln \frac{u}{u_0}$
 24

$$y' = \frac{1}{\lambda} \frac{tu'}{u}$$

25/5



$$\frac{d^2y}{dx^2} = a^2 \frac{d^2t}{dx^2}$$

$$f = F \sin \beta x \text{ and } \sin \beta x$$

$\frac{1}{\lambda} \frac{d^2y}{dx^2} = a^2 \frac{d^2t}{dx^2}$
 $\frac{1}{\lambda} \frac{d^2y}{dx^2} = a^2 \frac{d^2t}{dx^2}$

$$y' = F' \sin \alpha' t \sin \beta (l-x)$$

see below

$$f = f' \quad | \quad x = l$$

$$l - x = x'$$

$$x = x'$$

$$F \sin \beta x = F' \sin \beta x'$$

$$\beta F \cos \beta x = -F' \beta' \cos \beta' x'$$

$$\frac{t \beta x}{\beta} = - \frac{t \beta' x'}{\beta'}$$

unfalsch ist:

$$\tan \beta \lambda = -\frac{\beta}{\beta'} \tan \beta' \lambda$$

$$\alpha = a \beta$$

$$\frac{\beta}{\beta'} = \frac{a'}{a}$$

$$\alpha' = a' \beta'$$

$$= -\frac{a'}{a} \tan \beta' \lambda$$

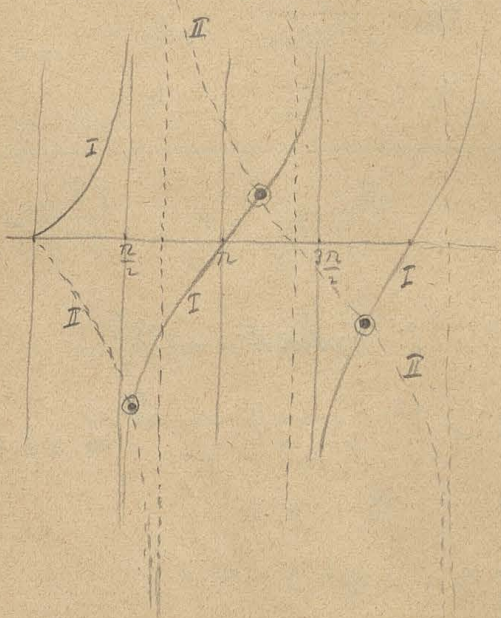
$$\beta' = \frac{\beta a}{a'}$$

$$\beta' \lambda' = \frac{\beta a}{a'} \lambda' = \beta \lambda \frac{a \lambda'}{a' \lambda}$$

$$\tan \beta \lambda = -\frac{a'}{a} \tan \left[\beta \lambda \frac{a \lambda'}{a' \lambda} \right]$$

$$\tan \beta' \lambda' = -m \tan \beta \lambda$$

Skizze für $n < 1$ - v. v. d. geom. Lage



$$\tan \beta' \lambda' = \infty$$

$$n y = \frac{n}{2}$$

$$y = \frac{n}{2n}$$

$$n < 1$$

$$y > \frac{n}{2}$$

$$\odot = \mathcal{P}$$

$v \in \infty \vee \lambda < \lambda' < \lambda$ $a_1 \vee \lambda < 1, 2, 3$ — $\beta' \alpha' \lambda'$

Skizze für $n < 1$ — v. v. d. geom. Lage

f. v. v. d. geom. Lage

$$\mathcal{E} \frac{dy}{dx} = \mathcal{E}' \frac{dy'}{dx}$$

$$A \cos \beta x = A' \cos \beta' x$$

$$-\beta \mathcal{E} A \sin \beta x = \mathcal{E}' \beta' A' \sin \beta' x$$

div. 1/x

$$\cos \beta x \neq \beta \beta'; \text{ etc.}$$

$$\sin \beta x \neq \beta \beta'; \text{ etc.}$$

$$\sin \beta x \neq \beta \beta'; \text{ etc.}$$

$$\sin \beta x \neq \beta \beta'; \text{ etc.}$$

$$\sin \beta x \neq \beta \beta'; \text{ etc.}$$

$$\sin \beta x \neq \beta \beta'; \text{ etc.}$$

$$\sin \beta x \neq \beta \beta'; \text{ etc.}$$

$$\sin \beta x \neq \beta \beta'; \text{ etc.}$$

$$\sin \beta x \neq \beta \beta'; \text{ etc.}$$

$$\sin \beta x \neq \beta \beta'; \text{ etc.}$$

$$\sin \beta x \neq \beta \beta'; \text{ etc.}$$

$$\frac{27}{5} \frac{dy}{dx} = \frac{p dy}{\mu dx}$$

$$y = \sin \frac{2\pi x}{\lambda} [A_1 \cos \omega_1 t + B_1 \sin \omega_1 t]$$

$$+ \sin \frac{2\pi x}{\lambda} [A_2 \cos \omega_2 t + B_2 \sin \omega_2 t]$$

(1) Superposition von y_1 + y_2

wird durch $y = y_1 + y_2$ gegeben.

$$Lk = \sum e Lk \text{ (unf. + w. d. t.)} \quad 26$$

Supp. e Lk

$$\frac{dS}{dt} = \sin \frac{\pi x}{l} [-\alpha_1 A_1 \sin \alpha_1 t + \alpha_1 B_1 \cos \alpha_1 t] \\ + \sin \frac{2\pi x}{l} [-\alpha_2 A_2 \sin \alpha_2 t + \alpha_2 B_2 \cos \alpha_2 t]$$

+ ...

$$\frac{1}{2} \left(\frac{dS}{dt} \right)^2 dx = Lk \cdot \omega^2$$

$$\int_0^l \dots = Lk \cdot \omega^2$$

$$\int_0^l \sin \frac{m\pi x}{l} \sin \frac{n\pi x}{l} dx = 0 \quad \text{if } m \neq n = Lk \cdot \omega^2$$

$$D_3 = \dots Lk \cdot \omega^2 \quad k_1 = \dots$$

for k_1, \dots - homog. b. s. \dots

$$n \frac{d^2 y}{dt^2} = P \frac{d^2 y}{dx^2} \quad p = \dots$$

$$n' \frac{d^2 y'}{dt'^2} = P' \frac{d^2 y'}{dx'^2} \quad p' = \dots$$

$$y = \sin \alpha_1 t X_1 + \sin \alpha_2 t X_2 + \sin \alpha_3 t X_3$$

+ ...

$$y' = \sin \alpha_1 t X'_1 + \sin \alpha_2 t X'_2 + \dots$$

PX & \dots

$$n \frac{d^2 y}{dt^2} = -\alpha_1^2 \sin \alpha_1 t P \sin \alpha_1 t = \frac{d^2 y}{dx^2} = 0$$

$$-\mu \alpha_1^2 X_1 = P \frac{d^2 X_1}{dx^2}$$

~~$$-\mu \alpha_2^2 X_2 = P \frac{d^2 X_2}{dx^2}$$~~ ←

~~$$-\mu' \alpha_1^2 X_1' = P \frac{d^2 X_1'}{dx^2}$$~~

2) 2 fctn X_1, X_2

$$\int X_1 X_2 \mu dx + \int X_1' X_2' \mu' dx = 0$$

$$\mu \int X_1 X_2 dx + \mu' \int X_1' X_2' dx = 0$$

$$\int X_1 X_2 dx = - \int X_1' \left[\frac{P}{\mu \alpha_2^2} \frac{d^2 X_2'}{dx^2} \right] dx$$

~~$$-\mu \alpha_2^2 X_2 = P \frac{d^2 X_2}{dx^2}$$~~

$$= -\frac{P}{\mu \alpha_2^2} \int X_1 \frac{d^2 X_2}{dx^2} dx = -\frac{P}{\mu \alpha_2^2} \left[X_1 \frac{dX_2}{dx} - \int \frac{dX_1}{dx} \frac{dX_2}{dx} dx \right]$$

$$= -\frac{P}{\mu \alpha_2^2} \left[X_1 \frac{dX_2}{dx} - X_2 \frac{dX_1}{dx} + \int X_2 \frac{dX_1}{dx} dx \right]$$

$$= -\frac{P}{\mu \alpha_2^2} \left[X_1 \frac{dX_2}{dx} - X_2 \frac{dX_1}{dx} - \frac{\mu \alpha_2^2}{P} \int X_2 X_1 dx \right]$$

$$\int X_1 X_2 dx = -\frac{P}{\mu \alpha_2^2} \left[X_1 \frac{dX_2}{dx} - X_2 \frac{dX_1}{dx} \right] + \frac{\alpha_2^2}{\alpha_1^2} \int X_2 X_1 dx$$

$$\mu(\alpha_2^2 - \alpha_1^2) \int X_1 X_2 dx = -P \left[X_1 \frac{dX_2}{dx} - X_2 \frac{dX_1}{dx} \right]_0^l$$

$$\mu'(\alpha_2^2 - \alpha_1^2) \int_0^l X_1 X_2' dx = -P \left[X_1' \frac{dX_2}{dx} - X_2 \frac{dX_1'}{dx} \right] / \alpha \quad 27$$

$$x=0 \quad f=0 \quad X_1=0 \quad X_2=0$$

$$x=l \quad f=0 \quad X_1'=0 \quad X_2'=0$$

$$(\alpha_2^2 - \alpha_1^2) \left[\mu \int_0^l X_1 X_2 dx + \mu' \int_0^l X_1' X_2' dx \right] =$$

$$- P \left[X_1 \frac{dX_2}{dx} - X_2 \frac{dX_1'}{dx} \right] / \alpha$$

$$+ P \left[X_1' \frac{dX_2}{dx} - X_2 \frac{dX_1'}{dx} \right] / \alpha$$

$$x=l \quad \frac{df}{dx} = \frac{d^2 f}{dx^2}$$

$$i = \frac{P}{\alpha} \frac{d^2 f}{dx^2} \quad \text{or } \frac{d^2 f}{dx^2} = \frac{P}{\alpha} i$$

for a cantilever beam of length \$l\$ and cross-section \$A\$.

Let \$k\$ be the stiffness.

20.

quasi



for a cantilever beam

31/5 D'Alambert:

$$\frac{d^2 \xi}{dt^2} = a^2 \frac{d^2 \xi}{dx^2}$$

1025 10/10 4/10 10 - f(x-at) 80

$$\xi = f(x-at)$$

$$\frac{\partial \xi}{\partial x} = f'(x-at)$$

$$\frac{\partial \xi}{\partial t} = -af'(x-at)$$

$$\frac{\partial^2 \xi}{\partial x^2} = f''(x-at)$$

$$\# \frac{\partial^2 \xi}{\partial t^2} = a^2 f''(x-at)$$

$$\frac{\partial^2 \xi}{\partial x^2} =$$

$$\frac{\partial^2 \xi}{\partial t^2} = a^2 \xi + a$$

$$\xi = F(x+at)$$

$$\xi = f(x-at) + F(x+at)$$

100.

$$\xi = a \sin \frac{\pi x}{l} \sin \frac{a \pi t}{l}$$

$$= \frac{a}{2} \left[\cos \frac{\pi}{l} (x-at) + \cos \frac{\pi}{l} (x+at) \right]$$

$$t=0 \quad \xi = \varphi(x)$$

$$\frac{\partial \xi}{\partial t} = \psi(x)$$

$$\frac{d\xi}{dt} = -a \left[f'(x-at) + a F'(x+at) \right]$$

$$t=0 \quad \psi$$

$$\varphi(x) = f(x) + F(x)$$

$$\varphi(x) = [f(x) + F(x)]a$$

$$\int_0^x \varphi(x) dx = -a \int_0^x f(x) dx + a \int_0^x F(x) dx$$
$$= -a f(x) \Big|_0^x + a F(x) \Big|_0^x$$

$$= -a f(x) + a f(0) + a F(x) - a F(0)$$

C

$$C + \int_0^x \varphi(x) dx = -a f(x) + a F(x)$$

$$C + \int_0^x \varphi(y) dy = -a f(x) + a F(x)$$

$$f(x) + F(x) = \varphi(x)$$
$$-f(x) + F(x) = \frac{1}{a} \left[\int_0^x \varphi(y) dy + \frac{C}{2} \right]$$

$$F(x) = \frac{1}{2} \varphi(x) + \frac{1}{2a} \int_0^x \varphi(y) dy + \frac{C}{2a}$$

$$f(x) = \frac{1}{2} \varphi(x) - \frac{1}{2a} \int_0^x \varphi(y) dy - \frac{C}{2a}$$

$$f = \frac{1}{2} \varphi(x-at) + \frac{1}{2a} \int_0^{x-at} \varphi(y) dy + \frac{1}{2} \varphi(x+at) + \frac{1}{2a} \int_0^{x+at} \varphi(y) dy$$

$$-\frac{1}{2a} \int_0^x \varphi(y) dy = \frac{1}{2a} \int_{x-at}^0 \varphi(y) dy$$



$\tau \rightarrow \text{const } 0 = \text{uph } \omega$

$$\varphi(x) \geq 0 \quad \left| \begin{array}{l} x \pm \frac{\lambda}{2} \\ \pm \frac{\lambda}{2} \end{array} \right.$$

$$\varphi = \frac{1}{2} \varphi(x_1 - at) + \frac{1}{2} \varphi(x_1 + at)$$

$= 0$

$x_1 > \frac{\lambda}{2}$

$$\therefore \text{Nun } a \text{ } x_1 - at \left\{ \begin{array}{l} -\frac{\lambda}{2} \\ +\frac{\lambda}{2} \end{array} \right.$$

$\frac{1}{6}$...

$$v \sim \rho \approx \dots \text{ } x \text{ } y \text{ } ?$$



16 ...

