9400

IV

Bibl. Jag.

82/53 I A 2 1

Praca laboratoryjna
w Glasgowie 1896/7

(a może Cambridge
1905/6 ?)

Die Gesetze des osmotischen Druckes lassen sich daher in kolloiden Lösungen ebensowohl nach zwei Methoden erforschen, welche beide statistischer Natur sind: mittels Beobachtung der Größe der Konzentrationsschwankungen oder der Sedimentationsverteilung im Schwerefeld. Erstere bietet jedoch den erheblichen Vorteil, daß sie auf jede kolloide Lösung mit sichtbaren Teilchen anwendbar ist, während letztere nur für gleichkörnige Hydrosole gute Resultate geben kann.

Anfangs schien es, als ob die Erfahrung dem widersprechen würde, indem Svedberg u. Inonye sowie Westgren mittels der Schwankungsmethode an einer ganzen Menge verschiedener kolloider Lösungen sehr bedeutende Abweichungen vom Boyleschen Gesetz konstatierten, während Westgren das exponentielle Gesetz bei der Sedimentation ganz gut bestätigt fand. So betrug z. B. in einem Goldsol (Radius $\varrho = 91\ \mu\mu$) bereits bei einer Volum-Konzentration von nur 5,20 der Wert $\frac{\beta}{\beta_0} = 0{,}677$; in einer Gummiguttlösung ($\varrho = 200\ \mu\mu$) war $\frac{\beta}{\beta_0} = 0{,}405$ für eine Volum-Konzentration $3{,}8 \cdot 10^{-4}$.

Andererseits hat aber Constatin in Perrins Laboratorium bei Gummiguttlösungen ($\varrho = 30\ \mu\mu$) nach beiden Methoden bis zu weit größeren Konzentrationen vollständige Übereinstimmung mit der idealen Kompressibilität konstatiert und hat erst bei Volum-Konzentrationen von über $^1/_{100}$ eine merkliche Verminderung des β erhalten, welche die Existenz einer Abstoßungssphäre um die Gummigutteilchen erweist, und zwar müßte letztere eine solche Wirkung ausüben, als ob der Teilchenradius 1,7 mal größer wäre als in Wirklichkeit.

Durch die von Westgren unlängst hier in Göttingen ausgeführte Arbeit ist dieser Widerspruch endgültig aufgeklärt worden, in dem der-

26/3 fresh surface oil lamp

5h 31.56

~~355~~

5h 37.55

zero 351) dark
353

(48)

401

♂
353 ← 42 15 (47)

400

♂
353 ← 44 5 (48)

401

5h 48.40 light on ♂ 1 min.

zero 355

49.40 ×

50.40 light stopping (45.5)

400.5

5h 52.25 ♂ light on 1 min.

535.5

53.25 × (45.0)

54.25 light off

400.5

5h 55.45 ♂ light on 5 min.

6h — 45 × 535.5 } 45.5
401.0

6h 2.55 ♂ light on } 15 min.

6h 17.55 354

18.55 light stopped } 46.3

400.3

6h 32.45 (×) 354 light on
33. 399.7 stop 45.7

6h 36.5 × 353.5 light on
399 45.5

fresh surface

6h 47.40 × 352.5 light on
398 45.5

6h 53.25 ×	352.0	42
	394	43.5
	437.5	42.5
	480	42
	522	42
	564	42
	606	41
	647	40.5
	687.5	40.5
	728	40
	768	39
	807	39
	846	38
	884	39
	923	38
	961.	

Light still on

7h 11.30 ~~zero~~ 354'5 ×

12 30 Light stopped

399'5
45̶0

7h 21.15 354.— glow lamp

22.15 459. stopped

---

glow lamp shining the whole night

27/3 zero 380 12h 8.—

379 9.—

12h 10.15 → × 380. oil lamp

11.15 440 ?

when insulated going on by itself. (× and charged up)

12h 15.15 × 385 dark

393 8 } insulation ?

401 8

7'5

18.15 408'5

18.30 (410'5) Light on

19.30 460 50

510 50

561 51

611 50

661 50

711. 50

708 47

806 48

852 46

899 47

945 46 —— light put out

→ 948

947'5 no insulation

947'5 loss

12h 36.10

389'5 × dark

396.0 6'5

402'5 6'5

408'0 5'5

460.
409'5
(50'5)

12h 39.25 409'5 Light on

40.25 " off

41.25 464.0 } 4.0

42.25 468.0 } 3.5

471'5

~~Light on~~

12h 45.40 389.— × dark

394.— 5.—

398 4.—

45.55 Light on

46.55 " off

47.55 451.0 } 4.

447.—
399
(48.)

455.0 } 3'5

458'5

12h 54.— 386.— × dark

390.— 4.—

394.— 4.—

55.15 Light on

443
395
(48)

446.—

448.5

451'5

27/3 Hg connected to 5

glow light on the whole time from 2h
zero slowly creeping up

1h 4.30 x 382 light on

382'5

~~fresh surface~~

1h 10.30 x 382 dark

386

390'5

12 45 light on

13 45 off

440'8

444

447

437
391'5
(45'5)

~~Hg cooled by water~~

1h 44.40 x 372 dark

374

380

47.40 383

1h 47.55 → light on

48.56 off

49.56 430

435

435

1h 54.30 371'5

375

378

56.45 light on

57.45 off

426

427'5

428'5

4h 40 50 zero x 366 light on

41 " 417'3 off

42 " 420.

414'5
366
(48'5)

4h 46.5 zero x 372

47.5
48.5 422.3 light off

49.5 424.3

420.3
372.0
(48.3)

fresh surface

4h 52.50 372'7

53.50

54.50 419.2

55.50 421.–

422'8

417.5
372'7
(44'8)

fresh surface ~~hea~~ (warmed up by hand)

5h 58 20 363 } light on
59.20
60 20 411'5

415.–

408'5
363
(45'5)

6h 19.20 [2?3 1]
366 dark | 40 Volts
366+
2 min. — light
441
442

440
366
74 : 2 = 37

6h 26.30 | 365 } 1 min. light
27.30
409
410.5

407'5
365
42'5

| 6'4 Volts on

6h 33 364 } 1 min light 28
392
392

6h 39.45 365 | 4'2 Volts on
389'5 24'5
389'5

6h 44.40 365 | 2'1 Volts on
377 12'0

6h 47.20 365
377 12'0

~~5h 38 365~~

4'2 Volts on

6h 53.30 366
390'5 (24'5)

~~6h 57 46 365~~ 6'4 Volts on

7h 0.30 366 } 1 min light
396.
(30.)

7h 4.40 365 40 Volts
414.–
417.

411
365
46

---

120 Volts

7h 13 .20 x 292
289'5
288'5

Needle charge 120 Volts

7h 36.20 590 6'4 Volts on
639
642
645

636
590
46
3
(43)

7h 42 15 x 592 (4'2 Volts on)
594'5
598
20 637'5

634'5
598'5
(36)

28/3

120 V.

7h 53 45 x 595 dark
601
606
612
618
light on — 620
55
— 675
685 — 55
10 — 10
695 (45)

28/3 1h 6 40 zero x 360'5 dark
360.0
360'2
360'3
360'2
360'2
360'1 without P.D.
1h 9 40
1h 10.20 glow lamp on
—
360'7
361'7
362'3
363'1
363'3 off
363'4
1h 17.20 363'5 light on ↕
4h 100 365'3
zero 350.

4.24 50 raised so that part of glass surface free
357
416
fresh surface
366
411 45
4.36.50 362'5 52
414'5
fresh surface
~~4.41~~ — 365
410.5 45'5
4.56.7 364 stone on
518.2
54.2
~~fresh surface~~
360
409
459
508
557

28/3

358
394
391
395
1/2 398

454

355
410) 55
409

356) 52'5
408'5

2 min light

359
411'5 52'5

29/3 glow lamp on solid crystals of Na Amalgam (?)

5	354	42'5
	396'5	39'5
	436	39
light shining	475	36
the whole time	511	

347 38
385

350	47
397	39'5
436'5	
475	38'5
511	36

~~24/3 11^h 19.6 surface not yet used, had been standing quiet the whole night~~

~~300~~

no needle change, ~~quietness~~ steadiness of needle?

11^h 26	326
43	319
46	315

11

5h 53.50 || 343   45.5
388.5
390.5
393
394
395.3
396.7   light put out
397.5   0.8
398   0.5
398.7   0.7

---

zero   355.5   5h 45 10
357
358.2
356.5   6h 3
363   9 — light on
364.5   10
366.4   — black paper cover
363
363.5
364   cover off
365.5
365
366.3
367.5   cover on
367
367.8
368.3   zero 352

with [illegible]   6h 38 15
351
389   37
—   77
465   37
502   38
540
    108 : 3 = 36
648
    69 : 2 = 34.5
717

new plate

---

zero 351   7h 1.20
382
31

Amalgam frozen by liquid air
402   88
490
—
48   398   54
452 ?   (bad contact)

---

45   398   100
488
586   98
677   91

22/3 oil lamp 6h 1 p.m. (burning the whole time)

299	64
363	57
420	51
471	51
522	48
570	49
619	47
666	46
712	

zero 300

Argand (oil shielded)

285	71
356	81
437	80
517	

Argand better position (screen [illegible])

278	78
356	86
442	82
524	81
605	81
686	83
769	81
850	85
935	

oil lamp 7h 41

290	49
339	52
391	51
442	50
492	50
542	46?
588	47
635	46
681	46
727	46
773	46
819	45
864	43
907	45
952	

662 : 14 = 47·3

glow lamp 7h 2 p.m.

256	38
294	51
345	50
395	43
438	45
483	40
523	43
566	
733	39
772	
892	41
933	
972	

167 : 4 = 42

120 : 3 = 40

972
256
716 : 17 = 42·1

240
660
1000

420
47m

6

23/3 11h 10m a.m. | surface shone on from 7h 2m p.m. (8 hours) by strong glow lamp | fresh surface 11h 56 a.m.

glow lamp → the same glow lamp

zero x	325	
	369	44
	416	47
	465	49
	519	54
	572	53
	622	50
	672	50
	724	52
	770	46
	821	51
	869	48
	920	51
	969	49
zero	329	

969
327
642 : 13 = 49.4
122
5

zero	332	
	374	42
	423	49
	469	46
	517	48
	564	47
	611	47
	659	48
	707	48
		242 : 5 = 48.4
	949	
	995	46
zero	334	

995
333
662 : 14 = 47.3
102
4

11h 35 zero — oil lamp (Argand shielded)

zero	328	
	372	44
	426	54
	482	56
	535	53
	585	50
	635	50
	685	50
	—	103
	788	52
	840	52
	893	53
	944	51
	994	50
zero	330	

994
329
665 : 12 = 55.4

→ oil lamp

zero	334	38
	372	43
	415	45
	460	42
	502	40
	542	
	625	
	748	
	828	
	944	
	981	
12h 32 p.m. zero	332	

208 : 5 = 41.6

83 : 2 = 41.5

123 : 3 = 41

40

116 : 3 = 38.7

37

981
333 :
648 : 16 = 40.5

7

~~glow lamp [illegible]~~ fresh surface

oil lamp (alone)

1^h 44.30

331
365 34
402 37
443 41
55 742 411:11 = 37'4
775 33
810 35
881 71:2 = 35'5
70:2 = 35
951
983

fresh surface

1^h 6.50 321 41
360 553:12 = 46'1
19 913 43
956 43
999
21.50 321
678 : 15 = 45'2

~~light [illegible] the whole time~~ (glow lamp near)

4^h 36.10 289 65 glow lamp (distant)
354 59
413
– 173 : 13
41 580

4^h 54.– glow lamp

325 58
383 53
436 53
489 } 424 : 8 = 53
5^h5 913
966 53

zero 325

oil lamp

5^h 13 31 325
371 46
419 48
25 865 446 : 10 = 44'6
907 42
948 41
991 43

zero 328 fresh surface

(5^h 34,30) 317 644 : 15'5 = 41'5 oil [illegible]
50 – 961 24
51.30

fresh surface glow lamp

17
15
155 5^h 58.– 322 32
354 301 : 6 = 50'0
655 42
697 128 : 3 = 42'7
9 – 825 53
878 49
927 48
30
05 12 975
25 : 13 = 52 321 zero

975
322
653 : 14 = 46'6
93
9

503 : 13 =
952
295
657 : 13 =
5^h 30 oil lamp
443 : 8 = 55'4
131 : 3 = 43'7
41'5
295 38 41 43
738
869
912
zero 332!
850
307
543
5^h 22
for 7 min!

12h 35
23/3 glow lamp

zero 330

371 41

474 103:2 = 51'5

522 48

569 47

620 51

718 98:2 = 49

763 45

808 45

899 91:2 = 45'5

984 85:2 = 42'5

984
325
659 : 14 = 47'1

put out 993 } insulation loss
992 per min.

zero 320

zero 320 12h 54 50 oil lamp as before

366 46

493 127 : 3 = 42'3

533 40

615 82 : 2 = 41

695 40

735 40

774 39

891 117 : 3 = 39

968 77 : 2 = 38'5

zero 310

968
315
653 : 16 = 40'8

Argand & oil lamp shielded

1h 15.0 zero 310
23.0 344

34 : 8 = 4'2

349 oil lamp on (Argand shield)

395 46

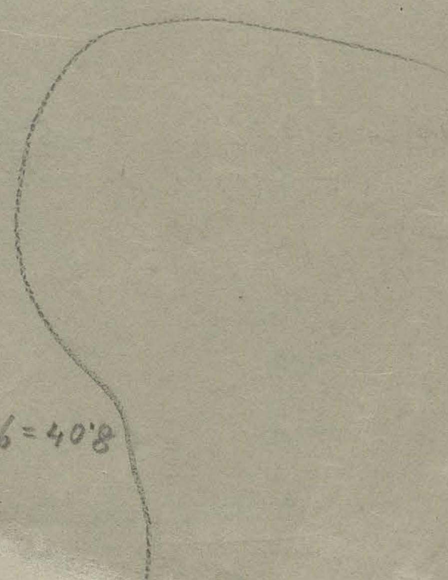
438 43

485 47

572 87 : 2 = 43'5

616 44

702 86 : 2 = 43



Durch die von Westgren unlängst hier in Göttingen ausgeführte Arbeit ist dieser Widerspruch endgültig aufgeklärt worden, in dem derselbe nachwies, daß die nach Svedbergscher Methode mit Hilfe des Spaltultramikroskops ausgeführten Teilchenzählungen bei hell leuchtenden Teilchen eine subjektive Fehlerquelle enthalten[1]), welche bei größerer Konzentration sehr stark ins Gewicht fällt und jene Abweichungen vorgetäuscht hatte, während die Zählungen, welche Westgren an einem zwischen Deckgläsern eingeschlossenen Präparat ausführte, noch für eine Goldsuspension ($\phi = 110\ \mu\mu$) von einer Konz. $= 2{,}3 \cdot 10^{-4}$ vollkommen normale Kompressibilität ergaben. Nach Analogie mit Costantins Resultaten zu schließen, wären Anomalien tatsächlich erst bei etwa 50 mal größeren Konzentrationen zu erwarten, welche sich in reinen Goldhydrosolen überhaupt kaum herstellen lassen dürften.

Das bisher Besprochene bezog sich auf die Abweichungen vom Boyleschen Gesetz, welche natürlicherweise vor allem die damit zusammenhängenden Erscheinungen der Schwankungsgröße

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1) Undeutliche Definition des beleuchteten Volums infolge seitlicher Zerstreuung des Lichtes. Ob dies auch für die von Lorenz u. Eitel an Tabakrauch gefundenen Abweichungen gilt, ist wohl erst durch weitere Untersuchungen zu entscheiden. Literaturnachweise, siehe S. [redacted] sowie die Zusammenstellung bei Th. Svedberg, Jahrb. d. Rad. u. Elektr. 10, 467, 1913.

9

102/13 IV 76

Notatki do jakiegoś przemówienia w Anglji

(około 1912)

?

Insbesondere hat Paine nachgewiesen, daß die Koagulationszeit, in Übereinstimmung mit unseren Formeln, umgekehrt proportional der Anfangskonzentration des Kolloids ist und daß sie umgekehrt proportional der 5. oder 6. Potenz der Elektrolytkonzentration variierte, was wir einer entsprechenden Änderung des Wirksamkeitskoeffizienten ε zuzuschreiben haben.

Letzteres kann aber natürlich kein allgemeines Gesetz sein, da bei stärkeren Zusätzen, in dem oben besprochenen Bereich der „raschen" Koagulation, die Koagulationsgeschwindigkeit von der Elektrolytkonzentration unabhängig wird. Sehr instruktiv sind in dieser Beziehung einige Zahlen, welche mir R. Zsigmondy gütigst mitgeteilt hat, wonach zur Erreichung eines bestimmten, an dem Farbumschlag Rot- Rotviolett kenntlichen Koagulationsgrades einer Goldlösung bei verschiedenen Elektrolytkonzentrationen c (Millimol $NaCl$ pro Liter) die nachstehenden Zeiten T (Sekunden) erforderlich waren:

c	5	10	20	50	100	150	200	300	500
T	> 150	12	7,2	7	7	6	6,5	7,5	7[1]

Ein dem Anfang dieser Messungsreihe angepaßtes Potenzgesetz müßte natürlich bei höheren Konzentrationen vollständig versagen.

Auch Freundlichs und Ishizakas Messungen bestätigten, wie gesagt, das Ähnlichkeitsprinzip, und zwar in bezug auf die Abhängigkeit vom Elektrolytzusatz, sonst sind sie aber zu einer quantitativen Kontrolle unserer Formeln nicht geeignet, da sie sich (wie auch Paines Messungen) nicht auf die Teilchenzahlen selbst, sondern auf andere Größen bezogen, wie Zähigkeit oder in anderen Untersuchungen gewisse Adsorptionserscheinungen, welche komplizierte und einstweilen unbekannte Funktionen der Zahl und Größe der Teilchen (bzw. deren Aggregate) sind.

Macht man betreffs der Abhängigkeit der

1) Allerdings könnte auch R von der Elektrolytkonzentration abhängen. Formell ist das aber mit einer Änderung des ε gleichwertig.

Somit könnte man annehmen, daß in Paines Versuchen alle Teilchen abgeschieden wurden, welche aus mehr als k-Primärteilchen bestanden (wo k eine große Zahl ist), während der Rest, bestehend aus

$$L = \nu_1 + 2\nu_2 + 3\nu_3 \ldots (k-1)\nu_{k-1}$$

Primärteilchen, in Lösung blieb. Werden hierin unsere Formeln (70) eingesetzt, so ergibt sich für die nicht koagulierte Menge, bei Benutzung der Abkürzung $\varepsilon\beta t = \alpha$, der Ausdruck:

$$L = 1 + \frac{(k+\alpha)^{k-1}}{(1+\alpha)^k},$$

welcher für große k gleichwertig ist mit:

$$L = 1 + \left[1 + \frac{1}{x}\right] e^{-\frac{1}{x}} \qquad (74)$$

wo x zur Abkürzung für die zur Zeit proportionale Größe

$$x = \frac{\alpha}{\nu} = \frac{\varepsilon\beta t}{\nu}$$

eingeführt ist.

Zeichnen wir uns diese Koagulationskurve auf, so überzeugen wir uns, daß sie tatsächlich ganz überraschend ähnlich mit den von Paine erhaltenen empirischen Kurven verläuft. Insbesondere muß eine große Zeit, die „Inkubationszeit", verstreichen, bevor sich die Teilchen soweit vergrößern, daß überhaupt ein merklicher Niederschlag erhalten wird, dann tritt bei dem Werte $x = \frac{1}{3}$, $t = 0{,}801$ ein Wendepunkt auf und von da an verläuft die Kurve konvex nach abwärts, um sich asymptotisch der Zeitachse anzuschmiegen. Der Unterschied besteht nur darin, daß der Übergang von der Inkubationszeit in die Koagulationskurve hier etwas allmählicher erfolgt als in Paines Kurven und die scharfe unnatür-

~~hypotheses & mental experiments~~

10

truthfulness
passion for accuracy, passion for facts
fanaticism of exactly
enthusiasm for truth

cautiousness of statement
suspended judgment
clearness of vision
sense of inter relatedness

biography Maxwell
Faraday etc.!

Cognition p. 50
law of nature = descriptive formula

there may be psychological & social generalisation which really tells us why that ...

but chemical and physical — only how

Huxley : science = game
rules of game

drudgery { Joule mech. equivalent
Andrews critical
Rayleigh Argon

|| potential energy

Kuhn p. 75 Natural history — Natural philosophy stage

und einstweilen unbekannte Funktionen der Zahl und Größe der Teilchen (bzw. deren Aggregate) sind.

Macht man betreffs der Abhängigkeit der Viskosität von der Teilchenzahl und -größe gewisse, ziemlich plausible Annahmen, so kann man die charakteristische, durch einen Wendepunkt gekennzeichnete Gestalt der Koagulationskurven Freundlichs ohne weiteres erklären, doch kommen da zu viel hypothetische Elemente ins Spiel, als daß man von einer zahlenmäßigen Kontrolle reden könnte und deshalb will ich auf diese Rechnungen hier nicht weiter eingehen.

Auch scheint es mir nicht rationell, aus derartigen Messungen die Differentialgleichung der Koagulationskinetik ableiten zu wollen, wie es Freundlich versucht, solange man nicht weiß, wie die Teilchenzahlen mit dem beobachteten Effekt zusammenhängen. Für erstere, nicht aber für den gemessenen Gesamteffekt, ist eine einfache Gesetzmäßigkeit zu erwarten. Dagegen sind derartige Messungen wohl geeignet, auf Grund des Ähnlichkeitsprinzips hochinteressante Aufschlüsse über die Abhängigkeit

1) Die Abweichungen von 7 Sek. bei 50—500 Millimol liegen innerhalb der Beobachtungsfehler.

der Übergang von der Inkubationszeit in die Koagulationskurve hier etwas allmählicher erfolgt als in Paines Kurven und die scharfe unnatürliche Ecke derselben vermeidet.

Wenn man diese wenigen bisher zu Gebote stehenden Kontrollversuche überblickt, gewinnt man wohl den Eindruck, daß die in Rede stehende Verallgemeinerung unserer, den Koagulationsmechanismus auf Brownsche Bewegungen zurückführenden Theorie, dem Wesen der Sache entspricht, und man kann wohl hoffen, daß dieselbe sich als Wegweiser bei weitergehenden Untersuchungen auf diesem bisher der Mathematik ganz unzugänglichen Gebiete nützlich erweisen dürfte.

(Eingegangen 1. September 1916.)

[I Ziele u. Wege d. math. Unterr.]

Im Sinne von Klein, gewinnt durch kommentiert 12

§1 Inhaltliche Grundl. Bildg. d. d. Math.

Meraner Vorschläge Stärkg. d. räuml. Anschauung §§ [illegible] funktionalen Denkens

1884 eher Inhalt als Form

§2 Funktionales Denken u. Anschauung

§3 Unterbau im math. U. : Fertigkeit im Rechnen u. Zeichnen

„selbst auf Kosten d. Gründlichkeit", ; scheinbare Ungründlichkeit = Voranstellen d. didaktischen Rücksichten vor verfrühten Wissenschaftlichkeit

§4 Reine u. angewandte Math.

Das Konkrete vor d. Abstrakten

Zuerst etwas sagen, dann darüber reden

§5 Pädagogische didakt. Darbietung: Sagen Sie einmal absolut langweilig (Herbart)

§6 Grundideen d. Lehrplangestaltung: Anpassung an die psychologischen Gegebenheiten d. Schülerseelen

Ungeplättete Gliederung, didaktische Einheiten

II Lehrproben, Lehrgänge, Lehrpläne

gegliedert nach Lebensalter

Einheitslehrplan für Gymn., Realsch. aller Länder, denn [illegible] Entschl.

Vorallem erste Einführg. ausführlich besprochen, höhere Weiterführg. [illegible]

§7 Zur Charakteristik d. Unterstufe

für 10–13 jährige Kinder gibt es noch keine math. Wissenschaft entsprechend die deduktive Math. [illegible]

wer ohne [illegible] logische Formulierung beibringen will, bringt nur leere Wörter bei. Keine Gedanken

Rechnen lernen! Raumlehre, Anschauen

§8 Zur Vorstufe d. Zahlenlehre sollte an die Volksschule anknüpfen

wo dort: Grundrechnen kleinerer, dann grösserer Zahlenkreis einzelne Zahlen als Individuen

arithm. Lehrtext wie er nicht sein soll

vernichtende Kritik: logische Fehler, Definitionssucht

in Wirklichkeit sollten die Kinder lernen die Wörter (Terminologie) zu gebrauchen, aber ohne dass Definitionen gegeben werden.

Lehrtextprobe, danke aus wirklichen Zahlen!

§9 D. mechanische Rechnen in d. untersten Klasse

fester Mechanismus [illegible] Rechengesetz, deutlich sprechen etc.

wie Griffe beim Exerzieren

§10 Dezimalbruch, gemeine Brüche Dezimalbrüche Vorkurs Tafeln p. 79 räumlichen Anschauung

§11 Weiteres.. gegen Proportionen

aber wohl Gedanke d. Proportionalität Textproben aus Lehrbüchern p. 86

§12 Zur Vorstufe d. Raumlehre

geom. Lehrtext wie er nicht sein soll ausführliche Kritik

§13 Skizzen zu einem Vorkurs ..

[illegible] ; [illegible] ; Zirkel Lineal, [illegible] Winkelsumme d Δ etc. manuelle Gebrauch

§14 [illegible] Jahr mehr Wert geom. [illegible], Symmetrie, Kongruenz

Übungen im Gelände, Konstruktion manuelle Betätigung

§15 Buchstabenrechnen Einführung mittels Kunststückes

Abgekürztes Rechnen, Irrationalzahlen Pythagoreischer Lehrsatz
Mittelstufe 14, 15 J.

§16 Euclid nur einige Beispiele zur Illustration d. Methode etc.

Gegen sog. wissensch. Lehre von d. Rechnungsoperationen

Dagegen viel Gleichungen u. zwar „Textgleichungen"

§17

§18 Erwecken des Verständnisses für logische Strenge aber mittels Modelle.

19 Nicht mit Sätzen über Lage der Geraden u. Ebenen u. s. w. beginnen sondern Vorkenntnisse d. Stereometrie

§20 Darstellende Geometrie! (Zeichnen von) Krystallformen Perspective
Elemente

§21 Vol. von krummflächigen Körpern, Cavalieri'sches Princip ([illegible])

Oberstufe 16–18 J.

§22 Funktionsbegriff

Grenzbegriff, complexe Zahlen etc. = höhere Analysis

§23 Lehrgebäude über Potenzen } Verbindung mit graph. Darst. Funktionsbegriff
§24 Logarithmen

Lehrgebäude d. Trigonometrie

§30 Für Behandlung d. diophantischen Gleichungen?

|| Prinzip d. Bevorzugung d. angewandten gegenüber d. reinen Math.

Weg mit Formalismus!

Excurse

Digressionen

102/53 IV 17

prawdopodobnie notatki
do współredakcji
dydaktycznego dzieła
prof. Alois. Höflera z
Wiednia — o naucza-
niu fizyki w szk. średn.

82/53

I A 14

14

Polaryzacja światła
w kryształach

gerufen wird. Aber die Erscheinungen des reversibeln Koagulationsgleichgewichts, welche z. B. Sven Odén an Schwefelhydrosolen studiert hat, gehen über den Geltungsbereich dieser Theorie vorläufig noch hinaus. Sie bildet also selbstverständlich nicht eine allseitige Aufklärung des ganzen Problems, sondern nur einen ersten Schritt auf diesem seitens theoretischer Begründung noch vollständig unbekannten Gebiete.

|bau

Vor allem lassen sich schon daraus gewisse Schlüsse ziehen, daß die Koagulation in unserer Theorie auf die Brownsche Molekularbewegung und auf die Existenz einer Wirkungssphäre R zurückgeführt wird, denn als Variable, von welchen der Koagulationsverlauf abhängt, kommen somit nur folgende drei in Betracht: der Radius R, die Teilchenzahl pro Volumeinheit n_0 und die Diffusionskonstante D, deren Dimensionen gegeben werden durch das Schema:

$$D \sim \frac{l^2}{t};\ n_0 \sim \frac{1}{l^3};\ R \sim l.$$

1) Eine ausführlichere Darstellung wird in der Zeitschr. f. phys. Chem. veröffentlicht werden.

1. Das hervorgehobene Teilche[n] und für sich eine ähnliche Brow[nsche Be]wegung aus wie die übrigen, es ko[mmt] für die Koagulation die relative Bewe[gung in] Betracht. Diesbezüglich läßt sich nu[n] nachweisen, daß die Relativverschiebung sich unabhängig voneinander bewegen[der Teil]chen ebenso erfolgt, wie die gewöhnlich[e Brown]sche Bewegungsformel (1) angibt, nur [mit dem] Unterschied, daß der Diffusionsko[effizient] gleich der Summe der Koeffizienten [der] Teilchen zu setzen ist. Allgemein g[ilt für] die Relativbewegung:

$$D_{ab} = D_a + D_b.$$

2. Unsere Formeln (61) (62) en[tsprechen] dem Falle, wo die Zahl ν_0 der Tei[lchen in] größerer Entfernung unverändert ble[ibt. In] Wirklichkeit kleben sie aber nicht nur [an dem] hervorgehobenen, sondern auch unter[einander] an. Von der Anzahl $4\pi D R \nu_0$ sind a[lso die] wegzustreichen, welche schon vor der [Zeit] koaguliert sind, somit ist die Zahl ν_0 du[rch die] Zahl ν_1 der zur Zeit t noch existierenden [Einzel]teilchen zu ersetzen. Ebenso kommen als [Koagula]tionskerne, wenn es sich um die Verei[nigung] von einfachen zu Doppel-Teilchen handel[t, nicht] sämtliche ν_0 sondern nur die noch frei[en Teil]chen ν_1 in Betracht.

15

$$\delta = \frac{2\pi}{\tau}\left[\frac{h}{v_0 \cos\beta_0} + \frac{h(\operatorname{tg}\beta_e - \operatorname{tg}\beta_0)\sin\alpha}{c} - \frac{h}{v_e \cos\beta_e}\right]$$

$$= \frac{2\pi h}{\tau}\left[\frac{\sin\alpha}{c}(\operatorname{tg}\beta_e - \operatorname{tg}\beta_0) + \right.$$

$$= \frac{2\pi h}{\tau}\left[\frac{h}{v_0} - \frac{h}{v_e} + \frac{h(\beta_e - \beta_0)\alpha}{c}\right]$$

$$\left[\frac{1}{v_0}\left[1 + \frac{\beta_0^2}{2}\right] - \frac{1}{v_e}\left[1 + \frac{\beta_e^2}{2}\right] + \frac{(\beta_e - \beta_0)\alpha}{c}\right]$$

$$v_e = o\left[1 - \frac{\alpha^2}{2}\frac{e^2(o^2 - e^2)}{c^2 o^2}\right]$$

$$\beta_e = \frac{e^2}{oc}\alpha$$

$$\beta_0 = \frac{o}{c}\alpha$$

$$\frac{1}{o}\left[1 + \frac{o^2\alpha^2}{2c^2}\right] - \frac{1}{o}\left[1 + \frac{\alpha^2}{2}\frac{e^2(o^2 - e^2)}{c^2 o^2}\right]\left[1 + \frac{e^4}{o^2 c^2}\frac{\alpha^2}{2}\right] + \left[\frac{e^2}{oc} - \frac{o}{c}\right]\frac{\alpha^2}{c}$$

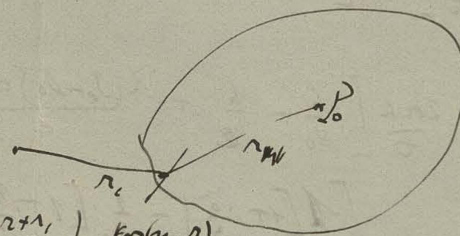
$$= \alpha^2\left\{\frac{o}{2c^2} - \frac{e^2(o^2 - e^2)}{2c^2 o^3} - \frac{e^4}{2o^3 c^2} + \frac{e^2}{oc^2} - \frac{o}{2c^2}\right\}$$

$$= -\left[\frac{e^2}{oc^2} + \frac{o}{c^2}\right]\frac{\alpha^2}{2}$$

$$4\pi u_0 + \int\left[\frac{1}{r}\frac{\partial u}{\partial n} - \cos(n r)\frac{\partial}{\partial n}\left(\frac{u}{r}\right)\right] dS = \int \frac{1}{r}\left(\nabla^2 u - \frac{\partial^2 u}{\partial t^2}\right) dv$$

$$u = \frac{A}{r_1}\cos 2\pi\left(\frac{t}{\tau} - \frac{r_1}{\lambda}\right)$$

$$\frac{\partial^2 u}{\partial t^2} \neq \frac{\partial^2 u}{\partial r^2} = \nabla^2 u$$



$$u_0 = \frac{1}{4\pi}\int \frac{1}{r}\frac{\partial u}{\partial n}\cos n r_1$$

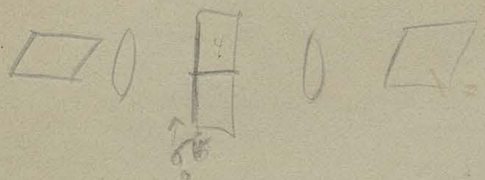
$$\frac{\partial u}{\partial n} = \frac{2\pi A}{\lambda r_1}\sin 2\pi\left(\frac{t}{\tau} - \frac{r+r_1}{\lambda}\right)\cdot\cos(n r)$$

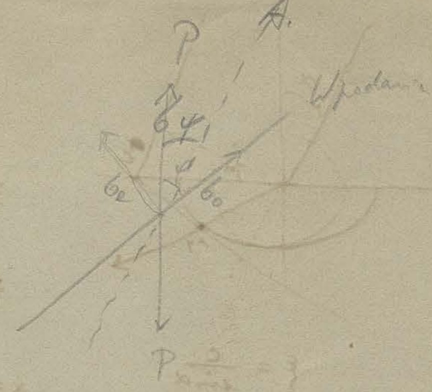
$$\frac{\partial}{\partial n}\left(\frac{u}{r}\right) = \frac{2\pi A}{\lambda r r_1}\sin 2\pi\left(\frac{t}{\tau} \ldots\right)$$

$$4\pi u_0 = \frac{2\pi A}{\lambda}\int \frac{1}{r r_1}\sin 2\pi\left(\frac{t}{\tau} - \frac{r+r_1}{\lambda}\right)\left[\cos n r + \cos(n r_1)\right] dS$$

$$\sigma''_x = \sigma'_e \sin\psi - \sigma'_o \cos\psi$$

$$= a \sin\psi \cos\psi \left[\sin\vartheta - \sin(\vartheta - \delta)\right]$$





$$\sigma_e = \sigma \sin\psi$$

$$\sigma_o = \sigma \cos\psi$$

$$\sigma'_o = a \cos\psi \sin \frac{2\pi}{\tau}(t - \varepsilon_o)$$

$$\sigma'_e = a \sin\psi \sin \frac{2\pi}{\tau}(t - \varepsilon_e)$$

$$1 + \cos\delta = 2\cos^2\frac{\delta}{2}$$

$$\sigma'_o \cos(\varphi - \psi) + \sigma'_e \sin(\varphi - \psi) =$$

$$= a \cos\psi \cos(\varphi-\psi) \sin\left(\frac{2\pi t}{\tau} - \varepsilon_o\right) + a \sin\psi \sin(\varphi - \psi) \sin\left(\frac{2\pi t}{\tau} - \varepsilon_e\right)$$

$$= a \cos\psi \cos(\varphi - \psi) \sin u + \sin\psi \sin(\varphi-\psi) \sin(u + \delta)$$

$$= a \sin u \left[\cos\psi \cos(\varphi-\psi) + \sin\psi \sin(\varphi - \psi) \cos\delta\right] + \cos u \left[\sin\psi \sin(\varphi-\psi) \sin\delta\right]$$

$$A^2 = \cos^2\psi \cos^2(\varphi-\psi) + \sin^2\psi \sin^2(\varphi - \psi) + 2\sin\psi\cos\psi \sin(\varphi-\psi)\cos(\varphi-\psi) \cdot \cos\delta$$

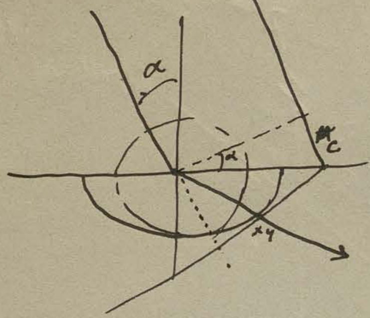
$$\tfrac{1}{2}\sin 2\psi \sin 2(\varphi - \psi) \cdot \cos\delta$$

$$\pm 2 \sin\psi \cos \dots \sin\psi + \psi \dots$$

$$\left[\sin\psi \sin(\varphi-\psi) + \cos\psi\cos(\varphi-\psi)\right]^2 - 2\sin\psi\cos\psi \dots \quad (1 - \cos\delta)$$

$$\cos^2\varphi - \sin 2\psi \sin 2(\varphi - \psi) \sin^2\frac{\delta}{2}$$

$$= \cos^2\varphi + \sin 2\psi \sin 2(\psi - \varphi) \sin^2\frac{\delta}{2}$$



$$\frac{x^2}{e^2} + \frac{y^2}{o^2} = 1$$

$$\frac{x\xi}{e^2} + \frac{y\eta}{o^2} = 1$$

$$\xi = \frac{c}{\sin\alpha}$$

$$\eta = 0$$

$$x = \frac{e^2 \sin\alpha}{c}$$

$$y = o\sqrt{1-\frac{x^2}{e^2}} = o\sqrt{1-\frac{e^2\sin^2\alpha}{c^2}}$$

$$\sin\beta = \frac{x}{\sqrt{x^2+y^2}} = \frac{\frac{e^2\sin\alpha}{c}}{\sqrt{\frac{e^4\sin^2\alpha}{c^2}+o^2-\frac{e^2o^2\sin^2\alpha}{c^2}}} = \frac{e^2\sin\alpha}{\sqrt{o^2c^2+e^2(e^2-o^2)\sin^2\alpha}}$$

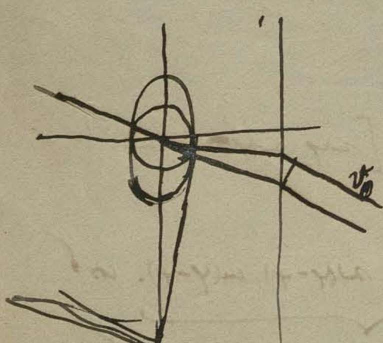
$$\sin^2\beta\left[o^2c^2+e^2(e^2-o^2)\sin^2\alpha\right] = e^4\sin^2\alpha$$

$$\sin^2\beta \cdot o^2c^2 = e^2\left[e^2\cos^2\beta + o^2\sin^2\beta\right]\sin^2\alpha$$

$$\frac{\sin\alpha}{\sin\beta} = \frac{oc}{e\sqrt{e^2\cos^2\beta+o^2\sin^2\beta}}$$

$$= \sqrt{o^2-\frac{o^2e^2}{c^2}\sin^2\alpha+\frac{e^4}{c^2}\sin^2\alpha}$$

$$= \sqrt{o^2-\frac{e^2(o^2-e^2)}{c^2}\sin^2\alpha}$$



$$\frac{\delta}{\cos\beta}$$

$$v_\beta = \sqrt{x^2+y^2} = \frac{x}{\sin\beta} = \frac{e^2\sin\alpha}{c\sin\beta}$$

$$= \frac{\delta \cdot c\sin\beta}{e^2\sin\alpha\cos\beta} = \frac{\delta\, e\, \sqrt{e^2\cos^2\beta+o^2\sin^2\beta}}{e^2\, o \quad \cos\beta}$$

$$= \frac{\delta}{o}\sqrt{e^2+o^2\operatorname{tg}^2\beta}$$

$$\frac{\delta}{o\cos\beta_0} + \frac{\delta(\operatorname{tg}\beta-\operatorname{tg}\beta_0)\sin\alpha}{c}$$

$X = Y = 0$ zatem ~~M~~ $N = 0$

$$\frac{\partial M}{\partial t} = \frac{\partial Z}{\partial x}$$

$$\frac{\partial L}{\partial t} = -\frac{\partial Z}{\partial y}$$

~~$$\frac{\partial X}{\partial x} + \frac{\partial Y}{\partial y} + \frac{\partial Z}{\partial z} = 0$$~~

$$\frac{\partial^2 Z}{\partial t^2} = a^2\left(\frac{\partial^2 Z}{\partial x^2} + \frac{\partial^2 Z}{\partial y^2}\right)$$

Z tylko zmienne w kierunkach $\perp z$

zatem kierunek największej zmienności określę w oś X: $\frac{\partial Z}{\partial y} = 0$

$$\frac{\partial^2 Z}{\partial t^2} = a^2 \frac{\partial^2 Z}{\partial x^2}$$

$N = L = 0$

$$\frac{\partial L}{\partial x} + \frac{\partial M}{\partial y} + \frac{\partial N}{\partial z} = 0$$

$$\frac{\partial^2 M}{\partial t^2} = a^2 \frac{\partial^2 M}{\partial x^2}$$

$$\frac{\partial X}{\partial t} = \frac{\partial M}{\partial z} - \frac{\partial N}{\partial y}$$

$$Z = A \sin \alpha\left(t - \frac{x}{a}\right)$$

$X, Y = 0$

$$M = B \sin \alpha\left(t - \frac{x}{a}\right)$$

z tego zaraz wypływa: $L = N = 0$

$$\frac{\partial M}{\partial t} = -\frac{\alpha A}{K\mu\, a} \cos \alpha\left(t - \frac{x}{a}\right)$$

$$M = \frac{A}{K\mu} \sin \alpha\left(t - \frac{x}{a}\right)$$

K elektromagnetycznie mierzone?:

$$K \frac{\partial X}{\partial t} \;--\; = \frac{\partial \beta}{\partial y}$$

$$\dim K = \frac{H}{E}$$

$$\dim \frac{K_m}{K_e} = \frac{H_m}{H_e} \cdot \frac{E_e}{E_m} = v^{-2}$$

$$\quad\;\; \overset{\shortparallel}{v^{-1}} \quad \overset{\shortparallel}{v^{-1}}$$

K_e dla eteru $= 1$

$(K_{em} = v^{-2}$

$$\frac{1}{\sqrt{\mu K}} = v$$

ogólnie: ~~$\sqrt{\mu K_e} = \sqrt{\mu_m K_m} \cdot \sqrt{\frac{K_e}{K_m}}$~~

~~$$c^2 = \frac{1}{\mu_m K_m} = \frac{1}{\mu_m K_e} \cdot \frac{K_e}{K_m}$$~~

$$= \frac{1}{\mu_m K_m} = \frac{1}{\mu_m K_{em}} \cdot \frac{K_{em}}{K_m}$$

$$\left(\frac{c}{v}\right)^2 = \frac{1}{n^2} = \frac{1}{K^2}$$

$$K \frac{\partial \mathfrak{E}}{\partial t} + 4\pi\lambda\, \mathfrak{E} = \operatorname{curl} \mathfrak{H} \qquad \frac{\partial}{\partial t} \Big| \quad \mu \frac{\partial \mathfrak{H}}{\partial t} = -\operatorname{curl} \mathfrak{E} \qquad \operatorname{curl}$$

$$\mu K \frac{\partial^2 \mathfrak{E}}{\partial t^2} + 4\pi\lambda\mu \frac{\partial \mathfrak{E}}{\partial t} = \nabla^2 \mathfrak{E}$$ Tak samo dla $\mathfrak{H}$

$$\frac{\partial^2 Z}{\partial t^2} + \frac{4\pi\lambda}{K} \frac{\partial Z}{\partial t} = a^2 \nabla^2 Z$$

$$Z = a e^{-\gamma x} \sin \alpha \left(t - \frac{x}{a}\right)$$

$$Y = X = 0$$

$\gamma =$

$a =$

Jeżeli λ duże w porówn z ...

