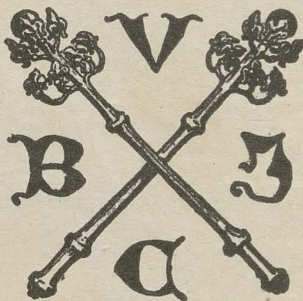




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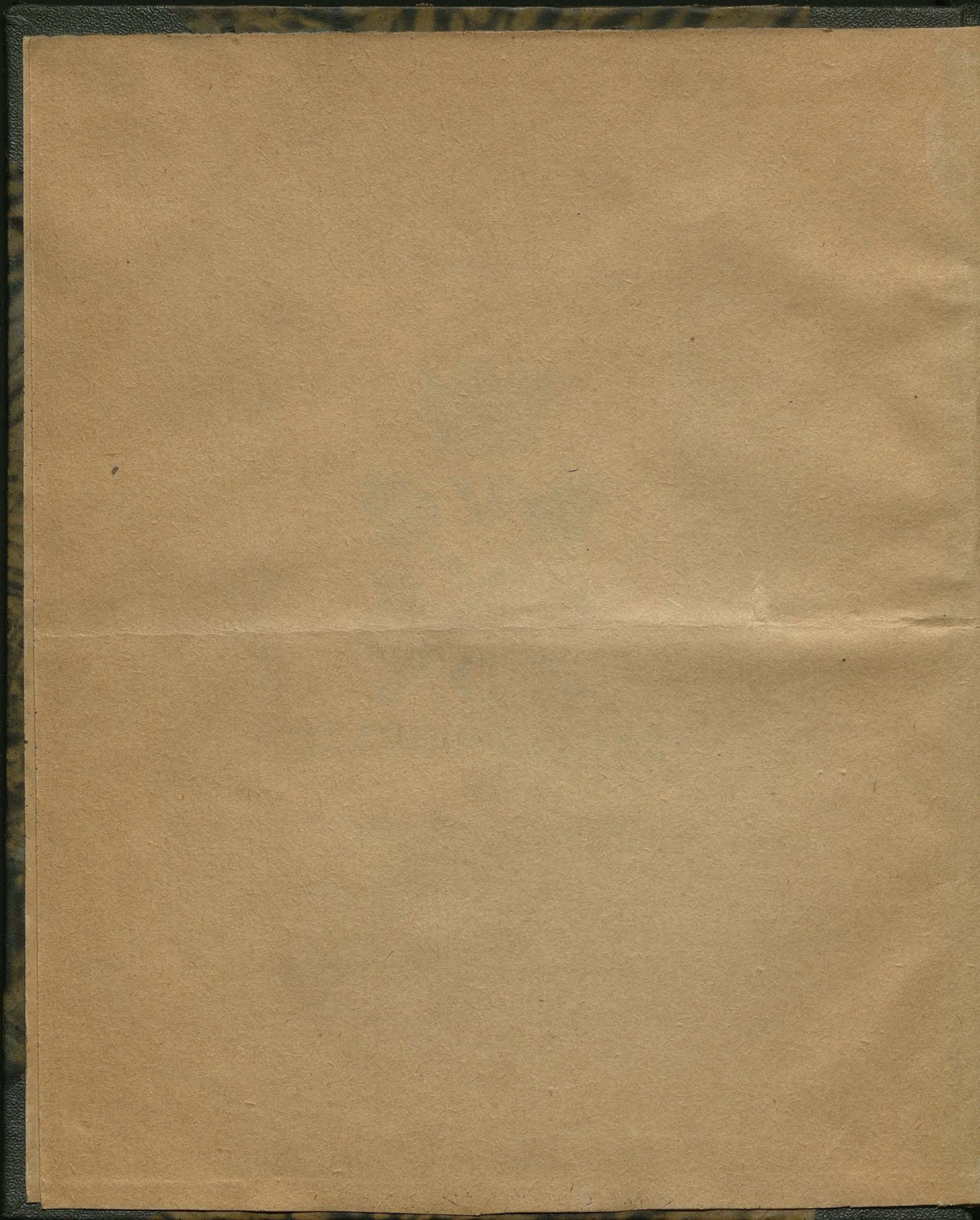
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10.