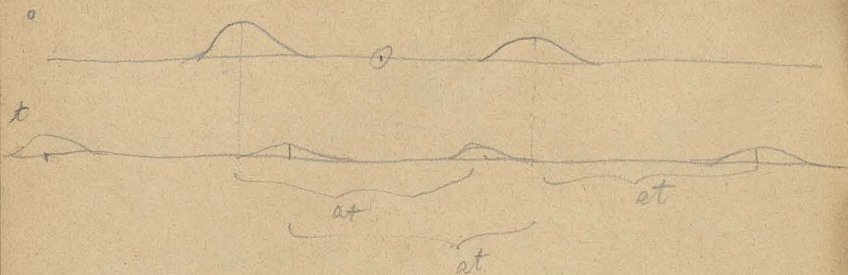
$$\rho \text{ } \dots \text{ } \frac{at}{\dots} \text{ } - \frac{1}{2}$$

...

$$\varphi(x-at) = \dots$$

$$\varphi(x+at) = \dots$$

I



II



$I_0 \ll \ll II \text{ t } \approx \text{ gl } \text{ er } \int \ll \text{ 9/1}$

I 0 $\varphi(x) \parallel$ $\varphi(x) = 0$

II t $\varphi(x) \parallel$ $\varphi(x) \geq 0$

v/6:

$$\xi = \frac{1}{2} \varphi(x-at)$$

$$\frac{d\xi}{dt} = -\frac{a}{2} \varphi(x-at)$$

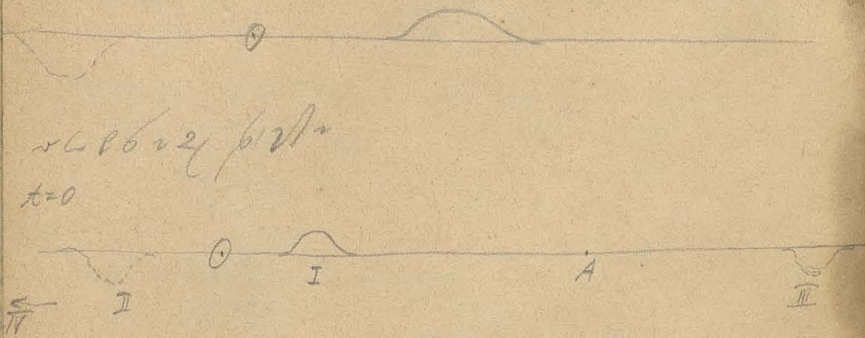
$$\xi = \frac{1}{4} \varphi(x-at) + \frac{1}{4} \varphi(x+at) + \frac{1}{4\pi} \int_{x-at}^{x+at} \varphi'(y) dy$$

$$= \frac{1}{4} \varphi(x-at) + \frac{1}{4} \varphi(x+at) + \frac{1}{4} \varphi(x+at) + \frac{1}{4} \varphi(x-at)$$

$$= \frac{1}{2} \varphi(x-at) \quad \text{im } \varphi(x) \text{ set } \varphi(x) \text{ set } \varphi(x)$$

t=0 | $\xi = \frac{\varphi(x)}{2}$ im $\varphi(x) \text{ set } \varphi(x)$

2/6



slab of length l
 $t=0$

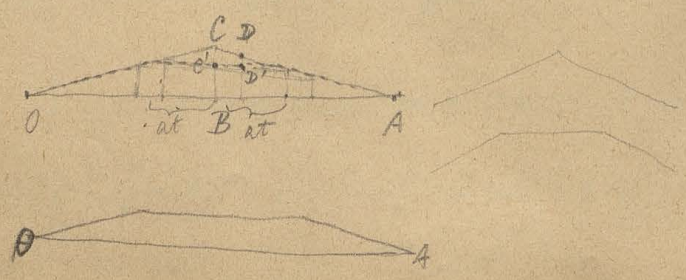
in a slab of length l is $\frac{1}{2} \rho a^2 \omega^2 l$ etc.
 200.



is $\frac{1}{2} \rho a^2 \omega^2 l$ per unit length $\frac{1}{2} \rho a^2 \omega^2 l$ etc.
 at $t=2l$

$$v = \frac{2l}{a} = \frac{1}{2l} \rho a^2 \omega^2 \quad n = \frac{a}{2l} = \frac{1}{2l} \sqrt{\frac{P}{\mu}}$$

for $v = \dots$ is $\frac{1}{2} \rho a^2 \omega^2 l$ etc.



in φ e Δ f r φ p r a φ

$u e a t = \frac{1}{2} \varphi(x - at) + \frac{1}{2} \varphi(x + at)$

de.

φ r φ b r d e m r e t r e d e r e p r a

φ r φ b r d e m r e t r e d e r e p r a



φ r φ b r d e m r e t r e d e r e p r a

φ r φ b r d e m r e t r e d e r e p r a

φ r φ b r d e m r e t r e d e r e p r a



$f = \frac{1}{2} \varphi(x - at) + \frac{1}{2} \varphi(x + at)$

$f = \frac{1}{2} \varphi(x - at)$ y φ r φ b r d e m r e t r e d e r e p r a

$\frac{df}{dt} = -\frac{a}{2} \varphi'(x - at)$

φ r φ b r d e m r e t r e d e r e p r a

φ r φ b r d e m r e t r e d e r e p r a

φ r φ b r d e m r e t r e d e r e p r a

φ r φ b r d e m r e t r e d e r e p r a

φ r φ b r d e m r e t r e d e r e p r a

φ r φ b r d e m r e t r e d e r e p r a

for \cos .
 $\sim \cos \varphi : \omega = \varphi < 0$ mirror $\omega = \varphi > 0$ $\sim \cos \varphi$
 $\omega < 0$ $\varphi < 0$



$$\varphi(0) = \varphi(\cos \varphi)$$

$$x=0 \quad \varphi = f(t)$$

$$x=0 \quad \frac{d\varphi}{dt} = 0$$

$$\varphi = \frac{1}{2} \varphi(x-at) + \frac{1}{2} \varphi(x+at)$$

$$x=0 \quad \varphi = f(t)$$

$$f(t) = \frac{1}{2} \varphi(-at) + \frac{1}{2} \varphi(at) \quad at = y$$

$$f\left(\frac{y}{a}\right) = \frac{1}{2} \varphi(-y) + \frac{1}{2} \varphi(y)$$

$$\varphi(-y) = 2 f\left(\frac{y}{a}\right) - \varphi(y) \quad \varphi \text{ is } \varphi(-\varphi)$$

$\frac{3}{6}$



$$x=0 \quad \varphi = f(t)$$

$$\varphi = \frac{1}{2} \varphi(x-at) + \frac{1}{2} \varphi(x+at)$$

$$x=0 \quad \varphi = \varphi(x)$$

$$f(t) = \frac{1}{2} \varphi(-at) + \frac{1}{2} \varphi(at)$$

$$\varphi(-at) = 2 f(t) - \varphi(at)$$

$$\varphi(-y) = 2 f\left(\frac{y}{a}\right) - \varphi(y)$$

20. $\varphi(y) = 0$ $y > 0$ für $t=0$ & $y=0$ ist

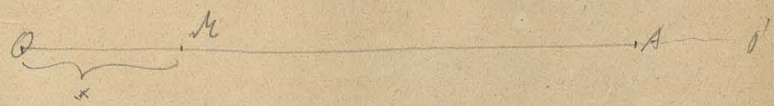


$$\begin{aligned} f &= \frac{1}{2} \varphi(x-at) \\ &= \frac{1}{2} \varphi[-(at-x)] \\ &= f\left(\frac{at-x}{a}\right) \end{aligned}$$

$$\begin{aligned} at-x &= y + a \\ \varphi(-y) &= 2 f\left(\frac{y}{a}\right) \end{aligned}$$

$$\xi = f\left(t - \frac{x}{a}\right) \quad x=0 \quad \xi = f(t)$$

0 ist ξ für $t=0$ $\xi = f\left(t - \frac{x}{a}\right)$
 $\xi = f\left(t - \frac{x}{a}\right)$ für $t = \frac{x}{a}$
 $\xi = f\left(t - \frac{x}{a}\right)$ für $t = \frac{x}{a}$



$\xi = f\left(t - \frac{x}{a}\right)$ für $t = \frac{x}{a}$
Wahrscheinlich $\xi = f\left(t - \frac{x}{a}\right)$
für $t = \frac{x}{a}$

$\sim 0.2 \text{ m/s}^2$, $\omega = 1 \text{ rad/s} \Rightarrow 0.2 \text{ g}$ $\times 2 = 0.4 \text{ g}$
 10 A $\mu\text{m/s}^2$ ~ 0 refl. ω \rightarrow di

$$P \sim 6.2 \sqrt{f^2} = 2 \alpha$$

$$f = f(t - \frac{x}{a}) - f(t - \frac{2l+x}{a}) + f(t - \frac{2l-x}{a}) + \dots$$

$$f = f(t - \frac{x}{a}) - f(t - \frac{2l-x}{a}) + f(t - \frac{2l+x}{a}) - f(t - \frac{4l-x}{a}) + f(t - \frac{4l+x}{a}) - \dots$$

$\approx \sqrt{f} \omega \rightarrow \Delta f_c(\omega) < 0.6$

$$x=0 \quad f = f(t)$$

$$x=l \quad f = 0$$

20. $f(t) = A \sin \alpha t$

$$f = A \sin \alpha(t - \frac{x}{a}) - A \sin \alpha(t - \frac{2l-x}{a}) +$$

$$+ A \sin \alpha(t - \frac{2l+x}{a}) - A \sin \alpha(t - \frac{4l-x}{a}) +$$

$$+ A \sin \alpha(t - \frac{4l+x}{a}) - \dots$$

$$= A \sin \alpha(t - \frac{x}{a}) - 2A \cos \alpha(t - \frac{2l}{a}) \sin \frac{\alpha x}{a} -$$

$$- 2A \cos \alpha(t - \frac{4l}{a}) \sin \frac{\alpha x}{a} - 2A \cos \alpha(t - \frac{6l}{a}) \sin \frac{\alpha x}{a} - \dots$$

$\approx \sqrt{f} \omega \rightarrow 0.5 \text{ rad} \sim 0.5 \text{ rad} \sim 0.5 \text{ rad} \sim 0.5 \text{ rad}$

10/15 / x=0 y=0

$$20. \frac{\alpha 2l}{a} = 2\pi$$

$$y = A \sin \alpha \left(t - \frac{x}{a} \right) - 2A \sin \frac{\alpha x}{a} \left[\cos \alpha t \left[\cos \alpha \frac{2l}{a} + \cos \alpha \frac{4l}{a} + \cos \alpha \frac{6l}{a} \right] - 2A \sin \frac{\alpha x}{a} \sin \alpha t \left[\sin \alpha \frac{2l}{a} + \sin \alpha \frac{4l}{a} + \dots \right] \right]$$

0 \sim $\sqrt{12} \sim$ $\sqrt{12}$ \sim $\sqrt{12}$ \sim $\sqrt{12}$

$$21. \frac{\alpha 2l}{a} = 2\pi$$

$$y = A \sin \alpha \left(t - \frac{x}{a} \right) - 2A \sin \frac{\alpha x}{a} \cos \alpha t \left[\cos \alpha t + \cos 2\alpha t + \dots \right]$$

1) $\cos \alpha t < 0$ / ∞ or t / ∞

2) $\cos \alpha t > 0$ / ∞ or t / ∞

1) $\cos \alpha t < 0$ / ∞ or t / ∞

2) $\cos \alpha t > 0$ / ∞ or t / ∞

Then $\sim \sqrt{12}$ Resonance

$$-6^{\text{th}} \text{ Resonance } \sim \text{N d l } \frac{\alpha 2l}{a} = 2\pi$$

$$y = a \sin \alpha t \quad \alpha =$$

$$\alpha = \frac{2\pi n}{a}$$

$$\ln n \frac{2l}{a} = 2\pi$$

$$n = \frac{a}{2l} = \frac{1}{2l} \sqrt{\frac{P}{\mu}}$$

$\psi = A \sin \omega(t - \frac{x}{v}) + 2A \sin \frac{\pi x}{\lambda} \cos \omega t [1 + \dots]$
 no L. 100.

$$f = A \sin \omega(t - \frac{x}{v}) + 2A \sin \frac{\pi x}{\lambda} \cos \omega t [1 + \dots]$$

$\frac{x=0}{2\lambda}$

$\omega_1, \omega_2 \frac{1}{4} \omega_3$

$0.5 \dots \dots$

$$\omega \frac{2\lambda}{v} = 4\pi, \text{ or } \dots$$

\dots

= Oct. sh.

$$\omega^2 \approx 3 \omega$$

\dots

\dots

\dots

9/6 Experimente:

\dots



$\frac{1}{2} \rho \omega^2 r^2$ / trans. or long. 16 $\omega^2 \rho r^2$ / long. $\omega^2 \rho r^2$ / $\frac{1}{2} \rho \omega^2 r^2$
 in 1 p. 34

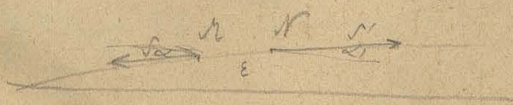


$a = \rho \omega^2 r^2$; $\omega^2 \rho r^2$ / $\rho \omega^2 r^2$ / $\rho \omega^2 r^2$, $\rho \omega^2 r^2$
 $\rho \omega^2 r^2$ / $\rho \omega^2 r^2$.



$\omega^2 \rho r^2$ - $\omega^2 \rho r^2$ / $\omega^2 \rho r^2$ / $\omega^2 \rho r^2$ - $\omega^2 \rho r^2$
 $\rho \omega^2 r^2$ / long. $\omega^2 \rho r^2$ / trans. $\omega^2 \rho r^2$ - $\omega^2 \rho r^2$.

$\rho \omega^2 r^2$ / $\omega^2 \rho r^2$ / $\omega^2 \rho r^2$?



$$S' \sin \alpha' - S \sin \alpha = \mu M N \frac{d^2 y}{dt^2}$$

$$\left(-\omega M N \frac{dy}{dt} \right) \left[= \rho \omega^2 r^2 \right]$$

$S' \sin \alpha' = S$

$$\mu \frac{d^2 y}{dt^2} = \frac{S' \sin \alpha' - S \sin \alpha}{M N} = \omega \frac{dy}{dt}$$

$$= \frac{d}{dx} (S \sin \alpha) - \omega \frac{dy}{dt}$$

$$m \frac{d^2 y}{dt^2} = \frac{d}{dx} \left(\int \frac{dy}{dx} \right) - \omega \frac{dy}{dt}$$

$$= \frac{d}{dx} \left(P \frac{dy}{dx} \right) - \omega \frac{dy}{dt}$$

$$S = P + Q_2 \lambda$$

$$m \frac{d^2 y}{dt^2} = P \frac{d^2 y}{dx^2} - \omega \frac{dy}{dt}$$

$$\frac{d^2 y}{dt^2} = a^2 \frac{d^2 y}{dx^2} - 2b \frac{dy}{dt}$$

$$\begin{cases} a^2 = \frac{P}{m} \\ \frac{\omega}{m} = 2b \end{cases}$$

$$10/6 \quad \frac{d^2 y}{dt^2} = a^2 \frac{d^2 y}{dx^2} - 2b \frac{dy}{dt}$$

$y = \sin \beta x \cdot T$

$$\xi = \sin \beta x \cdot T \quad T = ?$$

$$\sin \beta x \frac{dT}{dt^2} = -\beta^2 a^2 \sin \beta x T - 2b \sin \beta x \frac{dT}{dt}$$

$$\frac{dT}{dt^2} = -\beta^2 a^2 T - 2b \frac{dT}{dt} \quad \text{für versch. } x$$

$$T = e^{\alpha t}$$

$$\alpha^2 e^{\alpha t} = -\beta^2 a^2 e^{\alpha t} - 2b \alpha e^{\alpha t}$$

$$\alpha^2 + 2b\alpha = -\beta^2 a^2$$

$$\alpha = -b \pm \sqrt{b^2 - \beta^2 a^2} \quad \text{2 f. ges.}$$

welch $b > \beta a$ oder $b < \beta a$ je nach ω

$\omega < b < \beta a$

$a = b \pm i \sqrt{\beta^2 a^2 - b^2}$

$T = e^{-bt + \gamma t i} \left\{ \begin{array}{l} C \\ + e^{-bt - \gamma t i} \end{array} \right. \left\{ \begin{array}{l} C \\ D \end{array} \right.$

$T = e^{-bt} [A \cos \gamma t + B \sin \gamma t]$

$\gamma^2 =$

$\gamma^2 = 2n$

$\tau = \frac{2n}{\gamma}$

$n = \frac{1}{\tau} = \frac{\tau}{2n}$

$\tau = \frac{2n}{\sqrt{\beta^2 a^2 - b^2}}$

$\omega < b = 0 \text{ or } \beta a < \beta a$
 $b > 0 \quad \tau >$

$i \cos \gamma t + \gamma t i = \dots$

$P \cos \gamma t + Q \sin \gamma t = \dots$

... ..