to tak jak przewodnie wszystko

18

$$\sigma = r \sin\varphi + d\rho\delta \sin(\varphi+\varepsilon) + d\rho^3\delta \sin(\varphi+2\varepsilon) + \dots$$

$$= r\sin\varphi + d\rho\delta\Big\{ \sin\varphi\left[\cos\varepsilon + \rho^2\cos 2\varepsilon + \dots\right] + \cos\varphi\left[\sin\varepsilon + \rho^2 \sin 2\varepsilon + \dots\right]\Big\}$$

$$e^{i\varepsilon} + \rho^2 e^{2i\varepsilon} + \dots = \frac{e^{i\varepsilon}}{1-\rho^2 e^{i\varepsilon}} = \frac{\cos\varepsilon + i\sin\varepsilon \quad (\dots)}{(1-\rho^2\cos\varepsilon - \rho^2 i \sin\varepsilon)(1-\rho^2\cos\varepsilon + \rho^2 i\sin\varepsilon)}$$

$$= \frac{\cos\varepsilon\,(1-\rho^2\cos\varepsilon) - \rho^2\sin^2\varepsilon}{(1-\rho^2\cos\varepsilon)^2 + \rho^4\sin^2\varepsilon} + i\,\frac{\sin\varepsilon\,(1-\rho^2\cos\varepsilon) + \rho^2\sin\varepsilon\cos\varepsilon}{\dots}$$

$$= \frac{\cos\varepsilon - \rho^2}{1-2\rho^2\cos\varepsilon + \rho^4} + i\,\frac{\sin\varepsilon}{1-2\rho^2\cos\varepsilon+\rho^4}$$

$$\sigma = \sin\varphi\left[r + d\rho\delta\,\frac{\cos\varepsilon-\rho^2}{1-2\rho^2\cos\varepsilon+\rho^4}\right] + \cos\varphi\; d\rho\delta\,\frac{\sin\varepsilon}{1-2\rho^2\cos\varepsilon+\rho^4}$$

$$J = r^2 + 2d\rho\delta r\,\frac{\cos\varepsilon-\rho^2}{1-2\rho^2\cos\varepsilon+\rho^4} + (d\rho\delta)^2\,\frac{\dots}{\left[1-2\rho^2\cos\varepsilon+\rho^4\right]^2}$$

$d^2 = 1-r^2$:

$$\frac{1 - 2\rho^2\cos\varepsilon + \rho^4 \pm 2(1-r^2)\cos\varepsilon \mp 2r^2(1-r^2) + (1-r^2)^2}{\dots}$$

$\cos\varepsilon = 0$; $\cos\varepsilon = -1, +1$; $4r^4$; $4(1-r^2)^2$

$$= \frac{2 - 4r^2 + 4r^4 + 2\cos\varepsilon - 4r^2\cos\varepsilon}{} = 2\left\{\cos^2\frac{\varepsilon}{2} - 2r^2\cos^2\frac{\varepsilon}{2} + 2r^4\right\} = \frac{\cos^2\frac{\varepsilon}{2}(1-2r^2) + 2r^4}{}$$

wenn $d\rho = -$: $$\frac{2 \quad - 2\cos\varepsilon}{1-2\rho^2\cos\varepsilon+\rho^4} = 4\sin^2\frac{\varepsilon}{2}\, r^2$$

$d\rho\delta = +$: $$(1-r^2)^2 + 1 + \frac{2(1-r^2)\left[\cos\varepsilon - \rho^2\right]}{1-2\rho^2\cos\varepsilon+\rho^4} = 2 - 2r^2 + r^4$$

$$\alpha = \frac{1+\beta}{1-\beta}$$

$$\alpha - \alpha\beta = 1+\beta$$

$$\beta = \frac{\alpha-1}{1+\alpha} = -\frac{1-\alpha}{1+\alpha}$$

~~$= (\cos\Delta + i\sin\Delta)$~~ $=$ ~~$\sin^2\varphi +$~~

$$\frac{1-e^{i\Delta}}{1+e^{i\Delta}} = -\,i\,\frac{\cos\varphi\sqrt{\sin^2\varphi - n^2}}{\sin^2\varphi}$$

$$\frac{(1-e^{i\Delta})(1-e^{-i\Delta})}{-\;—} = \frac{1-e^{i\Delta}-e^{-i\Delta}+1}{-\;—}$$

$$= \frac{1-\cos\Delta}{1+\cos\Delta} = \left[\frac{\cos\varphi\sqrt{\sin^2\varphi-n^2}}{\sin^2\varphi}\right]^2$$

$$= \operatorname{tg}\frac{\Delta}{2} = \nearrow$$

$\varphi = \frac{\pi}{2}$, $\sin\varphi = n$ $\Big\}$ $\Delta = 0$

$$\frac{\partial}{\partial\varphi} = 0 = \frac{2n^2 - \sin^2\varphi(1+n^2)}{\sin^3\varphi\sqrt{\sin^2\varphi - n^2}}$$

$$\sin^2\varphi' = \frac{2n^2}{1+n^2}$$

$$\operatorname{tg}\frac{\Delta'}{2} = \frac{1-n^2}{2n}$$

nkl ? $n = \frac{1}{1{\cdot}51}$ $\varphi' = 51°20'$

$\Delta' = 45°76'$

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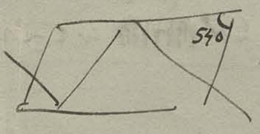
dla $\varphi = 48°37'$ $\Delta = 45°$

$54°37'$



Dla $n<1$

$$\cos\varphi = -i\sqrt{\frac{\sin^2\varphi}{n^2}-1}$$

Jakie znaczenie ampl. urojonej?

$$\sigma = A\sin\alpha\left(t-\frac{x}{v}\right) = A\,\frac{e^{i\varphi}+e^{-i\varphi}}{2i} = \mathcal{I}\left(A e^{i\varphi}\right)$$

to samo też możliwe rozwiązanie w razie gdyby A urojone

$$A = A_0 e^{i\delta} \qquad \sigma = A_0\,\mathcal{I}\left[e^{i(\varphi+\delta)}\right]$$

więc różnica fazy δ

$$= A_0 \sin(\varphi+\delta)$$

tak samo

$\sin\varphi$

$\sin(\varphi+\pi) = -\sin\varphi$

$\sin(\varphi+k\pi) = (-1)^k \sin\varphi$

$\sin(\varphi+\frac{\pi}{2})$ $\quad (-1)^{1/2}\sin\varphi$

~~$(\alpha+i\beta)\sin\varphi = \alpha\sin\varphi + \beta\cos\varphi$~~

$$A_0 e^{i\delta}\sin\varphi = (A\cos\delta + i A\sin\delta)\sin\varphi = a(\cos\delta\sin\varphi + \sin\delta\cos\varphi) = a\sin(\varphi+\delta)$$

$$B_s e^{i\delta_s} = \frac{\frac{i\cos\varphi}{\sqrt{\sin^2\varphi-n^2}}+1}{\frac{i\cos\varphi}{\sqrt{\sin^2\varphi-n^2}}-1}A_s$$

$$B_p e^{i\delta_p} = \frac{\frac{i\cos\varphi\cdot n}{\sqrt{\ldots}}+\frac{1}{n}}{\frac{i\cos\varphi\, n}{\sqrt{}}-\frac{1}{n}}A_p$$

} układając $A_s = A_p$

$(-\alpha+1)(\alpha+1) = 1-\alpha^2$

$(\alpha-1)(-\alpha-1) = 1-\alpha^2$

mnożąc przez $B_s e^{-i\delta_s}$ $= \dfrac{+\frac{\cos^2\varphi}{\sin^2\varphi-n^2}+1}{+\frac{\cos^2\varphi}{\sin^2\varphi-n^2}+1}B^2 = A_1$

$B_s = A_s$

$$e^{i(\delta_s-\delta_p)} = e^{i\Delta} = \frac{\left[\frac{i\cos\varphi}{\sqrt{}}+1\right]\left[\frac{+i\cos\varphi\, n}{\sqrt{}}+\frac{1}{n}\right]}{\left[\frac{i\cos\varphi}{\sqrt{}}-1\right]\left[\frac{i\cos\varphi\, n}{\sqrt{}}+\frac{1}{n}\right]} = \frac{\ldots - \frac{1}{n^2}}{\ldots}$$

$$= \frac{n^2(1-\cos^2\varphi)-\sin^2\varphi}{n} = \sin^2\varphi\,\frac{n^2-1}{n}$$

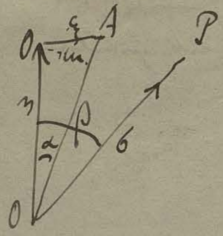
$$= \frac{\left[i\cos\varphi+\sqrt{}\right]\left[in\cos\varphi - \frac{1}{n}\sqrt{}\right]}{-} = \frac{-n\cos^2\varphi - \frac{\sin^2\varphi-n^2}{n} + i\left\{n\cos\varphi\sqrt{} - \frac{1}{n}\cos\varphi\sqrt{}\right\}}{-}$$

$$= \frac{\sin^2\varphi + i\cos\varphi\sqrt{}}{\sin^2\varphi - i\cos\varphi\sqrt{}} = \frac{1+\frac{i\cos\varphi}{\sin^2\varphi}\sqrt{}}{1-\ldots} =$$

$$\sin\varepsilon_1 = \frac{\beta_1}{\sqrt{\alpha_1^2+\beta_1^2}} \qquad \sin\varepsilon_2 = \frac{\beta_2}{\sqrt{\alpha_2^2+\beta_2^2}}$$

$$\cos\varepsilon_1 = \frac{\alpha_1}{\sqrt{\alpha_1^2+\beta_1^2}} \qquad \cos\varepsilon_2 = \frac{\alpha_2}{\sqrt{- -}}$$

$$\operatorname{tg}\frac{\Delta}{2} = \frac{1-\cos\Delta}{1+\cos\Delta} = \frac{1 - \frac{\alpha_1\alpha_2+\beta_1\beta_2}{\sqrt{(\alpha_1^2+\beta_1^2)(\alpha_2^2+\beta_2^2)}}}{1 + \cdot \text{—}}$$



$\xi = a \sin\gamma \sin\alpha$

$\eta = a \sin\gamma \cos\alpha$

$\sigma = a \sin(\varphi - \delta_0) \sin\alpha \sin(\beta-\alpha) + a \sin(\varphi - \delta_1) \cos\alpha \cos(\beta-\alpha)$

$$\frac{A^2}{a^2} = \left[\sin\alpha \sin(\beta-\alpha) + \cos\alpha \cos(\beta-\alpha) \cos\varepsilon\right]^2 + \left[\cos\alpha \cos(\beta-\alpha) \sin\varepsilon\right]^2$$

$$= \left[\sin\alpha \sin(\beta-\alpha) + \cos\alpha \cos(\beta-\alpha)\right]^2 + \left[\cos\alpha \cos(\beta-\alpha) \sin\varepsilon\right]^2 + 2 \underbrace{\sin\alpha \cos\alpha \sin(\beta-\alpha) \cos(\beta-\alpha)\left[\cos\varepsilon - 1\right]}_{-\sin 2\alpha \sin 2(\beta-\alpha) \sin^2\frac{\varepsilon}{2}}$$

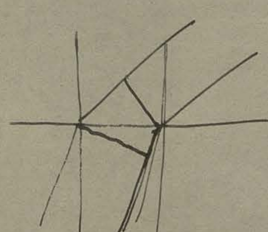
$/ \cos^2\beta$

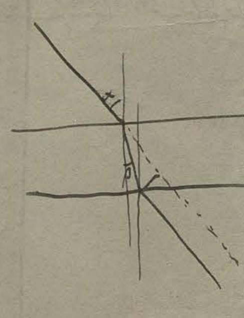
$$k_1^2 + k_2^2 + 2 k_1 k_2 \cos\delta \qquad k_1 = \cos\alpha \cos\beta$$

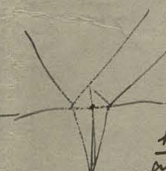
$$= (k_1 + k_2)^2 - 4 k_1 k_2 \sin^2\frac{\delta}{2} \qquad k_2 = \sin\alpha \sin\beta$$

$$J = \cos^2(\beta-\alpha) + \sin^2 2\alpha \sin^2\beta \sin^2\frac{\delta}{2}$$

$\alpha = \beta \qquad J_0 = 1 - \sin^2 2\alpha \sin^2\frac{\delta}{2} \qquad \Big| \ \alpha \perp \beta \qquad \sin^2 2\alpha \sin^2\frac{\delta}{2}$







$$\frac{\frac{h}{\cos\beta}\cos(\alpha-\beta)}{c} - \frac{n\frac{h}{\cos\beta}}{c}$$

$$= \frac{h}{c\cos\beta}\left[\sin\beta \cos(\alpha-\beta) - ~~\sin\alpha~~\right]$$

$\cos\alpha\cos\beta\sin\beta + \sin\alpha\sin^2\beta - \sin\alpha$

$- \sin\alpha \cos^2\beta$

$= \cos\beta\,(\cos\alpha\sin\beta - \sin\alpha\cos\beta)$

$= \cos\beta\ ~~\sin~~(\beta-\alpha)$

$$= \frac{h \sin(\beta-\alpha)}{c}$$

$$\frac{h}{c}\left[\sin(\beta-\alpha) - \sin(\beta'-\alpha)\right]$$

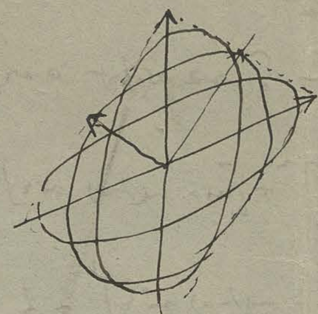
~~$\sin\beta\cos\alpha - \cos\beta\sin\alpha$~~

$(\sin\beta - \sin\beta')\cos\alpha - ~~\sin\alpha~~ (\cos\beta - \cos\beta')$

$\neq \left(\frac{\rho}{c} - \frac{e^2}{\rho c}\right) i \cos\alpha$

Drugi sposób rozwiązania: geometryczny

W tych wszystkich razach możliwe jest światło eliptyczne lub cyrkularne



~~$y = b \sin \alpha t$~~

$x = a \sin \alpha t$

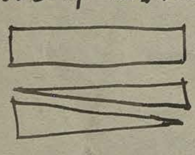
$y = b \sin(\alpha t + \delta) = b \cos \delta \sin \alpha t + b \sin \delta . \cos \alpha t$

$y - \frac{b}{a} x \cos \delta = b \sin \delta \cos \alpha t$

$$\left[\frac{y}{b \sin \delta} - \frac{x}{a \operatorname{tg} \delta}\right]^2 + \frac{x^2}{a^2} = 1$$

~~$\frac{x^2}{a^2 \sin^2 \delta} - \frac{2xy \operatorname{tg} \delta}{ab} + \frac{y^2}{b^2} = \sin^2 \delta$~~

Aby dowolne różnice fazy wytworzyć: Babinet Compensator



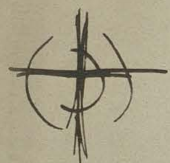
$\frac{1}{4}$ Und. Glimmer

$= 1/$

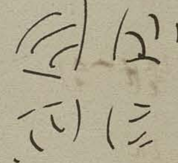
$X_n \cos x + Y_n \cos y + Z \cos z$

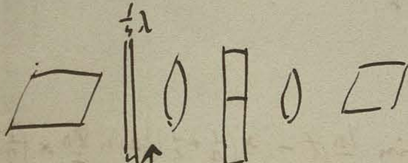
Npr.: $\varphi = \frac{\pi}{2}$

$A^2 = 1 + \sin^2 2\varphi \; \sin^2\frac{\delta}{2}$



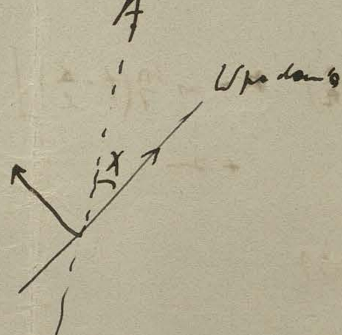
$\varphi = 0$: $A^2 = 1 - \sin^2 2\varphi \; \sin^2\frac{\delta}{2}$





$(P_0) = \xi = a \sin\frac{2\pi}{\tau} t$

$(P_e) = \eta = a \cos\frac{2\pi}{\tau} t$ } [illegible]



$\sigma = \sout{a \sin\frac{2\pi}{\tau} t - \varepsilon_0}$

$= a \sin(u - \varepsilon_0)\cos\chi + \cos(u - \varepsilon_e)\sin\chi$

$= a(\sin u \cos\chi + \cos(u + \delta)\sin\chi)$

$= \sin u\,(\cos\chi - \sin\chi \sin\delta) + \cos u \cos\delta \sin\chi$

$A^2 = (\cos\chi - \sin\chi \sin\delta)^2 + (\sin\chi\cos\delta)^2$

$= \cos^2\chi + \sin^2\chi - \sin 2\chi \sin\delta$

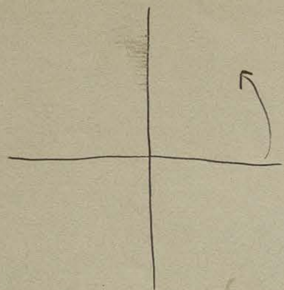
$= 1 - \sin 2\chi \sin\delta$

$\chi = 45°$: $1 - \sin\delta$

$-45°$ $\quad 1 + \sin\delta$

Npr. $\chi = 0, \frac{\pi}{2}$

[illegible]



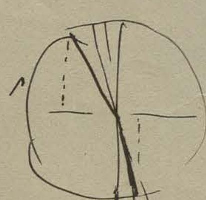
$\xi_1 = \frac{a}{2} \cos \frac{2\pi}{\tau}\left(t - \frac{x}{\lambda}\right)$ $\quad\quad$ $\xi_2 = -\frac{a}{2} \cos \frac{2\pi}{\tau} t$

$\eta_1 = \frac{a}{2} \sin \frac{2\pi}{\tau}$ $\quad\quad$ $\eta_2 = \frac{a}{2} \sin \frac{2\pi}{\tau}$

$\xi' = \frac{a}{2}\left[\cos \frac{2\pi}{\tau}\left(t - \frac{x}{\lambda}\right) - \cos \frac{2\pi}{\tau}\left(t - \frac{x}{\ell}\right)\right] = -a \sin \frac{2\pi}{\tau}\left(t - \frac{x}{2}\left(\frac{1}{\lambda} + \frac{1}{\ell}\right)\right) \sin \frac{\pi}{\tau} x \left(\frac{1}{\lambda} - \frac{1}{\ell}\right)$

$\eta' = \frac{a}{2}\left[\sin \quad + \sin \quad\right] = a \sin \; \text{---} \; \cos \frac{\pi}{\tau} x \left(\frac{1}{\lambda} - \frac{1}{\ell}\right)$

$\frac{\eta}{\xi} = - \operatorname{tg} \frac{\pi}{\tau} x \left(\frac{1}{\lambda} - \frac{1}{\ell}\right)$



$\sin(\varphi + \psi) \;\Big|\; \sin(\varphi - \psi)$

Tak zachowują się kryszt. jednoosiowe (hexagon, tetragonal
1-3 1-2

wapień, kwarzec, ~~[illegible]~~, mika, turmalin
apatyt, szmaragd
~~[illegible]~~ lód
+

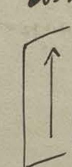
cyrkon (hyacinth)

granat, ałun, sól, fluoryt
dyament
[illegible] żadnych anomalii

trzyosiowe jednoskośne dwuskośne
dwuosiowe (rombowe, monokl., trikl.)

~~[illegible]~~ aragonit gips $CuSO_4$
topas ortoklas
siarka $FeSO_4$

Turmalin dichroityczny absorbuje o, przepuszcza tylko e



negat.

pozyt.

Szybkość światła w kierunkach osi optycznej

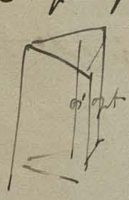
$CaCO_3$

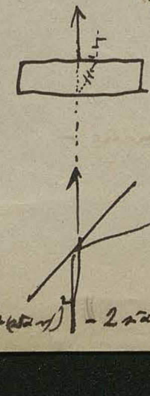
$\frac{v_o}{v} = 0'60295 \quad | \quad \frac{v_e}{v} = 0'67279$

$n_o = 1'658 \quad n_e = 1'487$

$SiO_2 \quad n_o = 1'544 \quad n_e = 1'553$

można zm. p. za pomocą pryzmatu

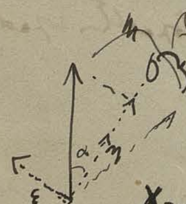




tylko o przejdzie
zaniknie

Płaszczyzna polaryzacji dla o — płaszczyzna zawierająca oś i pł. główna

promień o drga ⊥ do osi optycznej
promień e drga w płaszczyźnie głównej przekroju



$\xi = \sin(\varphi - \frac{\Delta\, 2\pi}{v_o \tau}) \sin\alpha$

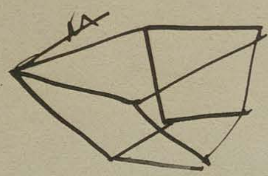
$\eta = \sin(\varphi - \frac{\Delta\, 2\pi}{v_e \tau}) \cos\alpha$

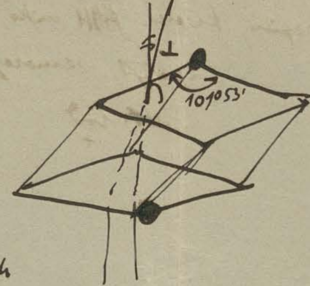
$X = -\xi \sin\beta + \eta \cos\beta$

$0 \quad = \sin(\varphi - \delta) \cos\alpha \cos\beta - \sin\varphi \sin\alpha \sin\beta$

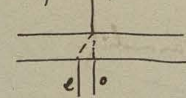
$A^2 = [\cos\alpha \cos\beta . \cos\delta - \sin\alpha \sin\beta]^2 + (\sin\delta \cos\alpha \cos\beta)^2 = (\cos\alpha\cos\beta)^2 + (\sin\alpha\sin\beta)^2 - 2 \sin\alpha \cos\alpha \sin\beta \cos\beta \cos\delta = \cos^2(\alpha+\beta) + \frac{\sin 2\alpha \sin 2\beta}{2}(1 - \cos\delta)$

Podw. załam. wszystkie kryształy z wyjątkiem regular.

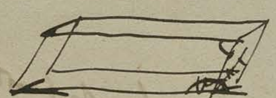


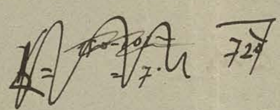


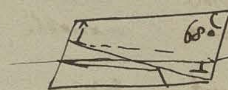
połączą się ze sobą – || do płaszcz. główn.



3 razy taki długi jak





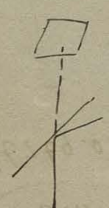


= płaszczyzna główna

2 razy albo tylko e

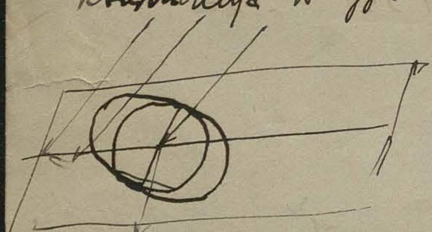
W świetle polaryzowanem przez odbicie:

ciemność nikola gdy płaszcz. wpad. św. = płaszcz. główna

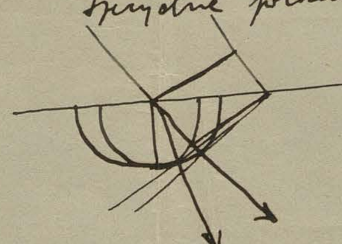


Erasmus Bartholinus (Duńczyk) 1670

Konstrukcya Huyghensa:



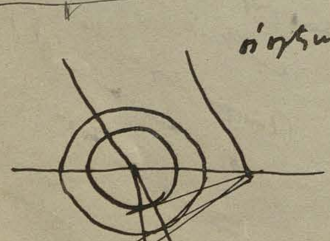
Szczególne położenia



powstają z płaszcz.

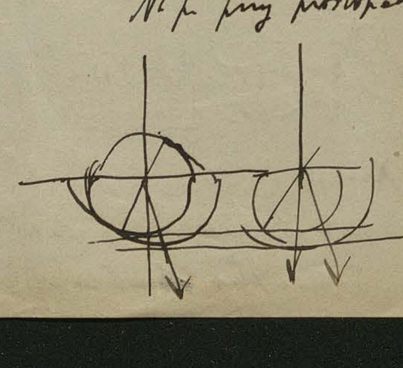
ale

oś optyczna



zawiera oś

Ale przy prostopadłem wpadaniu



$$A^2 = (\cos\alpha\cos\beta) + (\sin\alpha\sin\beta)^2 - 2\sin\alpha\cos\alpha\sin\beta\cos\beta\cos\delta =$$

$$= [\underbrace{\cos\alpha\cos\beta - \sin\alpha\sin\beta}_{\cos(\alpha+\beta)^2}]^2 + \sin 2\alpha \sin 2\beta \sin^2\frac{\delta}{2}$$

$\| \sin\alpha\cos\alpha [(\cos\delta - 1)^2 + \sin^2\delta]$ $\underbrace{2(1-\cos\delta)}_{4\sin^2\frac{\delta}{2}}$

23

Np. $\beta = -\alpha$

$$A^2 = 1 - \sin^2 2\alpha \sin^2\frac{\delta}{2}$$

jeśli $\alpha = 45^\circ$: $A^2 = 1 - \sin^2\frac{\delta}{2} = \cos^2\frac{\delta}{2}$

$$= 1 - \sin^2 \frac{\pi\Delta}{\lambda}(n_0 - n_e)$$

barwy jak cienkiej blaszki, zależne od Δ

ciemności dla: $\frac{\pi\Delta(n_0-n_e)}{\lambda} = \frac{\pi}{2}$

$$\lambda = 2\Delta(n_0 - n_e)$$

~~[illegible]~~ $\alpha + \beta = \frac{\pi}{2}$

$$A^2 = +\sin^2 2\alpha \sin^2\frac{\delta}{2}$$

$\alpha = 45^\circ$: $A^2 = \sin^2\frac{\delta}{2}$

Kręcąc analys. zmieniają się barwy

$\alpha = 0$: $A^2 = \cos^2\beta$ ~~[illegible]~~ białe

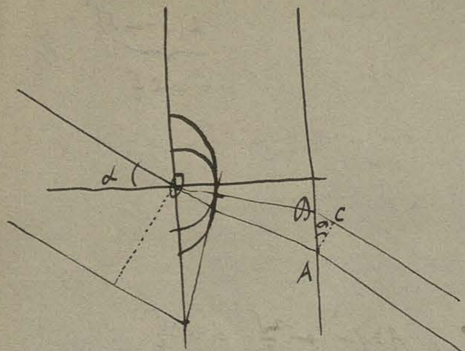
$\alpha = \frac{\pi}{2}$: $A^2 = \sin^2\beta + \sin 2\beta \sin^2\frac{\delta}{2} = \sin\beta[\sin\beta + \cos\beta(1-\cos\delta)]$

$\alpha = \frac{\pi}{4}$: $A^2 = \frac{1}{2}(\cos\beta - \sin\beta)^2 + \sin 2\beta \sin^2\frac{\delta}{2} = \frac{\cos^2\beta + \sin^2\beta - 2\cos\beta\sin\beta}{2} + \sin\beta\cos\beta(1-\cos\delta)$

$$= \frac{(\cos\beta + \sin\beta)^2}{2} - \sin\beta\cos\beta\cos\delta$$

To samo powtórzy się, jeśli oś nie leży w płycie lecz ukośnie, tylko wtedy opóźnienie dla δ inne, a zamiast 00 wtedy położenie ~~[illegible]~~ punktów głównych.

W spektroskopie: ~~[illegible]~~ ciemne pręgi w równych odstępach

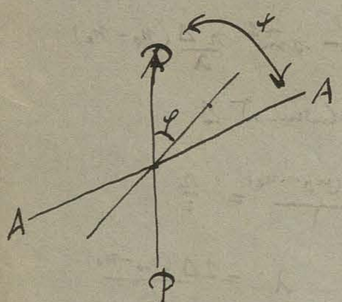


$$\delta = \frac{2\pi}{\tau}\left[\frac{OB}{v_0} + \frac{BC}{c} - \frac{OA}{v_e}\right]$$

$$= f_e(\Delta, \alpha)$$

$$= f(\Delta, 0) + \alpha \frac{\partial f}{\partial \alpha} + \alpha^2 \frac{\partial^2 \varphi}{\partial \alpha^2}$$

$$\left(= \frac{d}{2 v_0}(v_e^2 - v_0^2)\sin^2\alpha\right)$$



$$\xi = a\cos\varphi \sin(\vartheta\, [illegible])$$

$$\eta = a\sin\varphi \sin(\vartheta - \delta)$$

$$X = \xi \cos(\psi-\varphi) - \eta \sin(\psi-\varphi)$$

$$= a \sin\vartheta\left[\cos\varphi\cos(\psi-\varphi) - \sin\varphi\sin(\psi-\varphi)\cos\delta\right]$$

$$+ a\cos\vartheta\left[\sin\varphi\sin(\psi-\varphi)\sin\delta\right]$$

$$A^2 = \cos^2\varphi\cos^2(\psi-\varphi) - 2\sin\varphi\cos\varphi\sin(\psi-\varphi)\cos(\psi-\varphi)\cos\delta$$

$$+ \sin^2\varphi\sin^2(\psi-\varphi)$$

$$= \left[\cos\varphi\cos(\psi-\varphi) - \sin\varphi\sin(\psi-\varphi)\right]^2 + 2\sin\varphi\cos\varphi\sin(\psi-\varphi)\cos(\psi-\varphi)(1-\cos\delta)$$

$$= \cos^2\psi + \sin 2\varphi \sin 2(\psi-\varphi)\sin^2\frac{\delta}{2}$$

Oraz $\psi = 0$: $A^2 = 1 - \sin^2 2\varphi \sin^2\frac{\delta}{2}$

$\psi = \frac{\pi}{2}$: $A^2 = \sin^2 2\varphi \sin^2\frac{\delta}{2}$ $\begin{cases} = 0 \text{ dla } \varphi = 0 \\ \varphi = \frac{\pi}{2} \end{cases}$

itd.

lub dla $\delta = 2k\pi$

lösungen und den dabei auftretenden theoretischen Problemen brieflich Mitteilung gemacht hatte, habe ch eine mathematische Theorie der Koagulationskinetik ausgearbeitet, welche eine spezielle Anwendung der im Vorhergehenden entwickelten Theorie der Brownschen Bewegung bildet, und diese möchte ich Ihnen heute in einem ganz kurzen Abriß vorlegen[1]).

Von vornherein seien jedoch zwei einschränkende Bemerkungen vorausgeschickt:

1. Meine Theorie beansprucht nicht als vollständige Aufklärung der inneren Ursachen der Koagulation, d. h. der hierbei in Wirkung tretenden elektrischen oder kapillaren Kräfte, der Natur der elektrischen Doppelschichte auf der Oberfläche der Kolloidteilchen usw., zu gelten. Es ist eine sozusagen formale Theorie, aufgebaut auf einer mir von Prof. Zsigmondy vorgeschlagenen Annahme betreffs des Mechanismus der Koagulation, wonach sich jene Kräfte durch eine Wirkungssphäre vom Radius R ersetzen lassen, derart, daß die Brownsche Bewegung der Teilchen ungehindert vor sich geht, solange die Entfernung ihrer Mittelpunkte größer ist als R, daß jedoch zwei Teilchen sofort aneinander haften bleiben müssen, sobald ihre Mittelpunktsentfernung auf R herabsinkt.

2. Eben infolge dieser Annahme bezieht sich diese Theorie eigentlich direkt nur auf einen Grenzfall der Koagulations-Kinetik, d. i. die rasche iversible Koagulation, wie sie bei größeren Elektrolytkonzentrationen zustandekommt. Ich glaube, daß man sie mittels gewisser Modifkationen teilweise auch auf die langsame Koagulation ausdehnen kann, welche durch geringen, die elektrolytische Doppelschicht nicht vollständig entladenden Elektrolytzusatz hervorgerufen wird. Aber die Erscheinungen des reversibeln Koagulationsgleichgewichts, welche z. B. Sven Odén an Schwefelhydrosol studiert hat, gehen über den Geltungsbereich dieser

$$\int dt = 4\pi DRc\left[1+\frac{}{\sqrt{\pi Dt}}\right]dt$$

und für die von Anfang an abgeschiedene M

$$M = 4\pi DRc\left[t+\frac{2R\sqrt{t}}{\sqrt{\pi D}}\right].$$

Behufs Vereinfachung der Rechnu wollen wir schon an dieser Stelle eine An rung einführen, indem wir das zweite der rechten Seite, welches die $\sqrt{t}$ enthäl unwesentlich weglassen. Das heißt, daß wi Koagulationsverlauf in einem solchen Sta studieren, wo die Zeit t groß ist gegenübe Werte $\frac{R^2}{D}$. Das Anfangsstadium, welches diese Bedingung ausgeschlossen wird, b sich beispielsweise in Zsigmondys Vers auf nur 10^{-4} bis 10^{-3} Sekunden. Im ü könnte man die Rechnung auch ohne jen nachlässigung weiterführen, erhält aber ziertere, praktisch gleichwertige Formeln

Ersetzt man die Konzentration c du pro Volumeinheit entfallende Teilchenz so wäre die Anzahl der pro Zeiteinheit a hervorgehobenen Adsorptionskern ankl Teilchen: $4\pi RDv_0$, und die Zeit

$$T = \frac{1}{4\pi RDv_0} = \frac{1}{\beta},$$

welche wir weiterhin „Koagulationszeit wollen, würde dem Zeitpunkt entspre durchschnittlich gerade ein Teilchen hervorgehobenen haften bleibt.

3. Vervollständigte Berechn Koagulation.

Nun ist aber unsere Rechnung i Hinsicht zu verbessern:

1. Das hervorgehobene Teilche und für sich eine ähnliche Brow wegung aus wie die übrigen, es kö für die Koagulation die relative Bewe Betracht. Diesbezüglich läßt sich nun

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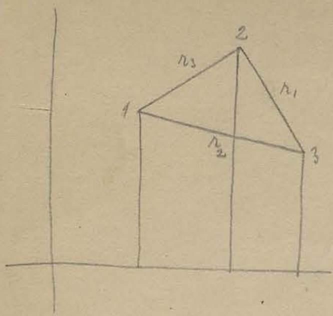
E. POLLY, IV. KAROLINENGASSE 23.

Problem der
drei Körper



$$X_1 = \frac{m_1 m_2}{r_3^2}\,\frac{x_2 - x_1}{r_3} + \frac{m_1 m_3}{r_2^2}\,\frac{x_3 - x_1}{r_2}$$

$$X_2 = -\frac{m_1 m_2}{r_3^2}\,\frac{x_2 - x_1}{r_3} + \frac{m_2 m_3}{r_1^2}\,\frac{x_3 - x_2}{r_1}$$

$$X_3 = -\frac{m_1 m_3}{r_2^2}\,\frac{x_3 - x_1}{r_2} - \frac{m_2 m_3}{r_1^2}\,\frac{x_3 - x_2}{r_1}$$

$$Y_1 = \frac{m_1 m_2}{r_3^2}\,\frac{y_2 - y_1}{r_3} - \frac{m_1 m_3}{r_2^2}\,\frac{y_1 - y_3}{r_2}$$

$$Y_2 = -\frac{m_2 m_1}{r_3^2}\,\frac{y_2 - y_1}{r_3} - \frac{m_2 m_3}{r_1^2}\,\frac{y_2 - y_3}{r_1}$$

$$Y_3 = \frac{m_1 m_3}{r_2^2}\,\frac{y_1 - y_3}{r_2} + \frac{m_2 m_3}{r_1^2}\,\frac{y_2 - y_3}{r_1}$$

Coordinaten des Kraftmittelpunktes:

$$\xi = \frac{m_1 x_1 r_1^3 + m_2 x_2 r_2^3 + m_3 x_3 r_3^3}{m_1 r_1^3 + m_2 r_2^3 + m_3 r_3^3}$$

$$\eta = \frac{m_1 y_1 r_1^3 + m_2 y_2 r_2^3 + m_3 y_3 r_3^3}{m_1 r_1^3 + m_2 r_2^3 + m_3 r_3^3}$$

$$\frac{d\xi}{dt} = \frac{\partial \xi}{\partial x_1}\frac{dx_1}{dt} + \frac{\partial \xi}{\partial x_2}\frac{dx_2}{dt} +$$

$$\frac{d\xi}{dt} = \frac{1}{[m_1 r_1^3 + m_2 r_2^3 + m_3 r_3^3]^2}\Big[(m_1 r_1^3 + m_2 r_2^3 + m_3 r_3^3) \times$$

$$\times (m_1 r_1^3 + 3 m_2 x_2 r_2 (x_1 - x_3) + 3 m_3 x_3 r_3 (x_1 - x_2)) -$$

$$-(m_1 x_1 r_1^3 + m_2 x_2 r_2^3 + m_3 x_3 r_3^3)(m_2 3 r_2 (x_1 - x_3) + m_3 3 r_3 (x_1 - x_2))\Big] \frac{dx_1}{dt} +$$

$$+ \cdots$$

$$= \frac{1}{[\quad]^2}\Big[m_1^2 r_1^6 + m_1 m_2 r_2^3 r_1^3 + m_1 m_3 r_1^3 r_3^3 +$$

$$+ 3 m_1 m_2 x_2 r_2 r_1^3 (x_1 - x_3) + \text{~~}3 m_2^2 x_2 r_2^4 (x_1 - x_3)\text{~~} + 3 m_2 m_3 x_2 r_2 r_3^3 (x_1 - x_3)$$

$$+ 3 m_1 m_3 x_3 r_3 r_1^3 (x_1 - x_2) + 3 m_2 m_3 x_3 r_3 r_2^3 (x_1 - x_2) + \text{~~}3 m_3^2 x_3 r_3^4 (x_1 - x_2)\text{~~}$$

$$- 3 m_1 m_2 x_1 r_2 r_1^3 (x_1 - x_3) - \text{~~}3 m_2^2 x_2 r_2^4 (x_1 - x_3)\text{~~} - 3 m_2 m_3 x_3 r_2 r_3^3 (x_1 - x_3)$$

$$- 3 m_1 m_3 x_1 r_3 r_1^3 (x_1 - x_2) - 3 m_2 m_3 x_2 r_2^3 r_3 (x_1 - x_2) - \text{~~}3 m_3^2 x_3 r_3^4 (x_1 - x_2)\text{~~}$$

$$= \frac{1}{[\quad]^2}\Big[m_1^2 r_1^6 + m_1 m_2 r_1^3 r_2^3 + m_1 m_3 r_1^3 r_3^3 + 3 \cdots +$$

$$+ 3 m_1 m_2 r_1^3 r_2 (x_1 - x_3)(x_2 - x_1) + 3 m_1 m_3 r_1^3 r_3 (x_1 - x_2)(x_3 - x_1) +$$

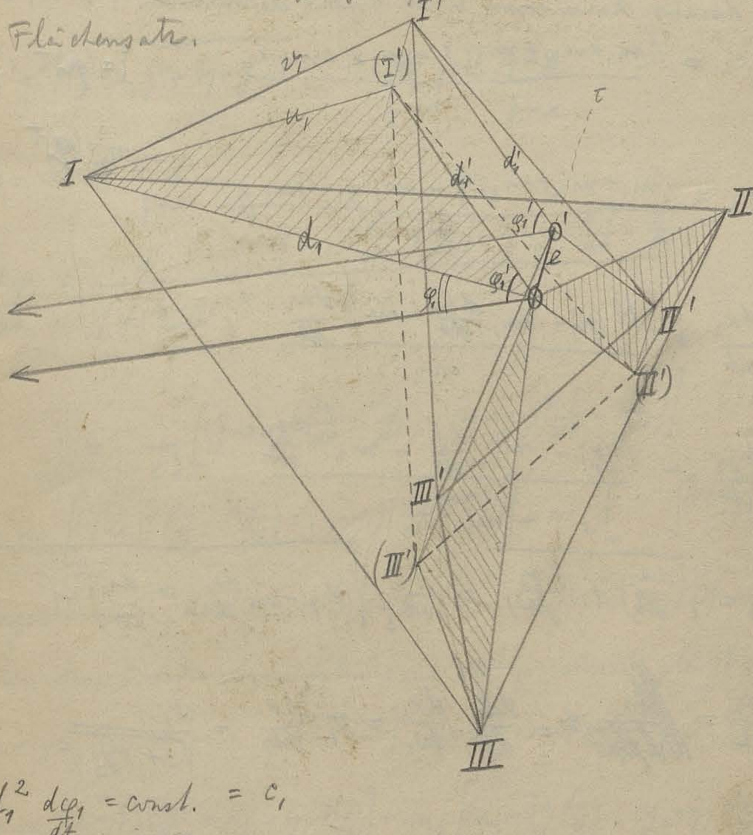
$$+ 3 m_2 m_3 r_2 r_3 \Big[r_3^2 (x_1 x_2 - x_2 x_3 - x_3 x_1 + x_3^2) + r_2^2 (x_1 x_3 - x_2 x_3 - x_1 x_2 + x_2^2)\Big]$$

$$= \frac{1}{[\quad]^2}\Big[\text{———} +$$

$$+3\Big\{ m_1 r_1^3 (m_2 r_2 + m_3 r_3)(x_1 - x_3)(x_2 - x_1) +$$

$$+ m_2 m_3 r_2 r_3 (x_3 - x_2)\left[r_3^2 (x_3 - x_1) + r_2^2 (x_1 - x_2)\right]\Big\}$$

Die Richtungen der Kräfte schneiden sich in einem Punkte; man kann also die Bewegung so betrachten, als ob eine Kraft [deren Größe durch jene Relat. gegeb.] von diesem, im Raume sich bewegenden Punkte ausginge, ohne Rücksicht auf die Wirkung der Massen untereinander; alles kann als eine relative Centralbewegung aufgefasst werden, also gilt der Flächensatz.



$$d_1^2 \frac{d\varphi_1}{dt} = \text{const.} = c_1$$

$$d_2^2 \frac{d\varphi_2}{dt} = \text{const} = c_2$$

$$d_3^2 \frac{d\varphi_3}{dt} = \text{const.} = c_3$$

Erhaltung des Schwerpunktes

$$\xi = \frac{m_1 x_1 + m_2 x_2 + m_3 x_3}{m_1 + m_2 + m_3} \qquad m_1 + m_2 + m_3 = M$$

$$\eta = \frac{m_1 y_1 + m_2 y_2 + m_3 y_3}{m_1 + m_2 + m_3}$$

daraus kann man einen Punkt eliminieren:

$$x_3 = \frac{(m_1 + m_2 + m_3)\xi - m_1 x_1 - m_2 x_2}{m_3} \qquad \xi = (p)t$$

$$\eta = (q)t$$

$$y_3 = \frac{(m_1 + m_2 + m_3)\eta - m_1 y_1 - m_2 y_2}{m_3}$$

$$\frac{dx_3}{dt} = \frac{M(p) - m_1 \frac{dx_1}{dt} - m_2 \frac{dx_2}{dt}}{m_3}$$

$$\frac{dy_3}{dt} = \frac{M(q) - m_1 \frac{dy_1}{dt} - m_2 \frac{dy_2}{dt}}{m_3}$$

$$v_3 = \sqrt{\left(\frac{dx_3}{dt}\right)^2 + \left(\frac{dy_3}{dt}\right)^2} = \frac{1}{m_3}\sqrt{\left(M p - m_1 \frac{dx_1}{dt} - m_2 \frac{dx_2}{dt}\right)^2 + (\quad)^2}$$

$$\frac{dx_1}{dt} = \frac{ds_1}{dt}\frac{dx_1}{ds_1} = v_1 \frac{dx_1}{ds_1} = \frac{v_1}{\sqrt{1 + \left(\frac{dy_1}{dx_1}\right)^2}}$$

statt ξ & η besser φ & $\mathfrak{z}$ zu setzen um Verwechslungen mit d. Coord. & Kraftmp. zu vermeiden

$$\frac{d\varphi}{dt} = \frac{1}{M}\left[m_1 \frac{dx_1}{dt} + m_2 \frac{dx_2}{dt} + m_3 \frac{dx_3}{dt}\right]$$

Erhaltung der Flächenräume.