# CONCLUSIO CIRCULI QUADRATURÆ NOVISSIMÆ ET BREVISSIMÆ.

§. 22. Scholion. Continuatione Quadraturæ novissimæ & brevissimæ jam impressa, sed nondum publicata, contigit mihi legere manuscriptum quoddam, cujus Autor Anonymus asserit, summam  $\frac{0 + m}{mo}$  (§. 3. 13.) innumeris modis posse resolvi in 2 partes,

adeoque non esse necesse, ut unus factor sit denominator unius partis, alter factor denominator alterius, & unitas numerator utriusque partis. Falsissimum esset, inquit, si diceretur, summam e. gr.  $\frac{36}{288} = \frac{24 + 12}{12 \times 24}$

non posse constare ex aliis partibus, quam ex  $\frac{1}{24}$  &  $\frac{1}{12}$ : Constat enim etiam ex partibus  $\frac{1}{48}$  &  $\frac{1}{16}$  (quoniam  $\frac{1}{48} + \frac{1}{16} = \frac{1}{48} + \frac{3}{48} = \frac{4}{48} = \frac{1}{12} = \frac{36}{432}$ ). Posset aliorum exemplorum magna copia addi.

Ad quod cum venia Cl. Anonymi ita respondeo: Partes  $\frac{1}{48}$  &  $\frac{1}{16}$  constituunt quidem summam  $\frac{1}{48}$  æqualem  $\frac{36}{432}$ , quia utrobique numerator habet ad denominatorem suum eandem rationem, nempe ut 1: 8; sed nunquam possunt constituere summam eandem  $\frac{36}{288}$ . Quæ sunt eadem, sunt etiam æqualia; sed non vice versa, quæ sunt æqualia, sunt etiam eadem. Nam etsi Rhomboides cum quadrato eandem basin & æqualem altitudinem habens, sit huic æqualis; absurdum tamen esset inde concludere, quadratum esse Rhomboidem, vel vice versa. Quodlibet par fractionum sequentium:  $\frac{1}{2}$  &  $\frac{1}{2}$ ;  $\frac{1}{4}$  &  $\frac{1}{4}$ ;  $\frac{1}{8}$  &  $\frac{1}{8}$ ;  $\frac{1}{16}$  &  $\frac{1}{16}$ ;  $\frac{1}{32}$  &  $\frac{1}{32}$ ;  $\frac{1}{64}$  &  $\frac{1}{64}$ ;  $\frac{1}{128}$  &  $\frac{1}{128}$ ;  $\frac{1}{256}$  &  $\frac{1}{256}$ ;  $\frac{1}{512}$  &  $\frac{1}{512}$ ;  $\frac{1}{1024}$  &  $\frac{1}{1024}$ ;  $\frac{1}{2048}$  &  $\frac{1}{2048}$ ;  $\frac{1}{4096}$  &  $\frac{1}{4096}$ ;  $\frac{1}{8192}$  &  $\frac{1}{8192}$ ;  $\frac{1}{16384}$  &  $\frac{1}{16384}$ ;  $\frac{1}{32768}$  &  $\frac{1}{32768}$ ;  $\frac{1}{65536}$  &  $\frac{1}{65536}$ ;  $\frac{1}{131072}$  &  $\frac{1}{131072}$ ;  $\frac{1}{262144}$  &  $\frac{1}{262144}$ ;  $\frac{1}{524288}$  &  $\frac{1}{524288}$ ;  $\frac{1}{1048576}$  &  $\frac{1}{1048576}$ ;  $\frac{1}{2097152}$  &  $\frac{1}{2097152}$ ;  $\frac{1}{4194304}$  &  $\frac{1}{4194304}$ ;  $\frac{1}{8388608}$  &  $\frac{1}{8388608}$ ;  $\frac{1}{16777216}$  &  $\frac{1}{16777216}$ ; 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ex utroque factore denominatoris, nisi  $\frac{1}{24}$  &  $\frac{1}{12}$ . Quod ut evidentius appareat, investigemus lunulam respondentem diametri quadrato = 1. Sit itaque hujus ratio ad illam excessiva, ut 4 :  $1\frac{1}{2}$  = 24 : 7, & defectiva ut 4 :  $\frac{2}{3}$  = 12 : 2 : erunt lunulae per utramque inventae  $\frac{7}{24}$  &  $\frac{2}{12} = \frac{64}{288}$  &  $\frac{48}{288}$ , quae ex se invicem ablatæ, relinquunt summam excessus & defectus  $\frac{24}{288}$ , cujus numerator est aggregatum ex utroque factore d + a = 12 + 24 denominatoris ad = 24. 