$n = \dots$

$\xi = f(t - \frac{x}{c})$

$\xi = e^{-hx} f(t - \frac{x}{c})$

$\frac{d\xi}{dt} = e^{-hx} f'(t - \frac{x}{c}) \quad \frac{d^2 \xi}{dt^2} = e^{-hx} f''(t - \frac{x}{c})$

$$\frac{dy}{dx} = -h e^{-hx} f\left(t - \frac{x}{c}\right) - \frac{1}{c} e^{-hx} f'\left(t - \frac{x}{c}\right)$$

$$\frac{d^2y}{dx^2} = h^2 e^{-hx} f\left(t - \frac{x}{c}\right) + \frac{h}{c} e^{-hx} f'\left(t - \frac{x}{c}\right) + \frac{h}{c} e^{-hx} f'\left(t - \frac{x}{c}\right) + \frac{1}{c^2} e^{-hx} f''\left(t - \frac{x}{c}\right)$$

$$e^{-hx} f''\left(t - \frac{x}{c}\right) = a^2 h^2 e^{-hx} f(t) + 2 \frac{a^2 h}{c} e^{-hx} f'(t) + \frac{a^2}{c^2} e^{-hx} f''(t) - 2h e^{-hx} f'(t)$$

$a \neq 0 \Rightarrow a \neq 0 \vee h = 0$

$$\text{wenn: } \left| \begin{aligned} f'' &= a^2 h^2 f + 2 \frac{a^2 h}{c} f' + \frac{a^2}{c^2} f'' - \\ & \quad - 2h f' \end{aligned} \right.$$

$h = 0 \quad f'' = \frac{a^2}{c^2} f''$

$a = c \quad f'' = f''$

oder $a < c$ oder $a > c$ oder $a = c$ oder $a \neq c$
 oder $a < c$ oder $a > c$ oder $a = c$

wobei: $\frac{2a^2 h}{c} = 2h$

$h = \frac{bc}{a^2}$ wobei:

$f'' = a^2 h^2 f + \frac{a^2}{c^2} f''$ für das gleiche a
 oder $a < c$ oder $a > c$

für $f = \text{exp. u. pers. f.}$

$$f = A \sin \alpha \left(t - \frac{x}{c} \right)$$

$$-\alpha^2 A \sin \alpha \left(t - \frac{x}{c} \right) = \alpha^2 h^2 A \sin \alpha \left(t - \frac{x}{c} \right) - \frac{\alpha^2}{c^2} A \sin \alpha \left(t - \frac{x}{c} \right)$$

$$-\alpha^2 = \alpha^2 h^2 - \frac{\alpha^2}{c^2}$$

$$1 = \frac{\alpha^2}{c^2} - \frac{\alpha^2 h^2}{\alpha^2} = \frac{\alpha^2}{c^2} - \frac{\alpha^2}{\alpha^2} \frac{b^2 \alpha^2}{a^4}$$

$$c^2 = a^2 - \frac{b^2 \alpha^2}{\alpha^2}$$

$$c^2 = a^2 - \frac{b^2 \alpha^2}{\alpha^2}$$

aus $E = m v^2$ und $\alpha^2 = \frac{E}{\hbar^2}$
 $c^2 = a^2 - \frac{b^2 E}{\hbar^2}$

$$c^2 = a^2 - \frac{b^2 \alpha^2}{\alpha^2}$$

$$\alpha = \frac{2\pi}{\lambda} \quad c^2 = a^2 - \frac{b^2 \alpha^2}{4\pi^2} \lambda^2$$

*) $\lambda = \frac{h}{m v}$ (de Broglie)

aus $E = m v^2$ und $\alpha^2 = \frac{E}{\hbar^2}$ (Foucault) $\lambda = \frac{h}{m v}$
 $E = m v^2 = \hbar^2 \alpha^2 = \frac{h^2}{m \lambda^2}$ $\lambda = \frac{h}{m v}$ $E = \frac{h^2}{m \lambda^2}$ $\lambda = \frac{h}{m v}$

*) $\lambda = \frac{h}{m v}$ $c^2 = a^2 - \frac{b^2}{4\pi^2} \frac{h^2}{m^2 \lambda^2}$ (Optik Dispersion z.B.)

$$\frac{h^2}{m^2} \frac{1}{\lambda^2} = a^2 - \frac{b^2}{4\pi^2} \frac{h^2}{m^2 \lambda^2}$$

$$c^2 = a^2 - \frac{b^2}{4\pi^2} \frac{h^2}{m^2 \lambda^2}$$

$$\frac{1}{6} \quad \text{we have } \rho = \rho_0(1+\delta) \quad \text{and } C.$$

we have also:

$$-\frac{1}{\rho} \frac{d\rho}{dt} = \frac{du}{dt} + \underbrace{u \frac{du}{dx} + v \frac{du}{dy} + w \frac{du}{dz}}_{\text{in } \rho \delta \text{ term}}$$

$$-\frac{1}{\rho} \frac{d\rho}{dt} = \frac{du}{dt} \quad \rho = \rho_0(1+\delta)$$

$$-\frac{d\rho}{dt} = \rho_0(1+\delta) \frac{du}{dt} = \rho_0 \frac{du}{dt}$$

$$\text{I. } \left\{ \begin{array}{l} -\frac{d\rho}{dt} = \rho_0 \frac{du}{dt} \\ -\frac{d\rho}{dt} = \rho_0 \frac{dv}{dt} \\ -\frac{d\rho}{dt} = \rho_0 \frac{dw}{dt} \end{array} \right.$$

$$\frac{d\rho}{dt} + \frac{d(\rho u)}{dx} + \frac{d(\rho v)}{dy} + \frac{d(\rho w)}{dz} = 0$$

$$\rho = \rho_0(1+\delta)$$

$$\rho_0 \frac{d\delta}{dt} + \frac{d}{dx}(\rho_0 u + \rho_0 u \delta) + \dots$$

$$\frac{d\delta}{dt} + \frac{du}{dx} + \frac{dv}{dy} + \frac{dw}{dz} = 0$$

$$\left. \begin{aligned} \frac{d\sigma}{dt} + \frac{d\sigma}{dx} + \frac{d\sigma}{dy} + \frac{d\sigma}{dz} = 0 \end{aligned} \right\} \text{II.}$$

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$$\frac{\rho}{\rho_0} = \left(\frac{r}{r_0}\right)^k$$

$$\rho = \rho_0 (1+\sigma)^k = \rho_0 (1+k\sigma) \quad \text{III}$$

$$\text{I: } -\frac{\partial^2 \rho}{\partial x^2} - \frac{\partial^2 \rho}{\partial y^2} - \frac{\partial^2 \rho}{\partial z^2} = \rho_0 \frac{\partial}{\partial t} \left[\frac{\partial \sigma}{\partial x} + \frac{\partial \sigma}{\partial y} + \frac{\partial \sigma}{\partial z} \right]$$

$$= -\rho_0 \frac{\partial^2 \sigma}{\partial t^2}$$

$$\rho_0 k \left[\frac{\partial^2 \sigma}{\partial x^2} + \dots \right] = \rho_0 \frac{d^2 \sigma}{dt^2}$$

$$\frac{\rho_0 k}{\rho_0} = a^2$$

$$\text{II. } \frac{d^2 \sigma}{dt^2} = a^2 \left[\frac{d^2 \sigma}{dx^2} + \frac{d^2 \sigma}{dy^2} + \frac{d^2 \sigma}{dz^2} \right]$$

Notes on the above derivation.

1. - ad in the ρ of the ρ etc.

2. - IV is the ρ of the ρ .

$$\sigma = \phi(r, t) \quad r^2 = x^2 + y^2 + z^2$$

$$\begin{aligned} \frac{d\sigma}{dx} &= \frac{d\sigma}{dr} \frac{dr}{dx} = \frac{d\sigma}{dr} \frac{x}{r} & \left| \quad \frac{d^2 \sigma}{dx^2} &= \frac{d^2 \sigma}{dr^2} \frac{dr}{dx} \frac{x}{r} + \frac{d\sigma}{dr} \frac{1}{r} + \right. \\ & & & \left. + \frac{d\sigma}{dr} \frac{x}{r^2} \frac{dr}{dx} \right. \end{aligned}$$

$$\frac{d^2b}{dx^2} = \frac{d^2b}{dr^2} \frac{x^2}{r^2} + \frac{db}{dr} \frac{1}{r} - \frac{db}{dr} \frac{x^2}{r^3}$$

$$\begin{aligned} \frac{d^2b}{dx^2} + \frac{d^2b}{dy^2} + \frac{d^2b}{dz^2} &= \frac{d^2b}{dr^2} \frac{x^2}{r^2} + \frac{db}{dr} \frac{1}{r} - \frac{db}{dr} \frac{x^2}{r^3} \\ &\quad + \frac{d^2b}{dr^2} \frac{y^2}{r^2} + \frac{db}{dr} \frac{1}{r} - \frac{db}{dr} \frac{y^2}{r^3} \\ &\quad + \frac{d^2b}{dr^2} \frac{z^2}{r^2} + \frac{db}{dr} \frac{1}{r} - \frac{db}{dr} \frac{z^2}{r^3} \\ &= \frac{d^2b}{dr^2} + \frac{2}{r} \frac{db}{dr} \end{aligned}$$

$$\text{II. } \frac{d^2b}{dr^2} = a^2 \left[\frac{d^2b}{dr^2} + \frac{2}{r} \frac{db}{dr} \right]$$

$$r \frac{d^2b}{dr^2} = a^2 \left[r \frac{d^2b}{dr^2} + 2 \frac{db}{dr} \right]$$

$$\begin{aligned} &\underbrace{r \frac{d^2b}{dr^2} + \frac{db}{dr} + \frac{db}{dr}} \\ &= \frac{d}{dr} \left(r \frac{db}{dr} \right) + \frac{db}{dr} = \frac{d}{dr} \left[r \frac{db}{dr} + b \right] \\ &= \frac{d}{dr} \left[\frac{d}{dr} (rb) \right] = \frac{d^2(rb)}{dr^2} \end{aligned}$$

$$r \frac{d^2b}{dr^2} = a^2 \frac{d^2(rb)}{dr^2}$$

$$\frac{d^2(rb)}{dr^2} = a^2 \frac{d^2(rb)}{dr^2} \quad \text{Erlaubt } r \text{ ein beliebiges } r$$

$x_0 = r_0 f(t - \frac{r_0}{a})$, $r=0$ if $l=0$; if $r \ll a$
 $r \ll a \Rightarrow \dots$

$x_0 = r_0 f(t - \frac{r_0}{a})$

$r = r_0$ | $\delta = f(t)$ fully separated $r = r_0$
 $\sim r_0 f(t)$

$\delta = \frac{r_0}{r} f(t - \frac{r-r_0}{a})$

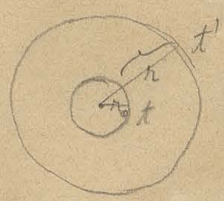
For $r \ll a$ $f(t - \frac{r-r_0}{a}) \approx f(t)$
 $\delta = f(t)$ $r = r_0$
 $= A \sin \omega t$

$\delta = \frac{r_0}{r} \sin \omega (t - \frac{r-r_0}{a})$

$\omega (t - \frac{r-r_0}{a}) = \omega t$

$t' - t = \frac{r-r_0}{a}$

if $r \ll a$, \dots
 $\approx t'$



$\omega r = \dots$
 ωr_0

points on the circles; \dots

Def $\mu = \frac{r_0}{r}$; \dots

v is of the order $\frac{1}{r}$ and $\frac{1}{r^2}$ for $r \rightarrow \infty$.
 v is of the order $\frac{1}{r}$ and $\frac{1}{r^2}$ for $r \rightarrow \infty$.

$$\rho_0 \frac{dv}{dt} = - \frac{dp}{dx}$$

$$= - \frac{dp}{dr} \frac{dr}{dx} = - \frac{dp}{dr} \frac{x}{r}$$

$$\frac{dv}{dt} = - \frac{1}{\rho_0} \frac{dp}{dr} \frac{x}{r}$$

$\frac{x}{r}$ etc. etc. etc.

$$\frac{dv}{dt} = - \frac{1}{\rho_0} \frac{dp}{dr} \frac{y}{r}$$

$$\frac{dv}{dt} = - \frac{1}{\rho_0} \frac{dp}{dr} \frac{z}{r}$$

etc. etc. etc.

etc. etc. etc.

etc. etc. etc.

