CONCLUSIO
CIRCULI QUADRATURÆ
NOVISSIMÆ ET BREVISSIMÆ.

§. 22. Scholion. Continuatione Quadraturæ novissimæ & brevissimæ *jam impressa, sed nondum publicata, contigit mihi legere manuscriptum quoddam, cujus Autor Anonymus asserit, summam* $\frac{o + m}{mo}$ (§. 3. 13.) *innumeris modis posse resolvi in 2 partes, adeoque non esse necesse, ut unus factor sit denominator unius partis, alter factor denominator alterius, & unitas numerator utriusque partis. Falsissimum esset, inquit, si diceretur, summam e. gr.* $\frac{36}{288} = \frac{24 + 12}{12 \times 24}$ *non posse constare ex aliis partibus, quam ex* $\frac{1}{24}$ & $\frac{1}{12}$: *Constat enim etiam ex partibus* $\frac{1}{40}$ & $\frac{1}{10}$ (*quoniam* $\frac{1}{40} + \frac{1}{10} = \frac{10}{400} + \frac{40}{400} = \frac{50}{400} = \frac{1}{8} = \frac{36}{288}$). *Posset aliorum exemplorum magna copia addi. Ad quod cum venia Cl.* Anonymi *ita respondeo: Partes* $\frac{1}{40}$ & $\frac{1}{10}$ *constituunt quidem summam* $\frac{50}{400}$ æqualem $\frac{36}{288}$, *quia utrobique numerator habet ad denominatorem suum eandem rationem, nempe ut* 1 : 8; *sed nunquam possunt constituere summam* eandem $\frac{36}{288}$. *Quæ sunt eadem, sunt etiam æqualia; sed non vice versa, quæ sunt æqualia, sunt etiam eadem. Nam etsi Rhomboides cum quadrato eandem basin & æqualem altitudinem habens, sit huic æqualis; absurdum tamen esset inde concludere, quadratum esse Rhomboidem, vel vice versa. Quodlibet par fractionum sequentium* $\frac{\frac{1}{2}}{42}$ & $\frac{1\frac{6}{24}}{12}$; $\frac{\frac{1}{3}}{24}$ & $\frac{1\frac{8}{24}}{12}$; $\frac{\frac{1}{4}}{24}$ & $\frac{1\frac{9}{24}}{12}$; $\frac{\frac{1}{6}}{24}$ & $\frac{1\frac{10}{24}}{12}$; $\frac{2\frac{4}{12}}{24}$ & $\frac{\frac{1}{3}}{12}$; $\frac{2\frac{6}{12}}{24}$ & $\frac{\frac{1}{4}}{12}$; $\frac{2\frac{8}{12}}{24}$ & $\frac{\frac{1}{6}}{12}$; $\frac{2\frac{9}{12}}{24}$ & $\frac{\frac{1}{8}}{12}$ *efficit quidem eandem summam, nempe* $\frac{36}{288}$: *nam fractiones* 1*mi paris sunt* $= \frac{6}{288} + \frac{30}{288} = \frac{36}{288}$, 2*di paris* $= \frac{4}{288} + \frac{32}{288}$; 3*tii* $\frac{3}{288} + \frac{33}{288}$; 4*ti* $\frac{2}{288} + \frac{34}{288}$; 5*ti* $\frac{28}{288} + \frac{8}{288}$; 6*ti* $\frac{30}{288} + \frac{6}{288}$; 7*mi* $\frac{32}{288} + \frac{4}{288}$; 8*vi* $\frac{33}{288} + \frac{3}{288}$: *attamen quoniam quævis fractionum priorum habet duplicem denominatorem; palam est, quamlibet esse fractionem fractionis. Sic* 1*ma est* $\frac{1}{2}$ *de* $\frac{1}{24} = \frac{1}{48}$, 2*da* $\frac{30}{24}$ *de* $\frac{1}{12} = \frac{30}{288}$, 3*tia* $\frac{1}{3}$ *de* $\frac{1}{24} = \frac{1}{72}$, 4*ta* $\frac{32}{24}$ *de* $\frac{1}{12} = \frac{32}{288}$, 5*ta* $\frac{1}{4}$ *de* $\frac{1}{24} = \frac{1}{96}$, 6*ta* $\frac{33}{24}$ *de* $\frac{1}{12} = \frac{33}{288}$, 7*ma* $\frac{1}{6}$ *de* $\frac{1}{24} = \frac{1}{144}$, 8*va* $\frac{34}{24}$ *de* $\frac{1}{12} = \frac{34}{288}$, *& ita porro. Jam cum numerator summæ per conditionem problematis* (§. 3. 13.) *debeat esse aggregatum ex utroque factore denominatoris; erit summa* 1*mi paris* $\frac{288 + 48}{48 \times 288}$, 2*di* $\frac{288 + 72}{72 \times 288}$, 3*tii* $\frac{288 + 96}{96 \times 288}$, 4*ti* $\frac{288 + 144}{144 \times 288}$, *& ita porro, consequenter ubique alia; ex quo manifestum est, summam* $\frac{36}{288}$ *innumeris modis quidem esse resolubilem in partes duas; sed nullas alias eam posse efficere talem, ut ejus numerator sit aggregatum*