Von aussen wirken auf den Schwerp. keine Kräfte, also muss die Summe der von den RV beschr. Flächenr. const. sein.

$$m_1 \rho_1^2 \frac{d\psi_1}{dt} + m_2 \rho_2^2 \frac{d\psi_2}{dt} + m_3 \rho_3^2 \frac{d\psi_3}{dt} = F$$

ρ = RV vom Schwerp. $\psi = \measuredangle$

$$\rho_1^2 = \sqrt{(\xi - x_1)^2 + (\eta - y_1)^2} \qquad \operatorname{tg} \psi = \frac{\eta - y}{\xi - x}$$

$$\psi_1 = \operatorname{arctg} \frac{\eta - y_1}{\xi - x_1}$$

$$\frac{d\psi_1}{dt} = \frac{1}{1 + \left(\frac{\eta - y_1}{\xi - x_1}\right)^2} \cdot \frac{(\xi - x_1)\left(\frac{d\eta}{dt} - \frac{dy_1}{dt}\right) - (\eta - y_1)\left(\frac{d\xi}{dt} - \frac{dx}{dt}\right)}{(\xi - x_1)^2}$$

$$= \frac{1}{\rho_1^2}\left[(\xi - x_1)\left(\frac{d\eta}{dt} - \frac{dy_1}{dt}\right) - (\eta - y_1)\left(\frac{d\xi}{dt} - \frac{dx}{dt}\right)\right]$$

$$F = m_1\left[(\xi - x_1)\left(\frac{d\eta}{dt} - \frac{dy_1}{dt}\right) - (\eta - y_1)\left(\frac{d\xi}{dt} - \frac{dx_1}{dt}\right)\right] + m_2\left[\quad\right] + m_3\left[\quad\right]$$

Angenommen der Schwerpunkt sei ruhend [sonst kann man dem Coordinatensystem eine entsprechende gleichförmige Bewegung ertheilen]; dann sind $\frac{d\xi}{dt} = 0$ $\frac{d\eta}{dt} = 0$

$$F = m_1\left[(x_1 - \xi)\frac{dy_1}{dt} - (y_1 - \eta)\frac{dx_1}{dt}\right] + m_2\left[(x_2 - \xi)\frac{dy_2}{dt} - (y_2 - \eta)\frac{dx_2}{dt}\right] +$$

$$+ m_3\left[(x_3 - \xi)\frac{dy_3}{dt} - (y_3 - \eta)\frac{dx_3}{dt}\right]$$

Bewegung des Kraftmittelpunktes.

$$\xi = \frac{m_1 x_1 r_1^3 + m_2 x_2 r_2^3 + m_3 x_3 r_3^3}{m_1 r_1^3 + m_2 r_2^3 + m_3 r_3^3}$$

$$\eta = \frac{m_1 y_1 r_1^3 + m_2 y_2 r_2^3 + m_3 y_3 r_3^3}{m_1 r_1^3 + m_2 r_2^3 + m_3 r_3^3}$$

$$\xi = f[x_1, x_2, x_3, y_1, y_2, y_3]$$

$$\frac{d\xi}{dt} = \frac{\partial \xi}{\partial x_1}\frac{dx_1}{dt} + \frac{\partial \xi}{\partial x_2}\frac{dx_2}{dt} + \frac{\partial \xi}{\partial x_3}\frac{dx_3}{dt} + \frac{\partial \xi}{\partial y_1}\frac{dy_1}{dt} + \frac{\partial \xi}{\partial y_2}\frac{dy_2}{dt} + \frac{\partial \xi}{\partial y_3}\frac{dy_3}{dt}$$

$$\xi = \frac{m_1 x_1 \left[(x_2-x_3)^2 + (y_2-y_3)^2\right]^{\frac{3}{2}} + m_2 x_2 \left[(x_1-x_3)^2 + (y_1-y_3)^2\right]^{\frac{3}{2}} + m_3 x_3 \left[(x_1-x_2)^2 + (y_1-y_2)^2\right]^{\frac{3}{2}}}{m_1 \left[(x_2-x_3)^2 + (y_2-y_3)^2\right]^{\frac{3}{2}} + m_2 \left[(x_1-x_3)^2 + (y_1-y_3)^2\right]^{\frac{3}{2}} + m_3 \left[(x_1-x_2)^2 + (y_1-y_2)^2\right]^{\frac{3}{2}}}$$

$$\frac{\partial \xi}{\partial x_1} = \frac{1}{\left[m_1 r_1^3 + m_2 r_2^3 + m_3 r_3^3\right]^2} \Big\{ \left[m_1 r_1^3 + m_2 r_2^3 + m_3 r_3^3\right]\left[m_1 r_1^3 + 3 m_2 x_2 (x_1-x_3) r_2 + 3 m_3 x_3 (x_1-x_2) r_3\right] - \left[m_1 x_1 r_1^3 + m_2 x_2 r_2^3 + m_3 x_3 r_3^3\right] \left[3 m_2 (x_1-x_3) r_2 + 3 m_3 (x_1-x_2) r_3\right] \Big\}$$

$$= \frac{m_1 r_1^3 + 3 m_2 x_2 r_2 (x_1-x_3) + 3 m_3 x_3 r_3 (x_1-x_2)}{m_1 r_1^3 + m_2 r_2^3 + m_3 r_3^3} - \frac{3\xi\left[m_2 r_2 (x_1-x_3) + m_3 r_3 (x_1-x_2)\right]}{m_1 r_1^3 + m_2 r_2^3 + m_3 r_3^3}$$

$$= \frac{m_1 r_1^3 + 3 m_2 r_2 (x_1-x_3)(x_2-\xi) + 3 m_3 r_3 (x_1-x_2)(x_3-\xi)}{m_1 r_1^3 + m_2 r_2^3 + m_3 r_3^3}$$

$$\frac{\partial \xi}{\partial x_2} = \frac{m_2 r_2^3 + 3 m_3 r_3 (x_2 - x_1)(x_3 - \xi) + 3 m_1 r_1 (x_2 - x_3)(x_1 - \xi)}{m_1 r_1^3 + m_2 r_2^3 + m_3 r_3^3}$$

$$\frac{\partial \xi}{\partial x_3} = \frac{m_3 r_3^3 + 3 m_1 r_1 (x_3 - x_2)(x_1 - \xi) + 3 m_2 r_2 (x_3 - x_1)(x_2 - \xi)}{m_1 r_1^3 + m_2 r_2^3 + m_3 r_3^3}$$

$$\frac{\partial \xi}{\partial y_1} = \frac{1}{[m_1 r_1^3 + m_2 r_2^3 + m_3 r_3^3]^2} \Big\{ [m_1 r_1^3 + m_2 r_2^3 + m_3 r_3^3][m_2 x_2 (y_1 - y_3) r_2 \cdot 3 +$$

$$+ m_3 x_3 (y_1 - y_2) r_3 \cdot 3] - [m_1 x_1 r_1^3 + m_2 x_2 r_2^3 + m_3 x_3 r_3^3][3$$

$$[3 m_2 (y_1 - y_3) r_2 + 3 m_3 (y_1 - y_2) r_3] \Big\}$$

$$= \frac{3}{[\quad]^2} \Big\{ [m_1 m_2 x_2 r_1^3 r_2 (y_1 - y_3) + \cancel{m_2^2 x_2 r_2^4 (y_1 - y_3)} +$$

$$+ m_2 m_3 r_3^3 r_2 x_2 (y_1 - y_3) + m_1 m_3 x_3 r_1^3 r_3 (y_1 - y_2) + m_2 m_3 r_2^3 r_3 x_3 (y_1 - y_2) +$$

$$+ \cancel{m_3^2 x_3 r_3^4 (y_1 - y_2)} - m_1 m_2 x_1 r_2 r_1^3 (y_1 - y_3) - \cancel{m_2^2 x_2 r_2^4 (y_1 - y_3)} -$$

$$- m_2 m_3 x_3 r_2 r_3^3 (y_1 - y_3) - m_1 m_3 x_1 r_1^3 r_3 (y_1 - y_2) + m_2 m_3 x_2 r_2^3 r_3 (y_1 - y_2) +$$

$$- \cancel{m_3^2 x_3 r_3^4 (y_1 - y_2)}]$$

$$= \frac{3}{[\quad]^2} \Big\{ [m_1 m_2 r_1^3 r_2 (x_2 - x_1)(y_1 - y_3) + \cancel{m_1 m_3}$$

$$+ m_1 m_3 r_1^3 r_3 (x_3 - x_1)(y_1 - y_2) + m_2 m_3 r_2 r_3^3 (x_2 - x_3)(y_1 - y_3) +$$

$$+ m_2 m_3 r_2^3 r_3 (x_3 - x_2)(y_1 - y_2)] \Big\}$$

oder einf.

$$= 3 \frac{m_2 x_2 r_2 (y_1 - y_3) + m_3 x_3 r_3 (y_1 - y_2) - \xi [m_2 r_2 (y_1 - y_3) + m_3 r_3 (y_1 - y_2)]}{m_1 r_1^3 + m_2 r_2^3 + m_3 r_3^3}$$

$$\frac{\partial \xi}{\partial x_1} + \frac{\partial \xi}{\partial x_2} + \frac{\partial \xi}{\partial x_3} = 1$$

$$\frac{\partial \xi}{\partial y_1} = 3\,\frac{m_2 r_2 (x_2 - \xi)(y_1 - y_3) + m_3 r_3 (x_3 - \xi)(y_1 - y_2)}{m_1 r_1^3 + m_2 r_2^3 + m_3 r_3^3}$$

$$\frac{\partial \xi}{\partial y_2} = 3\,\frac{m_3 r_3 (x_3 - \xi)(y_2 - y_1) + m_1 r_1 (x_1 - \xi)(y_2 - y_3)}{}$$

$$\frac{\partial \xi}{\partial y_3} = 3\,\frac{m_1 r_1 (x_1 - \xi)(y_3 - y_2) + m_2 r_2 (x_2 - \xi)(y_3 - y_1)}{}$$

$$\frac{\partial \xi}{\partial y_1} + \frac{\partial \xi}{\partial y_2} + \frac{\partial \xi}{\partial y_3} = 0$$

$$\frac{\partial \eta}{\partial x_1} = 3\,\frac{m_2 r_2 (y_2 - \eta)(x_1 - x_3) + m_3 r_3 (y_3 - \eta)(x_1 - x_2)}{m_1 r_1^3 + m_2 r_2^3 + m_3 r_3^3}$$

$$\frac{\partial \eta}{\partial x_2} = 3\,\frac{m_3 r_3 (y_3 - \eta)(x_2 - x_1) + m_3 r_3 (y_1 - \eta)(x_2 - x_3)}{m_1 r_1^3 + m_2 r_2^3 + m_3 r_3^3}$$

$$\frac{\partial \eta}{\partial x_3} =$$

$$\frac{\partial \eta}{\partial y_1} = \frac{m_1 r_1^3 + 3 m_2 r_2 (y_1 - y_3)(y_2 - \eta) + 3 m_3 r_3 (y_1 - y_2)(y_3 - \eta)}{m_1 r_1^3 + m_2 r_2^3 + m_3 r_3^3}$$

$$\frac{\partial \eta}{\partial y_2} =$$

$$\frac{\partial \eta}{\partial y_3} =$$

$$d_1^2 \frac{d\varphi_1}{dt} = c_1 = ?$$

[illegible] 0-X [illegible] 0X [illegible]:

$$d_1^2 = (\xi - x_1)^2 + (\eta - y_1)^2$$

$$tg\,\varphi_1 = \frac{y_1 - \eta}{\xi - x_1} \qquad (y_1, \eta, \xi, x_1) \;\; f(t)$$

$$\frac{d(tg\,\varphi_1)}{dt} = \frac{1}{\cos^2\varphi_1}\frac{d\varphi_1}{dt} = \frac{(\xi - x_1)\left(\frac{dy_1}{dt} - \frac{d\eta}{dt}\right) - (y_1 - \eta)\left(\frac{d\xi}{dt} - \frac{dx_1}{dt}\right)}{(\xi - x_1)^2}$$

$$\frac{d\varphi_1}{dt} = \frac{d(tg\,\varphi_1)}{dt}\cos^2\varphi_1 = \text{------}\;\cos^2\varphi_1$$

$$\cos^2\varphi_1 = \frac{1}{1 + tg^2\varphi_1} = \frac{1}{1 + \left(\frac{y_1 - \eta}{\xi - x_1}\right)^2} = \frac{(\xi - x_1)^2}{(\xi - x_1)^2 + (y_1 - \eta)^2}$$

$$\frac{d\varphi_1}{dt} = \frac{(\xi - x_1)\left(\frac{dy_1}{dt} - \frac{d\eta}{dt}\right) - (y_1 - \eta)\left(\frac{d\xi}{dt} - \frac{dx_1}{dt}\right)}{(\xi - x_1)^2 + (y_1 - \eta)^2}$$

$$d_1^2 \frac{d\varphi_1}{dt} = c_1 = (\xi - x_1)\left(\frac{dy_1}{dt} - \frac{d\eta}{dt}\right) - (y_1 - \eta)\left(\frac{d\xi}{dt} - \frac{dx_1}{dt}\right) = \frac{a_1}{m_1}$$

$$d_2^2 \frac{d\varphi_2}{dt} = c_2 = (\xi - x_2)\left(\frac{dy_2}{dt} - \frac{d\eta}{dt}\right) - (y_2 - \eta)\left(\frac{d\xi}{dt} - \frac{dx_2}{dt}\right) = \frac{a_2}{m_2}$$

$$d_3^2 \frac{d\varphi_3}{dt} = c_3 = (\xi - x_3)\left(\frac{dy_3}{dt} - \frac{d\eta}{dt}\right) - (y_3 - \eta)\left(\frac{d\xi}{dt} - \frac{dx_3}{dt}\right) = \frac{a_3}{m_3}$$

$$a_1 = m_1\left[\xi\frac{dy_1}{dt} - \xi\frac{d\eta}{dt} - x_1\frac{dy_1}{dt} + x_1\frac{d\eta}{dt} - y_1\frac{d\xi}{dt} + y_1\frac{dx_1}{dt} + \eta\frac{d\xi}{dt} - \eta\frac{dx_1}{dt}\right]$$

$$a_2 = m_2\left[\xi\frac{dy_2}{dt} - \xi\frac{d\eta}{dt} - x_2\frac{dy_2}{dt} + x_2\frac{d\eta}{dt} - y_2\frac{d\xi}{dt} + y_2\frac{dx_2}{dt} + \eta\frac{d\xi}{dt} - \eta\frac{dx_2}{dt}\right]$$

$$a_3 = m_3\left[\xi \qquad\qquad\qquad\qquad\right]$$

Siehe Erhaltung des Schwerpunktes:

$$a_1 + a_2 + a_3 = \xi M\frac{d\eta}{dt} - M\xi\frac{d\eta}{dt} -$$

$$a_1 + a_2 + a_3 = \xi M \frac{d\mathfrak{y}}{dt} - M\xi \frac{d\mathfrak{y}}{dt} - \left[m_1 x_1 \frac{dy_1}{dt} + m_2 x_2 \frac{dy_2}{dt} + m_3 x_3 \frac{dy_3}{dt}\right]$$

$$+ M\mathfrak{x} \frac{d\mathfrak{y}}{dt} - M\mathfrak{y}\frac{d\xi}{dt} + \left[m_1 y_1 \frac{dx_1}{dt} + m_1 y_2 \frac{dx_2}{dt} + m_3 y_3 \frac{dx_3}{dt}\right]$$

$$+ M\eta \frac{d\xi}{dt} - \eta M \frac{d\mathfrak{x}}{dt}$$

$$= \xi M \frac{d\mathfrak{y}}{dt} - \eta M \frac{d\mathfrak{x}}{dt} + M(\mathfrak{x} - \xi)\frac{d\mathfrak{y}}{dt} - M(\mathfrak{y} - \eta)\frac{d\xi}{dt} + \ldots$$

wenn man nun den Coordinatenanfangspunkt im Schwerpunkt nimmt ~~und diesen ruhend voraussetzt~~ so sind:

$$\frac{d\mathfrak{y}}{dt} = 0 \quad \frac{d\mathfrak{x}}{dt} = 0 \qquad \xi - \mathfrak{x} = \text{X-Coord. [illegible]}$$

$$\mathfrak{x} = 0 \quad \mathfrak{y} = 0 \qquad = p$$

$$\eta - \mathfrak{y} = q$$

$$\frac{dp}{dt} = \frac{d\xi}{dt} \qquad \frac{dq}{dt} = \frac{d\eta}{dt}$$

$$a_1 + a_2 + a_3 = M\left[q \frac{dp}{dt} - p\frac{dq}{dt}\right] + m_1\left[y_1 \frac{dx_1}{dt} - x_1 \frac{dy_1}{dt}\right] +$$

$$+ m_2\left[y_2 \frac{dx_2}{dt} - x_2 \frac{dy_2}{dt}\right] + m_3\left[y_3 \frac{dx_3}{dt} - x_3 \frac{dy_3}{dt}\right]$$

<u>also sind die durch den RV vom Schwerp. zum Kraftmittelpunkt zurückgelegten Flächenräume, × M mehr jenen, welche durch die RV zu den einzelnen Punkten zurückgelegt werden, gleich einer Constante.</u>

die sich aus dem Anfangszustande des Systems 34

bestimmt.

Da aber aus dem Gesetz der Erhaltung der Flächenräume folgt, dass die Summe der letzteren constant bleibt, so ist auch der durch den RV vom Schwerpunkt zum Kraftmittelpunkt zurückgelegte Flächenraum constant.

$$q^* \frac{dp}{dt} - p \frac{dq}{dt} = \text{constant} = F$$

$$K_1 = \frac{m_1 d_1 [m_1 r_1^3 + m_2 r_2^3 + m_3 r_3^3]}{r_2^3 \, r_3^3}$$

$$K_2 = \frac{m_2 d_2 [m_1 r_1^3 + m_2 r_2^3 + m_3 r_3^3]}{r_1^3 \, r_3^3}$$

$$K_3 = \frac{m_3 d_3 [m_1 r_1^3 + m_2 r_2^3 + m_3 r_3^3]}{r_1^3 \, r_2^3}$$

$$\frac{K_1}{m_1 d_1 r_1^3} + \frac{K_2}{m_2 d_2 r_2^3} + \frac{K_3}{m_3 d_3 r_3^3} = \frac{3[m_1 r_1^3 + m_2 r_2^3 + m_3 r_3^3]}{r_1^3 r_2^3 r_3^3}$$

$$\frac{m_1^2 d_1 r_1^3}{K_1 r_2^3 r_3^3} + \frac{m_2^2 d_2 r_2^3}{K_2 r_1^3 r_3^3} + \frac{m_3^2 d_3 r_3^3}{K_3 r_1^3 r_2^3} = 1$$

$$\cancel{\frac{m_1^2 K_2 K_3 d_1 r_1^6 + m_2^2 K_1 K_3 d_2 r_2^6 + m_3^2 K_1 K_2 d_3 r_3^6}{K_1 K_2 K_3 \, r_1^3 r_2^3 r_2^3} = 1}$$

$$K_1 : K_2 : K_3 = \frac{m_1 d_1}{r_2^3 r_3^3} : \frac{m_2 d_2}{r_1^3 r_3^3} : \frac{m_3 d_3}{r_1^3 r_2^3}$$

$$= m_1 d_1 r_1^3 : m_2 d_2 r_2^3 : m_3 d_3 r_3^3$$

$$\frac{K_1}{m_1 d_1 r_1^3} = \frac{K_2}{m_2 d_2 r_2^3} = \frac{K_3}{m_3 d_3 r_3^3} = \frac{m_1 r_1^3 + m_2 r_2^3 + m_3 r_3^3}{r_1^3 r_2^3 r_3^3}$$

$$K_1 + K_2 + K_3 = [m_1 d_1 r_1^3 + m_2 d_2 r_2^3 + m_3 d_3 r_3^3]\{(\uparrow)$$

für fixes C.S. (siehe Blätter)

Kraftfunction = U =

$$U = \frac{m_1 m_2}{r_3} + \frac{m_1 m_3}{r_2} + \frac{m_2 m_3}{r_1}$$ [für alle 3 P. gleich]

$$\frac{m_1}{2} v_1^2 - \frac{m_1}{2} v_{10}^2 = \frac{m_1 m_2}{r_3} - \frac{m_1 m_2}{r_{30}} + \frac{m_1 m_3}{r_2} - \frac{m_1 m_3}{r_{20}} + \frac{m_1 m_3}{r_3} - \frac{m_1 m_3}{r_{30}}$$

$$\left\{ \begin{aligned} \frac{m_1 v_1^2}{2} &= a_1 + \frac{m_1 m_2}{r_3} + \frac{m_1 m_3}{r_2} + \frac{m_2 m_3}{r_1} \\ \frac{m_2 v_2^2}{2} &= a_2 + \frac{m_1 m_2}{r_3} + \text{———} \\ \frac{m_3 v_3^2}{2} &= a_3 + \text{———} \end{aligned} \right\}$$

$$\frac{m_1 v_1^2}{2} - \frac{m_2 v_2^2}{2} = a_1 - a_2 \quad = \frac{m_1 v_{10}^2}{2} - \frac{m_2 v_{20}^2}{2}$$

$$\frac{m_2 v_2^2}{2} - \frac{m_3 v_3^2}{2} = a_2 - a_3 \quad = \text{—}$$

$$\frac{m_3 v_3^2}{2} - \frac{m_1 v_1^2}{2} = a_3 - a_1 \quad =$$

$$m_1 v_1 \frac{dv_1}{dt} - m_2 v_2 \frac{dv_2}{dt} = 0$$

$\frac{dv_1}{dt}$ = Beschleunigung in der Richtung der Bahn $= \frac{d^2 s}{dt^2}$

$\frac{dv_1}{dt} \cdot m_1$ = Kraftkompon. " " " " " $= B_1$

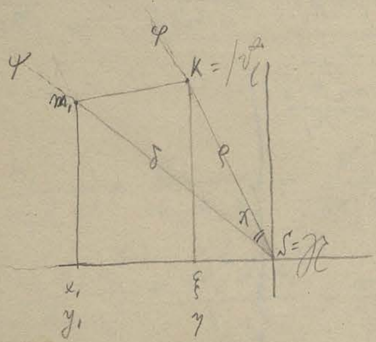
$$B_1 v_1 = B_2 v_2 = B_3 v_3$$

$$\left\{ \begin{aligned} v_1 : v_2 &= B_2 : B_1 \\ v_1 : v_3 &= B_3 : B_1 \\ v_2 : v_3 &= B_3 : B_2 \end{aligned} \right\} \quad v_1 : v_2 : v_3 = \frac{1}{B_1} : \frac{1}{B_2} : \frac{1}{B_3}$$

Flächenräume welche vom RV zwischen Kraftmittelp. und d. Punkten beschrieben werden: (Siehe Seite 15)

$$C_1 = \frac{a_1}{m_1} = \left[\xi \frac{dy_1}{dt} - y_1 \frac{d\xi}{dt} - \xi \frac{d\eta}{dt} + \eta \frac{d\xi}{dt} - x_1 \frac{dy_1}{dt} + y_1 \frac{dx_1}{dt} + \right.$$

$$\left. + x_1 \frac{d\eta}{dt} - \eta \frac{dx_1}{dt} \right]$$



$\xi = \rho \cos\varphi$

$\eta = \rho \sin\varphi$

$x_1 = \delta \cos\psi$

$y_1 = \delta \sin\psi$

$$\xi \frac{dy_1}{dt} - y_1 \frac{d\xi}{dt} + x_1 \frac{d\eta}{dt} - \eta \frac{dx_1}{dt} = ?$$

$$= \rho\cos\varphi \left[\frac{d\delta}{dt}\sin\psi + \delta\cos\psi\frac{d\psi}{dt}\right] - \delta\sin\psi\left[\frac{d\rho}{dt}\cos\varphi - \rho\sin\varphi\frac{d\varphi}{dt}\right] +$$

$$+ \delta\cos\psi\left[\frac{d\rho}{dt}\sin\varphi + \rho\cos\varphi\frac{d\varphi}{dt}\right] - \rho\sin\varphi\left[\frac{d\delta}{dt}\cos\psi - \delta\sin\psi\frac{d\psi}{dt}\right] =$$

$$= \frac{d\delta}{dt}\left[\rho\cos\varphi\sin\psi - \rho\sin\varphi\cos\psi\right] + \frac{d\psi}{dt}\left[\rho\delta\cos\varphi\cos\psi + \rho\delta\sin\varphi\sin\psi\right]$$

$$+ \frac{d\varphi}{dt}\left[\delta\rho\sin\varphi\sin\psi - \delta\rho\cos\varphi\cos\psi\right] + \frac{d\rho}{dt}\left[\delta\cos\psi\sin\varphi - \delta\sin\psi\cos\varphi\right] =$$

$$= -\frac{d\delta}{dt}\sin\chi\cdot\rho + \frac{d\psi}{dt}\cdot\rho\cdot\delta\cdot\cos\chi + \frac{d\varphi}{dt}\cdot\rho\cdot\delta\cdot\cos\chi + \frac{d\rho}{dt}\sin\chi\cdot\delta =$$

$=$

48

$$\frac{d^2x}{dt^2} = \frac{d^2r}{dt^2}\cos\varphi - 2\frac{dr}{dt}\sin\varphi\frac{d\varphi}{dt} - r\cos\varphi\left(\frac{d\varphi}{dt}\right)^2 - r\sin\varphi\frac{d^2\varphi}{dt^2}$$

$$\frac{d^2y}{dt^2} = \frac{d^2r}{dt^2}\sin\varphi + 2\frac{dr}{dt}\cos\varphi\frac{d\varphi}{dt} - r\sin\varphi\left(\frac{d\varphi}{dt}\right)^2 + r\cos\varphi\frac{d^2\varphi}{dt^2}$$

Уск. к ρ ½ е и $= \dfrac{du}{dt}$ | Уск. к ρ ½ ∘ $RN = \dfrac{d^2r}{dt^2} - r\left(\dfrac{d\varphi}{dt}\right)^2$

Уск. $\perp \rho$ ½ е и $= \dfrac{u^2}{\rho}$ | Уск. $\perp \rho$ ½ ∘ $RN = \dfrac{1}{r}\dfrac{d}{dt}\left(r^2\dfrac{d\varphi}{dt}\right)$

2 · уд. пл. $= y\dfrac{dx}{dt} - x\dfrac{dy}{dt} = r^2\dfrac{d\varphi}{dt}$

$u = \frac{ds}{dt} \qquad f = \frac{du}{dt} = \frac{d^2 s}{dt^2}$

$$f = \frac{d\left(\frac{u^2}{2}\right)}{ds}$$

$p = mg$

$$mu - mu_0 = \int_{t_0}^{t} P\,dt$$

$$m\frac{u^2}{2} - m\frac{u_0^2}{2} = \int_{s_0}^{s} p\,ds$$

$$= p(s - s_0) \quad \text{[illegible] const. } p$$

$\frac{dx}{dt} = \frac{ds}{dt}\cos\alpha = u\cos\alpha \quad \Big| \quad \frac{dy}{dt} = \frac{ds}{dt}\sin\alpha = u\sin\alpha$

$\frac{d^2x}{dt^2} = \frac{d^2s}{dt^2}\cos\alpha = \quad \Big| \quad \frac{d^2y}{dt^2} = \frac{d^2s}{dt^2}\sin\alpha =$

$$m\frac{d^2x}{dt^2} = \underbrace{m\frac{du}{dt}}_{p}\cos\alpha$$

$x = r\cos\varphi$

$y = r\sin\varphi$

$\frac{dx}{dt} = \frac{dr}{dt}\cos\varphi - r\sin\varphi\frac{d\varphi}{dt}$

$\frac{dy}{dt} = \frac{dr}{dt}\sin\varphi + r\cos\varphi\frac{d\varphi}{dt}$

[illegible] $RV = \frac{dr}{dt}$

$\perp$ [illegible] $RV = r\frac{d\varphi}{dt}$

[illegible] $= |v^2| = \left(\frac{ds}{dt}\right)^2 = \left(\frac{dr}{dt}\right)^2 + r^2\left(\frac{d\varphi}{dt}\right)^2$

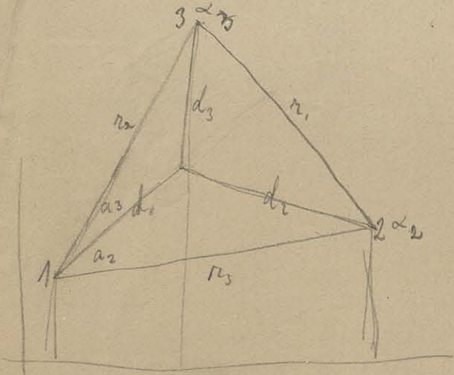
$$K_1 = \frac{m_1 d_1 \left[m_1 r_1^3 + m_2 r_2^3 + m_3 r_3^3\right]}{r_2^3 r_3^3}$$

$$K_1 = \frac{\partial U}{\partial d_1}$$

$$U = \left[d_1 \frac{\partial U}{\partial d_1} + d_2 \frac{\partial U}{\partial d_2} + \text{—}\right]$$

$$= \left[d_1 K_1 + d_2 K_2 + d_3 K_3\right]$$

$$U = \frac{m_1 r_1^3 + m_2 r_2^3 + m_3 r_3^3}{r_1^3 r_2^3 r_3^3} \left[m_1 r_1^3 d_1^2 + m_2 r_2^3 d_2^2 + m_3 r_3^3 d_3^2\right]$$



$$\xi = \frac{m_1 x_1 r_1^3 + m_2 x_2 r_2^3 + \text{—}}{m_1 r_1^3 + m_2 r_2^3 + \text{—}}$$

$$\xi - x_1 = \frac{m_2(x_2 - x_1) r_2^3 + m_3 (x_3 - x_1) r_3^3}{m_1 r_1^3 + m_2 r_2^3 + m_3 r_3^3}$$

$$d_1 \cos(\alpha_2 + a_2) =$$

$$= \frac{m_2 r_3 \cos\alpha_2 \, r_2^3 + m_3 r_2 \cos\alpha_3 \, r_3^3}{m_1 r_1^3 + m_2 r_2^3 + m_3 r_3^3}$$

$$\eta - y_1 = \frac{m_2 (y_2 - y_1) r_2^3 + m_3 (y_3 - y_1) r_3^3}{m_1 r_1^3 + \ldots}$$

$$d_1 \sin(\alpha_2 + a_2) = \frac{m_2 r_3 \sin\alpha_2 \, r_2^3 + m_3 r_2 \sin\alpha_3 \, r_3^3}{m_1 r_1^3 + \text{—}}$$

$$d_1^2 \left[\cos^2(\alpha_2 + a_2) + \sin^2(\alpha_2 + a_2)\right] = \frac{m_2^2 r_3^2 r_2^6 + m_3^2 r_2^2 r_3^6 +}{(m_1 r_1^3 + m_2 r_2^3 + m_3 r_3^3)^2}$$

$$+ 2 m_2 m_3 r_2^4 r_3^4 \cos(\alpha_2 - \alpha_3)$$

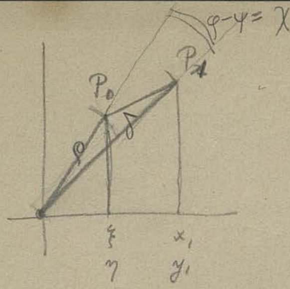
$$d_1 = \frac{\sqrt{m_2^2 r_3^2 r_2^6 + m_3^2 r_2^2 r_3^6 + 2 m_2 m_3 r_2^4 r_3^4 \cos(\alpha_2 - \alpha_3)}}{m_1 r_1^3 + m_2 r_2^3 + m_3 r_3^3}$$

~~$= \sqrt{m_2^2 r_3^2 r_2^6 + \ldots + 2 \ldots \cos(a_2 + a_3)}$~~

$$= \frac{r_2 r_3 \sqrt{m_2^2 r_2^4 + m_3^2 r_3^4 + 2 m_2 m_3 r_2^2 r_3^2 \cos(a_2 + a_3)}}{m_1 r_1^3 + m}$$

$$- \delta^2 \cos^2\psi \frac{d\psi}{dt} - \delta^2 \sin^2\psi \frac{d\psi}{dt}$$

$$= - \delta^2 \frac{d\psi}{dt}$$



$$\xi \frac{dy_1}{dt} - y_1 \frac{d\xi}{dt} + x_1 \frac{d\eta}{dt} - \eta \frac{dx_1}{dt}$$

$$\xi = \rho \cos\varphi$$
$$\eta = \rho \sin\varphi$$
$$x_1 = \delta \cos\psi$$
$$y_1 = \delta \sin\psi$$

$$x \frac{dy}{dt} - y \frac{dx}{dt}$$

$$x = r\cos\varphi$$
$$y = r\sin\varphi$$

$$r\cos\varphi\left[\frac{dr}{dt}\sin\varphi + r\cos\varphi\frac{d\varphi}{dt}\right] - r\sin\varphi\left[\frac{dr}{dt}\cos\varphi - r\sin\varphi\frac{d\varphi}{dt}\right] =$$

$$= r^2\cos^2\frac{d\varphi}{dt} + r^2\sin^2\frac{d\varphi}{dt} = r^2\frac{d\varphi}{dt}$$

$$\rho\cos\varphi\left[\frac{d\delta}{dt}\sin\psi + \delta\cos\psi\frac{d\psi}{dt}\right] - \delta\sin\psi\left[\frac{d\rho}{dt}\cos\varphi - \rho\sin\varphi\frac{d\varphi}{dt}\right]$$

$$+ \delta\cos\psi\left[\frac{d\rho}{dt}\sin\varphi + \rho\cos\varphi\frac{d\varphi}{dt}\right] - \rho\sin\varphi\left[\frac{d\delta}{dt}\cos\psi - \delta\sin\psi\frac{d\psi}{dt}\right]$$

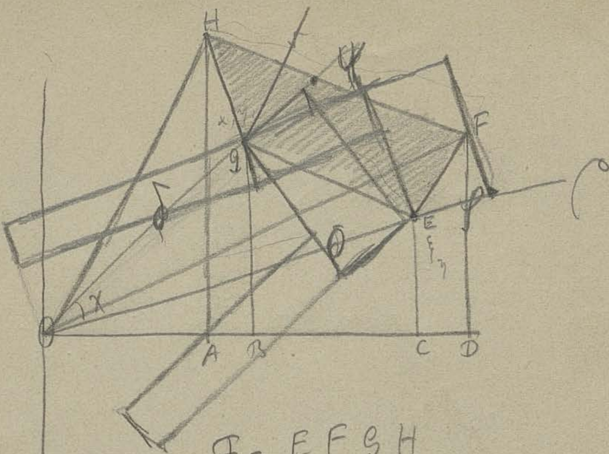
$$= \frac{d\delta}{dt}\left[\rho\cos\varphi\sin\psi - \rho\sin\varphi\cos\psi\right] + \frac{d\psi}{dt}\left[\rho\delta\cos\varphi\cos\psi + \right.$$

$$\left. + \rho\delta\sin\varphi\sin\psi\right] + \frac{d\varphi}{dt}\left[\delta\rho\sin\varphi\sin\psi + \delta\rho\cos\varphi\cos\psi\right]$$

$$+ \frac{d\rho}{dt}\left[\delta\cos\psi\sin\varphi - \delta\sin\psi\cos\varphi\right]$$

$$= -\frac{d\delta}{dt}\sin\chi\cdot\rho + \frac{d\psi}{dt}\cdot\rho\cdot\delta\cos\chi + \frac{d\varphi}{dt}\cdot\rho\cdot\delta\cos\chi$$

$$+ \frac{d\rho}{dt}\sin\chi\cdot\delta$$



$\mathcal{F} = EFGH$

$= ADFH - ABGH - BCEH - CDEF$

$=$

$= \frac{1}{2}\left[\xi + \frac{d\xi}{dt} - x_1 - \frac{dx_1}{dt}\right]\left[\eta + \frac{d\eta}{dt} + y_1 + \frac{dy_1}{dt}\right]$

$- \frac{dx_1}{dt}\left(y_1 + y_1 + \frac{dy_1}{dt}\right)\frac{1}{2} - \frac{1}{2}(\xi - x_1)(y_1 + \eta) - \frac{1}{2}$

$- \frac{1}{2}\frac{d\xi}{dt}\left(\eta + \eta + \frac{d\eta}{dt}\right)$

$= \frac{1}{2}\Big)\ \eta\xi + \eta\frac{d\xi}{dt} - \eta x_1 - \eta\frac{dx_1}{dt} + \xi\frac{d\eta}{dt} + \frac{d\xi}{dt}\frac{d\eta}{dt} -$

$- x_1\frac{d\eta}{dt} - \frac{dx_1}{dt}\frac{d\eta}{dt} + y_1\xi + y_1\frac{d\xi}{dt} - y_1 x_1 - y_1\frac{dx_1}{dt} +$

$+ \xi\frac{dy_1}{dt} + \frac{d\xi}{dt}\frac{dy}{dt} - x\frac{dy}{dt} - \frac{dx}{dt}\frac{dy}{dt} - 2y\frac{dx}{dt} - \frac{dy}{dt}\frac{dx}{dt}$

$- \xi y + xy - \xi\eta + x\eta - \eta\frac{d\xi}{dt} - \frac{d\eta}{dt}\frac{d\xi}{dt}$

$$K_1 : K_2 = m_1 d_1 r_1^3 : m_2 d_2 r_2^3$$

$$K_1 : m_1 d_1 r_1^3 = K_2 : m_2 d_2 r_2^3$$

$$\frac{K_1}{m_1 d_1 r_1^3} = \frac{K_2}{m_2 d_2 r_2^3} = \frac{K_3}{m_3 d_3 r_3^3}$$

$$m_1 \sqrt{\left(\frac{ds_1}{dt}\right)^2 - C_1} = m_2 \sqrt{\left(\frac{ds_2}{dt}\right)^2 - C_2}$$

$$m_1 v_1 \frac{dv_1}{dt} = m_2 v_2 \frac{dv_2}{dt}$$

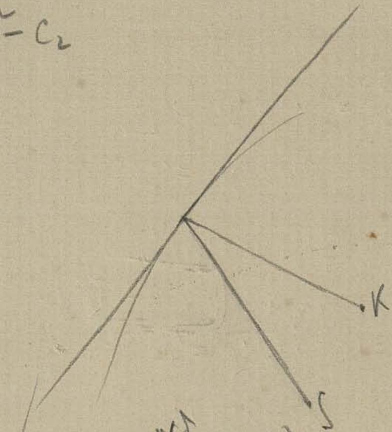
$$\frac{d^2 s}{dt^2}$$

$$v_1 B_1 = v_2 B_2$$

$$v_1 : v_2 = B_2 : B_1$$

$$m_1 v_1 .$$

$$\delta_1 v_1 \sin \lambda_1 + \delta_2 v_2 \sin \lambda_2 + \delta_3 v_3 \sin \lambda_3 = C$$



$$\delta_1 v_1 \sin \lambda_1$$

$$\delta_1 v_1 \sin \lambda_1 = \delta_1 \frac{v_1 B_2}{B_1} \sin \lambda_1$$

$$\Sigma m \frac{d^2 x}{dt^2}$$

$$\xi - x_1 = \frac{m_2 (x_2 - x_1) r_2^3 + m_3 (x_3 - x_1) r_3^3}{m_1 r_1^3 + m_2 r_2^3 + m_3 r_3^3}$$

$$f(d_1 d_3 r_2) = \frac{1}{2}\Big[(\xi - x_1)(\eta + y_1) + (x_3 - \xi)(y_3 + \eta) -$$

$$- (x_3 - x_1)(y_1 + y_3)\Big]$$

$$\xi \frac{dy_1}{dt} - y_1 \frac{d\xi}{dt} - \xi \frac{d\eta}{dt} + \eta \frac{d\xi}{dt} - x_1 \frac{dy_1}{dt} + y_1 \frac{dx_1}{dt}$$

$$+ x_1 \frac{d\eta}{dt} - \eta \frac{dx_1}{dt}$$

$$= \frac{d}{dt}\left(\frac{y_1}{\xi}\right) \cdot \xi^2 - \frac{d}{dt}\left(\frac{\eta}{\xi}\right) \cdot \xi^2 - \frac{d}{dt}\left(\frac{y_1}{x_1}\right) \cdot x_1^2 + \frac{d}{dt}\left(\frac{\eta}{x_1}\right) x_1^2$$

$$= \xi^2 \frac{d}{dt}\left(\frac{y_1 - \eta}{\xi}\right) + x_1^2 \frac{d}{dt}\left(\frac{\eta - y_1}{x_1}\right)$$

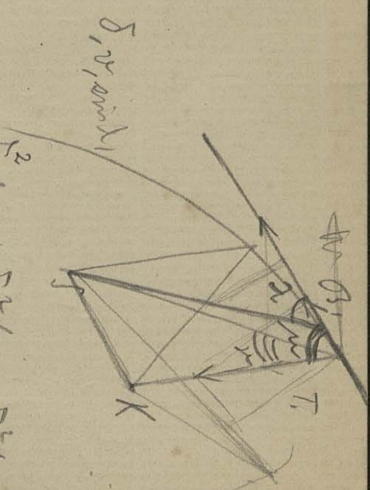
$$\vartheta_1 = K_1 \cos\mu$$

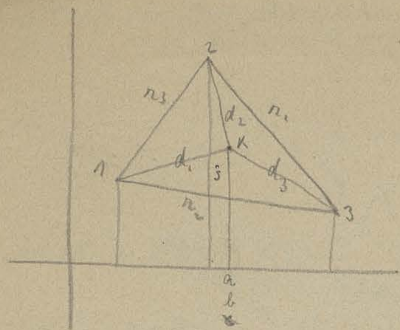
$$T_1 = K_1 \sin\mu = \frac{v_1^2 m_1}{\rho}$$

$$v_1 = \frac{\delta_1 \, d\varphi_1}{\sin\lambda_1} \quad v_2 = \quad v_3 =$$

$$v_1 : v_2 : v_3 = \frac{1}{P_1} : \frac{1}{P_2} : \frac{1}{P_3}$$

$$\delta_1^2 \frac{d\varphi_1}{dt} + \delta_2^2 \frac{d\varphi_2}{dt} + \delta_3^2 \frac{d\varphi_3}{dt} = C$$





$$\frac{d^2x_1}{dt^2} : \frac{d^2x_2}{dt^2} : \frac{d^2x_3}{dt^2} = m_1(a-x_1)r_1^3 : m_2(a-x_2)r_2^3 : m_3(a-x_3)r_3^3$$

$$\frac{d^2y_1}{dt^2} : \frac{d^2y_2}{dt^2} : \frac{d^2y_3}{dt^2} = m_1(b-y_1)r_1^3 : m_2(b-x_2)r_2^3 : m_3(b-x_3)r_3^3$$

$$\frac{d^2x_1}{dt^2} : \frac{d^2x_2}{dt^2} = \frac{dv_1}{dt} : \frac{dv_2}{dt} = \frac{dv_1}{dv_2} = \frac{m_1}{m_2}\,\frac{a-x_1}{a-x_2}\left(\frac{r_1}{r_2}\right)^3$$

$$\frac{d^2x_1}{dt^2} = \frac{m_1(a-x_1)\left[m_1r_1^3+m_2r_2^3+m_3r_3^3\right]}{r_2^3r_3^3} \qquad \frac{d^2y_1}{dt^2} = \frac{m_1(b-y_1)\left[m_1r_1^3+m_2r_2^3+m_3r_3^3\right]}{r_2^3r_3^3}$$

$$\frac{v_1}{v_2} = \frac{m_1}{m_2}\,\frac{a-x_1}{a-x_2}\left[\frac{(x_3-x_2)^2+(y_3-y_2)^2}{(x_3-x_1)^2+(y_3-y_1)^2}\right]^{\frac{3}{2}}$$

$$\frac{v_1}{v_3} = \frac{m_1}{m_3}\,\frac{a-x_1}{a-x_3}\left[\frac{(x_3-x_2)^2+(y_3-y_2)^2}{(x_2-x_1)^2+(y_2-y_1)^2}\right]^{\frac{3}{2}}$$

$$K_1 = \frac{m_1d_1r_1^3\left[m_1r_1^3+m_2r_2^3+m_3r_3^3\right]}{r_1^3r_2^3r_3^3}$$

$$K_1+K_2+K_3 = \frac{\left[m_1r_1^3+m_2r_2^3+m_3r_3^3\right]\left[m_1d_1r_1^3+m_2d_2r_2^3+m_3d_3r_3^3\right]}{r_1^3r_2^3r_3^3}$$

$$X_1 = \frac{\partial U_1}{\partial x_1} \qquad Y_1 = \frac{\partial U_1}{\partial y_1} \qquad U_1 = ?$$

$$X_1 = \frac{m_1m_2}{r_3^2}\,\frac{x_2-x_1}{r_3} + \frac{m_1m_3}{r_2^2}\,\frac{x_3-x_1}{r_2} \qquad Y_1 = \frac{m_1m_2}{r_3^2}\,\frac{y_2-y_1}{r_3} - \frac{m_1m_3}{r_2^2}\,\frac{y_1-y_3}{r_2}$$

Voraussetzung U_1 = homog. fc. vom mten Grad

$$U_1 = f(x_1, x_2, y_1, y_2, x_3, y_3)$$

$$mU_1 = x_1\frac{\partial U_1}{\partial x_1} + x_2\frac{\partial U_1}{\partial x_2} + y_1\frac{\partial U_1}{\partial y_1} + y_2\frac{\partial U_1}{\partial y_2} + x_3\frac{\partial U_1}{\partial x_3} + y_3\frac{\partial U_1}{\partial y_3}$$

./.