12 = 288. Quoniam igitur hæc summa est conflata ex excessu & defectu (§. 2.), & per demonstrata nullæ alicæ partes sunt assignabiles, quæ eam ita constituent, ut ejus numerator sit aggregatum ex utroque factore denominatoris nisi  $\frac{1}{24}$  &  $\frac{1}{12}$ ; debet harum altera necessario esse excessus, altera defectus. Jam cum lunula excessus sit  $\frac{7}{24}$  sit aggregatum ex vera & excessu; necesse est, ut ejus excessus sit vel  $\frac{1}{12}$ , vel  $\frac{1}{24}$ ; sed  $\frac{1}{12}$  est diversa,  $\frac{1}{24}$  autem ejusdem cum ea denominationis: ergo  $\frac{1}{24}$  utpote ejus pars homogenea debet necessario esse excessus, & per consequens  $\frac{1}{12}$  defectus; nam  $\frac{7}{24} - \frac{1}{24} = \frac{6}{24} = \frac{1}{4}$ ; vel  $\frac{7}{24} + \frac{1}{24} = \frac{8}{24} = \frac{1}{3}$  est utique lunula vera; quæ cum tam per rationes 4 :  $1\frac{1}{2}$  & 4 :  $\frac{2}{3}$ ; 4 :  $1\frac{1}{3}$  & 4 :  $\frac{2}{3}$ ; 4 :  $1\frac{1}{2}$  & 4 :  $\frac{1}{3}$ , quàm per innumeras alias exactissime determinetur; evidens est, problema IV demonstrans 2dam partem Theorematis §. 3tii, esse verò verius: consequenter problema III huic superstructum firmissimo inniti fundamento. Jam cum quantitates incognitæ proportionaliter crescentes (veluti diametri cum peripheriis; quadrata diametrorum cum lunulis, circulis & segmentis; cubi diametrorum cum sphaeris; quadrata linearum constantium cum sectionibus conicis, & quæ alia ejus generis) eadem methodo queant perfectissime determinari; dubitare non licet de usu amplissimo hujus solutionis. Me non monente autem facile intelligetur, nullatenus determinari posse e. gr. fractionem inter  $\frac{1}{2}$  &  $\frac{1}{3}$  cadentem, quam aliquis in mente habeat, quia limites  $\frac{1}{2}$  &  $\frac{1}{3}$  per nullam proportionem naturæ quæstionis conformem possunt investigari. Interim ne quid perfectioni hujus opusculi deesse videatur; lubet adhuc adjicere sequentia.

#### THEOREMA II.

§. 23. Circuli Ludolphinus, Metianus & Archimedeus peccant in excessu: 1mus  $\frac{12000}{32000}$ , 2dus  $\frac{6000}{14400}$ , 3tius  $\frac{2400}{448}$  quadrati diametri.

Demonstratio. Ratio Ludolphina quadrati diametri ad circulum 1000 : 785 multiplicata per 32 prodit = 32000 : 25120; assumptis deinde diametri quadrato = 1, & ratione defectiva 32 : 24, producit circuli  $\frac{25120}{32000}$  &  $\frac{24}{32} = \frac{803840}{1024000}$  &  $\frac{768000}{1024000}$ , quorum differentia, h. e. summa excessus & defectus (§. 2.) est  $\frac{33840}{1024000}$ , cujus numerator est = 120 d + a = 3840 + 32000. Ergo excessus est  $\frac{120}{32000}$  & defectus  $\frac{1}{32}$ , consequenter circulus verus  $\frac{25120}{32000} - \frac{120}{32000} = \frac{25000}{32000} = \frac{25}{32}$ , vel  $\frac{24}{32} + \frac{1}{32} = \frac{25}{32}$ . Ratio Metiana 452 : 355 multiplicata per 32, producit = 14464 : 11360, quæ cum defectiva 32 : 24 prodit circulos