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ex utroque factore denominatoris, nisi $\frac{1}{24}$ *&* $\frac{1}{12}$. *Quod ut evidentius appareat, investigemus lunulam respondentem diametri quadrato* $= 1$. *Sit itaque hujus ratio ad illam excessiva, ut* $4 : 1\frac{1}{6} = 24 : 7$, *& defectiva ut* $4 : \frac{2}{3} = 12 : 2$: *erunt lunulæ per utramque inventæ* $\frac{7}{24}$ *&* $\frac{2}{12} = \frac{84}{288}$ *&* $\frac{48}{288}$, *quæ ex se invicem ablatæ, relinquunt summam excessus & defectus* $\frac{36}{288}$, *cujus numerator est aggregatum ex utroque factore* $d + a = 12 + 24$ *denominatoris* ad $= 24 . 12 = 288$. *Quoniam igitur hæc summa est conflata ex excessu & defectu* (§. 2.), *& per demonstrata nullæ aliæ partes sunt assignabiles, quæ eam ita constituant, ut ejus numerator sit aggregatum ex utroque factore denominatoris nisi* $\frac{1}{24}$ *&* $\frac{1}{12}$; *debet harum altera necessario esse excessus, altera defectus. Jam cum lunula excess:* $\frac{7}{24}$ *sit aggregatum ex vera & excessu; necesse est, ut ejus excessus sit vel* $\frac{1}{12}$, *vel* $\frac{1}{24}$; *sed* $\frac{1}{12}$ *est diversæ,* $\frac{1}{24}$ *autem ejusdem cum ea denominationis: ergo* $\frac{1}{24}$ *utpote ejus pars homogenea debet necessario esse excessus, & per consequens* $\frac{1}{12}$ *defectus: nam* $\frac{7}{24} - \frac{1}{24} = \frac{6}{24} = \frac{1}{4}$; *vel* $\frac{2}{12} + \frac{1}{12} = \frac{3}{12} = \frac{1}{4}$ *est utique lunula vera; quæ cum tam per rationes* $4 : 1\frac{1}{5}$ *&* $4 : \frac{3}{4}$; $4 : 1\frac{1}{9}$ *&* $4 : \frac{9}{10}$; $4 : 1\frac{1}{12}$ *&* $4 : \frac{10}{11}$, *quàm per innumeras alias exactissimè determinetur; evidens est, problema IV demonstrans 2dam partem Theorematis §. 3tii, esse verò verius: consequenter problema III huic superstructum firmissimo inniti fundamento. Jam cum quantitates incognitæ proportionaliter crescentes (veluti diametri cum peripheriis; quadrata diametrorum cum lunulis, circulis & segmentis; cubi diametrorum cum sphæris; quadrata* linearum constantium *cum sectionibus conicis, & quæ alia ejus generis) eadem methodo queant perfectissime determinari; dubitare non licet de usu amplissimo hujus solutionis. Me non monente autem facile intelligetur, nullatenus determinari posse e. gr. fractionem inter* $\frac{1}{2}$ *&* $\frac{1}{3}$ *cadentem, quam aliquis in mente habeat, quia limites* $\frac{1}{2}$ *&* $\frac{1}{3}$ *per nullam proportionem naturæ quæstionis conformem possunt investigari. Interim ne quid perfectioni hujus opusculi deesse videatur; lubet adhuc adjicere sequentia.*

THEOREMA II.

§. 23. *Circuli* Ludolphinus, Metianus *&* Archimedeus *peccant in excessu: 1mus* $\frac{120}{32000}$, *2dus* $\frac{60}{14464}$, *3tius* $\frac{2}{448}$ *quadrati diametri.*