Wenn vorläufig vorausges: dass U für $\begin{cases}x_1\\y_1\end{cases}\begin{cases}x_2\\y_2\end{cases}\begin{cases}x_3\\y_3\end{cases}$ dasselbe ist so hat

man ~~was~~ nach Euler'schen Gleichungen:

~~[illegible]~~

$$m\,U = x_1 X_1 + x_2 X_2 + x_3 X_3 + y_1 Y_1 + y_2 Y_2 + y_3 Y_3$$

nach Vereinfachungen:

$$= \left(\frac{m_1 m_2}{r_3} + \frac{m_1 m_3}{r_2} + \frac{m_2 m_3}{r_1}\right)(-1)$$

} ~~a priori~~ ~~ersichtlich~~ ~~[illegible]~~

also wirklich homog. fu. u. z. vom -1 Grade: $m = -1$

$$(\pm)\, U = \frac{m_1 m_2}{\pm\sqrt{(x_1-x_2)^2+(y_1-y_2)^2}} + \frac{m_1 m_3}{\pm\sqrt{(x_1-x_3)^2+(y_1-y_3)^2}} + \frac{m_2 m_3}{\pm\sqrt{(x_2-x_3)^2+(y_2-y_3)^2}}$$

zur Probe:

$$X_1 = \frac{\partial U}{\partial x_1} = \frac{m_1 m_2 (x_2 - x_1)}{\sqrt{[(x_2-x_1)^2+(y_1-y_2)^2]^3}} + \text{———}$$

$$= \frac{m_1 m_2}{r_3^3}(x_2 - x_1) + \frac{m_1 m_3}{r_2^3}(x_3 - x_1)$$ also stimmt

$$\frac{m_1}{2} v_1^2 - \frac{m_1}{2} v_{10}^2 = \frac{m_1 m_2}{r_{30}} - \frac{m_1 m_2}{r_3} + \frac{m_1 m_3}{r_{20}} - \frac{m_1 m_3}{r_2} + \frac{m_2 m_3}{r_{10}} - \frac{m_2 m_3}{r_1}$$

$$\frac{m_1 v_1^2}{2} = a_1 + \frac{m_1 m_2}{r_3} + \frac{m_1 m_3}{r_2} + \frac{m_2 m_3}{r_1}$$

$$\frac{m_2 v_2^2}{2} = a_2 + \text{— — — — — —}$$

$$\frac{m_3 v_3^2}{2} = a_3 + \text{— — — — — —}$$

$$\frac{m_1 v_1^2}{2} - \frac{m_2 v_2^2}{2} = a_1 - a_2$$

$$\frac{m_1 v_1^2}{2} - \frac{m_3 v_3^2}{2} = a_1 - a_3$$

$$m_1 v_1^2 - 2a_1 = m_2 v_2^2 - 2a_2$$

$$v_1^2 = e^2 + u_1^2 - 2 e u_1 \cos(I')$$

$$(I') = \chi + 180 - \varphi_1' - (I I' 0) \qquad \cos(I I' 0) = \frac{u_1^2 + d_1'^2 - d_1^2}{2 u_1 d_1'}$$

$$\cos(I') = -\cos(\chi - \varphi_1' - (I I' 0))$$

$$\cos(I') = \sin(\chi - \varphi_1') \sin(I I' 0) - \cos(\chi - \varphi_1') \cos(I I' 0)$$

$$v_1^2 = e^2 + u_1^2 - 2 e u_1 \left[\sin(\chi - \varphi_1') \sin \ldots - \cos(\chi - \varphi_1') \cos \ldots\right]$$

$$\lim \cos(I I' 0) = \lim \frac{u_1^2 + d_1'^2 - d_1^2}{2 u_1 d_1'} = \lim \left\{\frac{u_1}{2 d_1'} + \frac{d_1'^2 - d_1^2}{2 u_1 d_1'}\right\} = \frac{u_1}{2 d_1}$$

$$\lim \sin(I I' 0) = \lim \sqrt{\frac{\cancel{4u_1^2 d_1'^2} - u_1^4 - d_1'^4 - d_1^4 + 2u_1^2 d_1'^2 + 2u_1^2 d_1^2 + 2 d_1'^2 d_1^2}{4 u_1^2 d_1'^2}}$$

$$= \lim \sqrt{\frac{2u_1^2(d_1'^2 + d_1^2) - u_1^4 - (d_1'^2 - d_1^2)^2}{4u_1^2 d_1'^2}} = \sqrt{1 - \frac{u_1^2}{4 d_1^2}}$$

$$\lim v_1^2 = e^2 + u_1^2 - 2 e u_1 \left[\sqrt{1 - \frac{u_1^2}{4 d_1^2}} \sin(\chi - \varphi) - \frac{u_1}{2 d_1} \cos(\chi - \varphi)\right]$$

Distanz ~~zw.~~ Schwerp. u. Kraftmittelp.:

$$\frac{(m_1 x_1 r_1^3 + m_2 x_2 r_2^3 + m_1 x_3 r_3^3)(m_1 + m_2 + m_3) - (m_1 x_1 + m_2 x_2 + m_3 x_3)(m_1 r_1^3 + m_2 r_2^3 + m_3 r_3^3)}{(m_1 r_1^3 + m_2 r_2^3 + m_3 r_3^3)(m_1 + m_2 + m_3)}$$

$$= \xi - (\xi)$$

$$\frac{(m_1 y_1 r_1^3 + m_2 y_2 r_2^3 + \;\;)(m_1 + m_2 + m_3) - (m_1 y_1 + m_2 y_2 + m_3 y_3)(m_1 r_1^3 + m_2 r_2^3 + \dots)}{(m_1 r_1^3 + \dots)(m_1 + m_2 + \;\;)}$$

$$= \eta - (\eta)$$

$$\Delta = \sqrt{[\xi - (\xi)]^2 + [\eta - (\eta)]^2} = \frac{1}{(m_1 + m_2 + m_3)(m_1 r_1^3 + m_2 r_2^3 + m_3 r_3^3)} \Big\{$$

$$Y_2X_1 - Y_1X_2 = -\frac{m_1^2 m_2 m_3}{r_2^3 r_3^3}(x_3-x_1)(y_2-y_1) - \frac{m_1 m_2^2 m_3}{r_1^3 r_3^3}(x_2-x_1)(y_2-y_3) -$$

$$-\frac{m_1 m_2 m_3^2}{r_1^3 r_2^3}(x_3-x_1)(y_2-y_3) - \frac{m_1 m_2^2 m_3}{r_1^3 r_3^3}(x_3-x_2)(y_2-y_1) + \frac{m_1^2 m_2 m_3 (x_2-x_1)(y_1-y_3)}{r_3^3 r_2^3} +$$

$$+\frac{m_1 m_2 m_3^2}{r_1^3 r_2^3}(x_3-x_2)(y_1-y_3) =$$

$$= -\frac{m_1^2 m_2 m_3}{r_2^3 r_3^3}\left[x_3y_2 - x_1y_2 - x_3y_1 + \cancel{x_1y_1} + x_2y_1 - \cancel{x_1y_1} - x_2y_3 + x_1y_3\right]$$

$$-\frac{m_1 m_2^2 m_3}{r_1^3 r_3^3}\left[\cancel{x_2y_2} - x_1y_2 - x_2y_3 + x_1y_3 + x_3y_2 - \cancel{x_2y_2} - x_3y_1 + x_2y_1\right]$$

$$-\frac{m_1 m_2 m_3^2}{r_1^3 r_2^3}\left[x_3y_2 - x_1y_2 - \cancel{x_3y_3} + x_1y_3 - x_3y_1 + x_2y_1 + \cancel{x_3y_3} - x_2y_3\right]$$

$$= -m_1 m_2 m_3\left[\;\cdots\;\right]\left[\frac{m_1}{r_2^3 r_3^3} + \frac{m_2}{r_1^3 r_3^3} + \frac{m_3}{r_1^3 r_2^3}\right]$$

$$\xi_{1,2} = \frac{\frac{m_1x_1}{(r_2r_3)^3} + \frac{m_2x_2}{(r_1r_3)^3} + \frac{m_3x_3}{(r_1r_2)^3}}{\frac{m_1}{(r_2r_3)^3} + \frac{m_2}{(r_1r_3)^3} + \frac{m_3}{(r_1r_2)^3}} = \frac{m_1x_1r_1^3 + m_2x_2r_2^3 + m_3x_3r_3^3}{m_1r_1^3 + m_2r_2^3 + m_3r_3^3}$$

$$\eta_{1,2} = \frac{Y_1}{X_1}\left[\xi_1 - x_1\right] + y_1 = \frac{Y_1}{X_1}\,\frac{(y_1-y_2)X_2Y_1 + X_1Y_2x_2 - \cancel{X_2Y_1x_1} - Y_2X_1x_1 + \cancel{Y_1X_2x_1}}{Y_2X_1 - Y_1X_2} + y_1 =$$

$$= \frac{y_1X_2Y_1^2 - y_2X_2Y_1^2 + X_1Y_1Y_2x_2 - X_1Y_1Y_2x_1 + y_1X_1^2Y_2 - y_1X_1X_2Y_1}{x_1(Y_2X_1 - Y_1X_2)}$$

$$\xi = \frac{y_1 m_1 + y_2 m_2}{m_1 + m_2} - \frac{y_2 m_2 + y_3 m_3}{m_2 + m_3} + \frac{(y_2 - y_1) m_2 + (y_3 - y_1) m_3}{(x_2 - x_1) m_2 + (x_3 - x_1) m_3}\,\xi_1 - \frac{(y_1 - y_3) m_1 + (y_2 - y_3) m_2}{(x_1 - x_3) m_1 + (x_2 - x_3) m_2}\,\xi_3$$

$$\frac{(y_2 - y_1) m_2 + (y_3 - y_1) m_3}{(x_2 - x_1) m_2 +} + \frac{(y_1 - y_3) m_1 +}{(x_1 - x_3) m_1 + (x_2 - x_3) m_2}$$

$$= \frac{(\eta_3 - y_1)(\xi_1 - x_1)(\xi_3 - x_3) + \xi_1 (\eta_1 - y_1)(\xi_3 - x_3) - \xi_3 (y_3 - y_3)(\xi_1 - x_1)}{(\eta_1 - y_1)(\xi_3 - x_3) - (y_3 - y_3)(\xi_1 - x_1)}$$

$\eta_3 \xi_1 - \eta_1 \xi_1 - y_3 x_1 + y_1 x_1 \mid \eta_1 \xi_3 - y_1 \xi_3 - \eta_1 x_3 + y_1 x_3 \mid y_3 \xi_1 - y_3 \xi_1 - y_3 x_1 + y_3 x_1$

$\eta_3 \xi_1 \xi_3 - \eta_1 \xi_1 \xi_3 - \eta_3 x_1 \xi_3 + y_1 x_1 \xi_3 - y_3 \xi_1 x_3 + \eta_1 \xi_1 x_3 + \eta_3 x_1 x_3 - y_1 x_1 x_3$

$+ \eta_1 \xi_1 \xi_3 - y_1 \xi_1 \xi_3 - \eta_1 \xi_1 x_3 + y_1 \xi_1 x_3 - \eta_3 \xi_1 \xi_3 + y_3 \xi_1 \xi_3 + \eta_3 x_1 \xi_3 - y_3 \xi_3 x_1$

$\eta_1 = \quad \xi_3 = \dfrac{x_1 m_1 + x_2 m_2}{m_1 + m_2} \qquad \xi_1 = \dfrac{x_2 m_2 + x_3 m_3}{m_2 + m_3}$

$$= \frac{(x_1 m_1 + x_2 m_2)(y_2 m_2 + y_3 m_3)}{(m_1 + m_2)(m_2 + m_3)} x_1 - \frac{(x_2 m_2 + x_3 m_3)(y_1 m_1 + y_2 m_2)}{(m_1 + m\,)(\quad)} x_3 + \frac{y_1 m_1 + y_2 m_2}{m_1 + m_2} x_1 x_3$$

$$- \frac{y_2 m_2 + y_3 m_3}{m_2 + m_3} x_1 x_3 + (y_3 - y_1) \frac{(x_1 m_1 + x_2 m_2)(x_2 m_2 + x_3 m_3)}{(\quad)(\quad)} + y_1 x_3 \frac{x_2 m_2 + x_3 m_3}{m_2 + m_3} - y_3 x_1 \frac{x_1 m_1 + x_2 m_2}{m_1 + m_2}$$

$x_1^2 y_2 m_1 m_2 + x_1 x_2 y_2 m_2^2 + x_1 y_3 m_1 m_3 + x_2 y_3 m_2 m_3 x_1 - x_2 x_3 y_1 m_1 m_2 - x_3^2 y_1 m_1 m_3 +$

$+ x_2 x_3 y_2 m_2^2 + x_3^2 y_2 m_2 m_3 + y_1 x_1 x_3 m_1 m_2 + y_2 x_1 x_3 m_2^2 + y_1 x_1 x_3 m_1 m_3 + y_2 x_1 x_3 m_2 m_3$

$- y_2 x_1 x_3 m_1 m_2 - y_3 x_1 x_3 m_1 m_3 - y_2 x_1 x_3 m_2^2 - y_3 x_1 x_3 m_2 m_3 + y_3 x_1 x_2 m_1 m_2 +$

$+ y_3 x_1 x_3 m_1 m_3 + y_3 x_2 x_3 m_1 m_3 + y_3 x_2^2 m_2^2 - y_1 x_1 x_2 m_1 m_2 - y_1 x_1 x_3 m_1 m_3 -$

$- y_1 x_2 x_3 m_1 m_3 - y_1 x_2^2 m_2^2 + y_1 x_2 x_3 m_1 m_2 + y_1 x_3^2 m_1 m_3 + y_1 x_3 x_2 m_2^2 + y_1 x_3^2 m_2 m_3 -$

$- y_3 x_1^2 m_1 m_2 - y^3 x_1 x_2 m_2^2 - y_3 x_1^2 m_1 m_3 - y_3 x_1 x_2 m_2 m_3$

$= x_1^2 m_1 [y_2 - y_3] + x_1 x_2 [-y_1 m_1 + y_2 m_2 + y_3 m_1 - y^3 m_2] + x_1 x_3 [y_1 m_1 + y_2 m_3 - y_2 m_1$

$- y_3 m_3] + x_2^2 [y_3 m_2 - y_1 m_2] + x_2 x_3 [-y_1 m_3 + y_1 m_1 m_2 + y_1 m_2 + y_2 m_2 + y_3 m_3] + x_3^2 [y_1 m_3 + y_2 m_3$

d_1

d_1

$K_1 =$

K_1

$$= x_1^2 m_1 [y_2 - y_3] + x_1 x_2 [m_1(-y_1+y_3) + m_2(y_2-y_3)] + x_1 x_3 [m_1(y_1-y_2) + m_3(y_2-y_3)] +$$

$$+ x_2^2 m_2 [y_3 - y_1] + x_2 x_3 [m_2(y_1+y_2) + m_3(-y_1+y_3)] + x_3^2 m_3 [y_1 + y_2]$$

$$\overline{m_2(m_1+m_2)(m_1+m_3)}$$

$$\left[\frac{y_2 m_2 + y_3 m_3}{m_2+m_3} - y_1\right]\left[\frac{x_1 m_1 + x_2 m_2}{m_1+m_2} - x_3\right] - \left[\frac{y_1 m_1 + y_2 m_2}{m_1+m_2} - y_3\right]\left[\frac{x_2 m_2 + x_3 m_3}{m_2+m_3} - x_1\right]$$

$$[y_2 m_2 + y_3 m_3 - y_1 m_2 - y_1 m_3][x_1 m_1 + x_2 m_2 - x_3 m_1 - x_3 m_2] - [y_1 m_1 + y_2 m_2 - y_3 m_1 - y_3 m_2][x_2 m_2 + x_3 m_3 - x_1 m_2 - x_1 m_3]$$

$$= y_2 m_2 x_1 m_1 + y_3 m_3 x_1 m_1 - y_1 m_2 x_1 m_1 - y_1 m_3 x_1 m_1 + y_2 m_2 x_2 m_2 + y_3 m_3 x_2 m_2 - y_1 m_2 x_2 m_2 - y_1 m_3 x_2 m_2$$

$$- y_2 m_2 x_3 m_1 - y_3 m_3 x_3 m_1 + y_1 m_2 x_3 m_1 + y_1 m_3 x_3 m_1 - y_2 m_2 x_3 m_2 - y_3 m_3 x_3 m_2 + y_1 m_2 x_3 m_2 + y_1 x_3 m_3 m_2$$

$$- y_1 m_1 x_2 m_2 - y_2 m_2 x_2 m_2 + y_3 m_1 x_2 m_2 + y_3 m_2 x_2 m_2 - y_1 m_1 x_3 m_3 - y_2 m_2 x_3 m_3 + y_3 m_1 x_3 m_3 + y_3 m_2 x_3 m_3$$

$$+ y_1 m_1 x_1 m_2 + y_2 m_2 x_1 m_2 - y_3 m_1 x_1 m_2 - y_3 m_2 x_1 m_2 + y_1 m_1 x_1 m_3 + y_2 m_2 x_1 m_3 - y_3 m_1 x_1 m_3 - y_3 m_2 x_1 m_3$$

$$= x_1 [y_2 m_1 + y_2 m_2 - y_3 m_1 - y_3 m_2 + y_2 m_3 - y_3 m_3] = x_1 [m_1(y_2-y_3) + m_2(y_2-y_3) + m_3(y_2-y_3)]$$

$$+ x_2 [y_3 m_3 - y_1 m_2 - y_1 m_3 - y_1 m_1 + y_3 m_1 + y_3 m_2] + x_2 [m_1(-y_1+y_3) + m_2(-y_1+y_3) + m_3(-y_1+y_3)]$$

$$+ x_3 [-y_2 m_1 + y_1 m_1 - y_2 m_2 + y_1 m_2 + y_1 m_3 - y_2 m_3] + x_3 [m_1(y_1-y_2) + m_2(y_1-y_2) + m_3(y_1-y_2)]$$

$$= [m_1 + m_2 + m_3][x_1(y_2-y_3) + x_2(+y_3-y_1) + x_3(y_1-y_2)]$$

$$(y_1-y_2)[m_1 x_1 x_3 \qquad] + (y_3 - y_1)[x_1 x_2 m_1 + x_2^2 m_2 + x_2 x_3 m_3] +$$

$$+ (y_2-y_3)[x_1^2 m_1 + x_1 x_2 m_2 + x_1 x_3 m_3]$$

$$= (y_1-y_2) x_3 [m_1 x_1 \qquad] + (y_3-y_1) x_2 [x_1 m_1 + x_2 m_2 + x_3 m_3) + (y_2-y_3) x_1 [\cdots$$

$$\xi = \frac{m_1 x_1 + m_2 x_2 + m_3 x_3}{m_1 + m_2 + m_3}$$

!

$$\delta_1 = \sqrt{(\xi - x_1)^2 + (\eta - y_1)^2} = \frac{\sqrt{[m_2(x_2-x_1)+m_3(x_3-x_1)]^2+[m_2(y_2-y_1)+m_3(y_3-y_1)]^2}}{m_1+m_2+m_3}$$

$$\xi - x_1 = \frac{m_2(x_2-x_1)+m_3(x_3-x_1)}{m_1+m_2+m_3}$$

$$D = \sqrt{(\xi - \xi)^2 + \eta}$$

$$\frac{m_1x_1r_1^3+m_2x_2r_2^3+m_3x_3r_3^3}{m_1r_1^3+m_2r_2^3+m_3r_3^3} - \frac{m_1x_1+m_2x_2+m_3x_3}{m_1+m_2+m_3}$$

$$\cancel{m_1^2x_1r_1^3}+\cancel{m_1m_2x_2r_2^3}+m_1m_3x_3r_3^3+m_1m_2x_1r_1^3+\cancel{m_2^2x_2r_2^3}+m_2m_3x_3r_3^3+m_1m_3x_1r_1^3+$$

$$+m_2m_3x_2r_2^3+\cancel{m_3^2x_3r_3^3}-\cancel{m_1^2x_1r_1^3}-m_1m_2x_2r_1^3-m_1m_3x_3r_1^3-\cancel{m_2m_1x_1r_2^3}-\cancel{m_2^2x_2r_2^3}-$$

$$-m_2m_3x_3r_2^3-m_1m_3r_3^3x_1-m_2m_3x_2r_3^3-\cancel{m_3^2x_3r_3^3}$$

$$= r_1^3[m_1m_2x_1+m_1m_3x_1-m_1m_2x_2-m_1m_3x_3]$$

$$+r_2^3[m_2m_3x_2-m_2m_3x_3]+r_3^3[m_1m_3x_3+m_2m_3x_3-m_1m_3x_1-m_2m_3x_2]$$

$$= r_1^3m_1[m_2(x_1-x_2)+m_3(x_1-x_3)]+r_2^3m_2[m_2x_2-m_3x_3]+$$

$$+r_3^3m_3[m_1(x_3-x_1)+m_2(x_3-x_2)]$$

$$= r_1^3[\cancel{m_1}m_2x_1+\cancel{m_1}m_3x_1-\cancel{m_1}m_2x_2-\cancel{m_1}m_3x_3]m_1$$

$$+r_2^3[m_1\cancel{m_2}x_2+\cancel{m_2}m_3x_2-m_1\cancel{m_2}x_1-\cancel{m_2}m_3x_3]m_2$$

$$+r_3^3[m_1\cancel{m_3}x_3+m_2\cancel{m_3}x_3-m_1\cancel{m_3}x_1-m_2\cancel{m_3}x_2]m_3$$

Mi

$$m_1r_1^3[m_2(x_1-x_2)+m_3(x_1-x_3)]$$

$$+m_2r_2^3[m_1(x_2-x_1)+m_3(x_2-x_3)]$$

$$+m_3r_3^3[m_1(x_3-x_1)+m_2(x_3-x_2)]$$

$$d = \sqrt{(\xi - x)^2 + (\eta - y)^2} \qquad \xi = \frac{m_1 x_1 r_1^3 + m_2 x_2 r_2^3 + m_3 x_3 r_3^3}{m_1 r_1^3 + m_2 r_2^3 + m_3 r_3^3}$$

$$d_1 = \frac{\sqrt{\left[m_2(x_2 - x_1) r_2^3 + m_3(x_3 - x_1) r_3^3\right]^2 + \left[m_2(y_2 - y_1) r_2^3 + m_3(y_3 - y_1) r_3^3\right]^2}}{m_1 r_1^3 + m_2 r_2^3 + m_3 r_3^3}$$!

$$K_1 = \sqrt{X_1^2 + Y_1^2} = \sqrt{\left[\frac{m_2 m_1}{r_3^3}(x_2 - x_1) + \frac{m_3 m_1}{r_2^3}(x_3 - x_1)\right]^2 + \left[\frac{m_2 m_1}{r_3^3}(y_2 - y_1) - \frac{m_3 m_1}{r_2^3}(y_1 - y_3)\right]^2}$$

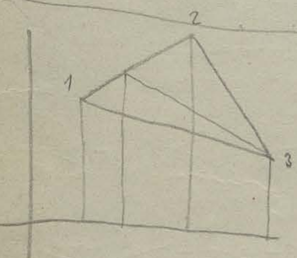
$$= \frac{m_1}{r_3^3 r_2^3} \sqrt{\left[m_2 r_2^3 (x_2 - x_1) + m_3 r_3^3 (x_3 - x_1)\right]^2 + \left[m_2 r_2^3 (y_2 - y_1) + m_3 r_3^3 (y_3 - y_1)\right]^2}$$

$$K_1 = \frac{m_1 d_1 \left[m_1 r_1^3 + m_2 r_2^3 + m_3 r_3^3\right]}{r_1^3 r_2^3 r_3^3}$$

$$r_1^2 = d_2^2 + d_3^2 - 2 d_2 d_3 \cos\alpha_1$$

$$(x_2 - x_1)(x_3 - x_1)$$
$$+ (x_3 - x_2)(x_1 - x_2)$$
$$+ (x_1 - x_3)(x_2 - x_3)$$

~~$x_2 x_3 - x_1 x_3 - x_2 x_1 + x_1^2 + x_1 x_3 - x_2 x_1$~~
~~$- x_2 x_3 + x_2^2 + x_1 x_2 - x_3 x_2 - x_1 x_3 + x_3^2$~~



$\mathcal{H}$?

$$(x_2 - \xi_3) : (\xi_3 - x_1) = m_1 : m_2$$

$$x_2 m_2 - \xi_3 m_2 = \xi_3 m_1 - x_1 m_1 \qquad \xi_3 = \frac{x_1 m_1 + x_2 m_2}{m_1 + m_2}$$

$$\eta - \eta_3 = \frac{\eta_3 - y_3}{\xi_3 - x_3}(\xi - \xi_3) \qquad \frac{\eta_3 - y_3}{\xi_3 - x_3} = \frac{(y_1 - y_3) m_1 + (y_2 - y_3) m_2}{(x_1 - x_3) m_1 + (x_2 - x_3) m_2}$$

$$\eta - \eta_1 = \frac{\eta_1 - y_1}{\xi_1 - x_1}(\xi - \xi_1)$$

$$\eta_3 - \eta_1 = \frac{\eta_1 - y_1}{\xi_1 - x_1}(\xi - \xi_1) - \frac{\eta_3 - y_3}{\xi_3 - x_3}(\xi - \xi_3)$$

$$\xi = \frac{\eta_3 - \eta_1 + \frac{\eta_1 - y_1}{\xi_1 - x_1}\xi_1 - \frac{\eta_3 - y_3}{\xi_3 - x_3}\xi_3}{\frac{\eta_1 - y_1}{\xi_1 - x_1} - \frac{\eta_3 - y_3}{\xi_3 - x_3}} = \frac{(\eta_3 - \eta_1)(\xi_1 - x_1)(\xi_3 - x_3) + \xi_1(\eta_1 - y_1)(\xi_3 - x_3) - \xi_3(\eta_3 - y_3)(\xi_1 - x_1)}{(\eta_1 - y_1)(\xi_3 - x_3) - (\eta_3 - y_3)(\xi_1 - x_1)}$$

$$\eta_{12} = \frac{Y_1}{X_1}\left[\xi_1 - x_1\right] + y_1 =$$

$$\xi_1 - x_1 = \frac{m_1 x_1 r_1^3 + m_2 x_2 r_2^3 + m_3 x_3 r_3^3 - m_1 x_1 r_1^3 - m_2 x_1 r_2^3 - m_3 x_1 r_3^3}{m_1 r_1^3 + m_2 r_2^3 + m_3 r_3^3}$$

$$= \frac{m_2 r_2^3 [x_2 - x_1] + m_3 r_3^3 [x_3 - x_1]}{m_1 r_1^3 + m_2 r_2^3 + m_3 r_3^3}$$

$$\frac{m_1 m_2^2 (x_2-x_1)(y_2-y_1) r_2^3}{r_3^3} - \frac{m_1 m_2 m_3 (x_2-x_1)(y_1-y_3) r_1^3}{r_1^3} + \frac{m_1 m_2 m_3 (x_3-x_1)(y_2-y_1) r_1^3}{r_1^3} -$$

$$- \frac{m_1 m_3^2 (x_3-x_1)(y_1-y_3) r_3^3}{r_2^3} + y_1 \left\{ \frac{m_1 m_2 (x_2-x_1) r_1^3}{r_3^3} + \frac{m_1 m_2^2 r_2^3 (x_2-x_1)}{r_3^3} + \frac{m_1 m_2 m_3 (x_2-x_1) r_1^3}{r_1^3} \right.$$

$$\frac{\left. + \frac{m_1 m_3 (x_3-x_1) r_1^3}{r_2^3} + \frac{m_1 m_2 m_3 (x_3-x_1) r_1^3}{r_1^3} + \frac{m_1 m_3^2 (x_3-x_1) r_3^3}{r_2^3} \right\}}{X_1 [\ldots\ldots}$$

m_1

$$= \frac{m_2^2 r_2^3}{r_3^3}\left[x_2 y_2 - x_1 y_2 - x_2 y_1 + x_1 y_1 + y_1 x_2 - y_1 x_1\right] - \frac{m_3^2 r_3^3}{r_2^3}\left[x_3 y_1 - x_1 y_1 - x_3 y_3 + x_1 y_3 - y_1 x_3 - y_1 x_1\right]$$

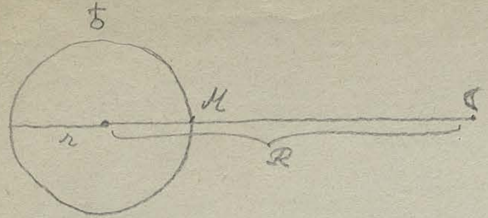
$$+ m_2 m_3 \left[-x_2 y_1 + x_1 y_1 + x_2 y_3 - x_1 y_3 + x_3 y_2 - x_1 y_2 - x_3 y_1 + x_1 y_1 + y_1 x_2 - y_1 x_1 + y_1 x_3 - y_1 x_1\right]$$

$$+ \frac{m_1 m_2 r_1^3}{r_3^3} y_1 (x_2 - x_1) + \frac{m_1 m_3 r_1^3}{r_2^3} y_1 (x_3 - x_1) \quad : \left[\frac{m_2}{r_3^3}(x_2 - x_1) + \frac{m_3}{r_2^3}(x_3 - x_1)\right] r_3^3 r_2^3$$

$$= \frac{m_2^2 r_2^6 y_2 [x_2 - x_1]}{r_1^3} + \frac{m_3^2 r_3^6 y_3 [x_3 - x_1]}{r_1^3} + m_2 m_3 \left[y_3 (x_2 - x_1) + y_2 (x_3 - x_1)\right] +$$

$$+ m_1 m_2 r_1^3 r_2^3 y_1 (x_2 - x_1) + m_1 m_3 r_1^3 r_3^3 y_1 (x_3 - x_1) \Big/ : \frac{\left[m_2 r_2^3 (x_2 - x_1) + m_3 r_3^3 (x_3 - x_1)\right]}{r_3^3 r_2^3} \left[m_1 r_1^3 + m_2 r_2^3 + m_3 r_3^3\right]$$

$$= \frac{m_1 r_1^3 y_1 + m_2 r_2^3 y_2 + m_3 r_3^3 y_3}{m_1 r_1^3 + m_2 r_2^3 + m_3 r_3^3}$$



$K = \frac{k.m.M}{r^2}$ $R = 60.2778\,r$

~~mittlere Distanz~~

Anz. ♁ Mittelp. : $M = \frac{1}{R^2} : \frac{1}{(R-r)^2} = r^2 - 2Rr + R^2 : R^2$

$\neq 1 - \frac{2r}{R} = 1 - \frac{2}{60.28}$

1 : 30.14 = 0.03318
958
54
24

1 : 0.96682

1 : 0.967 in mittlerer Distanz

1 : 0.033

im Perig. R = 0.9451 . 60.278
301390
241112
542502
56.96974

1 : 0.965

2 : 56.97 = 0.03510s
2909
60
3

1 : 0.035 im Perig.

für Sonne : (auf Mondmasse reduciert)

Anz. a ☉ ~~[illegible]~~ : Anz. a ☾ in mittl. Entf. $= \frac{1}{60.278^2 \cdot 858^2} : \frac{326800 \cdot 79.67}{20{,}000000^2}$

= 4,00 0.000,0 01.000 : - - - -

= 1 : 1739

= 0.57516 : 1.00000

3.51428
3.56032
1.90129
3.86626
12.84235
12.60206
0.24021

0.75979
0.57516
1739

Vorteil:

$1 : \frac{1}{(1 - 0.0168)^2} = 1 - 0.0336 : 1$
$= 0.9664 : 1$

0·57516 : 0·9664 = 0·5952

~~539~~

9196

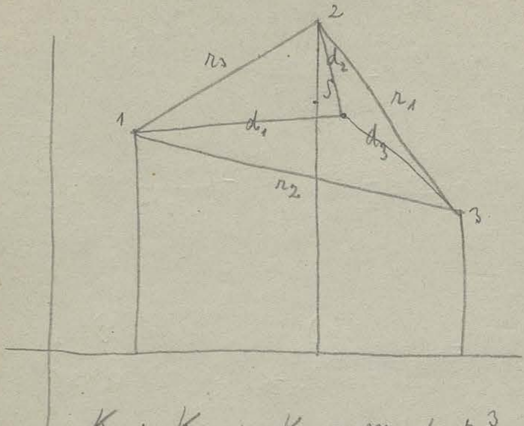
498

15

0·5952 : 1·0000 im Perihel

1 :

$$K_1 = \frac{m_1 d_1 [m_1 r_1^3 + m_2 r_2^3 + m_3 r_3^3]}{r_2^3 r_3^3}$$



$d_1^2 \frac{d\varphi}{dt}$

$m_1 \varrho_1^2 \frac{d\varphi_1}{dt}$

$$(\xi) = \frac{m_1 x_1 + m_2 x_2 + m_3 x_3}{m_1 + m_2 + m_3}$$

$$(\eta) = \frac{m_1 y_1 + m_2 y_2 + m_3 y_3}{m_1 + m_2 + m_3}$$

$$\varrho_1 = \frac{\sqrt{[m_2(x_2 - x_1) + m_3(x_3 - x_1)]^2 + [m_2(y_2 - y_1) + m_3(y_3 - y_1)]^2}}{m_1 + m_2 + m_3}$$

$$= \sqrt{}$$

$$K_1 : K_2 : K_3 = m_1 d_1 r_1^3 : m_2 d_2 r_2^3 : m_3 d_3 r_3^3$$

$$r_1 = \sqrt{d_2^2 + d_3^2 - 2 d_2 d_3 \cos\varphi_1} = \sqrt{r_2^2 + r_3^2 - 2 r_2 r_3 \cos\alpha_1}$$

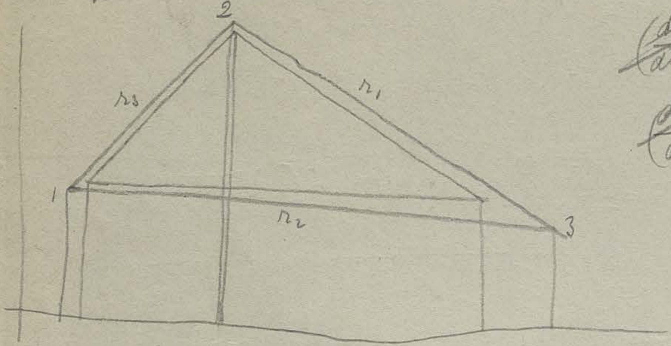
$$d_2^2 = r_3^2 + d_1^2 - 2 d_1 r_3 \cos\beta_2$$

$$r_2^2 = d_1^2 + d_3^2 - 2 d_1 d_3 \cos\varphi_2$$

$$r_3^2 = d_1^2 + d_2^2 - 2 d_1 d_2 \cos\varphi_3$$

$$m_1 \frac{d^2x_1}{dt^2} + m_2 \frac{d^2x_2}{dt^2} + m_3 \frac{d^2x_3}{dt^2} = 0$$

$$m_1 \frac{d^2y_1}{dt^2} + m_2 \frac{d^2y_2}{dt^2} + m_3 \frac{d^2y_3}{dt^2} = 0$$



$$r_3^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2$$

$$\frac{\partial r_3}{\partial(x_2 - x_1)} = \frac{x_2 - x_1}{r_3} = \frac{\partial r_3}{dx_1} \; ?$$

~~$\left(\frac{dv_1}{dt}\right)^2 + \frac{m_1}{2}\left(\frac{dv}{dt}\right)^2 = \frac{m_1 m_2}{r_3 - r_3'}$~~

~~$\left(\frac{dv_1}{dt}\right)^2 + \; = \frac{m_1 m_2 (r_3^2 - r_3')}{r_3^2} + \frac{m_1 m_3 (r_2^2 - r_2')}{r_2^2}$~~

$$X_1 = \frac{m_1 m_2}{r_3^2} \frac{\partial r_3}{\partial(x_2 - x_1)} + \frac{m_1 m_3}{r_2^2} \frac{\partial r_2}{\partial(x_3 - x_1)}$$

$$= \frac{m_1 m_2}{r_3^2} \frac{\partial r_3}{\partial x_1} + \frac{m_1 m_3}{r_2^2} \frac{\partial r_2}{\partial x_1} \; (?)$$

$$\left(\frac{dv_1}{dt}\right)^2 + \left(\frac{dv_2}{dt}\right)^2 + \left(\frac{dv_3}{dt}\right)^2 = m_1 m_2$$

$$\frac{m_1}{2}\int \frac{dv_2}{dt} \quad \frac{d\left(\frac{m_1 v_1^2}{2}\right)}{dt} = X_1 \frac{dx_1}{dt} + Y_1 \frac{dy_1}{dt} + \ldots$$

$$\frac{d\left(\frac{v_1^2}{2}\right)}{dt} = \frac{m_2}{r_3^2} \frac{\partial r_3}{\partial x_1} \frac{dx_1}{dt} + \frac{m_3}{r_2^2} \frac{\partial r_2}{\partial x_1} \frac{dx_1}{dt} + \frac{m_2}{r_3^2} \frac{\partial r_3}{\partial y_1} \frac{dy_1}{dt} + \frac{m_3}{r_2^2} \frac{\partial r_2}{\partial y_1} \frac{dy_1}{dt}$$

$$= \frac{m_2}{r_3^2}\left[\frac{\partial r_3}{\partial x_1} \frac{dx_1}{dt} + \frac{\partial r_3}{\partial y_1} \frac{dy_1}{dt}\right] + \frac{m_3}{r_2^2}\left[\frac{\partial r_2}{\partial x_1} \frac{dy_1}{dt} + \frac{\partial r_2}{\partial y_1} \frac{dy_1}{dt}\right]$$

$$\frac{1}{2} \frac{d(v_1)^2}{dt} = \frac{m_2}{r_3^2} \frac{dr_3}{dt} + \frac{m_3}{r_2^2} \frac{dr_2}{dt} \quad (?)$$

40 : 49 = ~~0.8~~   0.8 } 2   1.6

60 : 121 = 0.5   1.0

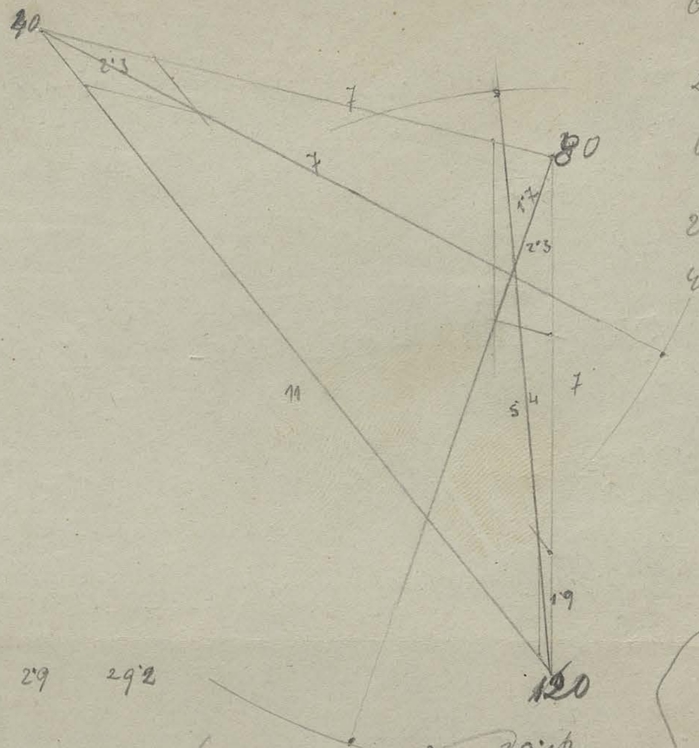
20 : 49 = 0.4 } 4   0.8

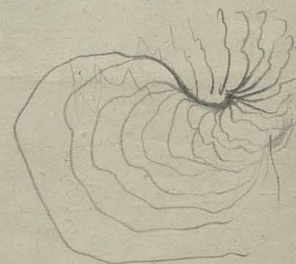
60 : 49 = 1.2   2.4

20 : 121 = 0.16   0.32

40 : 49 = 0.8   1.6

67





49   2.9   29.2

6	5.4	1.9	6
10	7	2.3	2
8	1.7	2.3	4

$5.4^2 = 29.16$

$7^2 = 49$

$1.7^2 = 3$

$w = \frac{m}{e^2}$ $e = \sqrt{\frac{m}{w}}$

$120 : 1.9 = \sqrt{63} = 8$ $160 : 2.3 = \sqrt{70} = 8.4$ $200 : 2.3 = \sqrt{87} = 9.3$