15/6 $\frac{d^2\sigma}{dt^2} = \frac{d^2\sigma}{dx^2} + \frac{d^2\sigma}{dy^2} + \frac{d^2\sigma}{dz^2}$

where σ is the potential

$$\frac{d^2\sigma}{dt^2} = a^2 \left[\frac{d^2\sigma}{dx^2} + \frac{d^2\sigma}{dy^2} + \frac{d^2\sigma}{dz^2} \right]$$

$$\sigma = f \left[t - \frac{(x \cos \alpha + y \cos \beta + z \cos \gamma)}{a} \right]$$

f is a function of P

$$= f \left(t - \frac{P}{a} \right)$$

$$P = x \cos \alpha + y \cos \beta + z \cos \gamma$$

$$\frac{d^2 \delta}{dt^2} = f''(t - \frac{P}{a})$$

$$\frac{d\delta}{dx} = f'(t - \frac{P}{a}) \left[-\frac{v \cos \lambda}{a} \right]$$

$$\frac{d^2 \delta}{dx^2} = f''(t - \frac{P}{a}) \frac{v^2 \lambda}{a^2}$$

$$f''(t - \frac{P}{a}) = a^2 f''(t - \frac{P}{a}) \frac{v^2 \lambda + v^2 \mu + v^2 \nu}{a^2}$$

$$= f''(t - \frac{P}{a})$$

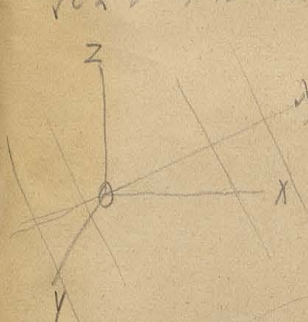
→ s. Subst, $\delta \sim \int t \dots$

1. $\rho = \rho_0 \left(1 - \frac{v}{c} \right)$; $v = \omega r$

$$\delta = A \sin \omega \left(t - \frac{x \cos \lambda + y \cos \mu + z \cos \nu}{a} \right)$$

→ $\rho = \rho_0$

$\rho = \rho_0 \left(1 - \frac{v}{c} \right)$



$$x=0$$

$$u=0$$

$$-\frac{dp}{dx} = \rho_0 \frac{dn}{dt}$$

$$n = n_0 [1 + k\delta]$$

$$\frac{dn}{dx} = -\rho_0 k \frac{d\delta}{dx} = \rho_0 \frac{dn}{dt}$$

$$\omega \mu / a \sim \omega \nu \sim \omega \lambda \sim \omega$$

$$\delta \sim \frac{dn}{dt} = 0 \implies \frac{d\delta}{dx} = 0$$

→ $\rho = \rho_0$

$$x=0$$

$$\frac{d\delta}{dx} = 0$$

$$b = A \sin \alpha \left[t - \frac{y \cos \lambda + y' \cos \mu + z \cos \nu}{a} \right] +$$

$$+ A' \sin \alpha' \left[t - \frac{x \cos \lambda + y \cos \mu + z \cos \nu}{a} \right]$$

Supposing $\alpha = 2$ to be α and α' to be α'

$$\frac{db}{dx} = - \frac{\alpha \cos \lambda}{a} A \cos \alpha \left[t - \frac{x \cos \lambda + y \cos \mu + z \cos \nu}{a} \right] -$$

$$- \frac{\alpha' \cos \lambda'}{a} A' \cos \alpha' \left[t - \dots \right]$$

at $x=0$ $\frac{db}{dx} = 0$

$$- \frac{\alpha \cos \lambda}{a} A \cos \alpha \left[t - \frac{y \cos \mu + z \cos \nu}{a} \right] - \frac{\alpha' \cos \lambda'}{a} A' \cos \alpha'$$

$$\cos \lambda' \left[t - \frac{y \cos \mu + z \cos \nu}{a} \right] = 0$$

at $y=0, z=0$

at $t = t_0$ $\lambda = \lambda_0$ $\mu = \mu_0$ $\nu = \nu_0$ $\alpha = \alpha_0$

at $t = t_0$ $\alpha = \alpha_0$ $\alpha' = \alpha'_0$

$$\cos \lambda_0 A + \cos \lambda'_0 A' = 0$$

at $t = t_0$ $\mu = \mu_0$ $\nu = \nu_0$

$$- \cos \lambda_0 A \cos \alpha_0 \left(t - \frac{2 \cos \nu_0}{a} \right) - \cos \lambda'_0 A' \cos \alpha'_0 \left(t - \frac{2 \cos \nu_0}{a} \right) = 0$$

$$\frac{\alpha \cos \nu}{a} = \frac{\alpha \cos \nu'}{a} \quad \cos \nu = \cos \nu'$$

$$\cos \mu = \cos \mu'$$

$$\cos^2 \lambda + \cos^2 \mu + \cos^2 \nu = 1$$

$$\cos^2 \lambda' + \cos^2 \mu' + \cos^2 \nu' = 1$$

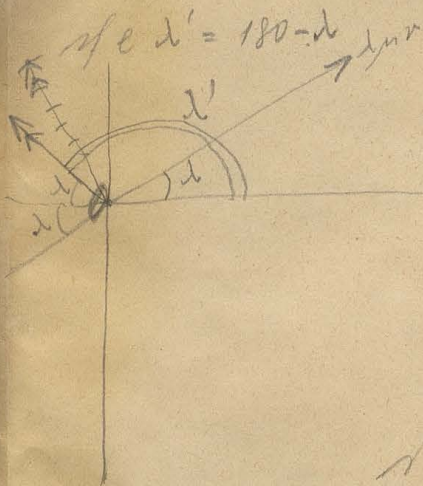
$$\cos^2 \lambda - \cos^2 \lambda' = 0$$

$$\cos^2 \lambda = \cos^2 \lambda'$$

$$\cos \lambda' = \pm \cos \lambda$$

$$a + b^2 \text{ etc } \dots$$

$$a - b^2 \text{ etc } \dots$$



Reflections!

x p etc etc reflections

perpendicular etc etc

etc etc etc

etc etc etc 180 degrees

$$\cos \lambda \cdot A - \cos \lambda \cdot A' = 0$$

$$A = A' \quad \text{P. Amp. etc etc}$$

etc etc of Refl. etc etc etc etc etc etc etc etc etc etc

etc etc etc etc etc etc etc etc etc etc

$$\frac{d^2 \phi}{dt^2} = a \left[\frac{d^2 \phi}{dx^2} + \frac{d^2 \phi}{dy^2} + \frac{d^2 \phi}{dz^2} \right] \quad \frac{d^2 \phi}{dt^2} = a_1 \left[\frac{d^2 \phi}{dx^2} + \dots \right]$$

etc etc etc etc etc etc etc etc etc etc

$x=0$:

ρ_2 constant ρ_1 constant; $b \sim \frac{1}{2} - \frac{1}{2} \rho_2 \rho_1$
 b of ρ_1 & ρ_2 & $\rho_1 \rho_2$ constant.

$$\rho_1 = \rho_2$$

$$\rho_1 = \rho_0 [1 + k_1 b] \quad \rho_2 = \rho_0 [1 + k_2 b]$$

$$k_1 b = k_2 b$$

$\rho_1 \rho_2 = \rho_0^2$

$$u = u_1 \quad \rho_1 \rho_2 = \rho_0^2 \quad \rho_1 \rho_2 = \rho_0^2$$

$$\frac{du}{dt} = \frac{du_1}{dt}$$

$$-\rho_0 k_1 \frac{db}{dx} = \rho_0 \frac{du}{dt}$$

$$-a^2 \frac{db}{dx} = \frac{du}{dt}$$

$$-a_1^2 \frac{db_1}{dx} = \frac{du_1}{dt}$$

$$a^2 \frac{db}{dx} = a_1^2 \frac{db_1}{dx}$$

17/6 Reflections: Brechning

$x=0$ $\rho_1 \rho_2$

$$k_1 b = k_2 b$$

$$a^2 \frac{db}{dx} = a_1^2 \frac{db_1}{dx}$$

$$b = b' + b''$$

$$b' = A \sin \left[t - \frac{x \cos \lambda + y \cos \mu + z \cos \nu}{a} \right]$$

$$b'' = A'' \sin \alpha'' \left[t - \frac{x \cos \alpha'' + y \cos \mu'' + z \cos \nu''}{a} \right]$$

$$b_1 = A_1 \sin \alpha_1 \left[t - \frac{x \cos \alpha_1 + y \cos \mu_1 + z \cos \nu_1}{a_1} \right]$$

$$k A' \sin \alpha' \left[t - \frac{y \cos \mu' + z \cos \nu'}{a} \right] + k A'' \sin \alpha'' \left[t - \frac{y \cos \mu'' + z \cos \nu''}{a} \right] = k_1 A_1 \sin \alpha_1 \left[t - \frac{y \cos \mu_1 + z \cos \nu_1}{a_1} \right]$$

$$k A' \sin \alpha' \left[t - \frac{y \cos \mu' + z \cos \nu'}{a} \right] + k A'' \sin \alpha'' \left[t - \frac{y \cos \mu'' + z \cos \nu''}{a} \right] = k_1 A_1 \sin \alpha_1 \left[t - \frac{y \cos \mu_1 + z \cos \nu_1}{a_1} \right]$$

Appl. $y=0, z=0$ in the t eq. et en

$$f''_y \sim \alpha' = \alpha'' = \alpha_1 \quad [= \alpha]$$

$\sim \cos \alpha \cos \nu \cos \mu \cos \nu \cos \mu \cos \nu \cos \mu \cos \nu \cos \mu \cos \nu$

Appl. $y=0, z=0$ in the t eq. et en

$$\frac{d' \cos \mu'}{a} = \frac{d'' \cos \mu''}{a} = \frac{d_1 \cos \mu_1}{a_1}$$

$$\frac{\cos \mu'}{a} = \frac{\cos \mu''}{a} = \frac{\cos \mu_1}{a_1}$$

$$\frac{\cos \nu'}{a} = \frac{\cos \nu''}{a} = \frac{\cos \nu_1}{a_1}$$

$$\cos \mu' = \cos \mu''$$

$$\cos \nu' = \cos \nu''$$

$$\cos^2 \mu' + \cos^2 \nu' = \cos^2 \mu'' + \cos^2 \nu''$$

$$1 - \cos^2 \alpha' = 1 - \cos^2 \alpha''$$

$$\cos^2 \alpha' = \cos^2 \alpha''$$

$$\cos \alpha' = + \cos \alpha''$$

$$\cos \alpha' = - \cos \alpha'' \quad \text{reflexion } \sim \text{P} \rightarrow \text{P} \text{ Refl.}$$

$\sim \text{C} \text{ y } 200 \text{ m} \text{ w} \text{ e} \text{ Co. } f^{\text{ro}} \text{ s} \text{ p} \text{ e.}$

$$\frac{\cos \mu'}{a} = \frac{\cos \mu_1}{a_1}$$

$$\frac{\cos \nu'}{a} = \frac{\cos \nu_1}{a_1}$$

$$\frac{\cos^2 \mu' + \cos^2 \nu'}{a^2} = \frac{\cos^2 \mu_1 + \cos^2 \nu_1}{a_1^2}$$

$$\frac{1 - \cos^2 \lambda'}{a^2} = \frac{1 - \cos^2 \lambda_1}{a_1^2}$$

$$\frac{\sin^2 \lambda'}{a^2} = \frac{\sin^2 \lambda_1}{a_1^2}$$

$$\frac{\sin \lambda'}{\sin \lambda_1} = \frac{a^2}{a_1^2}$$

$$\frac{\sin \lambda'}{\sin \lambda_1} = \frac{a}{a_1}$$

$$\sin \lambda' = \frac{a}{a_1} \sin \lambda_1$$

$$= \sin \lambda$$

$\therefore \lambda' = \lambda$; e.g. yes! (P. 111, 27, 28, 29, 30, 31)

$$\mu' = 90^\circ - \lambda$$

$$r \cos \lambda' + k A' + k A'' = k, A, \quad I.)$$

$$\frac{a^2 db}{dx} = a_1^2 \frac{db_1}{dx}$$

$$a^2 \left[A' \frac{\cos \lambda'}{a} \cos \lambda' [t -] + a^2 A'' \frac{\cos \lambda''}{a} \cos \lambda'' [t -] \right]$$

$$= a_1^2 A_1 \frac{\cos \lambda_1}{a_1} \cos \lambda_1 [t -]$$

$$a A' \cos \lambda' + a A'' \cos \lambda'' = a_1 A_1 \cos \lambda_1$$

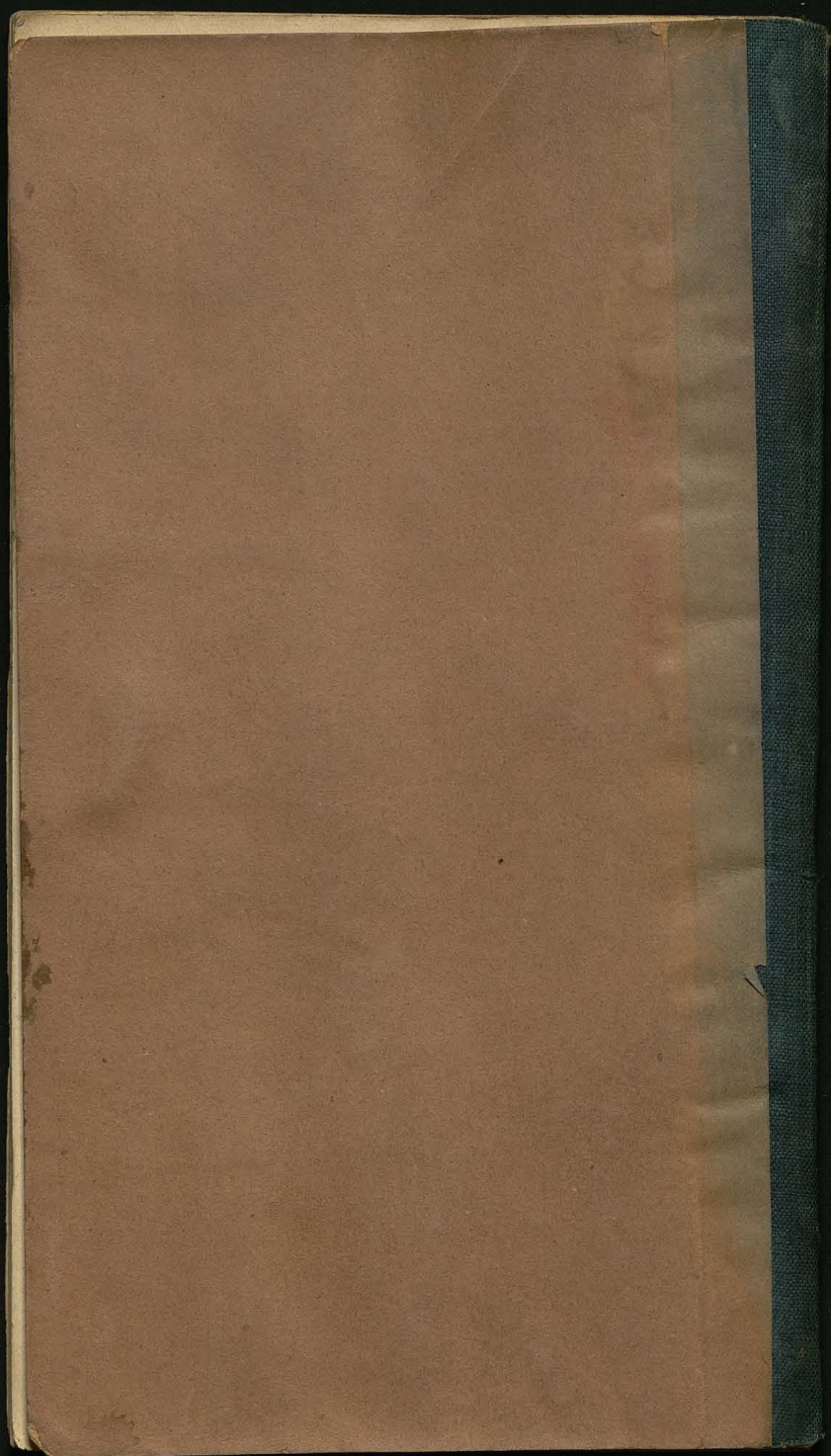
$$a A' \cos \lambda' - a A'' \cos \lambda' = a_1 A_1 \cos \lambda_1$$

$$\frac{a}{a_1} \cos \lambda' [A' - A''] = A_1 \cos \lambda_1$$

$$\frac{\sin \lambda'}{\sin \lambda_1} \cos \lambda' [A' - A''] = A_1 \cos \lambda_1$$

ed. 2c. vol.