Demonstratio. Ratio *Ludolphina* quadrati diametri ad circulum $1000 : 785$ multiplicata per 32 prodit $= 32000 : 25120$; assumtis deinde diametri quadrato $= 1$, & ratione defectiva $32 : 24$, prodeunt circuli $\frac{25120}{32000}$ & $\frac{24}{32} = \frac{803840}{1024000}$ & $\frac{768000}{1024000}$, quorum differentia, h. e. summa excessus & defectus (§. 2.) est $\frac{35840}{1024000}$, cujus numerator est $= 120\,d + a = 3840 + 32000$. Ergo excessus est $\frac{120}{32000}$ & defectus $\frac{1}{32}$, consequenter circulus verus $\frac{25120}{32000} - \frac{120}{32000} = \frac{25000}{32000} = \frac{25}{32}$, vel $\frac{24}{32} + \frac{1}{32} = \frac{25}{32}$. Ratio *Metiana* $452 : 355$ multiplicata per 32, producit $= 14464 : 11360$, quæ cum defectiva $32 : 24$ prodit circulos

culos $\frac{11360}{14464}$ & $\frac{24}{32} = \frac{363520}{462848}$ & $\frac{347135}{462848}$, quorum differentia seu summa exces. & defect. est $\frac{16384}{462848}$, cujus numerator est $= 60d + a = 1920 + 14464$: unde (per §. 16.) excessus est $\frac{60}{14464}$ & defectus $\frac{1}{32}$: consequenter circulus verus $\frac{11360}{14464} - \frac{60}{14464} = \frac{11300}{14464}$, & reductus per 452 ad terminos minimos $= \frac{25}{32}$. Ratio *Archimedea* 14: 11 multiplicata per 32, manifestat $= 448 : 352$, quæ cum defectiva prodit circulos $\frac{352}{448}$ & $\frac{24}{32} = \frac{11264}{14336}$ & $\frac{10752}{14336}$, qui ex se invicem ablati, relinquunt summam excessus & defectus $\frac{512}{14336}$, cujus numerator est $= 2d + a = 64 + 448$. Quare excessus est $\frac{2}{448}$, consequenter circulus verus $\frac{352}{448} - \frac{2}{448} = \frac{350}{448}$ & reductus per 14 ad terminos minimos $= \frac{25}{32}$.

§. 24. Scholion. *Ut usus amplissimus problematis V magis magisque patefiat, sit quadratum diametri* $= 1$, *ratio ejus ad circulum excessiva ut* $32 : 26$, *& defectiva, ut* $32 : 24\frac{1}{3} = 96 : 73$: *erunt circuli per utramque reperti* $\frac{26}{32}$ & $\frac{73}{96} = \frac{2496}{3072}$ & $\frac{2336}{3072}$, *quorum differentia, seu summa excessus & defectus, est* $\frac{160}{3072} = d + 2a = 96 + 64$, *adeoque excessus est* $\frac{1}{32}$ *& defectus* $\frac{2}{96}$, *consequenter circulus verus* $\frac{26}{32} - \frac{1}{32} = \frac{25}{32}$, *vel* $\frac{73}{96} + \frac{2}{96} = \frac{75}{96} = \frac{25}{32}$. *Sit deinde ratio excessiva* $32 : 25\frac{4}{5} = 160 : 129$, *& defectiva* $32 : 24$: *erunt circuli* $\frac{129}{160}$ & $\frac{24}{32} = \frac{4128}{5120}$ & $\frac{3840}{5120}$, *qui ex se ablati, relinquunt summam excessus & defectus* $\frac{288}{5120}$, *cujus numerator est* $= 4d + a = 128 + 160$: *unde excessus est* $\frac{4}{160}$ *& defectus* $\frac{1}{32}$: *consequenter circulus verus* $\frac{129}{160} - \frac{4}{160} = \frac{125}{160} = \frac{25}{32}$; *vel* $\frac{24}{32} + \frac{1}{32} = \frac{25}{32}$. *Sit denique ratio excessiva* $32 : 25\frac{3}{4} = 128 : 103$, *& defectiva* $32 : 24\frac{2}{7} = 224 : 170$: *erunt circuli* $\frac{103}{128}$ & $\frac{170}{224} = \frac{23072}{28672}$ & $\frac{21760}{28672}$, *quorum differentia prodit summam excessus & defectus* $= \frac{1312}{28672}$, *cujus numerator est* $= 3d + 5a = 672 + 640$. *Ergo excessus est* $\frac{3}{128}$ *& defectus* $\frac{5}{224}$: *consequenter circulus verus* $\frac{103}{128} - \frac{3}{128} = \frac{100}{128} = \frac{25}{32}$; *vel* $\frac{170}{224} + \frac{5}{224} = \frac{175}{225} = \frac{25}{32}$.

§. 25. *Corollarium.* Est ergo circulus ad quadratum diametri ut $\frac{25}{32} : 1 = 25 : 32$, consequenter diameter ad peripheriam, ut $8 : 25$ (§. 8.).

PROBLEMA VI.

§. 26. *Circulo par quadratum componere.*

Resolutio. 1mo. Circulus per 2 diametros secantes se ad angulos rectos, dividatur in 4 quadrantes. 2. Utraque diameter producatur 8va parte sui. 3. Ab extremitate unius diametri productæ, ad extremitatem alterius prolongatæ ducatur hypothenusa, quæ erit latus quadrati æqualis circulo.