$240 : 1.9 = \sqrt{126} = 12$ $240 : 2.3 = \sqrt{100} = 10$ $240 : 2.3 =$ 10

$$X_1 X_2 = - \left(\frac{m_1 m_2}{r_3^2}\right)^2 \left(\frac{x_2-x_1}{r_3}\right)^2 - \frac{m_1^2 m_2 m_3}{r_2^2 r_3^2}\,\frac{(x_3-x_1)(x_2-x_1)}{r_2 r_3} + \frac{m_1 m_2^2 m_3}{r_1^2 r_3^2}\,\frac{(x_3-x_2)(x_2-x_1)}{r_1 r_3}$$

$$+ \frac{m_1 m_2 m_3^2}{r_1^2 r_2^2}\,\frac{(x_3-x_1)(x_3-x_2)}{r_1 r_2}$$

$$X_2 X_3 = \frac{m_1^2 m_2 m_3}{r_2^2 r_3^2}\,\frac{(x_2-x_1)(x_3-x_1)}{r_2 r_3} - \frac{m_1 m_2 m_3^2}{r_1^2 r_2^2}\,\frac{(x_3-x_1)(x_3-x_2)}{r_1 r_2} + \frac{m_2^2 m_1 m_3}{r_1^2 r_3^2}\,\frac{(x_2-x_1)(x_3-x_1)}{r_1 r_3} -$$

$$- \frac{m_2^2 m_3^2}{r_1^4}\,\frac{(x_3-x_1)^2}{r_1^2}$$

$$X_1 Y_2 = - \frac{m_1^2 m_2^2}{r_3^4}\,\frac{(x_2-x_1)(y_2-y_1)}{r_3^2} + \frac{m_1^2 m_2 m_3}{r_2^2 r_3^2}\,\frac{(x_3-x_1)(y_2-y_1)}{r_2 r_3} - \frac{m_1 m_2^2 m_3}{r_1^2 r_3^2}\,\frac{(x_2-x_1)(y_2-y_3)}{r_1 r_3} -$$

$$- \frac{m_1 m_2 m_3^2}{r_1^2 r_2^2}\,\frac{(x_3-x_1)(y_2-y_3)}{r_1 r_2}$$

$$X_2 Y_1 = - \frac{m_1^2 m_2^2}{r_3^4}\,\frac{(x_2-x_1)(y_2-y_1)}{r_3^2} + \frac{m_1 m_2^2 m_3}{r_3^2 r_1^2}\,\frac{(x_3-x_2)(y_1-y_2)}{r_1 r_3} + \frac{m_1^2 m_2 m_3}{r_3^2 r_2^2}\,\frac{(x_2-x_1)(y_1-y_3)}{r_3 r_2}$$

$$- \frac{m_1 m_2 m_3^2}{r_1^2 r_2^2}\,\frac{(x_3-x_2)(y_1-y_3)}{r_1 r_2}$$

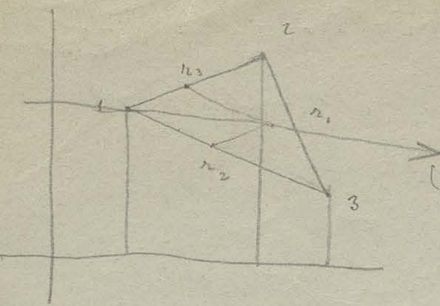
$$\xi_{1,2} = - \frac{m_1^2 m_2^2 (x_2-x_1)^2 (y_1-y_2)}{r_3^6} - \frac{m_1^2 m_2 m_3 (x_3-x_1)(x_2-x_1)(y_1-y_2)}{r_2^3 r_3^3} + \frac{m_1 m_2^2 m_3 (x_2-x_1)(x_3-x_2)(y_1-y_2)}{r_1^3 r_3^3}$$

$$+ \frac{m_1 m_2 m_3^2 (x_3-x_1)(x_3-x_2)(y_1-y_2)}{r_1^3 r_2^3} + \frac{m_1^2 m_2^2 (x_2-x_1)(y_2-y_1)\, x_2}{r_3^6} - m_1^2 m_2 m_3$$

$$- \frac{m_1^2 m_2 m_3 (x_3-x_1)(y_2-y_1)\, x_2}{r_2^3 r_3^3} - \frac{m_1 m_2^2 m_3 (x_2-x_1)(y_2-y_3)\, x_2}{r_1^3 r_3^3} - \frac{m_1 m_2 m_3^2}{r_1^2 r_2^2}\,\frac{(x_3-x_1)(y_2-y_3)\, x_2}{r_1 r_2}$$

$$+ \frac{m_1^2 m_2^2 (x_2-x_1)(y_2-y_1)\, x_1}{r_3^6} - \frac{m_1 m_2^2 m_3 (x_3-x_2)(y_2-y_1)\, x_1}{r_1^3 r_3^3} - \frac{m_1^2 m_2 m_3 (x_2-x_1)(y_1-y_3)\, x_1}{r_2^3 r_3^3}$$

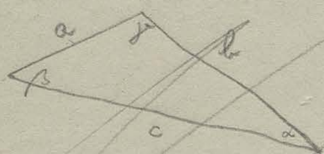
$$+ \frac{m_1 m_2 m_3^2 (x_3-x_2)(y_1-y_3)\, x_1}{r_1^3 r_2^3}$$



$$X_1 = \frac{m_2 m_1}{r_3^2} \frac{x_2 - x_1}{r_3} + \frac{m_3 m_1}{r_2^2} \frac{x_3 - x_1}{r_2}$$

$$X_2 = -\frac{m_2 m_1}{r_3^2} \frac{x_2 - x_1}{r_3} + \frac{m_2 m_3}{r_1^2} \frac{x_3 - x_2}{r_1}$$

$$X_3 = -\frac{m_1 m_3}{r_2^2} \frac{x_3 - x_1}{r_2} - \frac{m_2 m_3}{r_1^2} \frac{x_3 - x_2}{r_1}$$



$a, b, \gamma;\quad \operatorname{tg}\frac{\alpha-\beta}{2} = \frac{a-b}{a+b}\operatorname{ctg}\frac{\gamma}{2}$

$\operatorname{tg}\frac{\alpha+\beta}{2} = \frac{a+b}{a-b}\operatorname{ctg}\frac{\gamma}{2}$

$\alpha+\beta = 180 - \gamma$

$\frac{\alpha+\beta}{2} = 90° - \frac{\gamma}{2}$

$\frac{\alpha+\beta}{2} = \frac{\alpha-\beta}{2} + \beta = 90° - \frac{\gamma}{2}$

$\beta = 90° - \frac{\gamma}{2} - \frac{\alpha-\beta}{2}$

$$\operatorname{tg}\beta = \frac{\operatorname{ctg}\frac{\gamma}{2} - \operatorname{tg}\frac{\alpha-\beta}{2}}{1 - \operatorname{ctg}\frac{\gamma}{2}\cdot\operatorname{tg}\frac{\alpha-\beta}{2}}$$

$$\operatorname{tg}\beta = \frac{\operatorname{ctg}\frac{\gamma}{2}\left[1 - \frac{a-b}{a+b}\right]}{1 - \frac{a-b}{a+b}\left(\operatorname{ctg}\frac{\gamma}{2}\right)^2}$$

$$= \frac{2b\operatorname{ctg}\frac{\gamma}{2}}{a\left[1 - \operatorname{ctg}^2\frac{\gamma}{2}\right] + b\left[1 + \operatorname{ctg}^2\frac{\gamma}{2}\right]}$$

$$Y_1 = \frac{m_2 m_1}{r_3^2} \frac{y_2 - y_1}{r_3} - \frac{m_3 m_1}{r_2^2} \frac{y_1 - y_3}{r_2}$$

$$Y_2 = -\frac{m_2 m_1}{r_3^2} \frac{y_2 - y_1}{r_3} - \frac{m_2 m_3}{r_1^2} \frac{y_2 - y_3}{r_1}$$

$$Y_3 = \frac{m_1 m_3}{r_2^2} \frac{y_1 - y_3}{r_2} + \frac{m_2 m_3}{r_1^2} \frac{y_2 - y_3}{r_1}$$

$\operatorname{tg}\tau_1 = \frac{Y_1}{X_1}\quad \operatorname{tg}\tau_2 = \frac{Y_2}{X_2}\quad \operatorname{tg}\tau_3 = \frac{Y_3}{X_3}$

$\eta_1 - y_1 = \frac{Y_1}{X_1}\left[\xi_1 - x_1\right]$

$\eta_2 - y_2 = \frac{Y_2}{X_2}\left[\xi_2 - x_2\right]$

$y_1 - y_2 = \frac{Y_2}{X_2}\left[\xi_2 - x_2\right] - \frac{Y_1}{X_1}\left[\xi_1 - x_1\right]$

$(y_1 - y_2) X_2 X_1 = \left[Y_2 X_1 - Y_1 X_2\right]\xi - Y_2 X_1 x_2 + Y_1 X_2 x_1$

$$\xi_{1,2} = \frac{(y_1 - y_2) X_2 X_1 + X_1 Y_2 x_2 - X_2 Y_1 x_1}{Y_2 X_1 - Y_1 X_2}$$

$$\xi_{2,3} = \frac{(y_2 - y_3) X_3 X_2 + X_2 Y_3 x_3 - X_3 Y_2 x_2}{Y_3 X_2 - Y_2 X_3}$$

$$m(x_2-x_1)^2(y_2-y_1)+(x_2^2-x_1^2)(y_2-y_1)\cancel{(\ldots)} = (y_2-y_1)\;(x_2^2-2x_1x_2+x_1^2+x_2^2-x_1^2)$$

$$= 2x_2(y_2-y_1)(x_2-x_1)$$

$$-(x_3-x_1)(x_2-x_1)(y_1-y_2)-\cancel{(x_3-x_1)(y_2-y_3)x_2}+(x_2-x_1)(y_1-y_3)x_1$$

$$\cancel{=(x_3-x_1)\left[-x_2y_1+x_1y_1+x_2y_2-x_1y_2-x_2y_2+x_2y_1-y_2x_1+y_3x_1\right]}$$

$$=(x_3-x_1)(y_3-y_2)x_1 \quad = x_1\left[x_3y_1-\cancel{y_1x_1}-x_3y_2+x_1y_2-x_2y_1+\cancel{x_1y_2}+x_2y_3-x_1y_3\right]$$

$$(x_2-x_1)(x_3-x_2)(y_1-y_2)-(x_2-x_1)(y_2-y_3)x_2-\cancel{(x_3-x_2)(y_1-y_3)x_1}=$$

$$\cancel{(x_2-x_1)} \quad = x_2\left[x_3y_1-x_2y_1-x_3y_2+\cancel{x_2y_2}-\cancel{x_2y_2}+x_1y_2+x_2y_3-x_1y_3\right]$$

$$(x_3-x_1)(x_3-x_2)(y_1-y_2)-(x_3-x_1)(y_2-y_3)x_2+(x_3-x_2)(y_1-y_3)x_1$$

$$(x_3-x_1)\left[x_3y_1-x_2y_1-x_3y_2+\cancel{x_2y_2}-\cancel{x_2y_2}+x_2y_3\right]+(x_3y_1-x_2y_1-x_3y_3+x_2y_3)x_1$$

$$=x_3\left[x_3y_1-x_2y_1-x_3y_2+x_2y_3\right]+x_1\left[x_3[y_2-y_3]\right]$$

$$=x_3\left[x_3y_1-x_2y_1-x_3y_2+x_2y_3+x_1(y_2-y_3)\right]$$

$$(x_2-x_1)^2(y_1-y_2)-(x_2-x_1)(y_2-y_1)x_2+(x_2-x_1)(y_2-y_1)x_1$$

$$(y_1-y_2)\left[(x_2-x_1)^2-x_1x_2+x_2^2+x_1^2-x_1x_2\right]=0$$

$$\xi_{1,2}=\left[x_3y_1-x_2y_1-x_3y_2+x_2y_3+x_1y_2-x_1y_3\right]\left[\frac{m_1^2m_2m_3x_1}{r_2^3r_3^3}+\frac{m_1m_2^2m_3x_2}{r_1^3r_3^3}+\frac{m_1m_2m_3^2x_3}{r_1^3r_2^3}\right]$$

$$= - - \qquad (m_1m_2m_3)\left[\frac{m_1x_1}{(r_2r_3)^3}+\frac{m_2x_2}{(r_1r_3)^3}+\frac{m_3x_3}{(r_1r_2)^3}\right]$$

$$\xi_{2,3}=\left[x_1y_2-x_3y_2-x_1y_3+x_3y_1+x_2y_3-x_2y_1\right] \;—\; \left[\frac{m_2x_2}{(r_3r_1)^3}+\right.$$

$$K_1 = \frac{m_1 d_1 (m_1 r_1^3 + m_2 r_2^3 + m_3 r_3^3)}{r_2^3 r_3^3}$$

Voraussetzung V = homog. fc von nter Grad

~~$V = f(r_1, r_2, r_3, d_1, d_2, d_3)$~~

$$V = f(r_1, r_2, r_3) = f(d_1, d_2, d_3)$$

$$m V_1 = d_1 \frac{dV_1}{dd_1} + \ldots$$

?

$$= d_1 K_1 + d_2 K_2 + d_3 K_3$$

$$m V_1 = (m_1 d_1^2 r_1^3 + m_2 d_2^2 r_2^3 + m_3 d_3^2 r_3^3) \frac{(m_1 r_1^3 + m_2 r_2^3 + m_3 r_3^3)}{r_1^3 r_2^3 r_3^3}$$

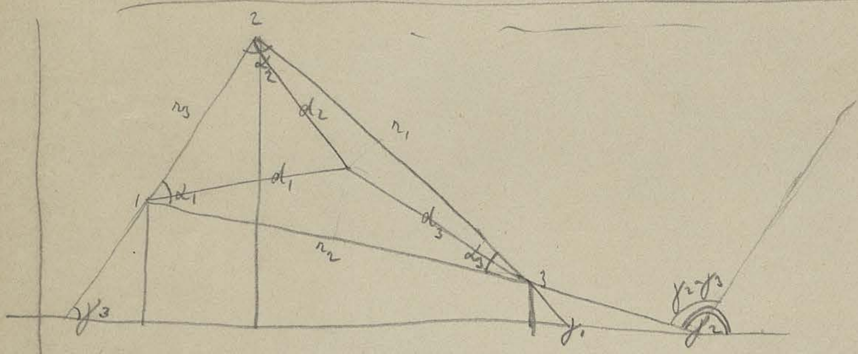
$$\left.\begin{aligned} &= m_1 r_1^3 r_2 r_3 \left[m_2^2 r_2^4 + m_3^2 r_3^4 - m_2 m_3 r_2 r_3 (r_2^2 + r_3^2 + r_1^2)\right] \\ &+ m_2 r_2^3 r_3 r_1 \left[m_3^2 r_3^4 + m_1^2 r_1^4 - m_3 m_1 r_3 r_1 (r_3^2 + r_1^2 - r_2^2)\right] \\ &+ m_3 r_3^3 r_1 r_2 \left[m_1^2 r_1^4 + m_2^2 r_2^4 - m_1 m_2 r_1 r_2 (r_1^2 + r_2^2 - r_3^2)\right] \end{aligned}\right\} : (m_1 r_1^3 + m_2 r_2^3 + \ )(r_1^3 r_2^2 r_3^3)$$

$$\left.\begin{aligned} &= m_1 m_2^2 r_1^2 r_2^4 + m_1 m_3^2 r_1^2 r_3^4 - m_1 m_2 m_3 r_1^2 r_2 r_3 (r_2^2 + r_3^2 - r_1^2) \\ &+ m_2 m_3^2 r_2^2 r_3^4 + m_2 m_1^2 r_2^2 r_1^4 - m_1 m_2 m_3 r_2^2 r_3 r_1 (r_3^2 + r_1^2 - r_2^2) \\ &+ m_3 m_1^2 r_3^2 r_1^4 + m_3 m_2^2 r_3^2 r_2^4 - m_1 m_2 m_3 r_3^2 r_1 r_2 (r_1^2 + r_2^2 - r_3^2) \end{aligned}\right\} : (m_1 r^3 + m_2 r_2^3 + m_3)(r_1^2 r_2 r_3)^2$$

$$d_1 = \frac{\sqrt{\left[m_2(x_2-x_1)r_2^3 + m_3(x_3-x_1)r_3^3\right]^2 + \left[m_2(y_2-y_1)r_2^3 + m_3(y_3-y_1)r_3^3\right]^2}}{m_1 r_1^3 + m_2 r_2^3 + m_3 r_3^3}$$

$$d_2 = \sqrt{\left[m_3(x_3-x_2)r_3^3 + m_1(x_1-x_2)r_1^3\right]^2 + \left[m_3(y_3-y_2)r_3^3 + m_1(y_1-y_3)r_1^3\right]^2}$$

$$d_3 = \sqrt{\left[m_1(x_1-x_3)r_1^3 + m_2(x_2-x_3)r_2^3\right]^2 + \left[m_1(y_1-y_3)r_1^3 + m_2(y_2-y_1)r_2^3\right]^2}$$



$$d_1 = \sqrt{\left[m_2 r_2^3 r_3 \cos\gamma_3 + m_3 r_3^3 r_2 \cos\gamma_2\right]^2 + \left[m_2 r_2^3 r_3 \sin\gamma_3 + m_3 r_3^3 r_2 \sin\gamma_2\right]^2}$$

$$= \sqrt{(r_2 r_3)\left\{m_2^2 r_2^4 \cos^2\gamma_3 + m_3^2 r_3^4 \cos^2\gamma_2 + 2 m_2 m_3 r_2^2 r_3^2 \cos\gamma_3 \cos\gamma_2 + m_2^2 r_2^4 \sin^2\gamma_3 + 2 m_2 m_3 r_2^2 r_3^2 \sin\gamma_3 \sin\gamma_2 + m_3 r_3^4 \sin^2\gamma_2\right\}}$$

$$= \frac{\sqrt{r_2 r_3\left[m_2^2 r_2^4 + m_3^2 r_3^4 + 2 m_2 m_3 r_2^2 r_3^2 \cos\alpha_1\right]}}{m_1 r_1^3 + m_2 r_2^3 + m_3 r_3^3}$$

$$r_1^2 = r_2^2 + r_3^2 - 2 r_2 r_3 \cos\alpha_1$$

$$\cos\alpha_1 = \frac{r_2^2 + r_3^2 - r_1^2}{2 r_2 r_3}$$

$$d_1 = \frac{\sqrt{r_2 r_3\left[m_2^2 r_2^4 + m_3^2 r_3^4 - m_2 m_3 r_2 r_3 (r_2^2 + r_3^2 - r_1^2)\right]}}{m_1 r_1^3 + m_2 r_2^3 + m_3 r_3^3}$$

$$d_1 = \frac{\sqrt{r_2 r_3\left[(m_2 r_2^2 + m_3 r_3^2)^2 - m_2 m_3 r_2 r_3\left([r_2 + r_3]^2 - r_1^2\right)\right]}}{m_1 r_1^3 + m_2 r_2^3 + m_3 r_3^3}$$

$$K_1 = m_1 \frac{\sqrt{r_2 r_3\left[m_2^2 r_2^4 + m_3^2 r_3^4 - m_2 m_3 r_2 r_3 (r_2^2 + r_3^2 - r_1^2)\right]}}{r_2^3 r_3^3}$$

84/53 I 17-30 73

Promemorianie

O promieniowaniu i analizie widmowej.

Jedna z najnowszych gałęzi fizyki; zasadn. ~~[illegible]~~ odkrycie Kirchhoffa dopiero 1860 i dopiero w ostatnich latach ogromne postępy.

Nawet nie wiadomo do czego to zaliczyć czy do optyki, czy termiki ?

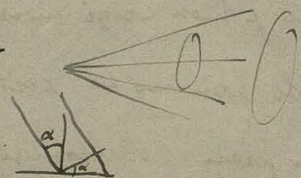
Zwykle analiza widmowa ati. do optyki ale to tylko szczeg. przyp. ogólnego prom. cieplnego, najściślejszy związek z zasadami termodyn., z drugiej strony związek z elektron. teorią

Dawniejsza (właściwa) optyka zajmuje się zjawiskami rozchodzenia się światła, przy czem źródło światła uważa już za coś danego. Skąd ono pochodzi to rzecz obojętna.

Otóż ~~[illegible]~~ to wszystko uważać będziemy za znane i interesować będzie nas właśnie sposób powstawania promieni i zanikania (absorpcji).

Historyczny rozwój z dwóch punktów widzenia optycznego i kalorycznego

Lambert (1760 Photometria) znał już prawo $J = \frac{c}{r^2}$

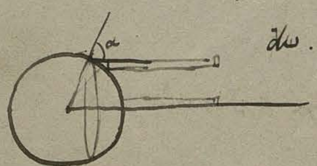


a dla nachylonych: $\frac{c \cos\alpha}{r^2}$

Takie samo prawo wyprowadził on dla wysyłania światła

prawo cosinusów Lamberta

i powołał się jako dowód że Słońce wydaje się równie jasne we wszystkich częściach



$$d\omega \cdot s = \frac{d\omega}{\cos\alpha} \cdot \sigma$$

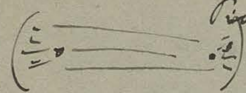
$$\sigma = s \cos\alpha$$

ale to oczywiście bardzo niedokładna ~~[illegible]~~ metoda, oczywiście to tylko przybliżenie prawdziwe jak późniejsze badania dośw. wykazały. Nadto dowód fałszywy!

Już Lambert twierdził na podstawie dośw. że promienie ciepła według tych samych praw się rozchodzą jak prom. światła i uważają także samo prawo dla ciepła prom.

Leslie przybliż. potwierdza prawo cosin.

Pictet (1790)



nie tylko ciepło ale też zimno

więc czy istnieje osobne promieniowanie zimna?

Przecież zimno tylko = brak ciepła

Zupełnie jasno zrozumiał to Prevost 1809

„ciało w otoczeniu o równej temp. zachowuje się jak jezioro do którego deszcz pada podczas gdy równie wiele wody paruje".

Przy nierównej temp. obserwujemy różnicę prom. w jednym a drugim kierunku.

Dulong i Petit 1817 zależność od temp. mat

Tak dalece jednak ciągle uważano prom. światła a ciepła jako coś różnego

dopiero powoli poznano, że każdy prom. świetlny jest też cieplnym.

Zwłaszcza prace Melloniego ~~1833–1845~~ od 1840 [illegible] i Knoblauch

Melloni, z Jaminem,

Fizeau & Foucault 1847 { interferencje zwierc. Fresnela, itd. 6 pewne pomiary długości fal / ugięciami

Knoblauch jeszcze obszerniej

Także polaryzacja Melloni Knoblauch i inni, Provostaye & Desains itd.

Skręt płaszcz. polaryzacji Desains

Już tylko interes historyczny

Hist. Wstęp: Lambert

Melloni

Stewart Kirchhoff-Bunsen, Clausius

{ Spectrosk. Lockyer, Plücker, Tyndall, ... Rowland Deslandres, ~~Langley~~

{ Teorya prom. ogólna. Langley, Michelson, Wien, Paschen, Lummer, Pringsheim, Planck, etc.

Światło = rodzaj prom. cieplnych

(A. Termoluminescencya) (B Inne lumin.

Prawo Kirchhoffa

T. zw. prawo Clausiusa, teorya Smol., prawo Lamberta

Dowody doświadcz. jakościowe (Exp.)

Dwa zadania: Prom. ciała czarnego, Absorpcya indywid.

Metody doświadcz. : Spectrosk.

β). { Ugięcie, siatki dyfr. / Michels. interfer. / Widma normalne etc., Rowland, Kayser } (Exp.)

γ). { Oko ludzkie, Eksp.. / Termo- / Bolometr / Radiometr Nichols-Rubens } (Exp.)

α). Ciało czarne idealne, realizacya jego (Exp.)

Langley

Paschen, Lummer-Pringsheim

Teorya: Zasada Dopplera, Ciśnienie promien. (Lebedew)

Michelson,

Wien - Planck, Rezultat teoret. - prakt.

Absorpcya :

prawo Kirchhoffa etc. ~~[illegible]~~ Widma łukowe, iskrowe, etc. (Exp.)

Doppler, Przesunięcie widmowe ciśnienia, Humphreys & Mohler

związek z spółcz. załamania, ~~teorya elektro~~

~~[illegible]~~ dysp. anormalna, ~~nowsze teorye~~ (Exp.)

teorye, Helmholtz, Drude etc.

Potwierdz. teoryi przez nowsze dośw. w widmie podczerwonem

Rezultaty co do gazów pierwiastków (Exp.)

Odkrycia nowych pierwiastków : In, Th, ... A, He, Ne, Xe, Kr., Rad. etc.

Prawo seryi : Balmer, Rydberg, Kayser & Runge

Zagadka teoret.

~~Widma~~ widma pasmowe (Exp.)

widma abs. ciał ciekłych i stałych , Jony (Exp.)

Barwy ciał (Exp.)

Praktyczne zastosowania analizy spektr.

Krew, Bessemer Pr., Barwniki anilinowe etc., Analiza chem. anorg. mokra i spektr. (Exp.)

Astronom. : skład ciał nieb.

prędkości ich

B. Luminescencye inne ciał. Fluorescencya (Exp.)

Phosphorescencya (Exp.)

Radyowanie

76

Ciśnienie promieniowania

Maxwell

Energia 

$$\sigma = a \sin 2\pi\left(\frac{t}{\tau} - \frac{x}{\lambda}\right)$$

$$\dot{\sigma} = a \frac{2\pi}{\tau} \cos 2\pi\left(\frac{t}{\tau} - \frac{x}{\lambda}\right)$$

$$\int \frac{\dot{\sigma}^2}{2}\, dx = \frac{4\pi^2 a^2}{\tau^2} \int$$

Promieniowanie z punktu widzenia termodynamiki

Odwracalne dopóki równowaga t.j. normalna gęstość prom. o jednakowej temp.

Newton $\alpha(t - t_0)$

Dulong Petit (1817) termom. stygnący w naczyniach

zgodzili że przewodn. ciepła znikomie się przy gęstości pow. (ciśn. zmniejsz. do 2 mm)

$S = ma^T$

Stefan (1879)

~~[illegible]~~ Rosetti, Schleiermacher, Paschen (Platyna do 1°0 − 6·4
C etc.
Pt)

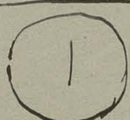
Boltzmann: ciało czarne Stichler 1884

Wien Lummer 1895

Lummer & Pringsheim 1897

Abs. temp.	wychylenie	Abs. temp. ber.	Δ
373·1	156	374·6	
723·0	3320	724·3	
868	6910	867·1	
1092	16400	1074	
1378	44700	1379	
1535	67800	1531	

Paschen 1899

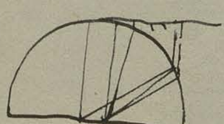


$$dW = a\frac{ds\,ds'}{r^2}\left(T^4 - T'^4\right)$$

$a = 1{\cdot}71 \cdot 10^{-5}$ ergs

$= 0{\cdot}408 \cdot 10^{-12}$ cal/sec

$ds, ds' \perp r$



~~[illegible]~~ $\sum ds'_{n} = r^2\pi$

$e = a\pi\, ds\left(T^4 - T'^4\right)$ N. p. $T = 373$, $T' = 273$ $e = 0{\cdot}01763$ gcal/sec

[illegible]

$$0{,}408 \cdot 10^{-12} \, \frac{ds}{r^2} \left(T^4 - T'^4\right) = \frac{3}{60}$$

[illegible]

$$ds = 2r^2\pi = \left(\tfrac{1}{2}\, 32'\right)^2 \pi : 2\pi$$

$$ds = r^2 \cdot \left(\tfrac{32'}{2}\right)^2 \pi$$

$$T = 6200^\circ$$

Kirchhoff 1859

emisja jakiegokolwiek ciała w danej barwie w danej temp. = emisja ciała dosk. cz. w tej samej temp. i barwie × absorpcja ciała ...

ważne wytłumaczenie do prom. cieplnego ; dla równowagi cieplnej

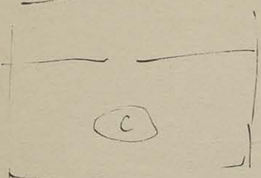
niezbędne do fotometrii, luminescencji, radiometrii, rotacji świetlówki

Prawo Tyle promieniuje ile pochłania

danej temp.

$1 = a + r + p + s$

dosk. czarne ciało które wszystko pochłania (wiązki, mierz. w środku=1 ...



1). ciało czarne w otoczeniu czarnym

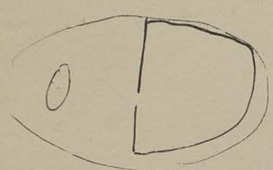
całkowite promieniowanie przez niezal. od radiacji i kontaktu

2). tak samo otoczone na barwy (żyłki) i fluorescencja ...

3). ciało nie czarne

4) zależność od zjaw. środka

W. [illegible]



1). promieniowanie wychodzące przez mały otwór = czarne

2). emisja ciała przen. = 0

3). zwierc. = 0

$r + s + t + a = 1$.

2). $I \xrightarrow{(r+s+t) E_0} = E_0 (1-a)$

zatem ażeby stały stan, z równ. $E = E_0 a$

3) [illegible]

1).

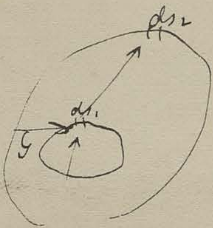
$$e = \int e_\lambda \, d\lambda \qquad E = \int A_\lambda e_\lambda \, d\lambda = \int A_\lambda e_{1\lambda} \, d\lambda = \dots$$

$$e_\lambda = e_{1\lambda} = e_{2\lambda} = \dots$$

$$e = \int e_\lambda \, d\lambda$$

Hg. tak, jako [illegible] $A = 1$ [illegible]

2).



$$e_\lambda = E_\lambda + S$$

$$S = (1 - A_\lambda)\, e_\lambda$$

$$e_\lambda = E_\lambda + (1 - A_\lambda)\, e_\lambda$$

$$E_\lambda = A_\lambda e_\lambda$$

Druhý důvod.

Draper

$$\frac{1}{\lambda} = A - \frac{B}{n^2}$$

Ogólnie $\frac{1}{\lambda} = A - \frac{B}{n^2} - \frac{C}{n^4}$ ~~Rydberg~~, Kayser & Runge

Runge & Paschen

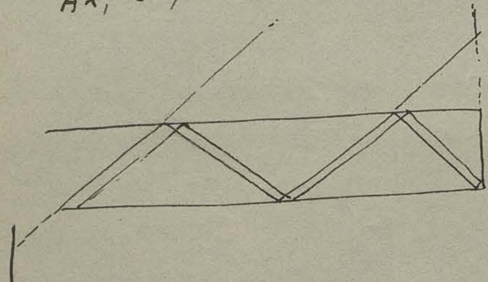
Rydberg $\frac{1}{\lambda} = A + \frac{B}{(m+\mu)^2}$

$$= A + \frac{B}{n^2} + \frac{C}{n^3} + \dots$$

I Li Na K 3 serye dubletów

II Mg Ca Sr, Zn Cd Hg 2 serye tripletów

III Al, In, Tl 2 serye podw.



80

$c=$

$$h\nu = 6{\cdot}5 \cdot 10^{-27} \cdot \frac{3 \cdot 10^{10}}{0{\cdot}001} = 6{\cdot}5 \cdot 3 \cdot 10^{-14}$$

$$= 2 \cdot 10^{-13}$$

$$\bar{L} = \frac{3}{2} k\theta = \frac{3}{2} \cdot 1{\cdot}3 \cdot 10^{-16} \cdot 273$$

$$= 5{\cdot}4 \cdot 10^{-14}$$

$$\alpha \int e^{-\beta(\xi^2+\eta^2+\zeta^2)} d\xi\, d\eta\, d\zeta \left[\log\alpha - \beta(\xi^2+\eta^2+\zeta^2)\right]$$

$$= \alpha \log\alpha \cdot \left(\frac{\pi}{\beta}\right)^{3/2} -$$

z Rektoratu Univ. 13/I 1912 L. 592

Jadwiga Falkowska

XIV 307/5 20/1

87

Tak trwało to jednak do połowy XVIII w. zanim identyczności poznano.

Linii Fraunhofera 1814

Sławne odkrycie analizy spektr. Kirchhoff i Bunsen 1860 stałe widm. ciężkie gorą linie o pewnym położeniu niezmiennym

prawo Kirchhoffa co do absorpcji — emisyji

Co prawda że częściowo to już dawniej wypowiedział Provostaye & Desains 1853

Balfour Stewart 1857

Ogromny rozwój analizy widm. sam Kirch. i B. badali wszelkie pierwiastki etc. minerały

i odkryli tym sposobem Rb, Cs

później Tl Crookes In Reich Ga Lecoq de Boisbaudran etc. etc.

Huggins, Lockyer, Angström pierwsze wyznaczenie sp. Słońca później bliższy ich okresy

Rowland 1888 atlas widma słon. fotograf. dyfr. siatki dyfr.

Deslandres 12m długie dokładność 0.000.01 μ

Regularności w widmach: Balmer, Rydberg, Kayser & Runge

Tym czasem promien. cieplne Stefan 1879 — Boltzmann

Klasyczne badanie Langleya 1886 bolometr rozkład ciepła w widmie rozmaitych ciał aż do tego 40°

Michelson 1887 ~~Rayleigh~~ Wien 1896, Planck

Lummer & Pringsheim —, Paschen etc.

Desains 1868 Therm Słońca

Langley 1883 28 μ

1888 28 μ

Tak że obecnie nawet temp. księżyca możemy mierzyć.

82

			Δ
α	6563.07		
β	4861.57	4861.52	+0.05
γ	4340.53	4340.63	−0.10
δ	4102.00	4101.90	+0.16
ε	3970.33	3970.22	+0.11
ζ	3889.15	3889.20	−0.05

Evershed (22/1 1898)

Angström unit = 10^{-7} mm = 10^{-8} cm

$$\frac{1}{\lambda_n} = \frac{1}{3647.2}\left(1 - \frac{4}{n^2}\right)$$

$$= 1.096750\left[\frac{1}{4} - \frac{1}{n^2}\right] \cdot 10^{-3} \quad (\text{AE})$$

n = 3 —— 32

$$\nu = N\left(\frac{1}{4} - \frac{1}{n^2}\right)$$ (I Nebenserie) breite Li.

Pickering ζ Puppis 1896

$$\nu = N\left[\frac{1}{4} - \frac{1}{(n+\frac{1}{2})^2}\right]$$ (II Nebenserie) scharfe u. mehr schwache

---

Hauptserie Rydberg scharfe starke Linien in U.V.

$$\nu = N\left[\frac{1}{(1\frac{1}{2})^2} - \frac{1}{n^2}\right] \quad n = 2, 3 \cdots$$

Na Wood fluorescence spectrum 48 lines of $D_1 D_2$ resonance previously known only 7 series

Li tak samo 3 serye

Na K Rb tak samo ale podwójne linie

$\nu_1 - \nu_2$ = const dla composantów Hartley & Julius w Nebenseryach

ale nie —

w II serye Rundbe... trypletý (Hauptserien nie mam)

Runge & Paschen effekt Zeemana ten sam dla serye

Balmer i Rydberg w optyce H. pomimo być   Ritz p. 170 – 173!

$$\frac{1}{\lambda} = N\left(\frac{1}{n^2} - \frac{1}{m^2}\right) \qquad \text{I Nb}$$

$$N\left(\frac{1}{n^2} - \frac{1}{(m+\frac{1}{2})^2}\right) \qquad \text{II N.}$$

istotnie Paschen wynalazł 1908:

$\lambda = 18751.3$ $\frac{1}{3^2} - \frac{1}{4^2} \mid 18751.6$

12817.6 $\frac{1}{3^2} - \frac{1}{5^2} \mid 12818.7$

dla innych:

Rydberg $\frac{1}{\lambda} = A + \frac{B}{(m+\mu)^2}$

K i R: $\frac{1}{\lambda} = A' + \frac{B'}{m^2} + \frac{C'}{m^4} + \dots$ $m = 2, 3$ [illegible]

Ritz: przytoczenie [illegible]:

$$\frac{1}{\lambda} = N\left\{\frac{1}{\left(n + a + \frac{b}{n^2}\right)^2} - \frac{1}{\left(m + a' + \frac{b'}{m^2}\right)^2}\right\}$$

N ta sama co dla wodoru

$m = 1\frac{1}{2}$ $n = 2, 3 \dots$ Hauptserie } więc wspólna linia $m = 1\frac{1}{2}$ $n = 2$!

$n = 2$ $m = 2\frac{1}{2}$ $3\frac{1}{2} \dots$ II Neben. scharfe Linie

$n = 2$ $m = 3$ 4 5 ...   I Neben. { diffuse Linie; zmienne a' b' } takie same $n = 3 \dots$

obie kończą się $m = \infty$ w tem samem miejscu

1 Nebenserie — 2 Nebenserie — 83

Na

n=3 | 5896.16
5890.19
$\Delta \sim \frac{1}{n^4}$

n=4 | 3303
3302

5 | 2853

6 | 2680

9 | 2511.77
58

3 | 8194.76
84.33

4 | 5688.26
5682.9

5 | 4983.5
4979.3

6 | 4668.45
4664.68

7 |

9 | 4323.94

4 | 6161.15
54.62

9 4342.44

$\lambda = A \frac{n^2}{n^2-4}$

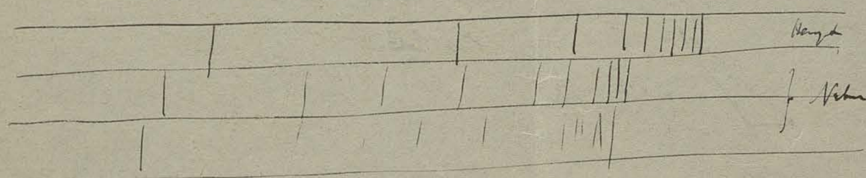
$\frac{1}{\lambda} = \ldots$

K. u. R. $\frac{1}{\lambda} = A - \frac{B}{n^2} - \frac{C}{n^4}$

Rydberg: $\frac{1}{\lambda} = A + \frac{B}{(m+\mu)^2}$

$= A + \frac{B}{m^2} + \frac{C}{m^3}$

(B to samo co u wodoru)



0	1	2	3	4	5	6	7	8
He 4	Li 7	Be 9	B 11	C 12	N 14	O 16	F 19	
Ne 20	Na 23	Mg 24	Al 27	Si = 28	P = 31	S = 32	Cl = 35.5	
A 40	K 39	Ca 40	Sc 44	Ti = 48	V = 51	Cr = 52	Mn = 55	Fe Ni Co
	Cu = 64	Zn 65	Ga 70	Ge = 72	As = 75	Se = 79	Br = 80	
						Mo = 96		Ru
Kr 82	Rb 85.4	Sr 87.6	Y 89	Zr = 90.6	Nb = 94	Te = 127	J = 127	
		In 114		Sn = 118.5	Sb = 120			
	Ag = 108	Cd 112						
X 128	Cs = 133	Ba = 137	La 138	Ce = 140	—	—		
	—	—	—	—	—	—		Os Ir Pt
	—	—	Yb = 173	—	Ta = 183	W = 184		
	Au = 197	Hg = 200	Tl = 204	Pb = 207	Bi = 208	—		
		Ra 225(?)		Th = 232	—	U = 240		

Humphreys & Mohler (1896)   15 atm. Linie linii. w skrzyni żelaznej

na 10 Atm przesunięcie linii żelaza $\frac{23}{1000}$ AE

5896.
↑
AE

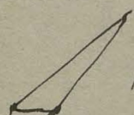
$\frac{d^2x}{dt^2} = \frac{eH}{m}\frac{dy}{dt}$ $x = A\cos\alpha t$ $-\alpha^2 A = -\alpha B c$ $\alpha = \frac{B}{A}c = \frac{A}{B}c$

$\frac{d^2y}{dt^2} = -\frac{eH}{m}\frac{dx}{dt}$ $y = B\sin\alpha t$ $-\alpha^2 B = -\alpha A c$ $A = B$

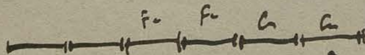
$\alpha = c$

$\nu = \frac{eH}{m}$

Ritz



$A = \frac{1}{n_1^2} - \frac{1}{n_2^2}$



f. 176

84

Hartley 1883 $\Delta\nu =$ [illegible]

$\sim a^2$ K. R.

Sn Pb As Sb Os Ru Pd Pt

$\nu = \frac{1}{\lambda}$

Sn:

3801.16	3175.13	−0.01
3330.71	2840.05	+0.01
2850.72	2483.49	+0.01

$$\frac{1}{3175.13 \cdot 10^{-8}} = \frac{1}{3801.16 \cdot 10^{-8}} + 5187.03$$

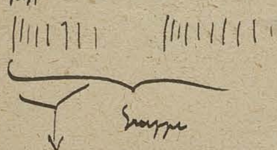
38000 · 10^{-8}

Zn 64.9 Cd 111.83

a	63.64
b	69
c	48
d	17
e	64.93
f	65.28
	15
	12
	16
	64.69
	64.77

Bandenspektra:

Kopf



Gruppe

$$\frac{1}{\lambda} = A + b n^2$$

Stickstoff Kopf 3914.6 63 Linien $\frac{1}{\lambda} = 255.454 + 0.0015335(n-1)^2$

Ganze Bande 160 Linien

Cyanbanden

Deslandres: $\frac{1}{\lambda} = A m^2 + B n^2 + C$

[illegible] $\Delta \sim \lambda$ Haupt : I N. : II N. = 1 : 2 : 4

N kostek = molekuł ^{10} na każdej P liczb

P komórek

1 2 3 4 - 10 kostka

okna ~~2 6 1 1 1 1 6 6 1 1~~

2 4 3 1 3 5 4 3 5 2

1 1
2 2
3 3
4 2
5 2
6 2
0

2 3 5

1 2 | 3 4 5 | 6 7 8 9 10

1 2 |

Rozkład w tabeli

1 1 1 1 1 1 1 - - 1 od mniejszych 1

$\frac{(\alpha+\beta+\gamma)!}{\alpha!\beta!\gamma!}$

~~Kombinacje z N elementów~~

1 2 1 1 - 1

2 1 1 -

1 1 2

~~Kombinacje~~ wyliczenie liczby permutacji

$$P = \frac{\# U_{ss}}{\varepsilon} = \frac{NU}{\varepsilon}$$

$$\frac{S_N}{N} = k\left\{\left[1+\frac{U}{\varepsilon}\right]\left[\lg N + \lg\left(1+\frac{U}{\varepsilon}\right)\right] - \lg N - \lg\frac{U}{\varepsilon} - N\frac{U}{\varepsilon}\lg N\frac{U}{\varepsilon}\right\}$$

$$\frac{S_N}{N} = S = k\left[\left(1+\frac{U}{\varepsilon}\right)\lg\left(1+\frac{U}{\varepsilon}\right) - \frac{U}{\varepsilon}\lg\frac{U}{\varepsilon}\right]$$

$$dS = k\frac{dU}{\varepsilon}\left\{1 + \lg\left(1+\frac{U}{\varepsilon}\right) - 1 - \lg\frac{U}{\varepsilon}\right] = k\frac{dU}{\varepsilon}\lg\frac{1+\frac{U}{\varepsilon}}{\frac{U}{\varepsilon}} = k\frac{dU}{\varepsilon}\lg\left(1+\frac{\varepsilon}{U}\right) = \frac{dU}{T}$$

$$dS = \frac{dU}{T}$$

$$1+\frac{\varepsilon}{U} = e^{\frac{\varepsilon}{kT}}$$

$$\frac{\varepsilon}{U} = \left(1-e^{\frac{\varepsilon}{kT}}\right)$$

$$U = \frac{\varepsilon}{1-e^{\frac{\varepsilon}{kT}}}$$

$$d\nu = \frac{c\,d\lambda}{\lambda^2}$$

$$\sigma = \frac{8\pi K}{c} = \frac{8\pi\nu^2}{c^3}U$$

N Rezonatorów = 3   P Elementów = 4

$\frac{8!}{2!\,6!} = 7.4$ (crossed out) $\frac{6!}{2!\,4!}$

112333 (crossed out)
111111 (crossed out)
111112 (crossed out)
111113 (crossed out)
111122 (crossed out)
111133 (crossed out)
111123

Rezonatory rozróżnialne ale elementy nie

004	013	112
040	031	121
400	103	211
	130	
	301	
	310	
	022	
	202	
	220	

ogólnie $\frac{(N+P-1)!}{(N-1)!\,P!}$ = liczba kombinacyj z powtórzeniem N elementów P klasy

$$\frac{N(N+1)\cdots(N+P-1)}{1.2\cdots P}$$

N = 2 ‖ P = 4 ‖ 04 13 22
40 31

$\frac{5!}{1!\,4!} = 5$

0 a b c d (crossed out)

0 a b c d

$P = \frac{UN}{\varepsilon}$

$$S_N = k \lg \frac{(N+P)!}{N!\,P!}$$

$$= k\{(N+P)\lg(N+P) - N\lg N - P\lg P\}$$

$$S = \frac{S_N}{N} = k\left\{\left(1+\frac{U}{\varepsilon}\right)\lg\left(1+\frac{U}{\varepsilon}\right) - \frac{U}{\varepsilon}\lg\frac{U}{\varepsilon}\right\}$$

= entropia średnia jednego rezonatora

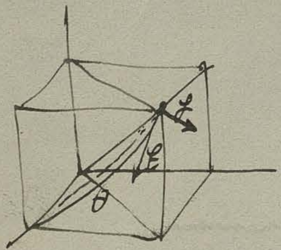
W razie drgań z:

$X = \frac{xz}{c^2r^3} \ddot{f}(t - \frac{r}{c})$ $\qquad L = -\frac{y}{c^2r^2}\ddot{f}$

$Y = \frac{yz}{..} \quad ..$ $\qquad M = \frac{x}{c^2r^2}\ddot{f}$

$Z = -\frac{x^2+y^2}{c^2r^3} \quad ..$ $\qquad N = 0$

$\mathfrak{E}R = \frac{2R}{c^2r^3} \qquad \mathfrak{H}R = -\frac{R}{c^2r^2}$



symetria względem z

więc w płaszczyźnie $y=0$

$X = \frac{xz}{c^2r^3}$ $\qquad L = 0$

$Y = 0$ $\qquad M = \frac{x}{c^2r^2}$

$Z = -\frac{x^2}{c^2r^3}$ $\qquad N = 0$

$\sqrt{X^2+Z^2} = \frac{x}{c^2r^2}$

$r \perp \mathfrak{E} \perp \mathfrak{H}$

$|\mathfrak{E}| = |\mathfrak{H}| = \frac{\sin\theta}{c^2r} \ddot{f}(t - \frac{r}{c})$

Prąd energii Poyntinga $= \frac{c}{4\pi}[\mathfrak{E}\,\mathfrak{H}] = \frac{\sin^2\theta}{4\pi c^3 r^2}(\ddot{f})^2$

$\sin\theta d\theta - \cos^2\theta \sin\theta d\theta$

$\cos\theta - \frac{\cos^3\theta}{3} =$

w całości

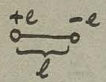
$\frac{2\pi}{4\pi c^3}\int_0^\pi \sin^3\theta d\theta \, (\ddot{f})^2 = \frac{1}{2c^3}\int_0^\pi \sin^3\theta d\theta$ $\qquad = \frac{4}{3}$

od chwili t do $t+T$: $\qquad = \frac{2}{3c^3}\int_t^{t+T} \ddot{f}^2(t - \frac{r}{c})dt$

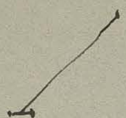
więc tym samym czasie przez kulę O: $\int_{t-\frac{r}{c}}^{t-\frac{r}{c}+T} \ddot{f}^2(t-\frac{r}{c}) = \int_t^{t+T} \ddot{f}^2(t)\,dt$

$= \dot{f}\ddot{f} - \int \dot{f}\dddot{f}\,dt$

Liniowy oscylator

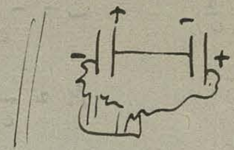


$el = f$ = moment elektryczny



siła = $X = -f\frac{x}{r^3}$

$Y = \dots$



e^2 energia elektrost.