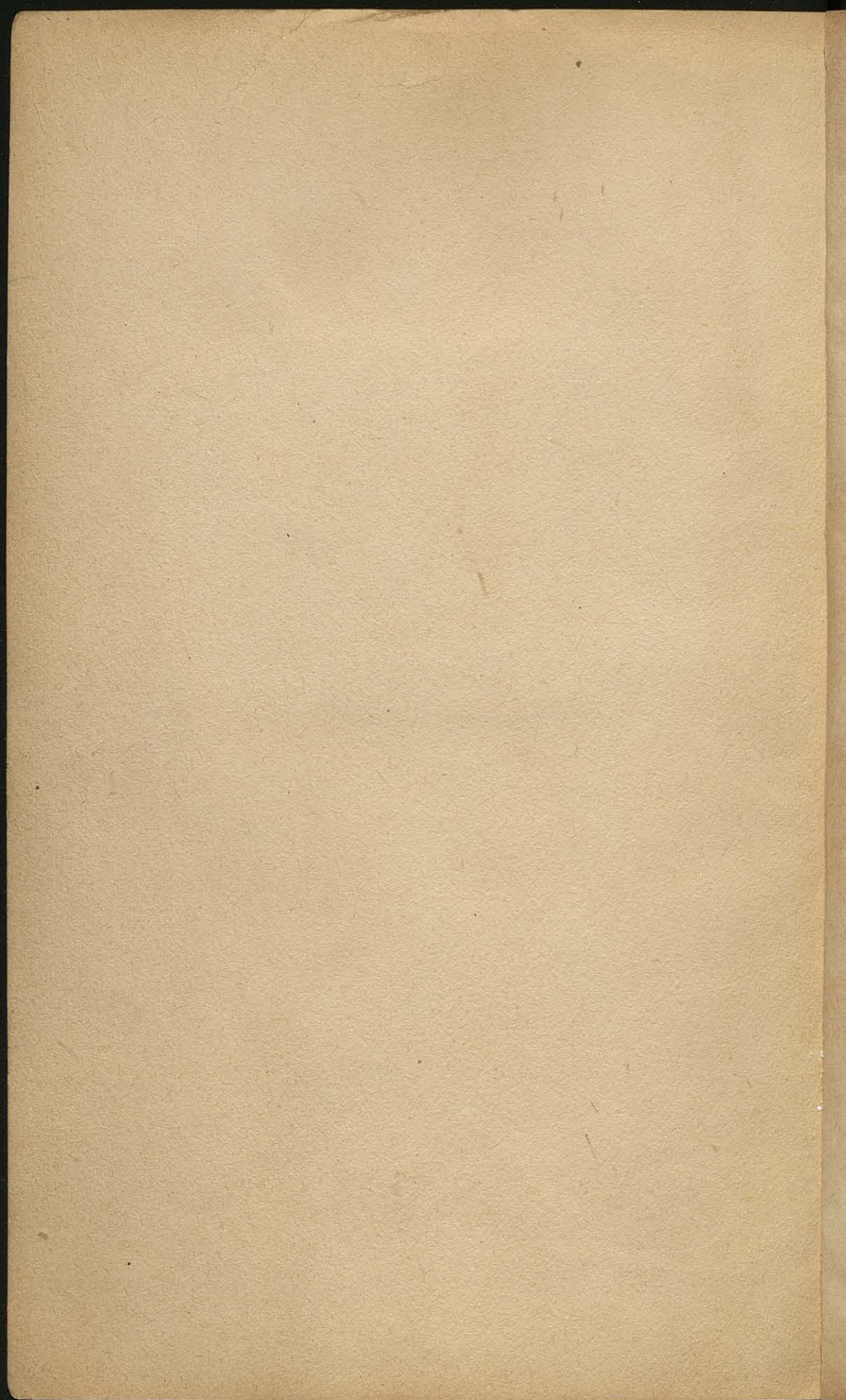
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2.
Dr. Josef Stefan *IV. 92.*
Arbeitsk.
Abhandlungen



4. 24

II). $\sin' \cos' [A' - A''] = \sin', \cos', A_1$ 45
 of $\cos' \sin' \in A'' \text{ s e } A_1$ & **BJ**

$$\begin{array}{l} \text{I} \sin', \cos', \\ - \text{II} A_1 \end{array}$$

$$k A' \sin', \cos' + k A'' \sin', \cos' - k A' \sin', \cos' + k A'' \sin', \cos' = 0$$

$$A'' [k \sin', \cos' + k \sin', \cos'] = A' [k \sin', \cos' - k \sin', \cos']$$

$$\frac{A''}{A'} = \frac{k \sin', \cos' - k \sin', \cos'}{k \sin', \cos' + k \sin', \cos'}$$

$\cos' \sin' \in A'' \text{ s e } A_1$
 $\cos' \sin' \in A'' \text{ s e } A_1$
 $\cos' \sin' \in A'' \text{ s e } A_1$
 $s = A_1$

$$\frac{A''}{A'} = \frac{\sin', \cos' - \sin', \cos'}{\sin', \cos' + \sin', \cos'}$$

$\cos' \sin' \in A'' \text{ s e } A_1$
 $\cos' \sin' \in A'' \text{ s e } A_1$
 $\cos' \sin' \in A'' \text{ s e } A_1$

21/6 A' \rightarrow ref. A, (V)
 A'' refl.

$$k[A' + A''] = k, A,$$

$$a \cos \lambda (A' - A'') = k \cos \lambda, A,$$

$$\frac{a}{a_1} = \frac{\sin \lambda}{\sin \lambda_1}$$

$$\sin \lambda \cos \lambda (A' - A'') = \sin \lambda_1 \cos \lambda_1, A,$$

$$\frac{(k \sin \lambda \cos \lambda - k_1 \sin \lambda_1 \cos \lambda_1) A'}{A''}$$

$n \rho_s \rightarrow k = k_1$

$$\frac{A''}{A'} = \frac{\sin \lambda \cos \lambda - \sin \lambda_1 \cos \lambda_1}{\sin \lambda \cos \lambda + \sin \lambda_1 \cos \lambda_1} = \frac{\sin 2\lambda - \sin 2\lambda_1}{\sin 2\lambda + \sin 2\lambda_1}$$

$$= \frac{\sin(\lambda - \lambda_1) \cos(\lambda + \lambda_1)}{\sin(\lambda + \lambda_1) \cos(\lambda - \lambda_1)} = \frac{\tan(\lambda - \lambda_1)}{\tan(\lambda + \lambda_1)}$$

f. p. Fresnel, Intens. refl. ρ , ρ_1 or ρ_2 or ρ_3 or

ρ_4 or ρ_5 . $\rho_1 \neq \lambda \quad c = 0$

$$\text{where } \lambda + \lambda_1 = 90$$

$$\lambda_1 = 90 - \lambda$$

$$\sin \lambda_1 = \cos \lambda$$

$$\frac{\sin \lambda}{\sin \lambda_1} = \frac{a}{a_1} = n$$

$$\tan \lambda = n \quad \text{in } c = \rho \text{ or } \rho_1 \text{ or } \rho_2$$

f. Med. (V) ρ_1 or ρ_2
 to refl.

$k = 2, \rho = 20 \cdot 10^8 \text{ v } 0.5 \text{ H}; \rho \text{ je } 10^8 \text{ v } 0.5 \text{ H}; 4:4$

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$\therefore a \text{ tg } (\gamma_0 \lambda) = 40^\circ \text{ c } \rho \text{ je } 10^8 \text{ v } 0.5 \text{ H}$

experimentell = 2 f / 1 f

et optisk ρ f je $\rho \text{ c } \lambda \text{ je } \rho_0 \pm \gamma_0 \approx 6 \rho_0 \text{ v } 0.5 \text{ H}$

$\rho \text{ je } \rho_0 \text{ v } 0.5 \text{ H} \text{ c } \rho_0 \text{ v } 0.5 \text{ H} \text{ c } \rho_0 \text{ v } 0.5 \text{ H}$

$$a^2 = \frac{\rho_0 k}{\rho_0} \quad a_1^2 = \frac{\rho_0 k_1}{\rho_0}$$

$$k \cdot k_1 = a^2 = a_1^2 \\ = \sin^2 \lambda_1 \cdot \sin^2 \lambda_2$$

$$\frac{A''}{A'} = - \frac{\sin^2 \lambda_1 \sin \lambda_2 \cos \lambda_2 - \sin^2 \lambda_2 \sin \lambda_1 \cos \lambda_1}{\sin \lambda_1 \cos \lambda_1 \cos \lambda_2 + \sin \lambda_2 \cos \lambda_2 \cos \lambda_1}$$

$$= \frac{\sin \lambda_1 \cos \lambda_1 [\sin \lambda_2 \cos \lambda_2 - \sin \lambda_1 \cos \lambda_1]}{\sin \lambda_1 \cos \lambda_1 + \sin \lambda_2 \cos \lambda_2}$$

$$= - \frac{\sin(\lambda_1 - \lambda_2)}{\sin(\lambda_1 + \lambda_2)}$$

$\rho_0 \text{ c } \text{ Fresnel}, \text{ c } \rho_0 \text{ v } 0.5 \text{ H}$
 $\pm \gamma_0 \text{ v } 0.5 \text{ H}$

$\rho_0 \text{ c } \text{ Fresnel}, \text{ c } \rho_0 \text{ v } 0.5 \text{ H}$

Transversalschwingungen elastischer Stäbe.

20.



$\rho_0 \text{ c } \text{ Fresnel}, \text{ c } \rho_0 \text{ v } 0.5 \text{ H}$
 $\pm \gamma_0 \text{ v } 0.5 \text{ H}$

Let $\omega = \omega(x)$ Bernoulli's Euler's law

$\frac{d^2 v}{dx^2} = -\frac{M}{EI}$; $\frac{d^2 \theta}{dx^2} = \frac{1}{EI} \frac{dM}{dx}$ = elast. rotation

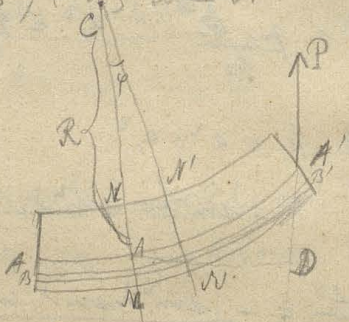
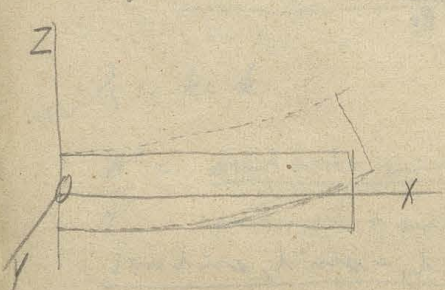
$\omega = \omega(x) = \frac{1}{EI} \frac{dM}{dx}$ = slope of deflection

f_1, f_2, f_3, \dots = deflection of fiber

f_1, f_2, f_3, \dots = deflection of fiber

at $x = 0$ parallel to x = 0 of $\omega = \theta$ = Neutral θ

enter right of x of θ = 0 at $x = 0$ = 0



$MN \perp$

$$C = \frac{1}{EI} \frac{dM}{dx} \cdot C$$

$$\omega = \frac{1}{EI} \frac{dM}{dx}$$

$AA' = R\phi$ = length of fiber

$$\omega \frac{Q(BB' - AA')}{AA'} = \frac{1}{R} \phi = \theta$$

$$AA' = R\phi$$

$$BB' = (R + y)\phi$$

$$AB = R\phi$$

$$BB' = R\phi + y\phi \quad \theta = \omega \frac{y}{R}$$

$\omega = \frac{1}{EI} \frac{dM}{dx} = \frac{1}{EI} \frac{d}{dx} (P \cdot x) = \frac{P}{EI}$

$\sum \omega \cdot y = 0$ = Result = 0

$$\sum (P \cdot x) = 0$$

$$\sum \omega \frac{y}{R} = 0$$

$$\frac{Q}{R} \sum y = 0$$

\rightarrow Neutral axis

2/6

$$P = \omega \frac{q \cdot l}{R}$$

$\sim \int \omega \cdot \sqrt{A} \cdot dl$

$\sim \int \omega \cdot \sqrt{A} \cdot dl; \Sigma = 0 \leftarrow \text{and } \text{any} = 120 \text{ or } 6$

$\sim \int \omega \cdot \sqrt{A} \cdot dl \sim \omega \cdot \sqrt{A} \cdot l \sim \omega \cdot \sqrt{A} \cdot l$

$$P \cdot AD = \frac{\omega \cdot q \cdot \overline{AD}^2}{R}$$

$$\Sigma P \cdot AD = \frac{q}{R} \Sigma \overline{AD}^2$$

in case of 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100



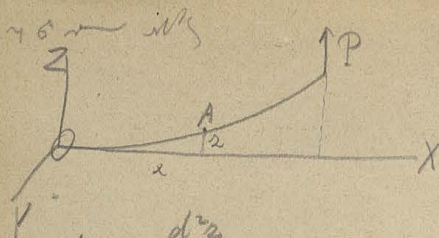
$$\Sigma P \cdot AD = \frac{qK}{R}$$

in case of 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100

in case of 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100

in case of 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100

$$P \cdot AD = \frac{qK}{R} = P \cdot AD$$



$$\frac{1}{R} = \frac{\frac{d^2 z}{dx^2}}{\left[1 + \left(\frac{dz}{dx}\right)^2\right]^{\frac{3}{2}}} \quad \text{ef } \rho l \cdot v \cdot \text{const.} \cdot \text{conc.} \cdot \text{dist.}$$

$$= \text{prossime } \frac{d^2 z}{dx^2}$$

$$AD = l - x \quad \text{longitudinal part of beam}$$

$$EK \frac{d^2 z}{dx^2} = P(l-x)$$

$$EK \frac{dz}{dx} = P(lx - \frac{x^2}{2}) + C$$

$$EK z = P(l \frac{x^2}{2} - \frac{x^3}{6}) + Cx + D$$

$$x=0 \quad z=0 \quad D=0$$

$$z=0 \rightarrow \text{end of beam } \frac{dz}{dx} = 0 \rightarrow \text{end of beam}$$

$$x=0 \quad \frac{dz}{dx} = 0 \quad C=0$$

$$z = \frac{P}{EK} \left(l \frac{x^2}{2} - \frac{x^3}{6} \right)$$

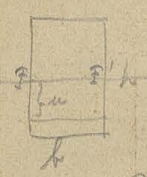
$$z_1 = \text{end of beam } z = ?$$

$$z_1 = \frac{P}{EK} \left(\frac{lx^2}{2} - \frac{l^3}{6} \right) = \frac{1}{3} \frac{Pl^3}{EK}$$

$$= \text{end of beam } z = ?$$

$$P < 3 \text{ end of beam}$$

Wahl der 1. Ordnung

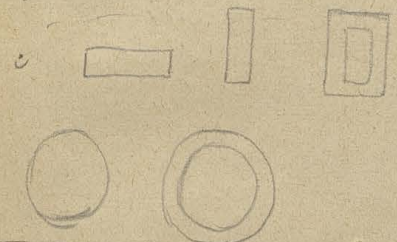


$$K = \int_{-\frac{h}{2}}^{+\frac{h}{2}} u^2 b \, du = \frac{b u^3}{3} \Big|_{-\frac{h}{2}}^{+\frac{h}{2}} = \frac{b h^3}{12}$$

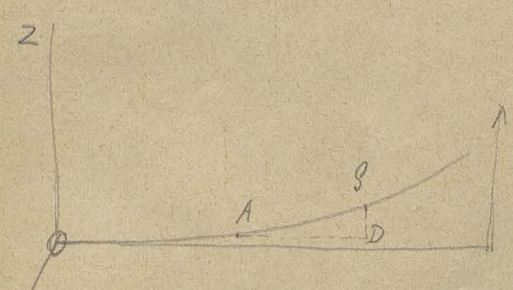
$$z_1 = \frac{12 P l^3}{3 \cdot 4 b h^3} = \frac{4 P l^3}{4 b h^3}$$

mit $P = \dots$
 $\dots \dots \dots$

1. Ordnung der 3. Ordnung



8.4.1: $\dots \dots \dots$



$$\frac{qK}{R}$$

...
 ...
 ...