Demonstratio. Quilibet Cathetus continet per constructionem 5 partes diametri: ergo quadrata 2 Cathetorum sunt $= 25 + 25 = 50$, consequenter quadratum hypothenusæ est etiam $= 50$. Atqui circulus, cujus diameter est 8 partium, est quoque $= 50$ (§. 8.). Ergo quadratum, cujus latus est dicta hypothenusa, est = circulo.

§. 27. *Corollarium.* Quod si ex circuli area data ($12\frac{1}{2}$) inveniendum sit quadratum ei æquale, extrahatur ex dimidia area

($6\frac{1}{4} = \frac{25}{4}$)

($6\frac{1}{4} = \frac{25}{4}$) radix quadrata ($\frac{5}{2}$); deinde jungantur 2 lineæ huic radici æquales ad angulum rectum, & ducatur hypothenuſa, quæ erit latus quadrati æqualis circulo, cujus area fuit data: nam quoniam quadrata cathetorum ($\frac{25}{4} \dagger \frac{25}{4} = \frac{50}{4} = 12\frac{1}{2}$) efficiunt aream datam circuli, & quadratum hypothenuſæ inventæ eſt = his quadratis cathetorum; palam eſt, idem quoque eſſe = circulo, cujus area fuit data.

THEOREMA III.

§. 28. *Sphæræ* Ludolphina, Metiana *&* Archimedea *peccant in exceſſu: 1ma* $\frac{36}{14400}$, *2da* $\frac{90}{32544}$, *3tia* $\frac{3}{1008}$ *cubi diametri.*

Demonſtratio. Ratio *Ludolphina* cubi diametri ad ſphæram 300 : 157 multiplicata per 48, producit æqualem 14400 : 7536; aſſumtis deinde cubo diam: = 1, & ratione defectiva 48 : 24, prodeunt ſphæræ $\frac{7536}{14400}$ & $\frac{24}{48} = \frac{361728}{691200}$ & $\frac{345600}{691200}$, quarum differentia prodit ſummam exceſſus & defectus $\frac{15128}{691200}$, cujus numerator eſt = 36 $d \dagger a$ = 1728 † 14400: ergo exceſſus eſt $\frac{36}{14400}$ & defectus $\frac{1}{48}$ (§. 16.), conſequenter ſphæra vera $\frac{7500}{14400} = \frac{25}{48}$. Ratio *Metiana* 678 : 355 multiplicata per 48, prodit = 32544 : 17040, quæ cum defectiva producit ſphæras $\frac{17040}{32544}$ & $\frac{24}{48} = \frac{817920}{1562112}$ & $\frac{781056}{1562112}$, quarum differentia manifeſtat ſummam exceſſus & defectus = $\frac{36864}{1562112}$, cujus numerator eſt = 90 $d \dagger a$ = 4320 † 32544: unde exceſſus eſt $\frac{90}{32544}$: conſequenter ſphæra vera $\frac{16950}{32544}$, & reducta per 678 ad terminos minimos = $\frac{25}{48}$. Ratio *Archimedea* 21 : 11 multiplicata per 48 dat = 1008 : 528, quæ cum defectiva prodit ſphæras $\frac{528}{1008}$ & $\frac{24}{48} = \frac{25344}{48384}$ & $\frac{24192}{84384}$, quarum differentia ſiſtit ſummam exceſſus & defectus = $\frac{1152}{48384}$, cujus numerator eſt = 3 $d \dagger a$ = 144 † 1008. Ergo exceſſus eſt $\frac{3}{1008}$: conſequenter ſphæra vera $\frac{525}{1008}$ & reducta per 21 ad terminos minimos = $\frac{25}{48}$.

§. 29. Scholion. *Numerator ſummæ exceſſus & defectus ſphærarum, quarum cubus diam.* = 1, *inventæ per rationes* 48 : $25\frac{2}{3}$ *&* 48 : $24\frac{2}{5}$, *eſt* = 2 d † 3 a; *repertæ per rationes* 48 : 26 *&* 48 : $24\frac{1}{4}$, *eſt* = d † 3 a; *& inveſtigatæ per rationes* 48 : $25\frac{5}{6}$ *&* 48 : 24, *eſt* = 5 d † a: *unde per* (§. 16.) *illico determinatur exceſſus & defectus, & ope utriusque ſphæra vera, quæ ubique prodit* = $\frac{25}{48}$.

§. 30. *Corollarium.* Eſt ergo ſphæra ad cubum diametri, ut $\frac{25}{48}$: 1 = 25 : 48: conſequenter diameter ad peripheriam, ut 8 : 25. (§. 8.).