$i^2 = \left(\frac{de}{dt}\right)^2$ magnet.

$\left.\begin{matrix}+e\\-e\end{matrix}\right\}$ jakoś połączone za pomocą jakiegoś mechanizmu

$U = \frac{K}{2} f^2 + \frac{L}{2}\dot{f}^2$ tak jak w obwodzie albo drganie mechaniczne

W razie jeśli energia zachowana:

$$dU = Kf\,df + L\dot{f}\,d\dot{f} = 0$$

$$Kf + L\ddot{f} = 0$$

$$f = C\cos(2\pi\nu_0 t - \vartheta);\quad \nu_0 = \frac{1}{2\pi}\sqrt{\frac{K}{L}}$$

Rozchodzenie się fali (elektr.) na zewnątrz

$$F = \frac{1}{r} f\left(t - \frac{r}{c}\right)$$

oś wibratora = Z

$$X = \frac{\partial^2 F}{\partial x \partial z} \qquad L = \frac{1}{c}\frac{\partial^2 F}{\partial y \partial t}$$

$$Y = \frac{\partial^2 F}{\partial y \partial z} \qquad M = -\frac{1}{c}\frac{\partial^2 F}{\partial x \partial t}$$

$$Z = \frac{\partial^2 F}{\partial z^2} - \frac{1}{c^2}\frac{\partial^2 F}{\partial t^2} \qquad N = 0$$

$$\frac{\partial X}{\partial t} = c\left(\frac{\partial N}{\partial y} - \frac{\partial M}{\partial z}\right) \qquad \frac{\partial L}{\partial t} = -c\left(\frac{\partial Z}{\partial y} - \frac{\partial Y}{\partial z}\right)$$

$$\frac{\partial Y}{\partial t} = \qquad \vdots$$

~~Edtt~~ wir jeśli $\frac{\dot{f}\ddot{f}}{c^3} \ll U = L\dot{f}^2$

$$\sqrt{K} f \sim \sqrt{L} \dot{f} \qquad \frac{\ddot{f}}{c^3} \ll L\dot{f}$$

$$\sqrt{L} \ddot{f} \sim \sqrt{K} \dot{f} \qquad \therefore \quad \sqrt{K}\dot{f} \ll \sqrt{L^3}\dot{f}$$

$$c^3\sqrt{\frac{L^3}{K}} \gg 1 \qquad c^3\sqrt{\frac{K}{L^3}} = \text{mało} = 6$$

$$\int dU = \frac{2}{3c^3}\dot{f}\dddot{f}$$

$$Kf + L\ddot{f} - \frac{2}{3c^3}\dddot{f} = 0$$

w razie [illegible] działa jeszcze [illegible] $E_z e \frac{dl}{dt} = E_z \dot{f}$

więc w takim razie:

$$Kf + L\ddot{f} - \frac{2}{3c^3}\dddot{f} = E_z \qquad \text{[illegible]}$$

$$f = Ae^{\alpha t} \dots = Ce^{\alpha t}\cos(\beta t - \vartheta) \qquad \nu_0 = \frac{\beta}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{K}{L}}$$

$$\alpha = -\frac{K}{3c^3L^2}$$

$$E_z = C\cos\left[2\pi\nu\left(t - \frac{x}{c}\right) - \vartheta\right]$$

$$K_0 = \sigma_0 \frac{c}{8\pi}$$

$$\bar{U} = \frac{c^2}{\nu_0^2}\bar{K}_0$$

↑ dla drgań o

$S = \lg W$

$S_1 + S_2 = f(W_1) + f(W_2) = f(W_1 W_2)$

$S = k \lg W$

$f(x\, y\, z\, \xi\, \eta\, \zeta)\ \underbrace{dx\,dy\,dz\,d\xi\,d\eta\,d\zeta}_{d\sigma}$

$W = \dfrac{N!}{\Pi(f\,d\sigma!)}$ ~~$\frac{N!}{1!\,2!\ldots}$~~

$N = \sum f\,d\sigma$

$S = k \lg N! - k \sum \lg (f\,d\sigma)!$ $\qquad n! = \left(\frac{n}{e}\right)^n \sqrt{2\pi n}$

$\lg n! = n(\lg n - 1)$

$= k \lg N! - k \sum f\,d\sigma [\lg(f\,d\sigma) - 1] \quad = \text{const} - k \sum f\,d\sigma \lg (f\,d\sigma) =$

$= \text{const} - k \int f \lg f\,d\sigma$

$\delta S = 0 \qquad \int (\lg f + 1)\,\delta f\,d\sigma = 0$

$\int \delta f\,d\sigma = 0$

$\int (\xi^2 + \eta^2 + \zeta^2)\,\delta f\,d\sigma = 0$

$\lg f + \beta(\xi^2 + \eta^2 + \zeta^2) = \text{const}$

$f = \alpha e^{-\beta(\xi^2 + \eta^2 + \zeta^2)}$

$V = \int dx\,dy\,dz$

$N = \int f\,d\sigma = V\alpha \int e^{-\beta(\xi^2 + \ldots)} d\xi\ldots = V\alpha \left(\frac{\pi}{\beta}\right)^{3/2}$

$U = \frac{m}{2} \int (\xi^2 + \eta^2 + \zeta^2) f\,d\sigma = \frac{Vm\alpha}{2} \int (\xi^2 + \eta^2 \ldots) e^{\ldots} d\xi \ldots$

$= \frac{3}{4} V m \frac{\alpha}{\beta} \left(\frac{\pi}{\beta}\right)^{3/2}$

$\alpha = \frac{N}{V} \left(\frac{3mN}{4\pi U}\right)^{3/2} \qquad \beta = \frac{3mN}{4U}$

$S = \text{const} + kN\left(\frac{3}{2} \lg U + \lg V\right)$

Gas jednoatomowy

Notamm: distribution 004 013 022 112

$\begin{Bmatrix} 0e & 1e & 2e & 3e & 4e \\ 2 & 0 & 0 & 0 & 1 \end{Bmatrix}$

~~Π 2 0 1 0 0~~

$013 = \begin{Bmatrix} 1 & 1 & 0 & 1 & 0 \end{Bmatrix}$

4 atomów 3 komory

abcd I II III

możliwe wypadki:

004	013	022	112
040	031	202	121
400	103	220	211
	130		
	301		
	310		

liczba 3! / 2!1!0!0!1!

3! / 1!1!0!1!0!

te kategorje są między sobą ~~[illegible]~~ jednakowo prawdopodobne w każdej klasie, ale nie napomnienie

bo n.p. 004 może być zrealizowane tylko w jeden sposób

a III

b III

c III

d III

$W_0 = \left(\frac{1}{3}\right)^4$

podczas gdy n.p. 022 w następujące sposoby

a	II	II	II	III	III	III
b	II	III	III	II	II	III
c	III	II	III	II	III	II
d	III	III	II	III	II	II

$W = 6.\left(\frac{1}{3}\right)^4 =$ ~~[illegible]~~

ogólnie $\frac{W}{W_0} = \frac{4!}{\alpha!\beta!\gamma!}$ | $\frac{4!}{0!2!2!} = 6$

$\frac{4!}{1.1.3!} = 4$

$\frac{4!}{1.1.2!} = 12$

Więc prawdop. klas:

$\left(\frac{1}{3}\right)^4 \quad 4\left(\frac{1}{3}\right)^4 \quad 6.\left(\frac{1}{3}\right)^4 \quad 12\left(\frac{1}{3}\right)^4$

w całości totalnie

$$\frac{3 + 4.6 + 3.6 + 3.12}{3^4} = \begin{matrix}45 \\ 36 \\ \frac{81}{3^4}\end{matrix} = 1$$

rozróżniamy komory, ale ~~[illegible]~~ indywidualność drobin jest obojętna

chodzi o ~~prawdop.~~ porównanie przy danej liczbie atom. i danej liczbie komór, ale dla różnych układów

Planck jednak w porównywaniu liczy liczbę tylko wszystkich kompleksów

tj. „kombinacyj” z powtórzeniem (bez względu na porządek)

I I I I	I II II II	II II II II	III III III III
I I I II	I II II III	II II II III	
I I I III	I II III III	II II III III	
I I II II	I III III III	II III III III	
I I II III			
I I III III			

$\} = 15$

ogólnie $\dfrac{(N+P-1)!}{P!\,(N-1)!}$

~~[illegible]~~ uważając je za równie prawdopodobne

a to może za miarę prawdopodob. że N elementów rozdziela się na P rezonatorów

więc nie liczby N P są dane tylko

liczba P jest dana N jest zmienne!

i porównuje się prawdop. dla różnych N

Ja jednak badam w takim razie wnioski co do tego jakie wypadki są równie prawdopodobne.

Chyba tak:

Dana liczba P rezonatorów, indywidualnie oznaczonych

Dla każdego z nich przypisujemy liczbę ~~[illegible]~~ energii wedle przypadku, ciągnąc z urny

liczby od 0 — M

potem porządkujemy wyniki wedle ilości energii otrzymanej

Najmniejsza energia 0, największa nP

Równie prawdop. będzie również np.

004 jak 005 albo 003

ale takich gdzie suma jest = 5 jest więcej niż takich gdzie suma = 4

Odnosi się w stronie więcej liczb kompleksyj

$$\frac{\frac{7!}{5!\,2!}}{\frac{6!}{4!\,2!}} = \frac{7}{5} \qquad \frac{\frac{6!}{2!\,4!}}{\frac{5!}{2!\,3!}} = \frac{6}{4} = \frac{3}{2}$$

003
030
300
102
120
111
201
210
012
021

Ale

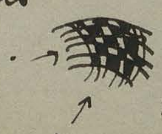
| Jeżeli jak u Plancka N, P dane to |
| nie ma w ogóle sensu mówić o prawdop.

Tak chodzi zatem o prawdop. otrzymania pewnej całkowitej ilości energii, jeśli każdy rezonator zupełnie przypadkowo więcej albo mniej może pobrać

[Czy to w pewnym obrazie nie będzie wyrażone przez interferencję [illegible] przypadkowego?]

Jedno źródło E = 1

Dwa źródła



E = 0
albo E = 4

Jak trzy źródła, to E = 9
1
1
1

$$\lg f = -\frac{N}{H\theta}(L+U) = \frac{1}{k}S$$

91

$J = \frac{C}{\lambda^5 \left(e^{\frac{c}{\lambda\theta}}-1\right)}$ $\frac{\partial J}{\partial \lambda} = 0$: $5 = \frac{c}{\lambda T}\,\frac{1}{1-e^{-\frac{c}{\lambda T}}}$ $c = 4.9651\,\lambda_m T$

$c = 14598$

Rubens – Kurlbaum

Fluorit 24.0 μ   31.6 μ   NaCl 51.2 μ

−273°

$$e = \frac{C}{\lambda^5}\left[e^{\frac{c}{\lambda\theta}}-1\right]^{-1} = \frac{C}{\lambda^5}\, e^{-\frac{c}{\lambda\theta}}\left[1-e^{-\frac{c}{\lambda\theta}}\right]^{-1} \neq \frac{C}{\lambda^5}\, e^{-\frac{c}{\lambda\theta}} \quad \text{[illegible] } \frac{\lambda\theta}{c} \gg 1$$

$$e = \frac{C}{\lambda^5}\left[1+\frac{c}{\lambda\theta}-1\right]^{-1} = \frac{C\theta}{c\,\lambda^4} \quad \text{Rayleigh } \frac{\lambda\theta}{c} \ll 1$$

$\frac{c}{\lambda\theta} = x$ $\lambda = \frac{c}{\theta x}$ $d\lambda = -\frac{c\,dx}{\theta x^2}$

$$\int J\,d\lambda = \frac{C\theta^4}{c^4}\int_0^\infty \frac{x^3\,dx}{e^x-1} = \frac{C\theta^4}{c^4}\int_0^\infty x^3\,dx\left(e^{-x}+e^{-2x}+e^{-3x}+ \cdots\right]$$

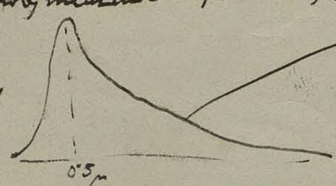
$$\int_0^\infty x^3 e^{-nx}\,dx = \frac{1}{n^4}\int (nx)^3 e^{-nx}\,d(nx) = \frac{1}{n^4}\int x^3 e^{-x}\,dx = \frac{3!}{n^4}$$

$$= \frac{C\theta^4}{c^4}\cdot 6\left[1+\left(\tfrac{1}{2}\right)^4+\left(\tfrac{1}{3}\right)^4+ \cdots\right]$$

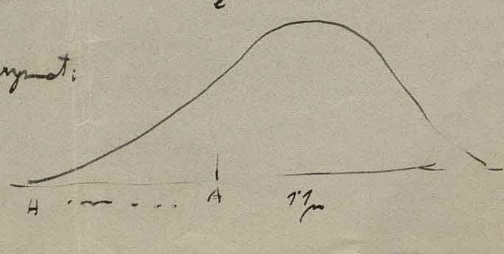
$$= 6.4938\,.\,\frac{C\theta^4}{c}$$

3 [illegible] temperature 1). θ^4 6490°C   2). $\lambda_m \theta$ $\theta = 5607$°C   3). $\sim \frac{1}{e^{\frac{c}{\lambda\theta}}-1}$ 4). $J_m \sim \theta^5$

Langley



[illegible] pigment:



92

$$f(u,v,w) = A e^{-\frac{N}{H\theta}\left[\frac{m}{2}(u^2+v^2+w^2) + \Phi\right]}$$

N. p. atmosfera

$\Phi = mgz$

$N = 8 \cdot 3 \cdot 10^7$

$H = 7 \cdot 10^{23}$

$$\frac{Nm}{H\theta} gz = \frac{gz}{R\theta}$$

$$E_\lambda^{d\lambda} = \frac{c^2 h}{\lambda^5} \frac{d\lambda}{e^{\frac{ch}{k\lambda\theta}} - 1}$$

$$\frac{c^2 h}{\lambda^5} e^{-\frac{ch}{k\lambda\theta}} =$$

$$\frac{ck\theta}{\lambda^4} \quad \text{Rayleigh}$$

$$k = 1{\cdot}35 \; 10^{-16} \left(\frac{erg}{\theta}\right) = \frac{2}{3}\left(m\frac{v^2}{2}\right)_{1^0} = \frac{H}{N}$$

$h = 6{\cdot}55 \cdot 10^{-27}$ ergsec

z tego mamy zatem $e = 9658 \cdot 3 \cdot 10^{10} \cdot \frac{k}{R} = 4{\cdot}69 \cdot 10^{-10}$

$\lambda = \frac{c}{\nu}$

$$E_\lambda^{d\nu} = \frac{h}{c^2} \frac{\nu^3 d\nu}{e^{\frac{N}{H\theta}(\nu h)} - 1}$$

Vibr. energy $\quad 0, \; \varepsilon, \; 2\varepsilon \ldots$

prawdop. $\quad 1 : e^{-\frac{\varepsilon}{k\theta}} : e^{-\frac{2\varepsilon}{k\theta}} \ldots$

$$1 + x + x^2 + x^3 \ldots = \frac{1}{1-x}$$

$$1 + 2x + 3x^2 + \ldots = \frac{1}{(1-x)^2}$$

N ~~elektronów~~ : $N = M\left[1 + e^{-\frac{\varepsilon}{k\theta}} + e^{-\frac{2\varepsilon}{k\theta}} + \ldots\right] = \frac{M}{1 - e^{-\frac{\varepsilon}{k\theta}}}$

energia całkowita : $E = M\varepsilon\left[e^{-\frac{\varepsilon}{k\theta}} + 2e^{-\frac{2\varepsilon}{k\theta}} + \ldots\right] = M\varepsilon \frac{e^{-\frac{\varepsilon}{k\theta}}}{(1 - e^{\ldots})^2}$

$$\frac{E}{N} = \varepsilon \frac{e^{-\frac{\varepsilon}{k\theta}}}{1 - e^{-\frac{\varepsilon}{k\theta}}} = \frac{\varepsilon}{e^{\frac{\varepsilon}{k\theta}} - 1}$$

$$E = \frac{N\varepsilon}{e^{\frac{\varepsilon}{k\theta}} - 1} = \frac{Nh\nu}{e^{\frac{h\nu}{k\theta}} - 1}$$

$$\sigma = \frac{\nu^2}{c^2} E = \frac{\nu^3}{\ldots} d\nu$$

$\nu = \frac{c}{\lambda}$

$d\nu = \frac{c}{\lambda^2} d\lambda \quad - \quad \lambda^5 \; d\lambda$

$h = 6{,}55 \cdot 10^{-27}$

$h\nu = 6{,}55 \cdot 10^{-27} \cdot \frac{c}{\lambda} = \left(\frac{10^{-4} \cdot 0{,}59}{3 \cdot 10^{10}}\right) = 3 \cdot 10^{-12}$ erg.

$\frac{6{,}5 \cdot 3 \cdot 10^{10} \cdot 10^{-27}}{5{,}9 \cdot 10^{-5}} = 3 \cdot 10^{-12}$

Jonnes

Laub [illegible] $\nu = 7 \cdot 10^{-9}$

[illegible] [illegible]

Ashtelungen

Ladenburg

Zeemant

Theorem $1 \cdot 10^{12}$

[illegible] $10^{4} \frac{erg}{cm^2} = 2{,}5 \cdot 10^{7}$ [illegible]

$\lambda = 0{,}200\,\mu$ $\nu = 6{,}3$

Ladenburg $V = 2$ Volt

Sposoby obserwacji

1). Oko ludzkie czer. 0·82 – 0·62
żółt – 0·56
ziel – 0·50
nieb – 0·45
fioł. – 0·38

2). Fluorescencja

3). Fotografia, w ogóle fotochemiczne efekty, aż do 0·1

Schumann kwarcowe soczewki, wodór, ewakuacja, płyty bez żelatyny

Ale nie tylko pozafioł.; sensibilizatory (farby) aż do 2·7
Eozyny, Cyanina, etc.

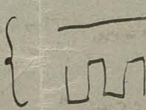
4). Cieplne (γ) Radyometr
Rubens & Nichols

5). Radiomikrometr Boys

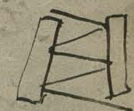
α). Stos termoelektr. Melloni (Thermometr)

Rubens: Linear Thermosäule 20 Fe-Konstantan
(10^{-6} ° = 1 mm) w złotej rezurgimi
Lebedew ewak.

β). Bolometr Langley 1881
czułość aż do 10^{-6} °



Lummer & Kurlbaum
bolometr (Pt 1 μ)
Tasktur $\Delta\theta = 10^{-7}$ °

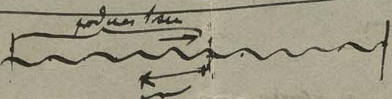


Aż do 60 μ Rubens Nichols (1897)
Rubens Aschkinass (1898)
Reststrahlen
Palnik Auera (bez siatki)
od Lymana 5 razy dłuższa $\lambda = 0{\cdot}061\,\mu$

300 μ Rubens & .. (1911) Quarz przepuszcza takie długie, ale inne zjawisko

Naj krótsze elektryczne Lampa, Lebedew 6 mm

Zasada Dopplera (1842)



$\lambda = c\tau$

$n = \frac{1}{\tau}$

$n' = n\left(1 + \frac{v}{c}\right) = \frac{1}{\tau'}$

$\frac{\tau'}{\tau} = \frac{1}{1 + \frac{v}{c}} \neq 1 - \frac{v}{c}$

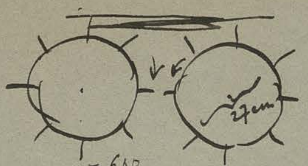
$\frac{\lambda'}{\lambda} = 1 + \frac{v}{c}$

Doświadczenie akustyczne Buys Ballot, Vogel (kometa?) Fizeau – koło posuwa(?) wozy(?) 1·051 1·050 Kong(?) dudnienie

Bielopolski

mikrometrem odbicie

fotograf.



30–50 obrotów / sek.

zwierciadła 2 × 10·5 × 0·3 cm

prędkość średnia 700 $\frac{m}{sec.}$

$$\frac{\frac{700 \cdot 12}{8.400}}{3 \cdot 10^8} = \pm 3 \cdot 10^{-5} \Big\} = \text{stosunkowe przesunięcie } \frac{d\lambda}{\lambda}$$

podwójny Na : D 0·5896156
0·5890188

10^{-3} więc $\frac{1}{30}$ część odstępu linii sodowych!

~~[illegible]~~

to byłoby jeszcze wygodnie dostrzegalne w lepszych spektroskopach, ale ta trudność że wygląd zamazany wskutek rozciągłości

zatem jedynie potwierdzenie jakościowe ilościowo ± 20%

Słońce Crew, Dunér $\left\{ \begin{array}{ll} 0^\circ & \text{---} \ 25{\cdot}46^d \\ 45^\circ & 30{\cdot}03 \\ 75^\circ & 38{\cdot}84 \end{array} \right.$

Kometa 1882 Thollon & Gouy 61–76 km spektr.
73 km astron.

Saturn Keeler (1895) obrót z prędkością wewnątrz większą

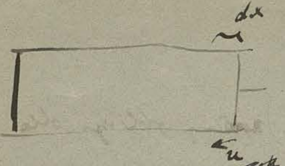
♀

Algol (1889) Vogel podwójny

Inne gwiazdy podwójne spektroskop.

Stark 1905 Kanalstrahlen H_2 N_2 Hg Na K $5-6 \cdot 10^7$ $\frac{e}{m}$ v tekście

Przy rozsunięciu tłoka zmienia się barwa i temperatura promieniowania [illegible] 94



$$\frac{\delta\lambda}{\lambda} = 2\frac{u}{c}\nu = 2\frac{dx}{\tau c}$$

$$\frac{d\lambda}{\lambda} = 2\frac{u}{c}\cdot\nu$$ (liczba odbić)

$$\nu = \frac{\frac{dx}{u}}{\frac{2x}{c}} = \frac{dx\, c}{2x\, u}$$

$$\frac{d\lambda}{\lambda} = \frac{dx}{x}$$

ale to tylko trzecia część promieni ulega przesunięciu $d\lambda$:

$$\frac{d\lambda}{\lambda} = \frac{1}{3}\frac{dx}{x}$$

Temperatura:

Gęstość promieniowania (Energia na $1\,cm^3$): $\psi(\theta)$ $\qquad \psi = c\,\theta^4$

$$E = q\,x\,\psi$$

po rozsunięciu tłoka: praca wykonana podczas rozsunięcia $\qquad dE = \pm q\frac{\psi}{3}(-dx)$

$$E' = q(x+dx)(\psi+d\psi)$$

$$E'-E = q\,x\,d\psi + q\,\psi\,dx = -q\frac{\psi}{3}dx$$

$$\frac{d\psi}{\psi} = -\frac{4}{3}\frac{dx}{x} = -4\frac{d\theta}{\theta}$$

$$\frac{1}{3}\frac{dx}{x} = -\frac{d\theta}{\theta}$$

zatem $\frac{d\lambda}{\lambda} = -\frac{d\theta}{\theta}$

$\lambda\theta = \text{const.}$

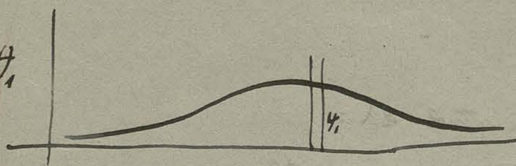
$$\lambda = \frac{\alpha}{\theta}$$

$$\frac{d\lambda}{d\theta} = -\frac{\alpha}{\theta^2}$$

Wien'sches Verschiebungsgesetz

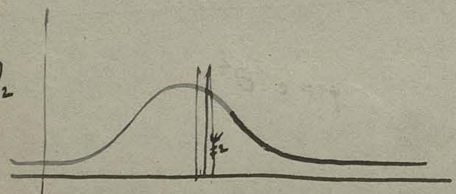
Krzywe promieniowania dla jednej temp. (dla ciała czarnego) ~~można~~ otrzymać dla jakiejś bądź innej:

θ_1



po zgęszczeniu:

θ_2



$$\lambda_2 : \lambda_1 = \theta_1 : \theta_2$$

$$d\lambda_2 : d\lambda_1 = \theta_1 : \theta_2$$

$$\int \psi_2 \, d\lambda = \int \psi_1 \, d\lambda = \theta_1^4 : \theta_2^4$$

zatem $\psi_2 : \psi_1 = \theta_2^5 : \theta_1^5$

Wien: $\psi(\lambda, \theta) = \dfrac{c_1 e^{-\frac{c_2}{\lambda\theta}}}{\lambda^5}$

$\lg \psi = \dots$

$\dfrac{\partial \psi}{\partial \lambda} = 0$ $\lambda_{max}\,\theta = \mathrm{const} = 29$[illegible] (μ)

słońce jest (Langley)

$\lambda_m = 0{\cdot}5\,\mu$

$\theta = 5774^\circ = 5501^\circ C$

Langley jest [illegible] [illegible] -2[illegible]$^\circ$

wzgl[illegible] -20°

$=$ [illegible] $12{\cdot}2\,\mu$

podług tego rachunek $10{\cdot}7\,\mu$

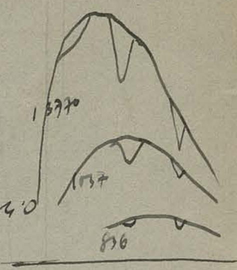
dla rzeczywistego [illegible]

Planck

Paschen $I = C \frac{e^{-\frac{c}{\lambda\theta}}}{\lambda^{\alpha}}$

α vyšlo 5·09

sadze, CuO 5·62



Lummer, Pringsheim [symbol] 1897 → α = 3·96

1μ – 6μ

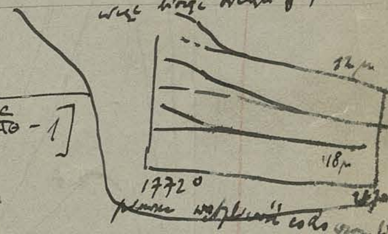
později po opravách α = 4·0

Paschen [symbol 1] 1897

ale nesouhlasí [illegible]

Lummer-Pringsheim 1900 až do 18μ ([illegible]) od −186° do 1383° || $\log I = \alpha - \frac{\beta}{\theta}$ (Isochromaty) [illegible]

[illegible]



Planck 1900 $I = \frac{C\lambda^{-5}}{e^{\frac{c}{\lambda\theta}} - 1} = \frac{C}{\lambda^5\left[e^{\frac{c}{\lambda\theta}} - 1\right]}$

Rubens & Kurlbaum 1900 − −188° až do 1500°

Paschen 1901

Reststrahlen Quarz 8·85μ

Flusspat λ=24, 31·6

Reststrahlen von Steinsalz λ = 51·2μ

$\lambda T \gg 1$

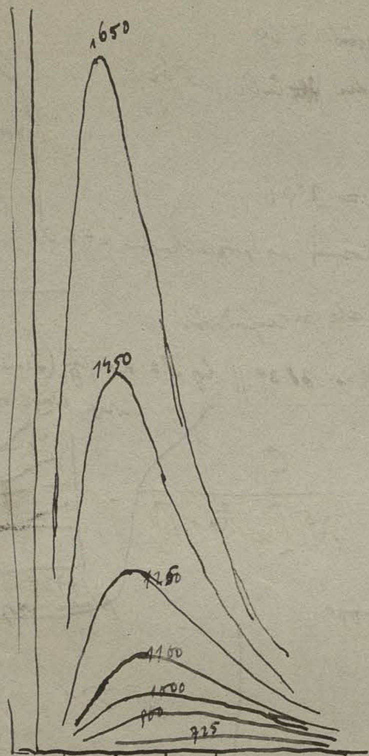
$I = \frac{cT e^{-\frac{c}{\lambda T}}}{\lambda^4}$

θ	I	Wien	Planck
−273		−121·5	−23·8
−188	−20·6	−107·5	−21·9
−80	−11·8	−48·0	−12·0
+20	0	0	0
250	34·0	63·5	30·4
500	64·5	96	63·8
750	98·1	118	93·2
1000	132·0	132	132
1500	196·8	147·5	200
1250°	164·5	141	166

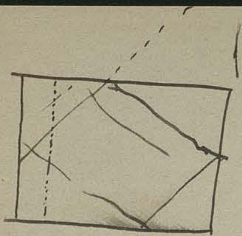
$C = 3{\cdot}7179 \cdot 10^{-5}$ (erg cm²/sec)

$c = 1{\cdot}4598$ (cm grad)

$\lambda_m \theta = 2940$ μ°



525	první žár ~~[illegible]~~
700	třešňově červená
1000	jasná
1200	jasně pomerančová
1300	bílo-žlutá
~~[illegible]~~	jaskravo



Jeśli promień nachylony pod kątem θ to skrócenie

$$\frac{d\lambda}{\lambda} = 2\frac{u}{c}\cos\theta$$ przy jednem odbiciu

~~podczas tak ...~~

ale podczas drogi dx będzie $\frac{c\,\Delta t}{x\cos\theta}$ odbić

więc $$\frac{d\lambda}{\lambda} = \frac{2u}{c}\cos\theta \cdot \frac{c\cos\theta}{x}\frac{dx}{u} = \frac{dx}{x}\cos^2\theta$$

przeciętnie zatem: $$\left(\overline{\frac{d\lambda}{\lambda}}\right) = +\frac{dx}{x}\frac{\int_0^{\frac{\pi}{2}} 2\pi\sin\theta\,d\theta\cos^2\theta}{\int_0^{\frac{\pi}{2}} 2\pi\sin\theta\,d\theta} = \frac{1}{3}\frac{dx}{x}$$

ale de facto nie rozdzielają się promienie na 3 kategorye, tylko jeśli ruch dostatecznie powolny to równomiernie (jeśli ściany mają ∞ gładkie i t. d.)

$$\frac{d\lambda}{\lambda} = +\frac{1}{3}\frac{\delta x}{x} \qquad \frac{d\theta}{\theta} = $$

Przez ~~Kl...~~ exp. adiab. ...

$\varphi = (\psi - d\psi)\,dx$

$\psi = \frac{\psi}{3}\,dx + \frac{d\theta}{\theta}$

$$\frac{d(\psi x)}{~~dx~~} = -\frac{\psi}{3}\,dx$$

$$\psi\,dx + x\,d\psi = -\frac{\psi}{3}\,dx$$

$$\frac{d\psi}{\psi} = -\frac{4}{3}\frac{dx}{x} = 4\frac{d\theta}{\theta} = -4\frac{d\lambda}{\lambda}$$

$\lambda\theta = \mathrm{const}$

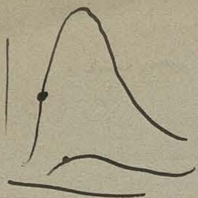
$f(\lambda,\theta) = \varphi\left(\frac{\lambda}{\theta}\right)$

$\frac{\psi}{\theta^4}$ to samo dla $\varphi(\theta)$

$$\frac{\psi(\theta)}{\theta^4} = \int_0^\infty \frac{\varphi(\lambda\theta)}{\theta^5}\,d(\lambda\theta)$$

$f(\lambda\theta)$

2887



$\psi_{\theta_0} = f_c(\lambda)$

$\psi =$ ~~[illegible]~~ $f(\theta)\,\varphi(\lambda\theta)$

~~[illegible] $\int f(\theta, \lambda\theta)\, d\lambda =$~~

$$\int_0^\infty f(\theta)\,\varphi(\lambda\theta)\,d\lambda = C\theta^4$$

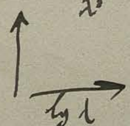
$$= \frac{f(\theta)}{\theta}\int_0^\infty \varphi(x)\,dx$$

$$\therefore f(\theta) \sim \theta^5$$

$$\therefore \psi = \theta^5 \varphi(\lambda\theta)$$

Mo tak[illegible] $\psi = \frac{\varphi(\lambda\theta)}{\lambda^5}$

Paschen



blachy pokryte sadzą

Fe_2O_3

CuO

Pt użył

$$J = c\,\frac{e^{-\frac{c_2}{\lambda\theta}}}{\lambda^\alpha} \quad / \alpha = 5{\cdot}3 - 6{\cdot}4 \ \text{Pt}$$

Wien 1896 $\alpha = 5$

Planck 1899

Czy Schwarzschild Kayser Rubens [illegible]

$\log\psi = \log\varphi(\lambda\theta) - 5\lambda$

$= \Phi(\lambda\theta) - 5\lambda$

$\log\psi + 5\lambda = \Phi(\lambda\theta)$

$= \Phi\,\log(\lambda\theta)$

$= \Phi(\log\lambda + \log\theta)$

więc [illegible] wzgl. λ, θ [illegible]

prawo co można skorzystać [illegible]

gęstość = $\int \dots = \frac{1}{\alpha c}$ 97

Gęstość promieniowania niezależna od źródła

Pewna gęstość [illegible] i pewien skład; gęstość charakteryzuje temperaturę prom. jeżeli skład odmienny od ∽ to można mówić o mieszaninie promieniowania o różnej temp.

Niemożliwość oziębienia jakimkolwiek sposobem do niższej temp. niż źródło promieni.

Ciśnienie promieniowania

Bartoli 1876: Gdyby nie ciśnienie to można by za pomocą tłoka i cylindra promieniowanie przenosić z ciała ciepl. do zimn. bez kompensaty

Maxwell 1873

Energia $W = \frac{1}{8\pi}(K Z^2 + \mu M^2)$ pro Vol. Einh.

$= 2\frac{K}{8\pi} A^2 \sin^2\alpha(t + \frac{x}{v})$

$P = W$ ↗

$\bar{P} = \bar{W} = \frac{K}{8\pi} A^2$

$Z = A \sin\alpha(t + \frac{x}{v})$

$X = Y = 0$

$M = A\sqrt{\frac{K}{\mu}} \sin\alpha(t + \frac{x}{v})$

$L = N = 0$

$K \frac{\partial Z}{\partial t} = c\left(\frac{\partial M}{\partial x} - \frac{\partial L}{\partial y}\right)$

$\mu \frac{\partial M}{\partial t} = -c\left(\frac{\partial X}{\partial z} - \frac{\partial Z}{\partial x}\right)$

$\mu K \frac{\partial^2 Z}{\partial t^2} = + c^2 \frac{\partial^2 Z}{\partial x^2}$

$v = \frac{c}{\sqrt{\mu K}}$

$\frac{\partial X}{\partial x} + \frac{\partial Y}{\partial y} + \frac{\partial Z}{\partial z} = 0$

$\frac{\partial L}{\partial x} + \frac{\partial M}{\partial y} + \frac{\partial N}{\partial z} = 0$

$\mu\alpha = c\frac{\alpha}{v}$

~~Transport energii (Poynting)~~

~~$= \frac{1}{4\pi}(Z M)$~~

~~$W = \frac{A^2}{4\pi}\sqrt{\frac{K}{\mu}} \sin^2\alpha(t + \frac{x}{v})$~~

~~$\bar{W} = \frac{A^2}{8\pi}\sqrt{\frac{K}{\mu}}$~~

Przepływ energii przez sek. = $v\bar{W} = \frac{2 \text{ gcal.}}{60}$ słońce

$\bar{W} = \frac{2 \cdot 4.2 \cdot 10^7}{60 \cdot 3 \cdot 10^{10}} = \frac{8.4 \cdot 10^{-5}}{1.8}$

$2\bar{P}$ pro m² = 1 dyn

$x = 0 \quad Z = 0$

$Z = A\left[\sin\alpha(t + \frac{x}{v}) + \sin\alpha(t - \frac{x}{v})\right] = 2A \sin\alpha t \cos\alpha\frac{x}{v}$

$M = A\sqrt{\frac{K}{\mu}}\left[\sin\alpha(t + \frac{x}{v}) - \sin\alpha(t - \frac{x}{v})\right] = 2A\sqrt{\frac{K}{\mu}} \cos\alpha t \sin\alpha\frac{x}{v}$

$W = \frac{K}{8\pi} Z^2 + \frac{\mu}{8\pi} M^2 = 4\frac{A^2 K}{8\pi}\left[\sin^2\alpha t \cos^2\alpha\frac{x}{v} + \cos^2\alpha t \sin^2 \dots\right]$

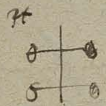
$\bar{W} = A^2 \frac{K}{4\pi}$

Prom. prostopad. padanie bez strat

absorb. P

zwierc. $2P$

Lebedew 1900 Nichols i Hull 1903 Ann. 12



jedna blaszka 0·1 mm

druga " 0·02 " ← siły radiometr. 5 razy tak duże ...

Poynting; Barlow siły styczne → pos. ciśn.

Poynting The pressure of light 1910

momenty obrotowe

reakcja źródła światła

Lebedew ciśnienie gaz absorbujący ; Komety! Arrhenius Schwarzschild etc!

Jeśli promieniowanie w wszystkich kierunkach

dS (zwierc.)

$$P = \frac{2\pi \sin\varphi\, d\varphi\, dS \cos\varphi}{c} \cdot \frac{\eta}{4\pi\alpha}\, 2\cos\varphi \;\Big|\; = \frac{4\pi e}{c}\,\frac{1}{3}$$

$$= \frac{dS.\eta}{\alpha c}\,\frac{1}{3} = \frac{u}{3}$$

Gęstość

98

Gęstość energii przy ... promieniu.

$$\eta \frac{\;}{4\pi r^2} e^{-\alpha r} \frac{r^2 d\omega\, dr}{c}$$

$$6 = \frac{\eta}{4\pi\alpha c}\int_0^{\pi} 2\pi \sin\theta \, d\theta = \frac{\eta}{\alpha c} =$$

$$e_{pow.} = \frac{\eta}{4\pi\alpha}$$

Ciśnienie promieni.

$$P = \frac{\eta}{4\pi\alpha c}\, 2\int_0^{\frac{\pi}{2}} 2\pi \sin\theta \cos^2\theta \, d\theta = \frac{\eta}{\alpha c}\cdot\frac{1}{3} = \frac{6}{3}$$

$x\, d6.$

$$d((x+\delta x)6) + \frac{6_1}{3} d\delta x = 0$$

$$d(x6) + \frac{6}{3} dx$$

$$U = v\, 6 = \text{const}\, v_0 6_0$$

$$\delta\varphi = v\, d6 + P\, dv = v\, d6 + \frac{6}{3} dv$$

$$\frac{\delta\varphi}{\theta} = dS$$

$$\frac{v}{\theta}\frac{\partial 6}{\partial\theta} = \theta\frac{\partial S}{\partial\theta}$$

$$\frac{v}{\theta}\frac{\partial 6}{\partial v} + \frac{6}{3} = \theta\frac{\partial S}{\partial v}$$

$$+\frac{1}{3}\frac{\partial 6}{\partial\theta} - \frac{\partial 6}{\partial\theta} = \frac{\partial S}{\partial v}$$

$$\theta_1:\; x\, 6_1 + \frac{6_1}{3} x$$

$$\frac{6_1}{3} dx = -x\, d6_1$$

$$dW = \frac{6_1}{3}(x+dx) - \frac{(6_1 - d6_1)}{3} = \frac{d6_1}{3} x$$

$$dW = \frac{4}{3}6_1 x \cdot \frac{d\theta}{\theta} = \frac{1}{3} x_1\, d6_1$$

φ

$$a f(\theta) + a \frac{f\theta_1}{3}$$

$$d[(a+x) f(\theta)] = -\frac{f}{3} dx \quad f(dx) + x\, df = 0$$

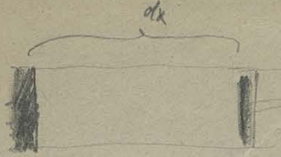
$a + d$

$(a+x)\cdot f$

$$4\frac{d\theta}{\theta} = \frac{d6}{6}$$

$$\lg 6 = \lg\theta^4$$

$$6 \sim \theta^4$$



(z [illegible] stron)

diff vol. energie $= \psi(T)$

[illegible] $p = \frac{\psi(T)}{3}$

1). T_1 $\quad q x \psi(T_1)$ $\quad q \frac{\psi(T_1)}{3} dx = W$

Ciepło wydatkowane ilość ciepła:

$(q\, dx)\, \psi(T_1) + q \cdot \frac{\psi(T_1)}{3} dx$

2). ~~[illegible]~~ zmiana temp. T_2 $\quad q \frac{\psi(T_2)}{3} dx = -W$

zyskuje

$q\, dx[\psi(T_1) - \psi(T_2)] + \psi(T_2)\, q\, dx$

$+ q \frac{\psi(T_2)}{3} dx$

3). zesunąć

odwracalność!

$$\frac{\psi(T_1) - \psi(T_2)}{3} = \frac{T_1 - T_2}{T_1} \frac{4}{3} \psi(T_1)$$

$$\frac{d\psi}{\psi} = 4 \frac{dT}{T}$$

$$\psi = a T^4 \qquad \text{Stefan (1879)}$$

1). Że wzgl. ~~[illegible]~~ musi wynikać odwrotnie wynika z Carnota, bo inaczej z ciepła zimnego za ciepło ciepło bez [illegible]. **Bartoli, Boltzmann**

wielkość teorię elektromagnet.

~~2.~~ przyjmuje $p = E$

słońce na powierzchni $\frac{2\ \text{cal}}{\text{min}}$ $\quad p = \frac{2 \cdot 4.2 \cdot 10^7}{3 \cdot 10^{10} \cdot 60} = \frac{4.2}{9} \cdot 10^{-4} = 5 \cdot 10^{-5}$

$\frac{1}{30} \frac{\text{gcal}}{\text{sec}}$

Pypsat   zdolnost rozsouz. ∞ AB $AB > \frac{\lambda}{d\lambda}$

$\frac{0{\cdot}0005889}{0{\cdot}000055} = 1\,cm$

100

D

excentrické stroje z všech [illegible]

Spectroskop.

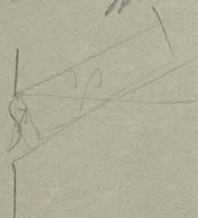
vidmo [illegible]

podrobka

~~[illegible]~~

à Vyso dráti

Lattké dyfr.



$\sin\beta = \frac{k\lambda}{b}$ rovnoměr.