culos  $\frac{11360}{14464}$  &  $\frac{24}{32} = \frac{363520}{462848}$  &  $\frac{347135}{462848}$ , quorum differentia seu summa excess. & defect. est  $\frac{16384}{462848}$ , cujus numerator est  $= 60d + a = 1920 + 14464$ : unde (per §. 16.) excessus est  $\frac{60}{14464}$  & defectus  $\frac{1}{32}$ : consequenter circulus verus  $\frac{11360}{14464} - \frac{60}{14464} = \frac{11300}{14464}$ , & reductus per 452 ad terminos minimos  $= \frac{25}{32}$ . Ratio *Archimedea* 14: 11 multiplicata per 32, manifestat  $= 448 : 352$ , quæ cum defectiva prodit circules  $\frac{344}{448}$  &  $\frac{24}{32} = \frac{11244}{448}$  &  $\frac{10752}{448}$ , qui ex se invicem ablati, relinquunt summam excessus & defectus  $\frac{512}{448}$ , cujus numerator est  $= 2d + a = 64 + 448$ . Quare excessus est  $\frac{2}{448}$ , consequenter circulus verus  $\frac{112}{448} - \frac{2}{448} = \frac{110}{448}$  & reductus per 14 ad terminos minimos  $= \frac{25}{32}$ .

§. 24. Scholion. Ut usus amplissimus problematis V magis magisque patefiat, sit quadratum diametri  $= 1$ , ratio ejus ad circum excessiva ut 32: 26, & defectiva, ut 32: 24 $\frac{1}{2}$  = 96: 73: erunt circuli per utramque reperti  $\frac{26}{32}$  &  $\frac{73}{96} = \frac{2090}{3072}$  &  $\frac{2336}{3072}$ , quorum differentia, seu summa excessus & defectus, est  $\frac{240}{3072} = d + 2a = 96 + 64$ , adeoque excessus est  $\frac{1}{32}$  & defectus  $\frac{2}{96}$ , consequenter circulus verus  $\frac{26}{32} - \frac{1}{32} = \frac{25}{32}$ , vel  $\frac{73}{96} + \frac{2}{96} = \frac{75}{96} = \frac{25}{32}$ . Sit deinde ratio excessiva 32: 25 $\frac{1}{2}$  = 160: 129, & defectiva 32: 24: erunt circuli  $\frac{129}{160}$  &  $\frac{24}{32} = \frac{4128}{5120}$  &  $\frac{3840}{5120}$ , qui ex se ablati, relinquunt summam excessus & defectus  $\frac{288}{5120}$ , cujus numerator est  $= 4d + a = 128 + 160$ : unde excessus est  $\frac{1}{160}$  & defectus  $\frac{1}{32}$ : consequenter circulus verus  $\frac{129}{160} - \frac{1}{160} = \frac{128}{160} = \frac{25}{32}$ ; vel  $\frac{24}{32} + \frac{1}{32} = \frac{25}{32}$ . Sit denique ratio excessiva 32: 25 $\frac{1}{4}$  = 128: 103, & defectiva 32: 24 $\frac{2}{7}$  = 224: 170: erunt circuli  $\frac{103}{128}$  &  $\frac{170}{224} = \frac{23072}{28672}$  &  $\frac{21760}{28672}$ , quorum differentia prodit summam excessus & defectus  $= \frac{1312}{28672}$ , cujus numerator est  $= 3d + 5a = 672 + 640$ . Ergo excessus est  $\frac{1}{128}$  & defectus  $\frac{5}{224}$ : consequenter circulus verus  $\frac{103}{128} - \frac{5}{224} = \frac{100}{128} = \frac{25}{32}$ ; vel  $\frac{170}{224} + \frac{5}{224} = \frac{175}{224} = \frac{25}{32}$ .

§. 25. Corollarium. Est ergo circulus ad quadratum diametri ut  $\frac{25}{32} : 1 = 25 : 32$ , consequenter diameter ad peripheriam, ut 8: 25 (§. 8.).