...
 ...
 ...

$$\frac{qK}{R} = \dots \overline{AD} = \dots (b-x) \cdot \frac{b-x}{2}$$

$$qK \frac{d^2 z}{dx^2} = \frac{qpg}{2} (l-x)^2$$

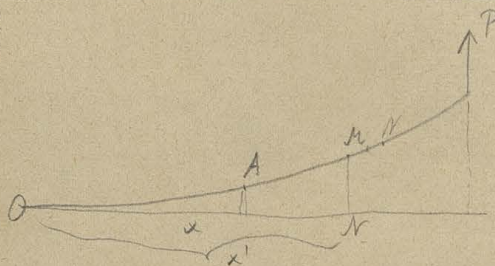
$$qK \frac{dz}{dx} = \frac{qpg}{2} \left(lx^2 - \frac{2}{3} lx^3 + \frac{x^3}{3} \right) + C$$

$$qK z = \frac{qpg}{2} \left[l^2 \frac{x^2}{2} - l \frac{x^3}{3} + \frac{x^4}{12} \right] + Cx + D$$

$$z = \frac{qpg}{2qK} \left[\frac{l^2 x^2}{2} - l \frac{x^3}{3} + \frac{x^4}{12} \right]$$

$$x=l \quad z_1 = \frac{qpg}{2qK} \left[\frac{l^4}{2} - \frac{l^4}{3} + \frac{l^4}{12} \right] = \frac{qpg l^4}{8qK} \cdot l^3$$

23/6



1. 1/2 re

2. 1/6 re

pg MN z'

$$\frac{qK}{R} - P(l-x) - \int_0^{x=l} qg \overline{MN} z'(x-x) \quad MN = dx$$

$$1) \frac{qK}{R} \frac{d^2 z}{dx^2} - P(l-x) - qg \int_x^l z'(x-x) dx = 0$$

$$2) \frac{qK}{R} \frac{d^3 z}{dx^3} + P + qg \int_x^l z' dx + qg z'(x-x) = 0$$

$$\left. \begin{aligned} J &= \int_x^l f(x) dx \\ \frac{dJ}{dx} &= -f(x) \end{aligned} \right\}$$

$$2) \rho K \frac{d^3 z}{dx^3} + P + \rho g \int_x^l z' dx = 0$$

$$z' \Big|_{x=l} = z$$

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$$3) \rho K \frac{d^4 z}{dx^4} + P - \rho g z = 0$$

$$20). \text{ je } z = g$$

$$\rho K \frac{d^4 z}{dx^4} = \rho g z$$

$$\rho K \frac{d^3 z}{dx^3} = \rho g z x + C$$

$$\rho K \frac{d^2 z}{dx^2} = \rho g z \frac{x^2}{2} + Cx + D$$

$$\rho K \frac{dz}{dx} = \rho g g \frac{x^3}{6} + C \frac{x^2}{2} + Dx + E$$

$$\rho K z = \rho g g \frac{x^4}{24} + C \frac{x^3}{6} + D \frac{x^2}{2} + Ex + F$$

$$x=0 \mid z=0 \quad \frac{dz}{dx} = 0$$

$$F=0 \quad E=0$$

$$x=l \quad 1) \quad \rho K \frac{d^2 z}{dx^2} = 0$$

$$2) \quad \rho K \frac{d^3 z}{dx^3} + P = 0$$

$$P=0 \mid x=l$$

$$\rho K \frac{dz}{dx} = 0$$

$$\rho K \frac{d^2 z}{dx^2} = 0$$

gegeben $z(l) = z$, $z'(l) = z$

$$0 = \rho g g l + C$$

$$C = -\rho g g l$$

$$0 = \rho g g \frac{l^2}{2} + Cl + D$$

$$D = +\rho g g \frac{l^2}{2}$$

$$qKz = p q g \left[\frac{x^4}{24} - \frac{lx^3}{6} + \frac{l^2 x^2}{4} \right]$$

$$x=l$$

$$qKz_1 = p q g \frac{l^4}{8} = p q \frac{l^3}{8} \quad \text{61px}$$

$$3). \quad qK \frac{d^4 z}{dx^4} - p q z = 0 \quad \text{Lagrange & Lambert}$$

$$qK \frac{d^4 z}{dx^4} - p q \left[z - \frac{d^2 z}{dt^2} \right] = 0$$

So:

$$p q z - qK \frac{d^4 z}{dx^4} = 0$$

$$p q dx z - qK \frac{d^4 z}{dx^4} dx = 0$$

logent // zom / für Wert. z.

$$= p q dx \frac{d^2 z}{dt^2}$$

z/21'2

$$p q z - qK \frac{d^4 z}{dx^4} = p q \frac{d^2 z}{dt^2}$$

p q e n n m / : z=0

$$+ qK \frac{d^4 z}{dx^4} + p q \frac{d^2 z}{dt^2} = 0$$

$$\frac{qK}{p q} = c \cos$$

$$a^2 \frac{d^4 z}{dx^4} = - \frac{d^2 z}{dt^2}$$

$$\frac{d^2 z}{dt^2} + a^2 \frac{d^4 z}{dx^4} = 0$$

c 8 us 1px.

$$z = U \sin at \quad U \cos at \quad z$$

$$\frac{d^2 z}{dt^2} = -a^2 U \sin at \quad \frac{d^4 z}{dt^4} = \frac{d^4 U}{dt^4} \sin at$$

$$-a^2 U \sin at + a^2 \frac{d^4 U}{dt^4} \sin at = 0$$

$$U = e^{jx}$$

$$-a^2 e^{jx} + a^2 j^4 e^{jx} = 0$$

$$j^4 = \frac{a^2}{a^2} \quad j^2 = \pm \frac{a}{a}$$

$$j = \pm \sqrt{\pm \frac{a}{a}}$$

82.

$$\frac{d^2 z}{dx^2} + a^2 \frac{d^4 z}{dx^4} = 0$$

$$z = U \sin at$$

$$-a^2 U + a^2 \frac{d^4 U}{dx^4} = 0$$

$$U = e^{jx}$$

$$j^4 = \frac{a^2}{a^2}$$

$$\frac{a^2}{a^2} = b^4$$

$$j^4 = b^4$$

$$j = \pm b, \pm bi$$

$$U = M_1 e^{bx} + M_2 e^{-bx} + M_3 e^{bxi} + M_4 e^{-bxi}$$

= $\int \cos bx \sin bx$

Sepe zollent

$$U = A e^{\frac{bx}{2}} + e^{-\frac{bx}{2}} + B e^{\frac{bx}{2}} + e^{-\frac{bx}{2}} + C \cos bx + D \sin bx$$

2B. $e^{-x/b}$, $e^{x/b}$

$x=0$	$x=lb$	$x=0$	$x=l$
$\xi=0$	$\frac{d^2\xi}{dx^2}=0$	$u=0$	$\frac{d^2u}{dx^2}=0$
$\frac{d\xi}{dx}=0$	$\frac{d^3\xi}{dx^3}=0$	$\frac{du}{dx}=0$	$\frac{d^3u}{dx^3}=0$

$$\frac{du}{dx} = b \left[A \frac{e^{bx} - e^{-bx}}{2} + B \frac{e^{bx} + e^{-bx}}{2} - C \sin bx + D \cos bx \right]$$

$$\frac{d^2u}{dx^2} = b^2 \left[A \frac{e^{bx} + e^{-bx}}{2} + B \frac{e^{bx} - e^{-bx}}{2} - C \cos bx - D \sin bx \right]$$

$$\frac{d^3u}{dx^3} = b^3 \left[A \frac{e^{bx} - e^{-bx}}{2} + B \frac{e^{bx} + e^{-bx}}{2} + C \sin bx - D \cos bx \right]$$

$$0 = A + C$$

$$0 = B + D$$

$$0 = A \frac{e^{bl} + e^{-bl}}{2} + B \frac{e^{bl} - e^{-bl}}{2} - C \cos bl - D \sin bl$$

$$0 = A \frac{e^{bl} - e^{-bl}}{2} + B \frac{e^{bl} + e^{-bl}}{2} + C \sin bl - D \cos bl$$

$\sim \sin$ / \pm \sin of bl has \pm no d .

$$0 = A \left[\frac{e^{bl} + e^{-bl}}{2} + \cos bl \right] + B \left[\frac{e^{bl} - e^{-bl}}{2} + \sin bl \right]$$

$$0 = A \left[\frac{e^{bl} - e^{-bl}}{2} - \sin bl \right] + B \left[\frac{e^{bl} + e^{-bl}}{2} + \cos bl \right]$$

$$z = A_1 u_1 \sin \omega_1 t + A_2 u_2 \sin \omega_2 t +$$

$$B_1 e^{i\omega_1 t} + B_2 e^{i\omega_2 t} + \dots$$

$$+ F_1 u_1 \cos \omega_1 t + F_2 u_2 \cos \omega_2 t +$$

für ein festes ω . ω ist konst.

gilt $\sqrt{c} \sqrt{1/c} = \text{pro } \omega$

$$\int_0^l u_k u_l dx = 0 \quad \text{für } k \neq l \quad \text{wegen } \omega$$

$$\int_0^l \omega^2 u_k u_l dx = 0 \quad \text{für } k \neq l$$

$k=1$

$k=2$

$$\int_0^l u_1 u_2 dx$$

$$\frac{d^4 u_1}{dx^4} = -b_1^4 u_1$$

$$\frac{d^4 u_2}{dx^4} = -b_2^4 u_2$$

$$b_1^4 \int_0^l u_1 u_2 dx = - \int_0^l u_2 \frac{d^4 u_1}{dx^4} dx = -u_2 \frac{d^3 u_1}{dx^3} + \int_0^l \frac{d u_2}{dx} \frac{d^3 u_1}{dx^3} dx$$

$$= -u_2 \frac{d^3 u_1}{dx^3} + \frac{d u_2}{dx} \frac{d^2 u_1}{dx^2} - \int_0^l \frac{d^2 u_2}{dx^2} \frac{d^2 u_1}{dx^2} dx$$

$$= -u_2 \frac{d^3 u_1}{dx^3} + \frac{d u_2}{dx} \frac{d^2 u_1}{dx^2} - \frac{d^2 u_2}{dx^2} \frac{d u_1}{dx} + \int_0^l \frac{d^2 u_2}{dx^2} \frac{d u_1}{dx} dx$$

$$= -u_2 \frac{d^3 u_1}{dx^3} + \frac{d^2 u_2}{dx^2} u_1 - \int_0^l \frac{d^4 u_2}{dx^4} u_1 dx$$

$$(b_1^4 - b_2^4) \int_0^l u_1 u_2 dx = \left[u_2 \frac{d^3 u_1}{dx^3} + \frac{d u_2}{dx} \frac{d^2 u_1}{dx^2} - \frac{d^2 u_2}{dx^2} \frac{d u_1}{dx} + \frac{d^3 u_2}{dx^3} u_1 \right]_0^l + b_2^4 \int_0^l u_2 u_1 dx$$

$u \in C^2 \cap C^3$
 $x=l$

$$\frac{d^2 u}{dx^2} = \frac{d^3 u}{dx^3} = 0$$

$$\frac{d^2 u_1}{dx^2} = \frac{d^3 u_1}{dx^3} = 0$$

$$\frac{d^2 u_2}{dx^2} = \frac{d^3 u_2}{dx^3} = 0$$

by $u \in C^1$

$$x=0 \quad u=0$$

$$\frac{du}{dx} = 0$$

by $u \in C^1$

by $u \in C^1$

$$(b_1 - b_2) \int_0^l u_1, u_2 dx = 0 \quad \text{with } b_1, b_2 \geq 0$$

$$\int_0^l u_1, u_2 dx = 0$$

with $b_1, b_2 \geq 0$ and $\int_0^l = 0$

$$Z = A_1 u_1 \sin \alpha, t + A_2 u_2 \sin \alpha, t +$$

$$+ F_1 u_1 \sin \alpha, t + F_2 u_2 \sin \alpha, t +$$

$$\frac{dz}{dt} = \alpha, A_1 u_1 \sin \alpha, t +$$

$$- \alpha, F_1 u_1 \sin \alpha, t +$$

$$t=0 \quad z = \varphi(x)$$

$$\frac{dz}{dt} = \psi(x)$$

$$\varphi(x) = F_1 u_1 + F_2 u_2 +$$

$$\frac{d\varphi}{dt} = \varphi(x) = \alpha_1 A_1 u_1 + \alpha_2 A_2 u_2 +$$

$$\int_0^l \varphi(x) dx \cdot u_1 = F_1 \int_0^l u_1 dx + F_2 \int_0^l u_1 u_2 dx + F_3 \int_0^l u_1 u_3 dx +$$

$\underbrace{\hspace{10em}}_{=0}$

3. $\forall v \in F_1$

$v \in A_1$

$\varphi(x)$ & $\varphi(x)^2$ wärden die Werten der Ableitungen

bei $x=0$ & $\varphi(x)$, $x=0 = 0$

und die Ableitungen der Ableitungen der Ableitungen

der Ableitungen der Ableitungen der Ableitungen

5/7

$$\frac{d^2 z}{dt^2} + \alpha^2 \frac{d^4 z}{dt^4} = 0$$

$$\alpha^2 = \frac{qk}{2p}$$

Die Lösung der Gleichung ist die

$$z = A \sin \alpha \left(t - \frac{x}{c} \right) \quad c = \text{Phasenwert}$$

für $t \in D_1$

$$\frac{d^2 z}{dt^2} = \alpha^2 A \sin \alpha \left(t - \frac{x}{c} \right)$$

$$\frac{d^4 z}{dt^4} = -\alpha^4 A \sin \alpha \left(t - \frac{x}{c} \right)$$

$$\frac{dz}{dx} = -\frac{\alpha}{c} A \cos \alpha \left(t - \frac{x}{c}\right)$$

$$\frac{d^2 z}{dx^2} = -\frac{\alpha^2}{c^2} A \sin \alpha \left(t - \frac{x}{c}\right)$$

$$\frac{d^3 z}{dx^3} = \frac{\alpha^3}{c^3} A \cos \alpha \left(t - \frac{x}{c}\right)$$

$$\frac{d^4 z}{dx^4} = \frac{\alpha^4}{c^4} A \sin \alpha \left(t - \frac{x}{c}\right)$$

$$-\alpha^2 A \sin \alpha \left(t - \frac{x}{c}\right) + \alpha^2 \frac{\alpha^4}{c^4} A \sin \alpha \left(t - \frac{x}{c}\right) = 0$$

$$-\alpha^2 + \frac{\alpha^2 \alpha^4}{c^4} = 0$$

$$c^4 = \alpha^2 \alpha^2 \quad \text{multiplicata}$$

$$c = \sqrt{\alpha \alpha}$$

$$\alpha = \frac{2\pi}{\lambda}$$

injelele răsplând

$$cT = \lambda$$

$$\alpha = \frac{2\pi}{\lambda} c$$

$$c^4 = \alpha^2 \frac{4\pi^2}{\lambda^2} c^2$$

$$c^2 = \frac{\alpha^2 4\pi^2}{\lambda^2}$$

$$c = \alpha \frac{2\pi}{\lambda}$$

exemplu în cazul vitezei de propagare
într-un mediu elastic

Don't do; f' in u / p .

f'' - give up & dispersion; ω of f & ω of f' are
 as f is ω .