$\frac{(k+\frac{1}{2})\lambda}{b}$ jasnost

nemnoho stejných b

$\sin\beta = \frac{k\lambda}{c}$ jasnost $= \frac{mk\lambda}{l}$

$mc = l$

$\sin(\beta + d\beta) = \frac{(mk+1)\lambda}{l}$

$d\beta\cos\beta = \frac{\lambda}{l}$

$d\beta \neq \frac{\lambda}{l} < \frac{k\,d\lambda}{c}$

$\frac{d\lambda}{\lambda} > \frac{c}{kl}\lambda = \frac{m}{k}\lambda$

$\sin(\beta + d\beta) = \frac{k(\lambda + d\lambda)}{c}$

$d\beta\cos\beta = \frac{k\,d\lambda}{c}$

0·5896156 μ
0·5890188 μ

$k = 1$: $\frac{d\lambda}{\lambda} =$ ~~0·000055~~ 1000

$\frac{1700}{mm}$

m do 100.000 !

vidmo slunce 12 m dlouhé ... 0·0001 μ

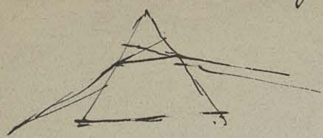
Rowland

síť drátěná
zrcadlo [illegible]
odrážení fotografie
(perlová matice)

mřížka vyhloubená

Pryzmat

Pozycja ~~[illegible]~~ odchyl. minimalnego

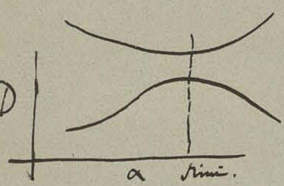


bo wtedy równe D dla nieco różnych –

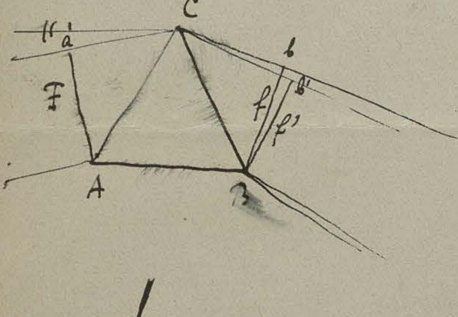
zatem to mi wyjdzie że promienie nieco różne

inaczej zależy widmo od ustawienia pryzmatu

że pozycja najm. odchyl. musi być max albo minim. to wynika z odwracalności prom.



Zdolność rozszczep.



dwa rodzaje światła λ λ'

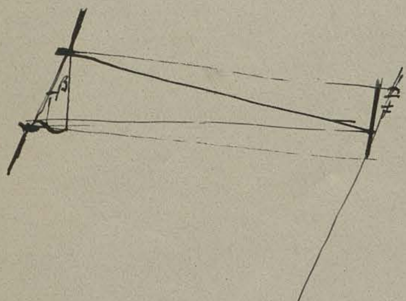
czas potrzebny jednakowy

$$n\,AB = aC + Cb$$

$$n'\,AB = aC + Cb'$$

$$(n'-n)\,AB = C(b'-b)$$

$$\alpha = \frac{Cb' - Cb}{Bb} = \frac{(n'-n)\,AB}{Bb}$$



ciemny prążek pierwszy:

$$\beta = \frac{\lambda}{Bb}$$

osobno tylko będzie widać jeśli $\alpha > \beta$

$$(n'-n)\,AB > \lambda$$

$$AB > \frac{\lambda}{n'-n}$$

podstawa zdolności rozszcz. szeroko, z drugiej strony obszerna

Flint

$n = 1{\cdot}650$

$dn = 0{\cdot}000055$

$\lambda = 0{\cdot}000589$ mm

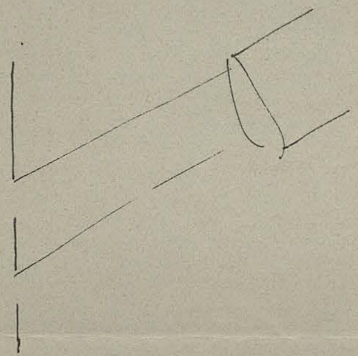
AB przynajmniej 1 cm

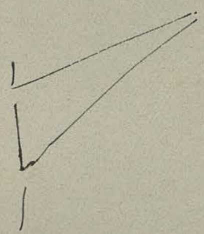
Widmo zależy od dyspersji substancji

Przyrównanie zapomocą widma pomocniczego — współcz. załam. $n_D - 1$

à Vision directe

	C	D	F	H	$\nu = \frac{n_F - n_C}{n_D - 1}$
Crown	1·515	1·518	1·524	1·533	0·0166
Flint	~~1·570~~	~~1·575~~	~~1·585~~	~~1·599~~	
	1·614	1·620	1·631	1·653	0·0276





$\beta = \frac{\lambda}{B}$

$\delta\beta = \frac{d\lambda}{b}$

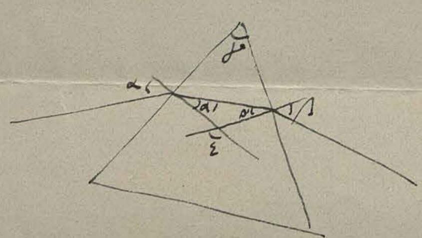
$\delta\gamma = \frac{\lambda}{B}$

$\delta\gamma > \delta\beta$

$\frac{d\lambda}{b} > \frac{\lambda}{B}$

$d\lambda > \frac{b\lambda}{B} = \frac{\lambda}{m}$

Na $\frac{d\lambda}{\lambda} = 0·001$ $m = 1000$



$\varepsilon = \alpha' + \beta' = 180 - \gamma$

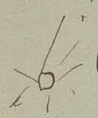
$\sin\alpha = n \sin\alpha'$

$\sin\beta = n \sin\beta'$

Gęstość energii

1 mg 1 cm²

$0{\cdot}0001 .\ r^2 = r^3$



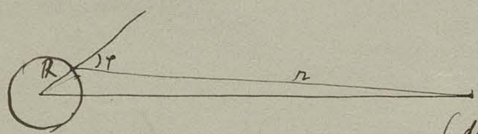
$$\rho = \int \frac{\eta\, d\sigma}{4\pi r^2}\,\frac{e^{-\alpha r}}{c} = \frac{\eta}{c}\int_0^\infty e^{-\alpha r}\, dr = \frac{\eta}{\alpha c}$$

podczas gdy dla zdolności emisyjnej [illegible] [illegible] $e = \frac{\eta}{4\alpha}$

więc $\rho = \frac{4e}{c}$

$\frac{\varepsilon\, d\omega}{\pi}$

$$\underbrace{i\int_0^{\frac{\pi}{2}} 2\pi \sin\varphi\, \pi\, d\varphi \cos\varphi}_{\Sigma\, i\, d\omega} = i\pi = e$$



$$\int df\, \frac{i\cos\varphi}{4\pi r^2} = \int_0^{\frac{\pi}{2}} \frac{2\pi \sin\varphi\, d\varphi \cos\varphi\, i R^2}{4\pi r^2} = \frac{i R^2}{4 r^2} = S$$

$$e = \frac{4\pi r^2}{R^2} S$$

$$\frac{4 . 10^{9} . 29}{0{\cdot}0013 . 29}$$

Graetz $\sigma = 1{\cdot}085 . 10^{-12}$ (1880)

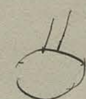
Schleiermacher Rosetti

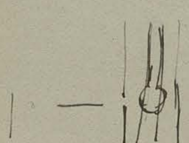
$$\frac{5{\cdot}32 . 10^{5}}{4{\cdot}2 . 10^{7}} = 1{\cdot}2\ 10^{-12}$$

Ostwald 1876

Abbot 1884

Schneebeli





871 1

Lummer – Pringsheim 1897

Paschen 1893

$E_{100} - E_0 = 0{\cdot}0176\ \frac{\mathrm{cal}}{\mathrm{sec}}$

3400

104

$$e^{-\frac{(x - x_0 - \beta x_0\tau - \gamma\tau)^2}{4\tau D}}$$

$$\frac{1}{(2\sqrt{\pi\tau D})}\int_{-\infty}^{+\infty} e^{-\frac{\left[\alpha - x_0(1+\beta\tau) - \gamma\tau\right]^2 + \left[x - \alpha(1+\beta\tau) - \gamma\tau\right]^2}{4\tau D}}\, d\alpha =$$

$$= \frac{1}{(2\sqrt{\pi\tau D})^2}\, e^{-\frac{\left[x_0(1+\beta\tau)+\gamma\tau\right]^2 + \left[x - \gamma\tau\right]^2}{4\tau D}} \int_{-\infty}^{+\infty} e^{-\frac{\alpha^2\left[1+(1+\beta\tau)^2\right] - 2\alpha\left[x_0(1+\beta\tau) + \gamma\tau + x(1+\beta\tau) - \gamma\tau(1+\beta\tau)\right]}{4\tau D}}\, d\alpha$$

(bracket over the coefficient of 2α: $(x+x_0)(1+\beta\tau) - \gamma\beta\tau^2$)

$$e^{-\frac{\left[x-\gamma\tau\right]^2 + \left[x_0(1+\beta\tau)+\gamma\tau\right]^2}{4\tau D} + \frac{\left[(x+x_0)(1+\beta\tau) - \gamma\beta\tau^2\right]^2}{\left[1+(1+\beta\tau)^2\right]4\tau D}} \sqrt{\frac{\pi \cdot 4\tau D}{1+(1+\beta\tau)^2}}$$

$$e^{-\frac{x_0^2(1+\beta\tau)^2 + 2\gamma\tau x_0(1+\beta\tau) + \gamma^2\tau^2 + x^2 - 2x\gamma\tau + x_0^2(1+\beta\tau)^4 + 2\gamma\tau x_0(1+\beta\tau)^3 + 2\gamma^2\tau^2(1+\beta\tau)^2}{4\tau D\left[1+(1+\beta\tau)^2\right]}}$$

$$+ x^2(1+\beta\tau)^2 - 2x\gamma\tau(1+\beta\tau)^2 - (x+x_0)^2(1+\beta\tau)^2 + 2(x+x_0)(1+\beta\tau)\gamma\beta\tau^2 - \gamma^2\beta^2\tau^4$$

$$2xx_0(1+\beta\tau)^2$$

~~$- x - x_0(1+\beta\tau)^2 - \gamma\tau(1+\beta\tau)$~~

$= e$

$+ 2x\gamma\beta^2\tau^3$

~~$+ 2x\gamma\beta\tau^2(1+\beta\tau)$~~

$$x^2 - 4x\gamma\tau - 2x\gamma\beta\tau^2 - 2x\gamma\beta^2\tau^3 + 2x\gamma\beta\tau^2 + 2x\gamma\beta^2\tau^3 - 2xx_0(1+\beta\tau)^2$$

$$+ 2\gamma\tau x_0 + 2\gamma\beta\tau^2 x_0 + 2\gamma\tau x_0(1+\beta\tau)^3 + 2x_0\gamma\beta\tau^2(1+\beta\tau) + x_0^2(1+\beta\tau)^4$$

$$+ 4\gamma^2\tau^2 + 4\beta\gamma^2\tau^3 + \gamma^2\beta^2\tau^4 - \gamma^2\beta^2\tau^4$$

$$+ 2\gamma\tau x_0\left[1+\beta\tau + (1+\beta\tau)^3 + \beta\tau(1+\beta\tau)\right] = (1+\beta\tau)^2 + (1+\beta\tau)^3 = (1+\beta\tau)^2(2+\beta\tau)$$

$$x - 2\gamma\tau + \gamma\beta^2\tau^3 - x_0(1+\beta\tau)^2$$

$$\int_{-\infty}^{+\infty} \frac{1}{2\sqrt{\pi D t}} \xi^2 e^{-\frac{\xi^2}{4Dt}} d\xi = 2Dt$$

$$\frac{1}{2\sqrt{\pi D t}} \int_l^\infty \xi e^{-\frac{\xi^2}{4Dt}} d\xi = 4Dt \int x e^{-x^2} dx = -4Dt \left. \frac{e^{-x^2}}{2} \right]_{\frac{l}{\sqrt{4Dt}}}^{\infty} = \frac{8Dt \cdot e^{-\frac{l^2}{4Dt}}}{8\sqrt{\pi D t}}$$

$$\overline{\xi^2} = 2Dt - \frac{4l}{\pi}\sqrt{Dt}\, e^{-\frac{l^2}{4Dt}} + \frac{2l^2}{\sqrt{\pi D t}} \int_l^\infty e^{-\frac{\xi^2}{4Dt}} d\xi$$

$$\frac{4l^2}{\sqrt{\pi}} \int_{\frac{l^2}{4Dt}}^{\infty} e^{-x^2} dx$$

$$2Dt \gg l^2$$

$$\lim_{t=\infty} \overline{\xi^2} = 2Dt - \frac{4l}{\pi}\sqrt{Dt} + 2l^2$$

$$\frac{l^2}{4Dt} = \beta^2 \qquad \beta = \frac{l}{2\sqrt{Dt}} \qquad 2\sqrt{Dt} = \frac{l}{\beta}$$

$$\overline{\xi^2} = 2Dt - \frac{2l^2}{\pi\beta} e^{-\beta^2} + \frac{4l^2}{\sqrt{\pi}} \int_\beta^\infty e^{-x^2} dx$$

$$\xi = \eta - 2l$$

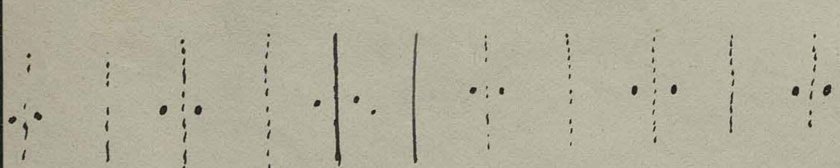
$$\int_{-l}^{\infty} \xi e^{-\frac{\xi^2}{4Dt}} d\xi + \int_{-l}^{\infty} \xi e^{-\frac{(2l+\xi)^2}{4Dt}} d\xi$$

$$+ \int_{+l}^{\infty} (\eta - 2l) e^{-\frac{\eta^2}{4Dt}} d\eta$$

$$\frac{\sqrt{Dt}}{\sqrt{\pi}} e^{-\frac{l^2}{4Dt}} - \frac{8l}{8\sqrt{\pi D t}} \int_{\frac{l}{2\sqrt{Dt}}}^{\infty} e^{-x^2} dx$$

$$\frac{l}{2\beta\sqrt{\pi}} e^{-\beta^2} - \frac{2\beta}{\sqrt{\pi}} \int_\beta^\infty e^{-x^2} dx$$

a



$$e^{-\xi^2} + e^{-(2a+\xi)^2} + e^{-(2a+2b+\xi)^2} + e^{-(4a+2b+\xi)^2} + e^{-(4a+4b+\xi)^2} + \dots$$

$$e^{-(2b-\xi)^2} + e^{-(2a+2b-\xi)^2} + e^{-(2a+4b-\xi)^2} + e \dots$$

$$e^{-\xi^2} + e^{-(2\alpha+\xi)^2} + e^{-(4\alpha+\xi)^2} + e^{-(6\alpha+\xi)^2} + \dots$$

$$e^{-(2\alpha-\xi)^2} + e^{-(4\alpha-\xi)^2} + e^{-(6\alpha-\xi)^2} + \dots$$

$$2n\alpha + \xi = \eta$$

$$2n\alpha - \xi = \eta$$

$$\xi = 2n\alpha - \eta$$

$$\int_{-\alpha}^{+\alpha} \xi^2 e^{-(2n\alpha+\xi)^2} d\xi = \int_{(2n-1)\alpha}^{(2n+1)\alpha} (\eta - 2n\alpha)^2 e^{-\eta^2} d\eta = \int \eta^2 e^{-\eta^2} d\eta - 4n\alpha \int \eta e^{-\eta^2} d\eta + 4\alpha^2 n^2 \int e^{-\eta^2} d\eta$$

$$\int_{-\alpha}^{+\alpha} \xi^2 e^{-(2n\alpha-\xi)^2} d\xi = -\int_{(2n+1)\alpha}^{(2n-1)\alpha} (2n\alpha - \eta)^2 e^{-\eta^2} d\eta = \uparrow$$

$$\overline{\xi^2} = 2\int_0^\infty \eta^2 e^{-\eta^2} d\eta - 4\alpha \sum_{n=1}^{\infty} n \int_{(2n-1)\alpha}^{(2n+1)\alpha} \eta e^{-\eta^2} d\eta + 4\alpha^2 \sum_1^\infty n^2 \int_{(2n-1)\alpha}^{(2n+1)\alpha} e^{-\eta^2} d\eta$$

~~$\overline{x-x_0^2} = \frac{1}{2}\sqrt{\frac{\beta}{\pi D(1-e^{-\beta t})}}\int (x-x_0)^2$~~

$\overline{x^2} = \frac{1}{2}\sqrt{\frac{\beta}{\pi D(1-e^{-\beta t})}} \frac{\int_{-\infty}^{+\infty} x^2 e^{-\frac{\beta(x-x_0e^{-\beta t})^2}{2D[1-e^{-2\beta t}]}}dx}{\int e^{-\,\cdots}\,dx}$

$x - x_0 e^{-\beta t} = z$

$\frac{\int_{-\infty}^{+\infty}[z + x_0e^{-\beta t}]^2 e^{-\gamma z^2}dz}{\int_{-\infty}^{+\infty} e^{-\gamma z^2}dz} = \overline{z^2} + \cancel{2x_0 e^{-\beta t}} + [x_0 e^{-\beta t}]^2$

$= \frac{1}{2\gamma} +$

$\overline{x^2} = \frac{2D[1-e^{-2\beta t}]}{2\beta} + x_0^2 e^{-2\beta t}$

$\overline{x} = \frac{\int [z + x_0e^{-\beta t}]e^{-\gamma z^2}dz}{\int e^{-\gamma z^2}dz} = x_0 e^{-\beta t}$

$\left.\frac{d\overline{|x|}}{dt} = \frac{e^{-2\beta t}\cdot 2\beta}{\sqrt{1-e^{-2\beta t}}}\sqrt{\frac{2D}{\pi\beta}}\right|_{t=0} = \infty$

somit wäre auch $T' = 0$ aber in Wirklichkeit kann $\frac{d\overline{|x|}}{dt}$ nur mittels Mittelungsgesetz gebildet werden (Ungenauigkeit der Br. M. F.) das kommt ein endlicher Wert für T'

β = Geschwindigkeit, wenn Abstand $x_0 = 1$

$= -B\,X_1$

$\lim_{t=\infty} \overline{x^2} = \frac{2D}{2\beta} = \frac{kB}{B X_1}$

$\frac{\overline{x^2}\cdot X_1}{2}$ = mittlere Arbeit $= \frac{k}{2}$

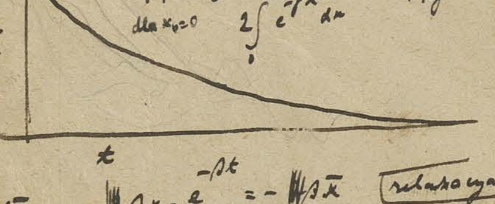
$= \frac{H\theta}{2N}$

$\overline{|x|} = 2\frac{\int_0^\infty x e^{-\gamma x^2}dx}{2\int_0^\infty e^{-\gamma x^2}dx} = \sqrt{\frac{1}{\pi\gamma}} = \frac{1}{\sqrt{\frac{\pi\beta}{2D(1-e^{-2\beta t})}}} = \sqrt{\frac{2D(1-e^{-2\beta t})}{\pi\beta}}$

dla $x_0 = 0$

~~unrichtig! sollte $\frac{1}{2}\frac{H\theta}{N}$ sein!~~

Stimmt! [illegible] 2β statt β?



$\frac{d\overline{x}}{dt} = -\beta x_0 e^{-\beta t} = -\beta\overline{x}$ relaxacya!

$dt = -\frac{1}{\beta}\frac{d\overline{x}}{\overline{x}}$

$dW = \sqrt{\frac{\beta}{2\pi D}}\, e^{-\frac{\beta x^2}{2D}}dx = \frac{dt}{T} = \frac{1}{T}\frac{dx}{\beta x}$

$T = \frac{e^{+\frac{\beta x^2}{2D}}\sqrt{2\pi D}}{x\beta\sqrt{\beta}}$

$T \gtrapprox \sqrt{2\pi\frac{\overline{x^2}}{x^2}}\frac{1}{\beta}e^{\frac{x^2}{2\overline{x^2}}}$

$W_1(x, x_0, \tau) = \frac{1}{2\sqrt{\pi\tau D}} e^{-\frac{[x - x_0(1+\beta\tau)]^2}{4\tau D}}$

między β ujemne!

$W_2(x, x_0, \tau) = \frac{1}{2\sqrt{\pi\tau D}\sqrt{1+(1+\beta\tau)^2}} e^{-\frac{[x - x_0(1+\beta\tau)^2]^2}{4\tau D[1+(1+\beta\tau)^2]}}$

$gS = 1 + g^n$

$1 + g + g^2 + \ldots g^{n-1} = S$

$g + \ldots\ g^n = Sg$

$g^n - 1 = S(g-1)$

$S = \frac{g^n - 1}{g - 1}$

$W_3(x, x_0, \tau) = \frac{1}{2\sqrt{\pi\tau D}\sqrt{1+(1+\beta\tau)^2[1+(1+\beta\tau)^2]}} e^{-\frac{[x - x_0(1+\beta\tau)^3]^2}{4\tau D[1+(1+\beta\tau)^2[1+(1+\beta\tau)^2]]}}$

$W_n(x, x_0, n\tau) = \frac{1}{2\sqrt{\pi\tau D}} \frac{1}{\sqrt{1 + \underset{g}{(1+\beta\tau)^2} + (1+\beta\tau)^4 + \cdots (1+\beta\tau)^{2(n-1)}}} e^{-\frac{[x - x_0(1+\beta\tau)^n]^2}{4\tau D\underbrace{[1+(1+\beta\tau)^2+\cdots\cdots(1+\beta\tau)^{2(n-1)}]}_{=\varphi_n}}}$

$\varphi_n = \frac{(1-\beta\tau)^{2n} - 1}{-\beta\tau} = 2n + \frac{2n(2n-1)}{2}\beta\tau + \cdots =$

$= \frac{g^{n+1}-1}{g-1} = \frac{(1-\beta\tau)^{2n+2} - 1}{(1+\beta\tau)^2 - 1} = \frac{e^{-2n\beta\tau} - 1}{2\beta\tau} = \frac{1 - e^{-2\beta t}}{2\beta t}$

$(1-\beta\tau)^n = (1-\beta\tau)^{-n\beta\tau\cdot\frac{1}{\beta\tau}} = e^{-n\beta\tau}$

$W_n(x, x_0, t) = \cancel{\frac{1}{2\sqrt{\pi\tau D}}\frac{1}{\sqrt{2n}}} e^{-\frac{[x - x_0 e^{-n\beta\tau}]^2}{4D\tau\cdot 2n}}$

$= \frac{1}{2\sqrt{2\pi t D}} e^{-\frac{[x - x_0e^{-\beta t}]^2}{8tD}}$

$\frac{e^{-\frac{[x - x_0 e^{-n\beta\tau}]^2 2\beta\tau}{4D\tau(1-e^{-2\beta t})}}}{2\sqrt{\pi\tau D\cdot\frac{(1-e^{-2\beta t})}{2\beta\tau}}}$

$\lim_{\tau=0} S = \lg\sqrt{\frac{\beta}{2\pi D}} - \frac{\beta}{2D}x^2$

$W(x, x_0, t) = \frac{\sqrt{2\beta}}{2\sqrt{\pi D(1-e^{-2\beta t})}} e^{-\frac{[x - x_0 e^{-\beta t}]^2 2\beta}{4D[1-e^{-2\beta t}]}} = \sqrt{\frac{\beta}{2\pi D(1-e^{-2\beta t})}} e^{-\frac{\beta[x - x_0e^{-\beta t}]^2}{2D[1-e^{-2\beta t}]}}$

określa ruch Browna przy istnieniu siły sprężystej dążącej ku środkowi [illegible], jeżeli punkt za czas $t=0$ [illegible] w x_0

9400

Samo wyjście w bliskości ściany

~~[illegible]~~

$$W(x, x_0, \tau)\,dx = \frac{1}{2\sqrt{\pi\tau D}}\left[e^{-\frac{(x-x_0)^2}{4\tau D}} + e^{-\frac{(x+x_0)^2}{4\tau D}}\right]dx$$

$$W(x, x_0, \tau)\,dx = \frac{1}{2\sqrt{\pi\tau D}}\left[e^{-\frac{(x-x_0+\gamma\tau)^2}{4\tau D}} + e^{-\frac{(x+x_0-\gamma\tau)^2}{4\tau D}}\right]dx$$

$$W(x, x_0, 2\tau)\,dx = dx\int_{\alpha=0}^{\infty} W(\alpha, x_0, \tau)\,d\alpha\; W(x, \alpha, \tau)$$

$$\int_0^\infty e^{-\frac{(\alpha-x_0+\gamma\tau)^2}{4\tau D} - \frac{(x-\alpha+\gamma\tau)^2}{4\tau D}}\,d\alpha + \int e^{\frac{-(\alpha-x_0+\gamma\tau)^2-(x+\alpha-\gamma\tau)^2}{4\tau D}}\,d\alpha\; +$$

$$+\int e^{\frac{-(\alpha+x_0-\gamma\tau)^2-(x+\alpha-\gamma\tau)^2}{4\tau D}}\,d\alpha + \int_0^\infty e^{\frac{-(\alpha+x_0-\gamma\tau)^2-(x-\alpha+\gamma\tau)^2}{4\tau D}}\,d\alpha$$

$$\int e^{-\left\{2\alpha^2+2\alpha(-x_0+\gamma\tau+x+\gamma\tau)+(x_0-\gamma\tau)^2+(x+\gamma\tau)^2\right\}}$$

$$\frac{m}{2}\frac{d^2}{dt^2}(x^2) + \frac{1}{2\beta}\frac{d(x^2)}{dt} = Fx - \gamma x + m\left(\frac{dx}{dt}\right)^2$$

$$\frac{m}{2}\frac{d(x^2)}{dt} + \frac{x^2}{2\beta} = \int xF\,dt - \gamma\int x\,dt + 2t\bar{L}$$

A w razie siły sprężystej:

$$\frac{m}{2}\frac{d(x^2)}{dt} + \frac{x^2}{2\beta} = \int xF\,dt - \gamma\int x^2\,dt + 2t\bar{L}$$

Więc $\frac{m}{2}\frac{d\overline{x^2}}{dt} + \frac{\overline{x^2}}{2\beta} + \gamma\int\overline{x^2}\,dt = 2t\bar{L}$

Równanie średnich wyrażenie dla $\overline{x^2}$! Czy się spełnia przy podstawieniu owym za drugą?

$$\overline{x^2} = \frac{D}{\beta}\left[1-e^{-2\beta t}\right] + x_0^2 e^{-4\beta t}$$

$$= k\left[1-e^{-2\beta t}\right] + x_0^2 e^{-4\beta t}$$

$D = k\beta$

$$\int\overline{x^2}\,dt = k\left[t + \frac{(e^{-2\beta t}-1)}{2\beta}\right] - \frac{x_0^2}{4\beta}\left(e^{-4\beta t}-1\right)$$

$$\frac{d}{dt}(\overline{x^2}) = -2\beta k e^{-2\beta t} - 4\beta x_0^2 e^{-4\beta t}$$

$$\frac{d^2}{dt^2}(\overline{x^2}) = -4\beta^2 k e^{-2\beta t} - 16\beta^2 x_0^2 e^{-4\beta t}$$

$$\frac{m}{2}\left[-2\beta k e^{-2\beta t} - 4\beta x_0^2 e^{-4\beta t}\right] + \frac{k}{2\beta}\left[1-e^{-2\beta t}\right] + \frac{x_0^2}{2\beta}e^{-4\beta t} + \gamma k t + \frac{k(e^{-2\gamma t}-1)}{2}$$

$$-\frac{x_0^2}{4}\left(e^{-4\gamma t}-1\right) = 2t\,k\,?$$

Przy istnieniu ciężkości opór siły sprężystej

$$W(\xi, x, \tau)\, d\xi = \frac{A}{\ } e^{-\frac{(\xi - \beta x \tau - \gamma \tau)^2}{4\tau D}} d\xi$$

$$= \frac{1}{2\sqrt{\pi \tau D}} e^{-\frac{[x - x_0(1+\beta\tau) - \gamma\tau]^2}{4\tau D}} dx$$

$$= \frac{1}{2\sqrt{\pi \tau D}} e^{-\frac{[x - x_0 - (x_0\beta + \gamma)\tau]}{4\tau D}} dx$$

$$e^{-\frac{1}{4\tau D}\left\{[\alpha - x_0(1+\beta\tau) - \gamma\tau]^2 + [x - \alpha(1+\beta\tau) - \gamma\tau]^2\right\}}$$

$$= e^{-\frac{1}{4\tau D}\left\{[x_0(1+\beta\tau)+\gamma\tau]^2 + [x - \gamma\tau]^2\right\} - \alpha^2[1 + (1+\beta\tau)^2] - 2\alpha\,[x_0(1+\beta\tau) + x(1+\beta\tau) - \gamma\beta\tau^2]}$$

$$= e^{-\frac{1}{4\tau D}\left\{\frac{[x - x_0(1+\beta\tau)^2]^2 + 2\gamma\tau[x_0(1+\beta\tau) - x] + 2\gamma\tau^2}{[1 + (1+\beta\tau)^2]}\right\}}$$

$$= e^{-\frac{1}{4\tau D}}\left\{ x - x_0(1+\beta\tau) \right.$$

$$-2\gamma\beta\tau^2 x_0(1+\beta\tau) - 2\gamma\beta\tau^2 x(1+\beta\tau) + \gamma^2\beta^2\tau^4$$

$$\frac{\left\{[x_0(1+\beta\tau)+\gamma\tau]^2 + [x - \gamma\tau]^2\right\}[1 + (1+\beta\tau)^2] + [x_0 + x(1+\beta\tau)]^2(1+\beta\tau)^2}{1 + (1+\beta\tau)^2}$$

$$= \frac{[x - x_0(1+\beta\tau)^2]^2}{1 + (1+\beta\tau)^2} + 2\gamma\tau[x_0(1+\beta\tau) - x] + 2\gamma\tau^2$$

$$W_1 = e^{-\frac{[x - x_0(1+\beta\tau)]^2 - 2\gamma\tau[x - x_0(1+\beta\tau)] + \gamma^2\tau^2}{\ }}$$

$$W_2 = e^{-\frac{x x_0 (1+\beta\tau)^2}{1 + (1+\beta\tau)^2} - 2\gamma\tau[x - x_0(1+\beta\tau)] + 2\gamma\tau^2}$$

$$x_0^2(1+\beta\tau)^2 + 2\gamma\tau x_0(1+\beta\tau) + x^2 - 4x\gamma\tau - 2x\beta\gamma\tau^2$$

$$+ x_0^2(1+\beta\tau)^4 + 2\gamma\tau x_0(1+\beta\tau)^3 + x^2(1+\beta\tau)^2$$

$$\left\{x_0^2(1+\beta\tau)^2 + x^2(1+\beta\tau)^4 + 2x x_0(1+\beta\tau)^3 - 2\gamma\beta\tau^2 x_0(1+\beta\tau) - 2\gamma\beta\tau^2 x(1+\beta\tau) + \gamma^2\beta^2\tau^4\right\}$$

$$+ x_0^2(1+\beta\tau)^2 + x^2(1+\beta\tau)^2 + \gamma^2\tau^4 + 2x x_0(1+\beta\tau)^2 - 2x_0\gamma\beta\tau^2(1+\beta\tau)$$

$$+ 4\gamma\tau^2 + 4\gamma^2\beta\tau^3 + \gamma^2\beta^2\tau^4$$

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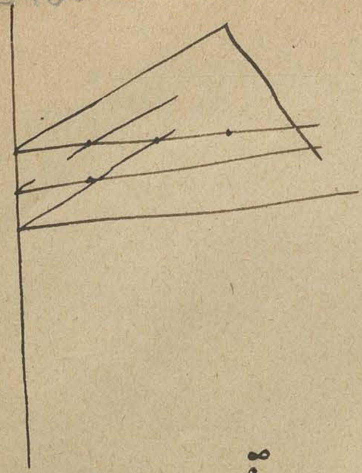

$$\overline{W}_n(x) = \frac{1}{2\lambda}\int_{x-\lambda+\varepsilon}^{x+\lambda+\varepsilon} \overline{W}_{n-1}(\alpha, x_0)\, d\alpha \qquad \text{dla } x > \lambda-\varepsilon$$

$$\overline{W}_n(x) = \frac{1}{2\lambda}\int_0^{x+\lambda+\varepsilon} \overline{W}_{n-1}(\alpha, x_0)\, d\alpha + \int_0^{\lambda-\varepsilon-x} \overline{W}_{n-1}(\alpha, x_0)\, d\alpha \qquad 0 > x < \lambda-\varepsilon$$

$$= \frac{1}{2\lambda}\left[\int_{\lambda-\varepsilon+x}^{x+\lambda+\varepsilon} \ldots\ldots + 2\int_0^{\lambda-\varepsilon+x-} \ldots\right]$$

$$\overline{W}_1(\alpha, x_0) = \frac{1}{\pi}\int_0^\infty \frac{\sin \varrho\lambda}{\varrho\lambda} \cos \varrho(\alpha - x_0 + \varepsilon)\, d\varrho \qquad \text{dla } \alpha > \lambda-\varepsilon$$

$$\overline{W}_2(\alpha, x_0) = \frac{1}{\pi}\int_0^\infty \left(\frac{\sin \varrho\lambda}{\varrho\lambda}\right)^2 \cos \varrho(\alpha - x_0 + 2\varepsilon)\, d\varrho$$

$$\overline{W}_3(\alpha, x_0) = \frac{1}{\pi}\int_0^\infty \left(\frac{\sin \varrho\lambda}{\varrho\lambda}\right)^3 \cos \varrho(\alpha - x_0 + 3\varepsilon)\, d\varrho$$

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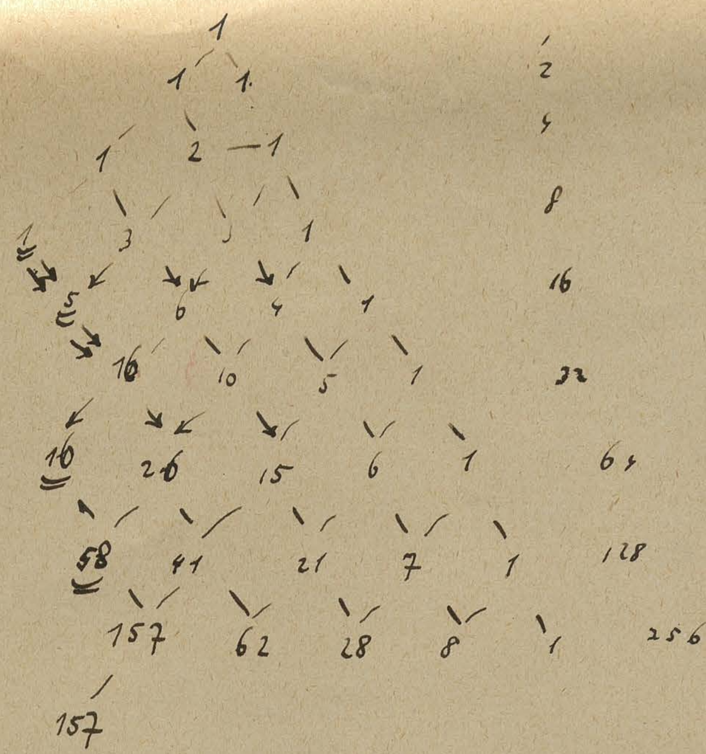
$$\overline{W}_2(\alpha, x_0) = \frac{1}{2\lambda}\left\{\frac{1}{\pi}\int_0^\infty \frac{\sin \varrho\lambda}{\varrho\lambda}\, d\varrho \int_0^{x+\lambda+\varepsilon} \cos \varrho(\alpha - x_0 + \varepsilon)\, d\alpha + \int_0^{\lambda-\varepsilon+x} \cos \varrho(\alpha - x_0 + \varepsilon)\, d\alpha\right\} \qquad 0 < \alpha < \lambda-\varepsilon$$

$$= \frac{1}{2\pi}\int_0^\infty \frac{\sin \varrho\lambda}{(\varrho\lambda)^2}\left[\sin \varrho(\alpha - x_0 + \lambda + 2\varepsilon) + \sin \varrho(x_0 - \varepsilon) + \sin \varrho(\lambda - x_0 + x) + \sin \varrho(x_0 - \varepsilon)\right]$$



$$\frac{\partial c}{\partial t} = D\frac{\partial^2 c}{\partial x^2} + v\frac{\partial c}{\partial x} \qquad \frac{\partial c}{\partial x} = 0 \Big|^{x=0} \quad \text{also} \int_0^\infty c\,dx = \text{const}$$

$$\int e^{-\frac{x^2}{Dt}}$$

$$\frac{1}{t} \qquad \ldots\left[\left(\frac{x}{t}-\alpha\right)^2 + \frac{1}{t}\right] + v\left(\frac{x}{t}-\alpha\right)$$

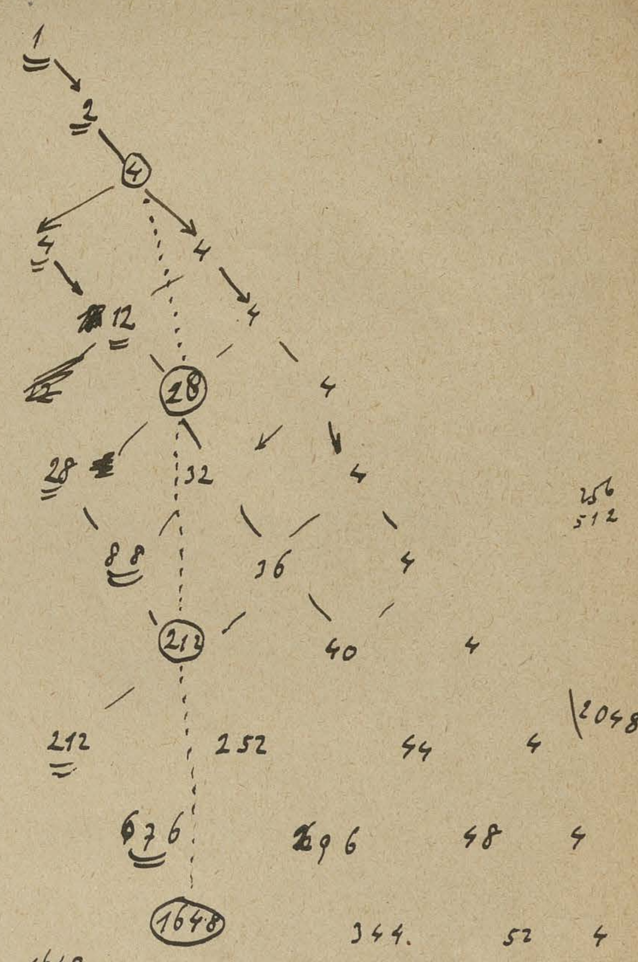
$$\frac{e^{-\frac{x^2}{t}-\alpha x}}{\sqrt{t}}$$

$2\alpha D = v$

$4D\alpha^2 = \alpha v$

$$1 \quad \frac{4}{8} \quad \frac{28}{64} \quad \frac{212}{512} \quad \frac{1648}{4096}$$

$$1 \cdot \frac{1}{2} \cdot \frac{7}{16} \cdot \frac{53}{128} \cdot \frac{107}{256}$$



$$\int_0^\infty \bar{W}(\alpha, x_0, \tau)\, W(x, \alpha, \tau)\, d\alpha =$$

$$\bar{W}(x, x_0, n\tau) = \int_0^\infty \bar{W}(\alpha, x_0, (n-1)\tau)\, W(x, \alpha, \tau)\, d\alpha$$

$$= \int \left[W(\alpha, x_0, n\tau) - \frac{\partial W}{\partial \tau}(\alpha, x_0, n\tau)\, \bar{W}(x, \alpha, \tau) \right] \bar{W}(x, \alpha, \tau)\, d\alpha$$

$$= \underbrace{W(x, \infty, \tau)}_{=0} \int_0^\infty \underbrace{\bar{W}(\alpha, x_0, (n-1)\tau)\, d\alpha}_{=1} - W(x, 0, \tau) \int_0^0 \; - \int_0^\infty d\alpha\, \frac{\partial W(x, \alpha, \tau)}{\partial \alpha} \int_0^\alpha \bar{W}(\alpha, x_0, (n-1)\tau)\, d\alpha$$

$$\int_0^\infty e^{-\frac{(x - x_0 + \gamma\tau)^2}{4\tau D}}\, dx + B \int_0^\infty e^{-\frac{(x + x_0 + \gamma\tau)^2}{4\tau D}}\, dx = \int_0^\infty e^{-\frac{(x - x_0)^2}{4\tau D}} - \gamma\tau$$

$$e^{-\frac{(x - x_0)^2}{4\tau D} - \frac{\gamma}{2D}(x - x_0)}$$

$$\bar{W}_n(x, x_0) = \int_0^\infty \bar{W}_{n-1}(\alpha, x_0)\, \bar{W}_1(x, \alpha)\, d\alpha$$

$$\bar{W}_2(x, x_0) = \int_0^\infty e^{-\alpha^2 + 2\alpha(x + x_0) - (x_0 - \gamma\tau)^2 - (x + \gamma\tau)^2}\, d\alpha = e^{-x^2 - x_0^2 - 2\gamma\tau(x - x_0)} \int e^{-\alpha^2 + 2\alpha(x + x_0)}\, d\alpha$$

$$-(x - \beta + \gamma\tau)^2$$

$$\bar{W}_2(\beta, x_0) = \int e^{-\alpha^2 + 2\alpha(\beta + x_0) - (x_0 - \gamma\tau)^2 - (\beta + \gamma\tau)^2}\, d\alpha \qquad \bar{W}_1(x, \beta) = e$$

$$\bar{W}_3(x, x_0) = \int d\beta\, d\alpha\; e^{-\alpha^2 + 2\alpha(\beta + x_0) - (x_0 - \gamma\tau)^2 - (\beta + \gamma\tau)^2 - (x - \beta + \gamma\tau)^2}$$

$$p_1(z) = \frac{1}{\pi} \int_0^\infty dq \int_{-\infty}^{+\infty} p_1(\beta) \cos q(z - \beta)\, d\beta$$

$$p_1(z, x_0) = \frac{1}{\pi} \int_0^\infty dq \int_{0}^{+\infty} \left[e^{-(\beta - x_0 + \gamma\tau)^2} + e^{-(\beta + x_0 - \gamma\tau)^2} \right] \cos q(z - \beta)\, d\beta$$

$$p(x, z) = \frac{1}{\pi} \int_0^\infty dq \int_0^\infty \left[e^{-(\alpha - z + \gamma\tau)^2} + e^{-(\alpha + z - \gamma\tau)^2} \right] \cos q(x - \alpha)\, d\alpha$$

Hipoteza:

$$W(x, x_0, \tau) = \frac{e^{-\frac{(x - x_0 + \gamma\tau)^2}{4\tau D}}}{\int_0^\infty e^{-\frac{(x - x_0 + \gamma\tau)^2}{4\tau D}}\, dx}$$

Wtedy, [illegible] $\lim_{\tau = \infty} = e^{-\frac{x}{\cdot\cdot}}$.