#### PROBLEMA VI.

§. 26. Circulo par quadratum componere.

*Resolutio.* 1mo. Circulus per 2 diametros secantes se ad angulos rectos, dividatur in 4 quadrantes. 2. Utraque diameter producat 8va parte sui. 3. Ab extremitate unius diametri productæ, ad extremitatem alterius prolongatæ ducatur hypothenusa, quæ erit latus quadrati æqualis circulo.

*Demonstratio.* Quilibet Cathetus continet per constructionem 5 partes diametri: ergo quadrata 2 Cathetorum sunt  $= 25 + 25 = 50$ , consequenter quadratum hypothenusæ est etiam  $= 50$ . Atqui circulus, cujus diameter est 8 partium, est quoque  $= 50$  (§. 8.). Ergo quadratum, cujus latus est dicta hypothenusa, est  $=$  circulo.

§. 27. Corollarium. Quod si ex circuli area data ( $12\frac{1}{2}$ ) inveniendum sit quadratum ei æquale, extrahatur ex dimidia area ( $6\frac{1}{4} = \frac{25}{4}$ )

( $6\frac{3}{4} = \frac{25}{4}$ ) radix quadrata ( $\frac{5}{2}$ ); deinde jungantur 2 lineæ huic radici æquales ad angulum rectum, & ducatur hypothenusa, quæ erit latus quadrati æqualis circulo, cujus area fuit data: nam quoniam quadrata cathetorum ( $\frac{25}{4} + \frac{25}{4} = \frac{50}{4} = 12\frac{1}{2}$ ) efficiunt aream datam circuli, & quadratum hypothenusæ inventæ est = his quadratis cathetorum; palam est, idem quoque esse = circulo, cujus area fuit data.

T H E O R E M A III.

§. 28. *Sphæræ Ludolphina, Metiana & Archimedea peccant in excessu: 1ma*  $\frac{36}{14400}$ , *2da*  $\frac{90}{32544}$ , *3tia*  $\frac{3}{1038}$  *cubi diametri.*