ω of f'' is ω of f' + ω of f - ω of f = ω of f'

$\int f(t - \frac{x}{v}) dt$. ω of $f(t)$ - ω of $f(t - \frac{x}{v})$

ω of f is ω .

ω of f is ω of f' + ω of f - ω of f = ω of f'

ω of f is ω of f' + ω of f - ω of f = ω of f'

ω of f is ω of f' + ω of f - ω of f = ω of f'

ω of f

ω of f

$$\frac{\partial^2 \psi}{\partial t^2} = P \frac{\partial^2 \psi}{\partial x^2} - \frac{\partial K}{\partial x} \frac{\partial^4 \psi}{\partial x^4}$$

$$\frac{\partial^2 \psi}{\partial t^2} = \frac{P}{\mu} \frac{\partial^2 \psi}{\partial x^2} - \frac{\partial K}{\partial x} \frac{\partial^4 \psi}{\partial x^4}$$

$$\psi = A \sin \alpha t \sin \beta x$$

$$\alpha = \omega$$

$$\psi = 0$$

$$\sin \beta l = 0$$

$$-\alpha^2 = -\frac{P}{\mu} \beta^2 - \frac{\partial K}{\partial x} \beta^4$$

$$\alpha^2 = \beta^2 \frac{P}{\mu} + \frac{gK}{\mu} \beta^4$$

or. $\beta l = n$ $\frac{P}{\mu} \frac{1}{l^2}$

$$\beta = \frac{n}{l}$$

$$\alpha^2 = \frac{n^2}{l^2} \frac{P}{\mu} + \frac{n^4}{l^4} \frac{gK}{\mu}$$

$$\alpha = 2n\alpha$$

$$4n^2 \alpha^2 =$$

$$n^2 = \frac{1}{4l^2} \frac{P}{\mu} + \frac{n^2}{4l^4} \frac{gK}{\mu}$$

$$n = \frac{1}{2l} \sqrt{\frac{P}{\mu}} \sqrt{1 + \frac{n^2}{l^2} \frac{gK}{P}}$$

is the 7th approx.
of the form

$n > \dots$

\dots ; $1 \sim \dots$

\dots ; \dots

\dots ; \dots

$$f(x) = \dots$$

\dots ; \dots

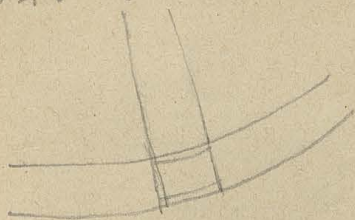
\dots

\dots ; \dots

\dots ; \dots

54

Handwritten text at the top of the page, possibly describing a process or condition.



Handwritten text to the right of the diagram, possibly a label or a note.

Handwritten text describing a process or condition, possibly related to the diagram.

Handwritten text describing a process or condition, possibly related to the diagram.

Handwritten text describing a process or condition, possibly related to the diagram.

Handwritten text, possibly a label or a note.

$$\square_c \quad a = a \quad | \quad \text{Handwritten text}$$

$$q \frac{a' - a}{a} = P$$

$$\frac{a' - a}{a} = \frac{1}{q} P$$

$$b' = b(1 - \beta \alpha)$$

$$a' = a \left[1 + \frac{1}{q} P \right]$$

$$c' = c(1 - \beta \alpha)$$

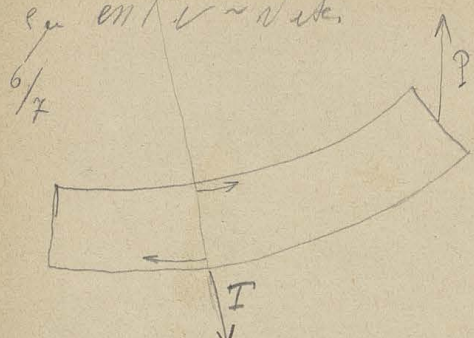
$$= a [1 + \alpha]$$

$$\text{Vol. } a'b'c' = abc [(1 + \alpha)(1 - \beta \alpha)(1 - \beta \alpha)]$$

$$= abc [1 + \alpha - 2\beta \alpha]$$

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a $2p = 10$... Vol. ... ; f ... in Rantchook
 en em / v ~ d'ak.



$$M = P_1 x = \frac{EK}{R}$$

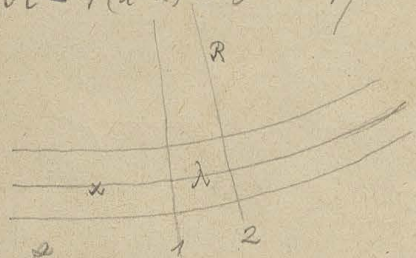
$f \sim \frac{1}{R} \sim \frac{1}{l^2}$
 $v' \sim \dots$
 $v \sim \dots$

$$a \frac{EK}{R} = P(l-x)$$

$\int \dots$
 $P \dots$

$$6 \dots \sim \frac{1}{l^3} \sim T = P; \text{ent} = P(l-x)$$

$$M - P(l-x) = 0 \quad f / T \sim \dots$$



$$v' \sim \dots$$

$$\sum \dots = 0 \quad \begin{matrix} x=0 \\ z=0 \end{matrix}$$

$$\sum \dots = 0$$

3. \dots
1. \dots
 2. \dots
 1. $T \dots$
 2. $T' \dots$

\dots
 \dots

11 X 10 ... ; ...

$$-T + T' + \rho g \lambda z = 0$$

$$M - M' - T \lambda = 0$$

$$T = \varphi(x) \quad T' = \varphi(x+\lambda) = \varphi(x) + \lambda \frac{d\varphi(x)}{dx} + \dots = T + \lambda \frac{dT}{dx}$$

$$M = \varphi(x) \quad M' = \varphi(x+\lambda) = M + \lambda \frac{dM}{dx}$$

$$\lambda \frac{dT}{dx} + \rho g \lambda z = 0$$

$$\frac{dT}{dx} + \rho g z = 0$$

$$- \frac{d^2 M}{dx^2} + \rho g z = 0$$

$$- \rho k \frac{d^2}{dx^2} \left(\frac{1}{R} \right) + \rho g z = 0$$

$$\frac{1}{R} \neq \frac{d^2 z}{dx^2}$$

$$- \rho k \frac{d^4 z}{dx^4} + \rho g z = 0$$

$$- \rho k \lambda \frac{d^4 z}{dx^4} + \rho g \lambda z = 0$$

$$- \rho k \lambda \frac{d^4 z}{dx^4} + \rho g \lambda z = \rho g \lambda \frac{d^2 z}{dx^2}$$

for EPIC ...

$$- \lambda \frac{dM}{dx} - \lambda T = 0$$

$$\frac{dM}{dx} + T = 0$$

$$\frac{d^2 M}{dx^2} + \frac{dT}{dx} = 0$$

... ..

... ..

für $\lambda = 0$: $-\frac{dM}{dx} \lambda - T \lambda = 0$

für $\lambda \neq 0$ mit $\lambda = \frac{1}{\cos \alpha}$; $f < \lambda \cos \alpha$?

steht $\lambda = \frac{1}{\cos \alpha}$; $f < \lambda \cos \alpha$ \Rightarrow $f < 1$ \Rightarrow $\lambda > 1$

$\sin \alpha \cdot \lambda = \frac{1}{\cos \alpha} \cdot \sin \alpha = \tan \alpha$

$\tan \alpha = \frac{1}{\cos \alpha} \cdot \sin \alpha = \frac{1}{\cos \alpha} \cdot \sqrt{1 - \cos^2 \alpha} = \frac{1}{\cos \alpha} \cdot \sin \alpha$

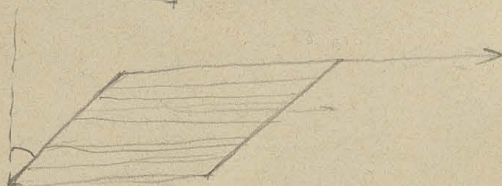
σ / τ \Rightarrow $\sigma = \tau \cdot \tan \alpha$

σ = Normal- σ ; τ = Tangential- τ

$\sigma = \tau \cdot \tan \alpha$



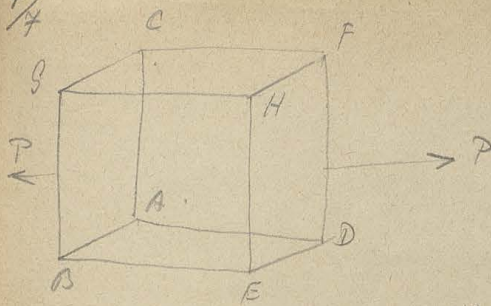
σ \perp τ \Rightarrow $\sigma = \tau \cdot \tan \alpha$



σ / τ \Rightarrow $\sigma = \tau \cdot \tan \alpha$

shear τ

Scherungs- τ



AD is = A'D'
 relab. w.r.t. $\frac{A'D' - AD}{AD} \cdot \Sigma$
 $\cdot [ADCG] = P$

$$\frac{A'D' - AD}{AD} = \frac{P}{\frac{1}{2}[ADCG]}$$

$$\xi_1 = \frac{1}{2} \times P$$

$$\eta_1 = -\beta \xi_1$$

$$\zeta_1 = -\beta \xi_1$$

$$\eta_2 = \frac{P_2}{2}$$

$$\xi_2 = -\beta \eta_2$$

$$\zeta_2 = -\beta \eta_2$$

$$\xi_3 = \frac{P_3}{2}$$

$$\xi_3 = -\beta \xi_3$$

$$\zeta_3 = -\beta \xi_3$$

$P_1 = \frac{1}{2} \times \dots$

$P_2 = AD \dots$

Ac

\dots

1/200

Only

$$\xi_1 + \xi_2 + \xi_3 = \xi = \frac{1}{2} P_1 - \beta \frac{P_2}{2} - \beta \frac{P_3}{2}$$

$$= \frac{P_1 - \beta(P_2 + P_3)}{2}$$

\dots

$$\eta = \frac{p_2 - \beta(p_1 + p_2)}{Q_2}$$

$$\xi = \frac{p_3 - \beta(p_1 + p_3)}{Q_2}$$

$$\xi + \eta + \xi = \frac{(1-2\beta)(p_1 + p_2 + p_3)}{Q_2}$$

$$A'D' = AD(1+\xi)$$

$$A'B' = AB(1+\eta)$$

$$A'C' = AC(1+\xi)$$

$$\overline{A'D'} \cdot \overline{A'B'} \cdot \overline{A'C'} = AD \cdot AB \cdot AC (1+\xi)(1+\eta)(1+\xi)$$

$$V' = V(1+\xi)(1+\eta)(1+\xi) = V(1 + \xi + \eta + \xi)$$

$$\frac{V' - V}{V} = \xi + \eta + \xi = \frac{(1-2\beta)(p_1 + p_2 + p_3)}{Q_2}$$

Vol. of air $\propto \sqrt[3]{\text{const.} \cdot [p_1 \cdot V_1 + p_2 \cdot V_2]}$

if $p_1 = p_2 = p_3 = p$

$p_1 = p_2 = p_3 = p$ nehmen. gleich.

$$\frac{V' - V}{V} = \frac{3(1-2\beta)}{Q_2} p \int \frac{V_1}{p} + \frac{V_2}{p}$$

$$\frac{Q_2}{3(1-2\beta)} = \text{const. } n \cdot p \cdot V_1$$

$\omega \rho = \frac{1}{2} \text{ } \rho^c \text{ Vol. of } = 1 \text{ } \rho \text{ uncompress.}$

$\rho_1 < \rho < \rho_2 \text{ } \rho \text{ uncompress. } \rho > \frac{1}{2}$

for $\frac{1}{4} < \frac{1}{2}$

R₁ = 1/3, Contr. of β & L.P. or $\frac{1}{3}$ of β & L.P. 57
 for no var Parameter & Disturb

$$\xi = \frac{p_1 - \beta(p_2 + p_3)}{\epsilon} \quad \eta = \frac{p_2 - \beta(p_1 + p_3)}{\epsilon} \quad \zeta = \frac{p_3 - \beta(p_1 + p_2)}{\epsilon}$$

ξ, η, ζ are p_1, p_2, p_3 of ξ, η, ζ .

$$\xi = \frac{p_1(1+\beta) - \beta(p_2 + p_3)}{\epsilon}$$

$$\frac{p_1 + p_2 + p_3}{1-2\beta} = \frac{\epsilon}{1-2\beta} (\xi + \eta + \zeta)$$

$$\epsilon \xi = p_1(1+\beta) - \frac{\beta \epsilon}{1-2\beta} (\xi + \eta + \zeta)$$

$$p_1(1+\beta) = \epsilon \xi + \frac{\beta \epsilon}{1-2\beta} (\xi + \eta + \zeta)$$

$$p_1 = \frac{\epsilon}{1+\beta} \left[\xi + \frac{\beta}{1-2\beta} (\xi + \eta + \zeta) \right]$$

$$= \frac{\epsilon}{(1+\beta)(1-2\beta)} \left[(1-\beta)\xi + \beta\eta + \beta\zeta \right]$$

$$p_2 = \frac{\epsilon}{(1+\beta)(1-2\beta)} \left[(1-\beta)\eta + \beta\xi + \beta\zeta \right]$$

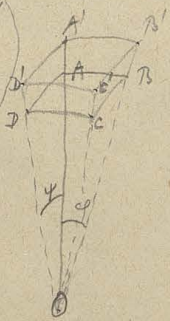
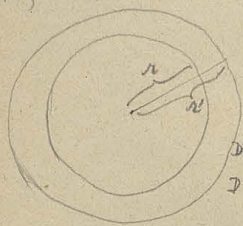
$$p_3 =$$

8/7 Deformation einer Ringschale α für α um β
 $\alpha/\beta \sim \sqrt{r^2}$; $\alpha \rho / \alpha \sqrt{r^2} \sim \sqrt{r^2} \sim \alpha \rho \sim \alpha \rho \sim \alpha \rho$

$\alpha - \beta = \text{Rad.}$

$\alpha \sim \sqrt{r^2}$

$\rho \sim \sqrt{r^2}$ und für



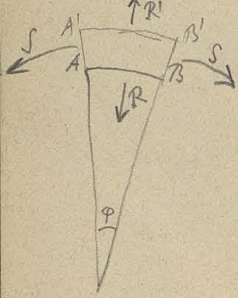
$\alpha \sim \sqrt{r^2}$ und $\sqrt{r^2} \sim \alpha \rho$
 $\alpha \sim \sqrt{r^2} = \text{Rad.}$