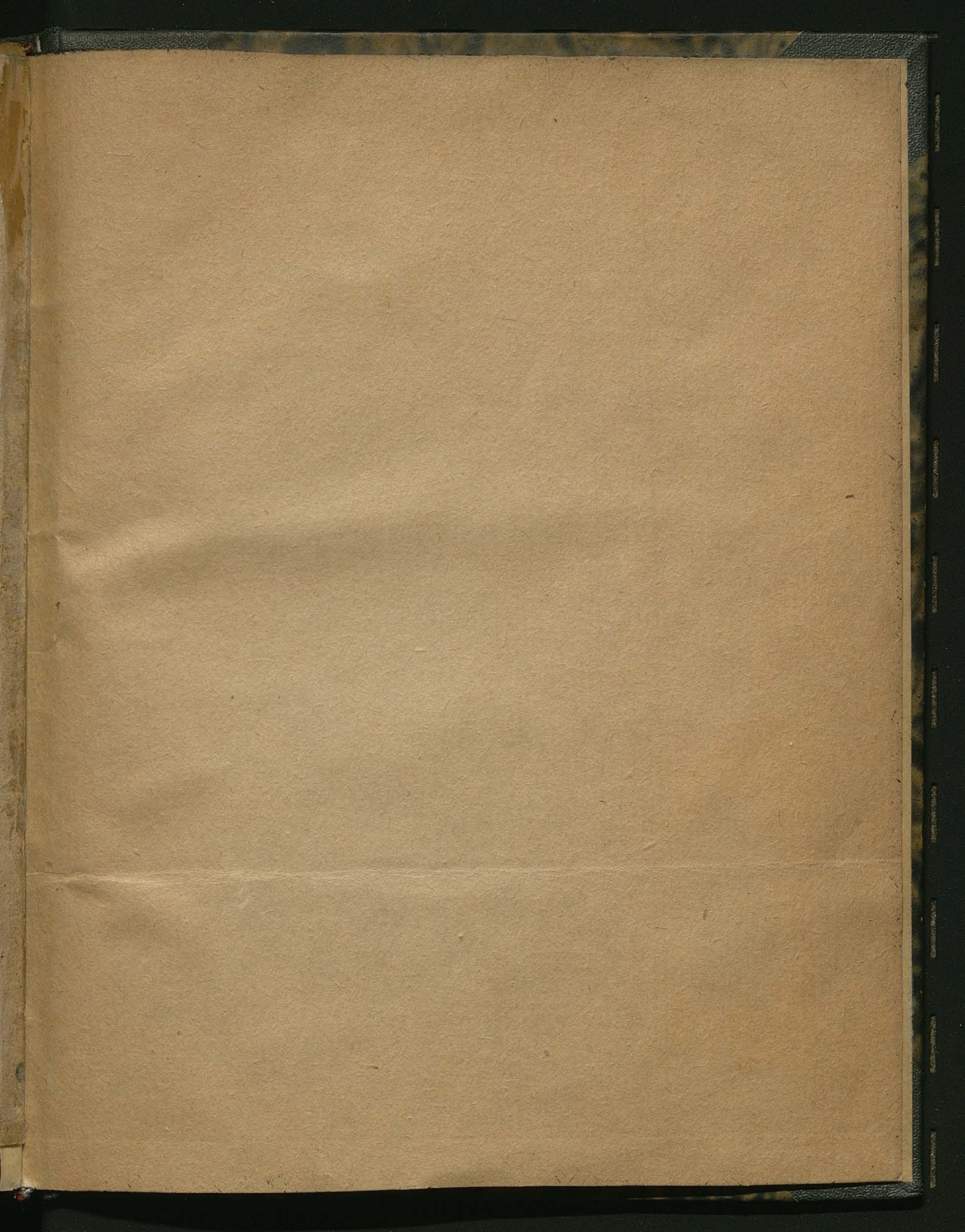
*Demonstratio.* Ratio *Ludolphina* cubi diametri ad spheram 300: 157 multiplicata per 48, producit æqualem 14400: 7536; assumtis deinde cubo diam: = 1, & ratione defectiva 48: 24, producit spheræ  $\frac{7536}{14400}$  &  $\frac{24}{48} = \frac{361128}{591200}$  &  $\frac{345600}{591200}$ , quarum differentia prodit summam excessus & defectus  $\frac{15128}{591200}$ , cujus numerator est =  $36d + a = 1728 + 14400$ : ergo excessus est  $\frac{36}{14400}$  & defectus  $\frac{1}{48}$  (§. 16.), consequenter spheræ vera  $\frac{7500}{14400} = \frac{25}{48}$ . Ratio *Metiana* 678: 355 multiplicata per 48, prodit = 32544: 17040, quæ cum defectiva producit spheræ  $\frac{17040}{32544}$  &  $\frac{24}{48} = \frac{817920}{1562112}$  &  $\frac{761056}{1562112}$ , quarum differentia manifestat summam excessus & defectus =  $\frac{136864}{1562112}$ , cujus numerator est =  $90d + a = 4320 + 32544$ : unde excessus est  $\frac{90}{32544}$ : consequenter spheræ vera  $\frac{16950}{32544}$ , & reducta per 678 ad terminos minimos =  $\frac{25}{48}$ . Ratio *Archimedea* 21: 11 multiplicata per 48 dat = 1008: 528, quæ cum defectiva prodit spheræ  $\frac{528}{1008}$  &  $\frac{24}{48} = \frac{25344}{48384}$  &  $\frac{24192}{48384}$ , quarum differentia sistit summam excessus & defectus =  $\frac{1152}{48384}$ , cujus numerator est =  $3d + a = 144 + 1008$ . Ergo excessus est  $\frac{3}{1008}$ : consequenter spheræ vera  $\frac{1005}{1008}$  & reducta per 21 ad terminos minimos =  $\frac{25}{48}$ .