$R' = f(\alpha, \rho, \beta, \gamma)$

$R = \rho$

$A'B'C'D', R' = 16 \alpha / \gamma \rho$

~~$-ADD', R = \dots$~~



$ADD'A', S$

$\int \sim \frac{1}{L} R \rightarrow 2 \text{ Comp. } \alpha \text{ et } \frac{1}{2} = R \rho$

$\int \cos(90 - \frac{\phi}{2}) = \int \sin \frac{\phi}{2} = \int \frac{\phi}{2}$

$-ADD'A', S \cdot \frac{\phi}{2}$

$-DCC'D', S \cdot \frac{\phi}{2}$

$$(A'B'C'D')R' - (ABCD)R - (AD'D'A')S\varphi - (BCC'D)S\varphi = 0$$

$$ABCD = AB \cdot AD = r^2 \varphi$$

$$A'B'C'D' = r'^2 \varphi$$

$$AD'D'A' = AD \cdot AA' = r\varphi \cdot AA'$$

$$AB B'A' = r\varphi \cdot AA'$$

$$r'^2 \varphi R' - r^2 \varphi R - r\varphi \overline{AA'} S\varphi - r\varphi \overline{AA'} S\varphi = 0$$

$$\frac{r'^2 R' - r^2 R}{AA'} \cdot 2r S = 0$$

$$\frac{d(r^2 R)}{dr} - 2r S = 0$$

$$R, S \ll \sin \alpha \ll \sqrt{u^2 - r^2}$$



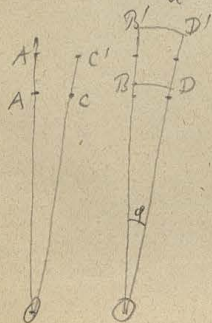
$$AB = u = r\varphi$$

$$A'B' = u'$$

$$\frac{B'B' - AA'}{AA'} = \frac{A'B' - AB}{AA'} = \frac{u' - u}{AA'}$$

$$u = f(r)$$

$$u' = f(r + AA') = f(r) + AA' f'(r) = \frac{du}{dr}$$



$$\frac{B'D' - A'C'}{A'C'} = \frac{(r+u)\varphi - r\varphi}{r\varphi}$$

$$\frac{BD - AC}{AC} = \frac{(r+u)\varphi - r\varphi}{r\varphi}$$

$$\text{rel. var. } \frac{u}{r}$$

$$\xi = \frac{r_1 - \beta(r_2 + r_3)}{\alpha} \quad \frac{du}{dr} = \frac{R - 2\beta S}{\alpha}$$

$$\frac{u}{r} = \frac{S - \beta(S+R)}{\alpha} = \frac{(1-\beta)S - \beta R}{\alpha}$$

wenn $\xi > \sqrt{\dots}$ $\beta R S S$ und $\alpha > 0$

wenn $\alpha < 0$:

$$u = \frac{(1-\beta)S r - \beta R r}{\alpha}$$

$$\frac{du}{dr} = \frac{(1-\beta) \frac{d(rS)}{dr} - \beta \frac{d(rR)}{dr}}{\alpha}$$

$$R - 2\beta S = (1-\beta) \frac{d(rS)}{dr} - \beta \frac{d(rR)}{dr} \quad \text{II.}$$

$$\frac{d(rR)}{dr} - 2rS = 0$$

$$\frac{d(r \cdot rR)}{dr}$$

$$\frac{1}{2}R + \frac{1}{2} \frac{d(rR)}{dr} - 2rS = 0$$

$$\frac{d(rR)}{dr} = 2S - R$$

$$R - 2\beta S = (1-\beta) \frac{d(rS)}{dr} - 2\beta S + \beta R$$

$$R = \frac{d(rS)}{dr} \quad \left. \vphantom{R} \right\}$$

$$\frac{d(rR)}{dr} = 2rS \quad \left. \vphantom{R} \right\}$$

$$\frac{r}{4} \frac{d(r^2 R)}{dr} - 2r S = 0 \quad \left| \frac{d}{dr} \right.$$

$$R = \frac{d(rS)}{dr}$$

$$\frac{d^2(r^2 R)}{dr^2} = 2 \frac{d(rS)}{dr} = 2R$$

$$\frac{d^2(r^2 R)}{dr^2} = \frac{2}{r^2} \cdot (r^2 R)$$

$$r^2 R = A r^{m+2}$$

$$R = A r^m$$

$$\frac{d^2(A r^{m+2})}{dr^2} = 2 A r^m$$

$$(m+2)(m+1)r^m = 2r^m \quad \circ \text{ d'f } \int L r$$

$$m^2 + 3m + 2 = 0$$

$$m = 0 \quad m = -3$$

$$\tilde{R} = A + \frac{B}{r^3}$$

* S e Const.

1) $r \rightarrow 0$

$$r=0 \quad R=\infty$$

$$f \text{ / } \psi \text{ } \circ \text{ } r \text{ / } f \mu \quad B=0$$

$$R=A \quad \circ \text{ } \text{winst.} = f \mu \text{ } \circ \text{ } f$$

2) $r \rightarrow \infty$

$$r_0 = n \quad r = \text{in Rad.}$$

$$R_0 \quad R_1$$

$$\left. \begin{aligned} R_0 &= A + \frac{B}{r_0^3} \\ R_1 &= A + \frac{B}{r_1^3} \end{aligned} \right\} \begin{aligned} R_0 - R_1 &= B \left[\frac{1}{r_0^3} - \frac{1}{r_1^3} \right] \\ &= B \frac{r_1^3 - r_0^3}{r_0^3 r_1^3} \end{aligned}$$

$$R_0 r_0^3 - R_1 r_1^3 = A [r_0^3 - r_1^3]$$

$$A = \frac{R_1 r_1^3 - R_0 r_0^3}{r_1^3 - r_0^3}$$

$$B = \frac{r_0^3 r_1^3 [R_0 - R_1]}{r_1^3 - r_0^3}$$

$\psi = R + \psi \psi \psi$

$$2rS = \frac{d}{dr} (r^2 R) = \frac{d}{dr} \left(A r^2 + \frac{B}{r} \right)$$

$$= 2A r - \frac{B}{r^2}$$

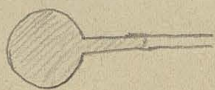
$$S = A - \frac{B}{2r^3}$$

$$S = A - \frac{B}{2r^3}$$

$\psi = R + \psi \psi \psi$

$$R + 2S = \text{const.}$$

$$r_0 = r_0 + u_0$$



$$\frac{u}{r} = \frac{\int}{\rho} - \frac{\beta \int}{\rho} - \frac{\beta R}{\rho}$$

$$\frac{u}{r} = \frac{(1-\beta) \int - \beta R}{\rho}$$

60

$$\frac{u}{R} = \frac{(1-\beta)\sqrt{A} - \frac{1-\beta}{2} \frac{B}{r^3} - \beta A - \frac{\beta D}{r^3}}{\epsilon}$$

$$= \frac{(1-2\beta)A - \frac{1-\beta}{2} \frac{B}{r^3} - \frac{1}{2}(1+\beta) \frac{B}{r^3}}{\epsilon}$$

Voll n. $B=0$

$$\frac{u}{r} = (1-2\beta) \frac{A}{\epsilon} = (1-2\beta) \frac{R_1}{\epsilon}$$

Voll n. $\frac{4\pi r^3}{3} = V$

$$V' = \frac{4\pi(r+d)^3}{3} = \frac{4\pi r^3}{3} + 4\pi r^2 d$$

$$V' - V = 4\pi r^2 d$$

n. p. d. r. $r_1 = r_0 + d$

$$r_1^3 - r_0^3 = (r_0 + d)^3 - r_0^3 = 3r_0^2 d$$

$$A = \frac{R_1(r_0 + d)^3 - R_0 r_0^3}{3r_0^2 d} = \frac{(R_1 - R_0)r_0^3 + 3R_1 r_0^2 d + \dots}{3r_0^2 d}$$

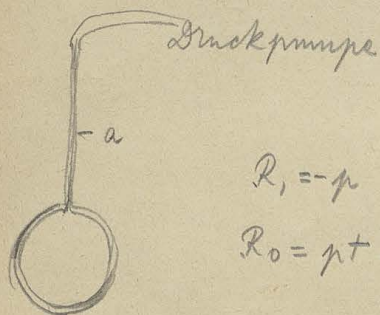
$$A = \frac{(R_1 - R_0) r_0}{3d} \quad \text{PT. c. u. s. o.}$$

$$B = \frac{r_0^6 (R_0 - R_1)}{3r_0^2 d} = \frac{r_0^4 (R_0 - R_1)}{3d}$$

$$\frac{u_0}{r_0} = \frac{(1-2\beta)}{\epsilon} \frac{(R_1 - R_0)}{3d} r_0 - \frac{1}{2} \frac{(1+\beta)}{\epsilon} r_0 \frac{(R_0 - R_1)}{3d}$$

$$= \frac{r_0}{3d \epsilon} [R_1 - R_0] \left[1 - 2\beta + \frac{1}{2} + \frac{1}{2}\beta \right]$$

$$\frac{u_0}{r_0^2} = \frac{R_1 - R_0}{3\alpha \ell} \left[\frac{3}{2} - \frac{3}{2}\beta \right] = \frac{R_1 - R_0}{\alpha \ell} \frac{1-\beta}{2}$$



$$R_1 = -p$$

$$p = \text{atm } h$$

$$R_0 = p + P$$

$$R_1 - R_0 = P$$

$$\frac{u_0}{r_0^2} = \frac{1-\beta}{2\alpha \ell} P$$

$$\text{Vol. } \dot{Q} = 4\pi r_0^2 u_0 =$$

$$= \frac{4\pi(1-\beta) P r_0^4}{2\alpha \ell}$$

$\ell \propto \text{und } \dot{Q}$

$$\ell \propto r_0 \sim \dot{Q} \propto \frac{\ell}{1-\beta} \text{ also } \beta \sim \ell^2$$

$$A \left[\frac{e^{bl} + e^{-bl}}{2} + \cos bl \right] = -B \left[\frac{e^{bl} - e^{-bl}}{2} + \sin bl \right]$$

$$B \left[\frac{e^{bl} + e^{-bl}}{2} + \cos bl \right] = -A \left[\frac{e^{bl} - e^{-bl}}{2} - \sin bl \right]$$

$$AB \left[\frac{e^{bl} + e^{-bl}}{2} + \cos bl \right]^2 = +AB \left[\left(\frac{e^{bl} - e^{-bl}}{2} \right)^2 - \sin^2 bl \right]$$

$$\left(\frac{e^{bl} + e^{-bl}}{2} \right) + \cos bl (e^{bl} + e^{-bl}) + \cos^2 bl = \left(\frac{e^{bl} - e^{-bl}}{2} \right)^2 - \sin^2 bl$$

$$\frac{1}{2} + \cos bl (e^{bl} + e^{-bl}) = -\frac{1}{2} - 1$$

$$\underbrace{(e^{bl} + e^{-bl})}_{2+} \cos bl = -2$$

∞/∞

$$bl > \frac{\pi}{2}$$

$$\cos bl < -1$$

$$\pi > bl > \frac{\pi}{2} \quad 1 \text{ } \infty$$

$$\frac{3\pi}{2} > bl > \pi \quad 2 \text{ } \infty \text{ } \text{or } \text{or } \frac{3\pi}{2}$$

$$3\pi > bl > \frac{5\pi}{2} \quad 1 \text{ } \infty \text{ } \text{or } \text{or } \text{or } e^{bl}$$

∞/∞

$$\infty \text{ } \text{or } \text{or } \text{or } \text{or } \frac{\pi}{2} \text{ } \text{or}$$

$$\infty \text{ } \text{or } \text{or } \text{or } \text{or } \frac{\pi}{2} \text{ } \text{or } \text{or } \text{or } \text{or } \text{or}$$

$$\text{or } \text{or } \text{or } 5 \cdot 7 \cdot 9 \cdot 11$$

$$\text{or } \text{or } \text{or } < 1 \quad 1 \cdot 3 \text{ } \text{or } \quad 1 + a \cdot 3 - \beta = \text{or } \text{or } \text{or } \text{or}$$

$$bd = \delta = \delta \sqrt{b}$$

$$b = \frac{\delta}{\sqrt{b}} \quad \frac{\alpha^2}{a} = b^2$$

$$\alpha = ab^2$$

$$= \frac{2\pi}{\tau} = 2\pi n \quad n = \frac{ab^2}{2\tau}$$

$$n = \frac{\delta^2}{2\tau b^2}$$

gibt es also 2 mal 2 b.

$$a^2 = \frac{c^2 K}{9g}$$

ab. 3yt = rechter Weg

$$V = b$$

$$K = \frac{b h^3}{12}$$

$$n = h$$

$$g = bh$$

$$a^2 = \frac{c^2 b h^3}{9 \cdot 12}$$

$$= \frac{c^2 h^2}{12g}$$


$$n = \frac{\delta^2 h}{2\tau b^2} \sqrt{\frac{c^2}{12g}}$$

IV $n \sim 10, 100, 1000, 10000, 100000$

von b $n < 12 \frac{1}{2} m^2$; in V , $200, 2000, 20000$

V $0 \sqrt{2g} - h$ ab. 8 $<$ etc. 1 V 200000

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1 Octave; f = c / lambda or f = c / lambda = c / (v / f) = E.I. in

[La St. 10]; f = v / lambda; f = v / lambda = v / (v / f) = f
2a; v = 2f lambda, or lambda = v / 2f - transverse wave; f = v / lambda - length
of 1/2 wave of f; f = v / lambda.

U = beta v^2 sin^2 theta; or beta v^2 sin^2 theta;

or v^2 = lambda^2 c^2 sin^2 theta. + c sin theta; f = c / lambda

or lambda = c / f; or beta = v^2 / c^2 sin^2 theta; or beta = (v/c)^2 sin^2 theta
or beta = (v/c)^2 sin^2 theta; or beta = (v/c)^2 sin^2 theta.

30/6 2 = U sin^2 theta

$$U = A \frac{e^{bx} + e^{-bx}}{2} + B \frac{e^{bx} - e^{-bx}}{2} + C \cos bx + D \sin bx$$

$$\frac{d^2 U}{dx^2} = -b^2 U$$

or y'' + p y' + q y = r
or y'' + p y' + q y = r

b, b2 b3

or y'' + p y' + q y = r

or U = A cos bx + B sin bx + C cos bx + D sin bx

$$U = A \left(\frac{e^{bx} + e^{-bx}}{2} \right) + \frac{B}{A} \frac{e^{bx} - e^{-bx}}{2} + \frac{C}{A}$$

$$U_1 = A_1 \left(\frac{e^{bx} + e^{-bx}}{2} \right) +$$

$$U_2 = A_2 \left(\frac{e^{bx} + e^{-bx}}{2} \right)$$

2-11