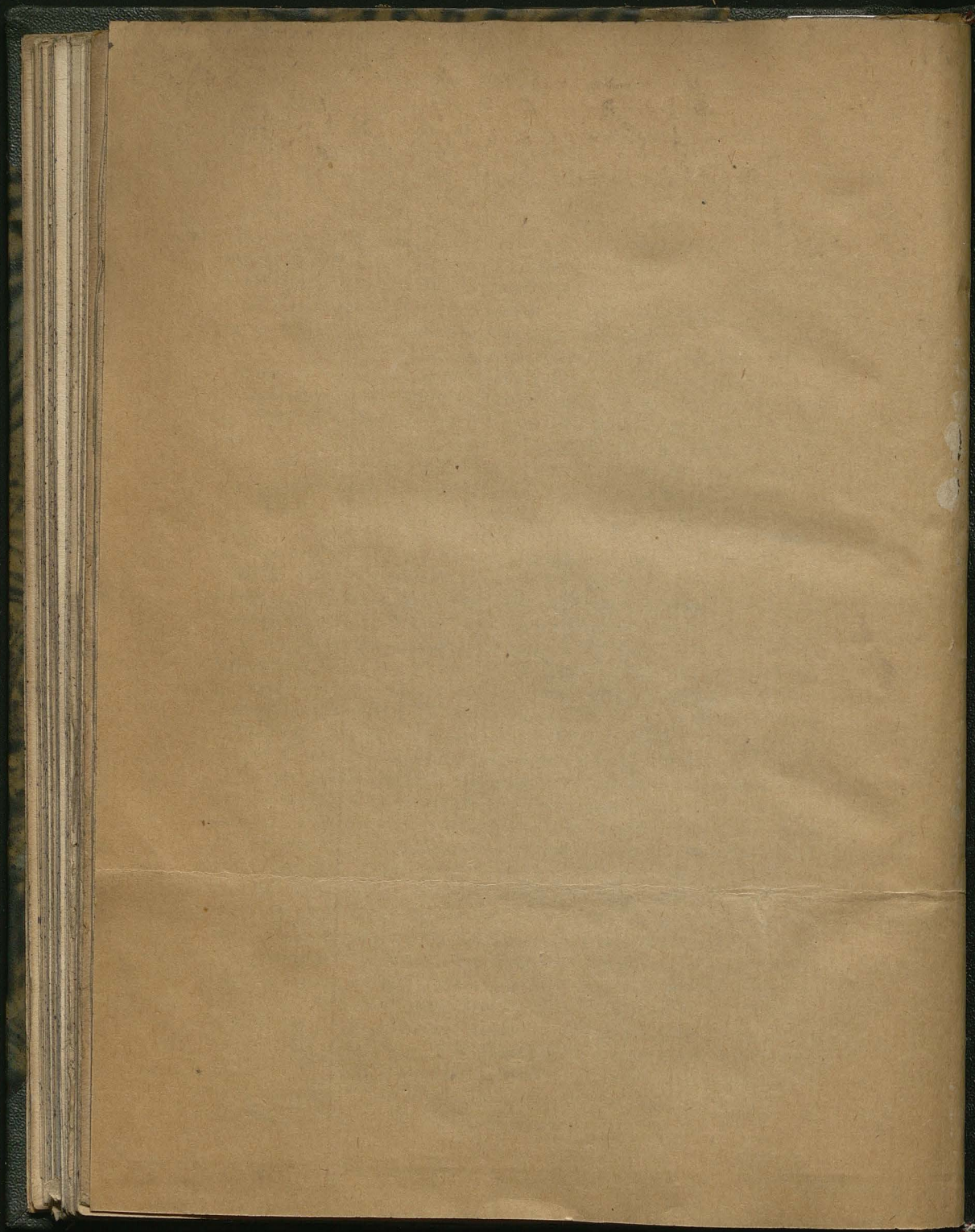
§. 29. *Scholion.* Numerator summæ excessus & defectus spherarum, quarum cubus diam. = 1, inventæ per rationes 48: 25 & 48: 24, est =  $2d + 3a$ ; repertæ per rationes 48: 26 & 48: 24, est =  $d + 3a$ ; & investigatæ per rationes 48: 25 & 48: 24, est =  $5d + a$ : unde per (§. 16.) illico determinatur excessus & defectus, & ope utriusque spheræ vera, quæ ubique prodit =  $\frac{25}{48}$ .

§. 30. *Corollarium.* Est ergo spheræ ad cubum diametri, ut  $\frac{25}{48}$ : 1 = 25: 48: consequenter diameter ad peripheriam, ut 8: 25. (§. 8.).









Biblioteka Jagiellońska



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