$$\int e^{-\frac{(x - x_0)^2}{4\tau D} - \frac{\gamma}{2D}(x - x_0)}\, dx \qquad A\, e^{-\frac{\gamma}{2D}x_0 + \frac{(\gamma\tau)^2}{4\tau D}} \int_0^\infty e^{-\frac{(x - x_0 + \gamma\tau)^2}{4\tau D}}\, dx = 1$$

$$A = \frac{e^{\frac{\gamma}{2D}x_0 - \frac{(\gamma\tau)^2}{4\tau D}}}{\int_0^\infty \cdots}$$

$$W(x,x_0,\tau)\,dx = A\left[e^{-\frac{(x-x_0+\gamma\tau)^2}{4\tau D}}dx + e^{-\frac{(x+x_0-\gamma\tau)^2}{4\tau D}}dx\right]$$

$$\int_0^\infty e^{-\frac{(x-x_0)^2}{\mu}}dx + \int_0^\infty e^{-\frac{(x+x_0)^2}{\mu}}dx$$

$$\int_{x_0}^\infty e^{-\frac{\xi^2}{\mu}}d\xi + \int_{-x_0}^\infty e^{-\frac{\xi^2}{\mu}}d\xi \quad = -\int_{+x_0}^{-\infty} d\xi$$

$$W(x,x_0,2\tau)\,dx = \int_0^\infty W(\alpha,x_0,\tau)\,dx\; W(x,\alpha,\tau)$$

$$= A^2\int_0^\infty e^{-(\alpha-x_0+\gamma\tau)^2-(x-\alpha+\gamma\tau)^2} + e^{-(\alpha-x_0\gamma\tau)^2-(x+\alpha-\gamma\tau)^2} + e^{-(\alpha+x_0-\gamma\tau)^2-(x-\alpha+\gamma\tau)^2} + e^{-(x+\alpha-\gamma\tau)^2-(\alpha+x_0-\gamma\tau)^2}\cdot d\alpha$$

$$\int_0^\infty e^{-\alpha^2+2\alpha(x+x_0)-(x_0-\gamma\tau)^2-(x+\gamma\tau)^2}d\alpha + \int e^{-\alpha^2+2\alpha(x_0-x)-(x_0-\gamma\tau)^2-(x-\gamma\tau)^2}d\alpha +$$

$$+\int_0^\infty e^{-\alpha^2+4\alpha\gamma\tau-2\alpha(x_0-x)-(x_0-\gamma\tau)^2-(x+\gamma\tau)^2}d\alpha + \int_0^\infty e^{-\alpha^2+4\alpha\gamma\tau-2\alpha(x+x_0)-(x-\gamma\tau)^2-(x_0-\gamma\tau)^2}d\alpha$$

W razie małego τ:

$$e^{-\frac{(x-x_0)^2}{4\tau D}-\frac{\gamma}{2D}(x-x_0)-\frac{\gamma^2\tau}{4D}}$$

$$= e^{-\frac{(x-x_0)^2}{4\tau D}}\left[1-\frac{\gamma}{2D}(x-x_0)+\frac{\gamma^2}{4D^2}(x-x_0)^2-\frac{\gamma^2\tau}{4D}\right] -$$

$$+ e^{-\frac{(x+x_0)^2}{4\tau D}}\left[1+\frac{\gamma}{2D}(x+x_0)+\frac{\gamma^2}{4D^2}(x+x_0)^2-\frac{\gamma^2\tau}{4D}\right]$$

$$\int_0^\infty e^{-\frac{(\alpha-x_0)^2}{4\tau D}}\left[1-\frac{\gamma}{2D}(\alpha-x_0)+\frac{\gamma^2}{4D^2}(\alpha-x_0)^2-\frac{\gamma^2\tau}{4D}\right]e^{-\frac{(x-\alpha)^2}{4\tau D}}\left[1-\frac{\gamma}{2D}(x-\alpha)+\frac{\gamma^2}{4D^2}(x-\alpha)^2-\frac{\gamma^2\tau}{4D}\right]$$

$$\int_0^\infty e^{-\alpha^2+2\alpha\beta}d\alpha = e^{+\beta^2}\int_0^\infty e^{-(\alpha-\beta)^2}d\alpha = e^{\beta^2}\int_\beta^\infty e^{-x^2}dx \qquad \alpha-\beta = x,\ \alpha = x+\beta$$

$$e^{(x+x_0)^2-(x_0-\gamma\tau)^2-(x+\gamma\tau)^2}\int_{x+x_0}^\infty \cdots + e^{(x_0-x)^2-(x_0-\gamma\tau)^2-(x-\gamma\tau)^2}\int_{x_0-x}^\infty \cdots$$

$$e^{2xx_0+2\gamma\tau(x_0-x)-2\gamma^2\tau^2}\int_{x+x_0}^\infty\cdots + e^{-2xx_0+2\gamma\tau(x_0+x)-2\gamma^2\tau^2}\int_{x_0-x}^\infty\cdots$$

$\|\ e^{(x+\gamma\tau)(x_0-\gamma\tau)}$

$-(x-\gamma\tau)(x_0-\gamma\tau)$

$-(x_0-\gamma\tau)(x+\gamma\tau)$

$(x-\gamma\tau)(x_0-\gamma\tau)$

$$+e^{(2\gamma\tau-x_0+x)^2-(x_0-\gamma\tau)^2-(x+\gamma\tau)^2}\int_{x-x_0+2\gamma\tau}^\infty \quad + e^{(2\gamma\tau-x-x_0)^2-(x-\gamma\tau)^2-(x_0-\gamma\tau)^2}\int_{2\gamma\tau-x-x_0}$$

$$\int_{-\infty}^{x_0-x-2\gamma\tau} \qquad \int_{-\infty}^{x_0+x-2\gamma\tau}$$

$$e^{2\gamma^2\tau^2-2\gamma\tau(x_0-x)-2xx_0}\left[\int_{-\infty}^{x_0-x}e^{-\alpha^2}d\alpha-2\gamma\tau\,e^{-(x_0-x)^2}\right] + e^{2\gamma^2\tau^2-2\gamma\tau(x+x_0)+2xx_0}\left[\int_{-\infty}^{x_0+x}-2\gamma\tau\,e^{-(x_0+x)^2}\right]$$

$$W_2(x,x_0) = \int_{-\infty}^{+\infty} \frac{4c^2}{\pi^2} \Big[\frac{dq}{q} \sin q(\alpha - x_0 + \varepsilon) \frac{dp}{p} \sin p(x - \alpha + \varepsilon) \cos q\lambda \cos p\lambda$$

$$+ \frac{dq\,dp}{p\cdot q} \sin q\left(\alpha + x_0 \frac{\lambda-\varepsilon}{\lambda+\varepsilon} + \varepsilon\right) \sin p(x - \alpha + \varepsilon) \cos q\lambda \cos p\lambda \Big]$$

$$= \frac{2c^2}{\pi} \int_{\alpha-\lambda-\varepsilon}^{\alpha+\lambda-\varepsilon} d\alpha \int_0^\infty \frac{dq}{q} \left[\sin q(x - \alpha + \varepsilon) + \sin q\left(\alpha + \alpha\frac{\lambda-\varepsilon}{\lambda+\varepsilon} + \varepsilon\right) \right] \cos q\lambda$$

$$= \frac{2c^2}{\pi} \int^\infty \frac{dq}{q} \cos q\lambda \quad \cos q(\alpha$$

$$W_2(x,x_0) = \frac{1}{2\lambda} \int_{-\infty}^{+\infty} d\alpha \; \frac{2}{\pi} \int_0^\infty \frac{dq}{q\lambda} \sin q\lambda \left[\sin q(\alpha - x_0 + \varepsilon) + \sin q\left(\alpha + x_0 \frac{\lambda-\varepsilon}{\lambda+\varepsilon} + \varepsilon\right) \right]$$

$$= \frac{1}{\pi} \int_0^\infty \frac{dq}{\,} \frac{\sin q\lambda}{2q^2\lambda^2} \left[\sin q(x + \lambda + \varepsilon - x_0 + \varepsilon) - \sin q(x - \lambda + \varepsilon - x_0 + \varepsilon) \right.$$

$$= \left[\sin q(x - x_0 + 2\varepsilon + \lambda) - \sin q(x - x_0 + 2\varepsilon - \lambda) \right]$$

$$= 2\cos q(x - x_0 + 2\varepsilon) \sin q\lambda$$

$$= \frac{1}{\pi} \int dq \frac{\sin^2 q\lambda}{(q\lambda)^2} \left[\cos q(x - x_0 + 2\varepsilon) + \cos q\left(x + x_0 \frac{\lambda-\varepsilon}{\lambda+\varepsilon} + 2\varepsilon\right) \right]$$

$$W_n(x,x_0) = \frac{1}{\pi} \int_0^\infty dq \left(\frac{\sin q\lambda}{q\lambda} \right)^n \left[\cos q(x - x_0 + n\varepsilon) + \cos q\left(x + x_0 \frac{\lambda-\varepsilon}{\lambda+\varepsilon} + n\varepsilon\right) \right]$$

$$e^{-\frac{n q^2 \lambda^2}{6}}$$

$$\frac{(x - x_0 + n\varepsilon)^2}{n\lambda^2}$$

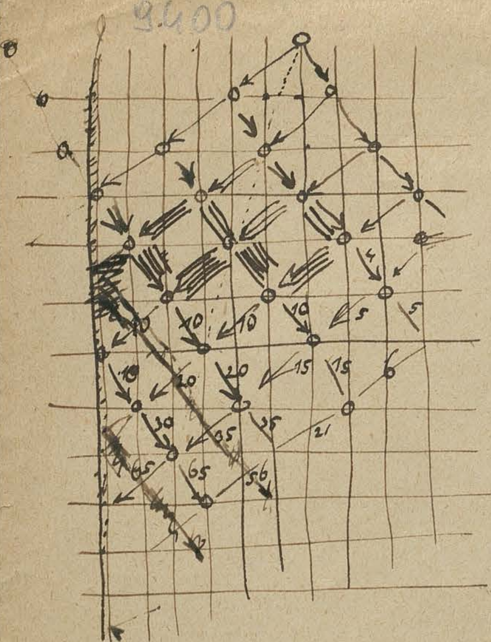
$n\lambda = ct$

$c\lambda = D$

$$\cdots \left\{ e^{-\frac{(x - x_0 + \gamma t)^2}{4Dt}} + e^{-\frac{(x + x_0 f(t) + \gamma t)^2}{4Dt}} \right\}$$

$$\frac{\lambda - \varepsilon}{\lambda + \varepsilon} = \frac{c - \gamma}{c + \gamma}$$

9400



$$W_n(x, x_0) = \int_0^\infty W_{n-1}(\alpha, x_0)\, W(x, \alpha)\, d\alpha$$

$$W_1(x, x_0) = c \Big|_{x = x_0 - \lambda - \varepsilon}^{x = x_0 + \lambda - \varepsilon} = \frac{1}{2\lambda}$$

pod warunkiem: $x_0 > \lambda + \varepsilon$

$$W_1(x, x_0) = 2c \Big|_{x=0}^{x=}$$

Uzupełniając sobie po drugiej stronie ściany obraz dowolnemu widzimy, że zastępuje zupełnie efekt ściany, zatem stąd całkiem ogólnie będzie:

nie prawda! bo wtedy byłby punkt czyli z prawej strony ku lewej! nie byłoby wtedy zupełnie!

$$W(x, \alpha) = c \Big|_{x = \alpha - \lambda - \varepsilon}^{x = \alpha + \lambda - \varepsilon}$$

$$W_0(x, x_0) = 1 \quad \begin{array}{c} x = -x_0 \frac{\lambda - \varepsilon}{\lambda + \varepsilon} \\ \hline x = x_0 \\ \hline \end{array}$$

$$W_1(x, x_0) = c = \frac{1}{2\lambda} \Big|_{x = x_0 - \lambda - \varepsilon}^{x = x_0 + \lambda - \varepsilon} \quad \Big|_{x = -x_0 \frac{\lambda - \varepsilon}{\lambda + \varepsilon} - \lambda - \varepsilon}^{x = -x_0 \frac{\lambda - \varepsilon}{\lambda + \varepsilon} + \lambda - \varepsilon}$$

$$= \frac{1}{\pi} \int_0^\infty d\varrho \int_{-\infty}^{+\infty} W_1(z) \cos \varrho (x - z)\, dz = \frac{c}{\pi} \int d\varrho \left\{ \int_{x_0 - \lambda - \varepsilon}^{x_0 + \lambda - \varepsilon} \cos \varrho (x - z)\, dz + \int_{-x_0 \frac{\lambda - \varepsilon}{\lambda + \varepsilon} - \lambda - \varepsilon}^{\cdots} \cos \varrho (x - z)\, dz \right\}$$

$$W_1(x, x_0) = -\frac{c}{\pi} \int \frac{d\varrho}{\varrho} \left[\sin \varrho (x - x_0 - \lambda + \varepsilon) - \sin \varrho (x - x_0 + \lambda + \varepsilon) + \sin \varrho \left(x + x_0 \tfrac{\lambda - \varepsilon}{\lambda + \varepsilon} - \lambda + \varepsilon\right) - \sin \varrho \left(x + x_0 \tfrac{\lambda - \varepsilon}{\lambda + \varepsilon} + \lambda + \varepsilon\right) \right]$$

$$W(x, \alpha) = -\frac{c}{\pi} \int \frac{d\varrho}{\varrho} \left[\sin \varrho (x - \alpha - \lambda + \varepsilon) - \sin \varrho (x - \alpha + \lambda + \varepsilon) \right]$$

$$W_1(x, x_0) = +2\frac{c}{\pi} \int \frac{d\varrho}{\varrho} \left[\cos \varrho (x - x_0 + \varepsilon) + \cos \varrho \left(x + x_0 \tfrac{\lambda - \varepsilon}{\lambda + \varepsilon} + \varepsilon\right) \right] \sin \varrho \lambda$$

$$W(x, \alpha) = 2\frac{c}{\pi} \int \frac{d\varrho}{\varrho} \cos \varrho (x - \alpha + \varepsilon) \sin \varrho \lambda = c \Big|_{x = \alpha - \lambda - \varepsilon}^{x = \alpha + \lambda - \varepsilon}$$

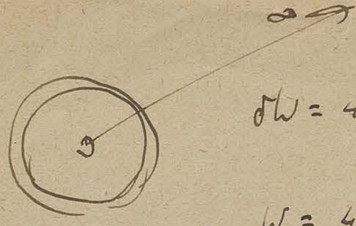
$$W_1(x, x_0) = \frac{1}{\pi} \int_0^\infty \frac{\sin \varrho \lambda}{\varrho \lambda} \left[\cos \varrho (x - x_0 + \varepsilon) + \cos \varrho \left(x + x_0 \tfrac{\lambda - \varepsilon}{\lambda + \varepsilon} + \varepsilon\right) \right]$$

81/53 I A 9

„Ruchy Boawne"

81/53 V A 6 113

Włoskowatości



$$\delta W = 4\pi\rho m\, r^2 f(r)\, dr$$

$$W = 4\pi\rho m \int_0^\infty r^2 f(r)\, dr$$

falsch!

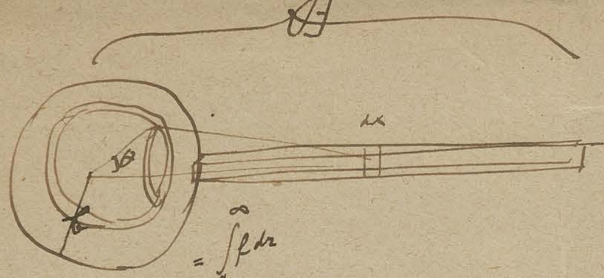
$$I = \int p\, dv + 2\pi\rho \underbrace{\int_0^\infty r^2 f(r)\, dr}$$

pro 1g. $p(v_1 - v_2)$

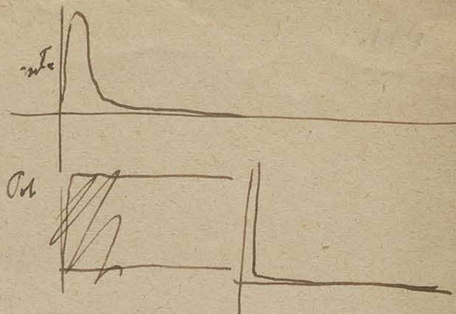
$$= -\int_0^\infty x \frac{d\psi}{dr}\, dr = \int_0^\infty \psi(r)\, dr = \frac{a}{2\pi}$$

$$\underbrace{\qquad\qquad}_{= a\rho}$$

$$I = \int p\, dv + a\rho$$



$$= \int_r^\infty f\,dr$$



$m m' \chi(r)$ = Pot... $- m m' \chi(r)$ = ...

$$-2\pi\rho^2\, d\sigma\, dx\, u^2 \sin\vartheta\, du\, d\vartheta \; \frac{d\chi(r)}{dx}$$

pido=

$$dA = 2\pi\rho^2\, d\sigma \int_0^b u\,du \int_b^\beta dx \frac{d}{dx} \underbrace{\int_0^\pi \chi(r)\, u \sin\vartheta\, d\vartheta}_{} \qquad \int_r^\infty r\,\chi(r)\,dr = \psi(r)$$

$$u\,x \sin\vartheta\, d\vartheta = r\,dr \qquad = \frac{1}{x}\left[\psi(x-u) - \psi(x+u)\right]$$

$$\int_b^\beta = \frac{1}{\beta}\left[\psi(\beta-u) - \psi(\beta+u)\right] - \frac{1}{b}\left[\psi(b-u) - \psi(b+u)\right]$$

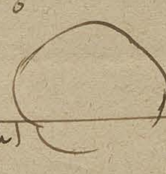
$$\quad \overset{\shortparallel}{0} \qquad \overset{\shortparallel}{0} \qquad\qquad \overset{\shortparallel}{0}$$

$$dA = \frac{2\pi\rho^2\, d\sigma}{b} \underbrace{\int_0^b u\,du\, \psi(b-u)}_{} \qquad z = b-u$$

$$= b\int_0^b \psi\,dz - \int_0^b z\,\psi\,dz \qquad \psi(\infty) = 0$$

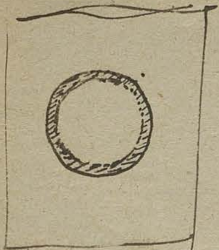
$$p_i = a\rho^2 - \frac{2\alpha\rho^2}{b} \qquad a = 2\pi\int_0^\infty \psi(z)\,dz \qquad \alpha = \pi\int_0^\infty z\,\psi(z)\,dz$$

ofstvie $-\alpha\rho^2\left(\frac{1}{R_1} + \frac{1}{R_2}\right)$ venii pouvrcutivi



$+\alpha\rho^2\left(\frac{1}{R}\frac{1}{R}\right)$

dr



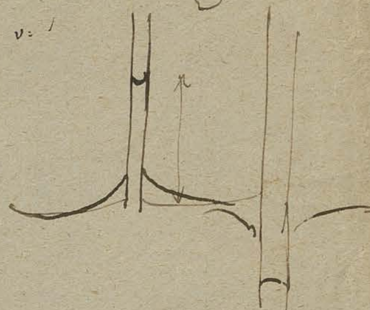
$4\pi R^2 dR$

$\Delta p = \frac{-2\alpha}{\rho g R}$

$N d\vartheta + \alpha\, dx$

$\delta\varphi = dU + \alpha\, dx + Ap\, dv$

$\left[\frac{2}{R}\cdot\alpha \frac{\;}{dx} + Ap\right]\sigma\, dx$



$\Omega = 4\pi R^2$

$d\Omega = 8\pi R\, dR$

$dR = \frac{d\Omega}{8\pi R}$

$dVol = \frac{R}{2}\, d\Omega$

2 sposoby p ... : w całości praca $Ap\, dv$

rozdział na kropli $A\left[p' + \frac{2\alpha}{R\sigma}\right] dv$

$p' = p \quad \frac{2\alpha}{R\sigma}$

$p \geqq \frac{\alpha}{r}$

Wilhelmy

Adhäsions platten

$g\rho \frac{z_1^2 - z_2^2}{2}$

$g\rho r z = \alpha\left(\frac{1}{\lambda} + \frac{1}{r_2}\right)$

Tropfen

$z = \text{const} - \frac{2\alpha}{\rho g} \text{const}$

~~... $h = 15{\cdot}33 - 0{\cdot}0286\, t$~~ (1 mm = r)

$\alpha = \alpha_0(1 - ct)$

H_2O	c = 0·0019
Alk.	0·0022
H_2SO_4	0·0025
Äther	0·005

$\frac{dy}{d}$

117

$\frac{1}{R_1} = \infty$

$\frac{ds}{d\varphi} = \frac{dy}{dy}\frac{ds}{d\varphi} =$

$\alpha \frac{1}{R_2} = \frac{\frac{d^2y}{dx^2}}{\sqrt{1+}^{3/2}} \quad \alpha = y$

$y \frac{dy}{} = \alpha \, dy \, \frac{d^2y}{dx^2} =$

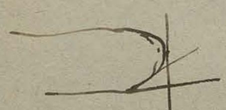
$\frac{y^2}{2} = \frac{\alpha}{\sqrt{1+\left(\frac{dy}{dx}\right)^2}} = \alpha \cos\varphi + const$

$0 = \qquad \alpha \cos\varphi_0 + c$

$y^2 = \frac{2\alpha}{s}(1-\cos\varphi)$

$y = \sqrt{\frac{2\alpha}{s}}\sqrt{1-\cos\varphi}$

$= \sqrt{a}\sqrt{\ }$



$Y = \sqrt{a^2} = a$

α H_2O = 78 0°

74 20°

59 100°

2 = 15'4

Hg 436 Alk 25 Ether 19

6'1 5'4 mm

$tg\varphi = \frac{dy}{dx}$

$\frac{d\varphi}{dx\cos^2\varphi} = \frac{d^2y}{dx^2}$

$y = \alpha \frac{d\varphi}{dx \cos^2\varphi} \cdot \cos^3\varphi$

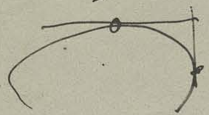
$= \alpha \cos\varphi \frac{d\varphi}{dx} = \alpha\cos\varphi \frac{d\varphi}{dy} tg\varphi$

$y^2 = a \cos\varphi + h$

$= a \sin\varphi \frac{d\varphi}{dy}$

$$z = \alpha \cos\gamma \frac{\partial y}{\partial z} = = \alpha \frac{\frac{d^2 z}{dx^2}}{\sqrt{\quad}^{\,3}} = \alpha \frac{d}{dx}\frac{1}{\sqrt{\quad}} = \alpha$$

$$\frac{z^2}{2} = \alpha\,(h - \cos\gamma)$$



$$\Big\}\; y_0 = \sqrt{\alpha}$$

$y = 0 \quad z = 0 \quad h = 1$

$$\frac{z^2}{2} = \alpha\,(1 - \cos\gamma) = \alpha \sin\frac{\gamma}{2}$$

$$z = \sqrt{\alpha}\,\sqrt{1-\cos\gamma}$$

2

$$r = r_0 + x$$

$$z = \frac{1}{r_0 + x}\frac{\partial}{\partial x}\,(r_0 \dots)$$

$= \dot{x}$

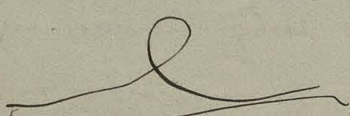
$$z = \sin\frac{\gamma}{2}$$

$\sin\gamma$

$$z^2 \alpha \;\big(= \frac{1}{\sqrt{1+\left(\frac{dz}{dx}\right)^2}}\big)$$

$$\sqrt{1+\left(\frac{dz}{dx}\right)^2} = \frac{1}{z^2\alpha}$$

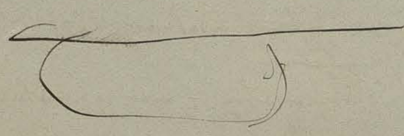
$$\left(\frac{dz}{dx}\right) = \sqrt{\frac{1}{(z^2\alpha)^2} - 1}$$

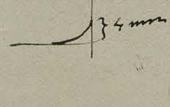


Doświadczenie przy ścianie płaskiej tekstowo

$$h = \sqrt{2\alpha} = \sqrt{\frac{2\alpha}{\rho g}} = \text{woda}\ \sqrt{\frac{160}{980}} = \sqrt{0{\cdot}16}$$

$$= 0{\cdot}4\ \text{cm}$$





szerszą akcję zewnętrzną poprowadzić własnymi środkami.

Ustępując obecnie z tego miejsca, cieszę się że skonstatować trzeba od szeregu lat trwający pomyślny rozwój naszego towarzystwa, uwydatniający się nawet naocznie w grafikonie Dr. Romera w częstym przyroście liczby członków i ciągłym wzroście dochodów. Życzę aby rozwój w tych kierunkach i nadal trwał, aby nam z czasem umożliwił szerszą akcję zewnętrzną koło krzewienia u nas nauk przyrodniczych i aby w równej mierze się doskonaliła nasza wewnętrzna działalność naukowa, która nie zależy tak dalece od organizacji towarzystwa, jak przedewszystkiem od zapału i naukowej kwalifikacji pojedyńczych jego członków!

Jeśli nie ma sił zewnętrznych tylko nap. pow.

~~$\Delta \int \frac{1}{R}$~~ $\Delta F = 0$

$$p = K + \alpha\left(\frac{1}{R_1} + \frac{1}{R_2}\right)$$

$$\int \left(\frac{1}{R_1} + \frac{1}{R_2}\right) v \, df = 0$$

---

błonki mydlane, różne ciecze o jednakowej gęstości

Jeśli błonka mydl. ~~jedna~~ $p = 2\alpha\left(\frac{1}{R_1} + \frac{1}{R_2}\right)$

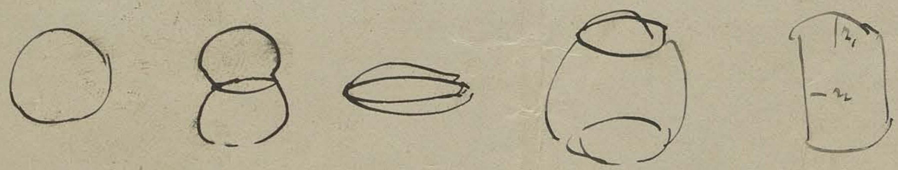
jeśli wszystkie części atmosf. w komunikacji:

$$p = 2\alpha\left(\frac{1}{R_1} + \frac{1}{R_2}\right) = 0$$

---

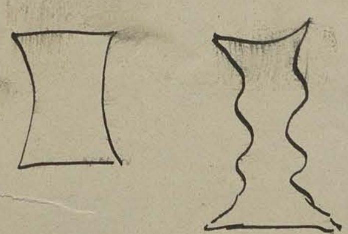
ciecze o jednak. gęstości Plateau (1843–1863) oliwa – woda + alk.

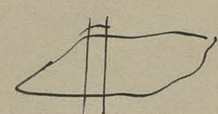
~~$\int v\left(\frac{1}{R_1} + \frac{1}{R_2}\right) df$~~ $\int v \, df = 0$ dodatkowy warunek



$\frac{1}{r_2} = \frac{2}{r_1}$

$r_1 = 2 r_2$





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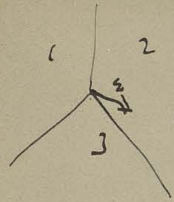
wpływ ciężkości

$$\int\left(\frac{1}{R_1} + \frac{1}{R_2}\right) v \, df + v \, df \, g (z - h)(\rho_1 - \rho_2) = 0$$

$$\int v \, df = 0$$

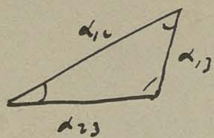
$$\frac{1}{R_1} + \frac{1}{R_2} + (\rho_1 - \rho_2) g z = 0$$

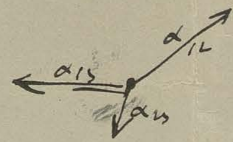


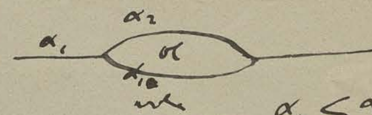
$$\int [\alpha_{12} \cos(\varepsilon\, n_{12}) + \alpha_{13} \cos(\varepsilon\, n_{13}) + \alpha_{2n} \ldots]\, \varepsilon\, dl = 0$$

$\alpha_{12} \cos \ldots = 0$

$\alpha_{12} : \alpha_{23} : \alpha_{13} = \sin\varphi_{12} : \sin\varphi_{13} \ldots$







$\alpha_1 < \alpha_2 + \alpha_{12}$

jeśli $\alpha_1 > \ldots$

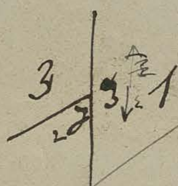
to rozpływanie

Rayleigh [illegible] błonka

[illegible] oleju

$\delta = 1.6 - 0.3$

10^{-6} cm



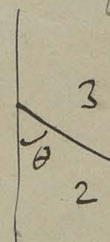
~~α_{12} [illegible] $\alpha_{12}\cos\varphi + \alpha_{13} - \alpha_{23} = 0$~~

$\cos\varphi\, \alpha_{23} + \alpha_{12} - \alpha_{13} = 0$

$$\cos\varphi = \frac{\alpha_{13} - \alpha_{12}}{\alpha_{23}}$$

$\alpha_{13} < \alpha_{12} + \alpha_{23}$

Constance de Rendézkels! [illegible]



$$\cos\theta = \frac{\alpha_{13} - \alpha_{12}}{\alpha_{23}}$$

H_2O 8.25 α_1

olej $\alpha_2 = 3.76$

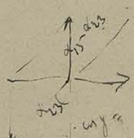
$\alpha_{12} = 2.09$

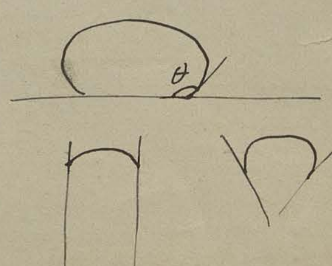
$\theta_{Hg} = 138^\circ$

to samo z punktu napięcia powierzchni

tak samo jak inny.

$F_1 \cos\theta = F_2$





$$R = \frac{ds}{d\theta} = \frac{ds}{dr}\,\frac{dr}{d\theta} \qquad \operatorname{tg}\theta = \frac{dz}{dr}$$

$$\frac{1}{\cos^2\theta}\,\frac{d\theta}{dr} = \frac{d^2z}{dr^2}$$

$$\frac{\left(1+\frac{dz}{dr}\right)^{3/2}}{\frac{d^2z}{dr^2}} = R$$

$$\sin\theta = \frac{\frac{dz}{dr}}{\sqrt{1+\left(\frac{dz}{dr}\right)^2}}$$

$$\frac{1}{R_1} = \frac{\frac{d^2z}{dr^2}}{\sqrt{1+\left(\frac{dz}{dr}\right)^2}^{\,3}} \qquad \frac{1}{R_2} = \left(\frac{r}{\sin\theta}\right) = \frac{\frac{dz}{dr}}{r\sqrt{\cdots}}$$

$$\frac{1}{R_1}+\frac{1}{R_2} = \frac{1}{r}\,\frac{d}{dr}\left(\frac{r\,\frac{dz}{dr}}{\sqrt{1+\left(\frac{dz}{dr}\right)^2}}\right)$$

$$\frac{1}{r}\,\frac{\partial}{\partial r}(r\sin\theta) =$$

$$\frac{\sin\theta}{r} + \cos\theta\,\frac{d\theta}{dr}$$

$$\frac{\frac{d^2z}{dr^2}}{\sqrt{}}$$

$$z = \frac{a^2}{2}\,\frac{1}{r}\,\frac{\partial}{\partial r}(r\sin\theta)$$

Rozwiązanie θ jest krzywizną

$$r = r_0 + x$$

Capillar

$$z = \frac{a^2}{2}\,\frac{\partial(\sin\theta)}{\partial x}$$

$$dz = \operatorname{tg}\theta\,dx$$

Tropfen

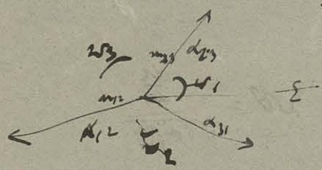
$$z^2 = -a^2\cos\theta + \text{const}$$

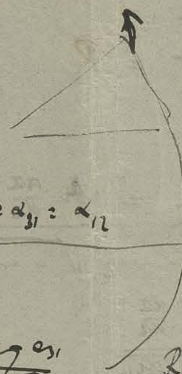
$$2\pi r\alpha = p$$

$$z^2 = 2a^2\left(h - \cos^2\frac{\theta}{2}\right)$$

$$\int \xi \left[\alpha_{12}\cos(n_{12}\,\xi) + \alpha_{23}\cos(n_{23}\,\xi) + \alpha_{31}\cos(n_{31},\xi)\right] ds = 0$$

(bracketed sum) $= 0$





$$\sin\omega_1 : \sin\omega_2 : \sin\omega_3 = \alpha_{23} : \alpha_{31} : \alpha_{12}$$

$n_{23} = -n_{31}$

~~$\cos\omega_2 = \frac{\alpha_{23}\ \alpha_{31}}{\alpha_{12}}$~~

Randwinkel

$$\alpha_{12}\cos\omega_2 + \alpha_{23} - \alpha_{31} = 0$$

$$\cos\omega_2 = \frac{\alpha_{31} - \alpha_{23}}{\alpha_{12}}$$

$$z = \frac{a^2}{2}\left(\frac{1}{R_1} + \frac{1}{R_2}\right)$$

~~$\frac{1}{R_1} = \frac{\frac{\partial^2 z}{\partial x^2}}{\left[1 + \left(\frac{\partial z}{\partial x}\right)^2\right]^{3/2}}$~~

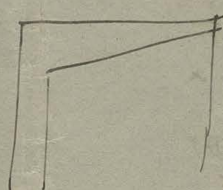
$\frac{1}{R_2} =$,

---

przy wielkiej krzywiźnie:

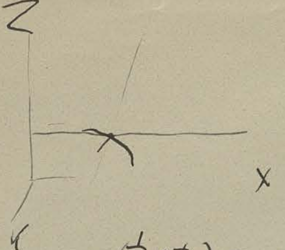
dla ścian zwilżających ścianę

$z = -\frac{a^2}{R}$



$z = -\frac{a^2}{\delta}$ = ---

$R = \frac{r}{\cos\theta}$



$$\alpha = \frac{\frac{\partial z}{\partial x}}{\sqrt{1+\left(\frac{\partial z}{\partial x}\right)^2+\left(\frac{\partial z}{\partial y}\right)^2}} \qquad \beta = \frac{\frac{\partial z}{\partial y}}{\sqrt{}}$$

$$\left(\frac{1}{R_1}+\frac{1}{R_2}\right) = -\left(\frac{\partial \alpha}{\partial x}+\frac{\partial \beta}{\partial y}\right)$$

$$r = \sqrt{x^2+y^2}$$

$$\frac{\partial z}{\partial x} = \frac{x}{r}\frac{dz}{dr} \qquad \frac{\partial z}{\partial y} = \frac{y}{r}\frac{dz}{dr}$$

$$\frac{\partial}{\partial x}\left[\frac{\frac{x}{r}\frac{dz}{dr}}{\sqrt{1+\left(\frac{dz}{dr}\right)^2}}\right] + \frac{\partial}{\partial y} \cdots$$

$$= 2\,\frac{\frac{1}{r}\frac{dz}{dr}}{\sqrt{}} + r\frac{d}{dr}\left(\frac{\frac{1}{r}\frac{dz}{dr}}{\sqrt{}}\right)$$

$$\parallel$$

$$r\left[-\frac{1}{r^2}\frac{dz}{dr}+\frac{1}{r}\frac{d^2z}{dr^2}\right]$$

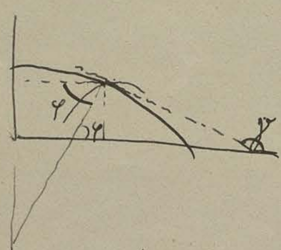
$$= \cdots$$

$$= \frac{1}{r}\frac{d}{dr}\left(\frac{r\frac{dz}{dr}}{\sqrt{}}\right) = \frac{1}{r}\frac{d}{dr}\left(r\sin\psi\right)$$

$$\cos\psi = \frac{\frac{dz}{dr}}{\sqrt{1+\frac{dz}{dr}}}$$

$$\cos\psi\,\frac{d\psi}{dr} = \frac{\frac{d^2z}{dr^2}}{\sqrt{}} - \frac{\left(\frac{dz}{dx}\right)^2\frac{d^2z}{dr^2}}{\sqrt{}^{\,3}}$$

$$= \frac{\frac{d^2z}{dr^2}}{\sqrt{}^{\,3}}$$



$$\frac{\frac{d^2z}{dr^2}}{\sqrt{1+\left(\frac{dz}{dr}\right)^2}^{\,3}} + \frac{\sin\psi}{r}$$

$$R_2 = \frac{r}{\cos\varphi} = \frac{r}{\sin\psi}$$

$$\varphi + (\pi-\psi) = \frac{\pi}{2}$$

$$\varphi = \psi - \frac{\pi}{2}$$

$$\cos\psi\,\frac{d\psi}{dr}$$

$$\cos\psi\,\frac{d\psi}{dr} + \frac{\sin\psi}{r}$$

$$\frac{d\psi}{ds} = \frac{\frac{d\psi}{dr}}{\cos\psi}$$

$$Z = \frac{\alpha}{r}\frac{d}{dr}\left(r\sin\psi\right)$$

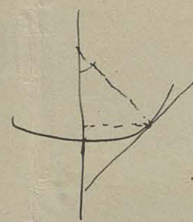
48 h. 2

u bardzo duża, z dużo $\neq z_0$

$$z_0 r = \alpha^2 \frac{d\, r \sin\theta}{dr}$$

$$z_0 r + \frac{c}{r} = \alpha^2 \sin\gamma$$

$$z_0 r = \alpha^2 \sin\gamma$$



kula

$$\text{promień} = \frac{\alpha^2}{z_0}$$

$$\frac{1}{r}\int_0^r z\, r\, dr = \alpha^2 \sin\theta = \frac{1}{r}\int_0^r (z_0+\zeta)\, r\, dr$$

$$= \frac{r}{2} z_0 + \frac{1}{r}\int_0^r \zeta\, r\, dr$$

$$r\zeta = R(1-\cos\gamma) r = \frac{\alpha^2}{z_0}(1-\cos\gamma)\frac{\alpha^2 \sin\gamma}{z_0}$$

$$\frac{1}{\sin\theta}\int_0^\theta (1-\cos\theta)\sin\theta\cos\theta\, d\theta$$

$$z_0 r = 2\alpha^2 \sin\gamma - \left(\frac{2\alpha^2}{z_0}\right)^2 \left[\sin\gamma - \frac{2}{3}\frac{1-\cos^3\gamma}{\sin\gamma}\right]$$

$$z_0 R = 2\alpha - \frac{1}{3}\left(\frac{2\alpha}{z_0}\right)^2$$

$$+ 2\alpha - \frac{R^2}{3}$$

$$z_0 \neq \frac{2\alpha}{R} - \frac{R}{3}$$

siła tarcia (?)

$$\alpha\, 2\pi R \sin\gamma = R^2 \pi \rho g z - \frac{2}{3}R^3\pi$$

$$z = \frac{2\alpha \cos\gamma}{\rho R}$$

woda

H_2O	$\alpha = 79$
Hg	480
	26
alk.	20
?	33
oliwa	33
CS_2	

$\psi = \frac{R\theta}{p}$

$$\delta\left(4\pi r^2 \alpha + \frac{e^2}{2r}\right) + \int 4\pi r^2 dr \int_p^P p\, dv = 0$$

$$R\theta \lg \frac{P}{p}$$

$$R\theta \lg \frac{P}{p} = \frac{1}{4\pi r^2} \frac{d}{dr}\left(4\pi r^2 \alpha + \frac{e^2}{2r}\right)$$

$$= \frac{2\alpha}{r} - \frac{e^2}{8\pi r^4}$$

$$c = \sqrt[3]{\frac{e^2}{16\pi\alpha}}$$

$H_2O \quad \alpha = 76$

$e =$

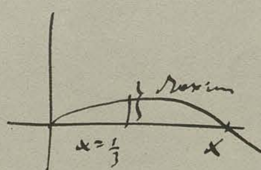
$$= 2\alpha\left(\frac{1}{r} - \frac{c^3}{r^4}\right) = \frac{2\alpha}{c}\left(\frac{c}{r} - \frac{c^4}{r^4}\right)$$

$$= \frac{2\alpha}{c}(x - x^4)$$

$r = \frac{1}{3.2}\, 10^{-7}$

$e = 3.10^{-10}$

$$\sqrt[3]{\frac{9.10^{-20}}{16.3.80}} = \sqrt[3]{\frac{10^{-20}.2}{8.16}} = \sqrt[3]{\frac{10^{-21}}{400}} =$$



[illegible] przekroczył

$$R\theta \lg \frac{P}{p} = \frac{2\alpha}{c}\, 0.471$$

$$\frac{p}{P} = 5.3$$

$\frac{v_2}{v_1}$ 1.25

1.31 [illegible]

1.38 [illegible]

CTR Wilson 4-5

~~$T = \frac{p}{\rho} = \rho^{k-1}$~~

$$\frac{T}{T} = \left(\frac{v}{v}\right)^{k-1} = (1.25)^{0.4}$$

$= 1.1 =$ ~~[illegible]~~ .

$20° - (-10)$

Sidrowny ○ $p = \frac{2\alpha}{R}$ $\alpha = 80$

dla $R = 10^{-}$

~~Condensat~~ δ/β $p = P - \frac{\alpha}{R}\frac{\rho_x}{\rho_w}$

Rayleigh zatrzymanie ruchu kamfory

0·81 mg ~~π~~ $\left(\frac{84}{4}\right)^2 \pi \,/\, \mu m^2$ $\delta = 1{\cdot}6 . 10^{-8}$ cm

	δ	
0·40 mg	δ = 0·81	no effect
0·52	1·06	barely perceptible
0·65	1·32	not quite enough
0·78	1·58	just enough

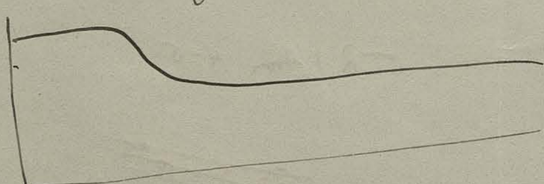
Hg α = 55·03

H_2O 8·25

α_{12} = 42·58

Rayleigh weight of drops IV p. 420

Pockels Rayleigh.



123

~~woda (Knopp)~~ t.

$\alpha = 79\,(1 - 0{\cdot}002\,t)$

	H_2O		Alk.		Äther
0°	79	0°	26	0°	20
100°	62	75°	19	35°	16

woda ucieka od miejsc ogrzanych

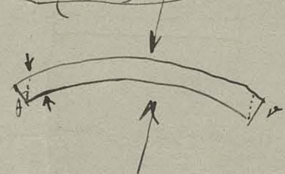
$CO_2 \quad h = 26{\cdot}04 - 0{\cdot}825\,t$

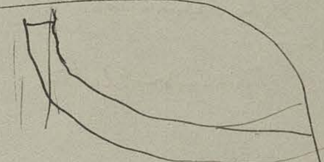
$= 0 \qquad t = 31{\cdot}5$

$\alpha\,(Mv)^{2/3}$ molekulare Oberfl. Energie — Eötvös Ramsay & Shields

$= k(\vartheta - t - d) \qquad k = 2$

$(d = 6)$





$\int \left(\frac{1}{R_1} + \frac{1}{R_2}\right) dx\, dy$

$\cos xz = 1$

$= \left[\left(\frac{1}{R_1} + \frac{1}{R_2}\right) df \cos\ldots\right] = \int \frac{\Delta F}{v} \cos(x z) = \int dl \cdot \frac{dl\, v \cos(n z)}{v}$

$= dl$

~~...~~ $2\, dw z = f\, l\, \alpha$

Odwrotnie pływanie ciał, pająków

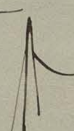
Roztwory Quincke wzrastające $\alpha = 7\cancel{7} + 0{\cdot}16\,y$

opornymi wpływ $CHCl_3$, Äther etc. działa — zupełnie pokrywa się warstwą powierzchowną

Kamfora na wodzie

Skórki ~~emulsyjne~~ wstwory koloid.

Range of molec. Forces Quincke



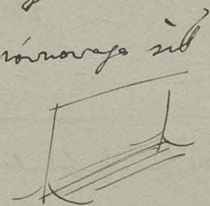
$l > 0{\cdot}000054$ mm H_2O/Ag/ metal

$$z = a \frac{\frac{d^2z}{dx^2}}{\sqrt{1+\left(\frac{dz}{dx}\right)^2}^{\,3}} = a \frac{d}{dx}(\sin\varphi)$$

$\int z\,dx = a \sin\varphi \Big|^{2}_{1}$ dla cieczy zwilżającej : $= a = \frac{\alpha}{\rho g}$

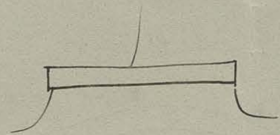
więc ciężar $\rho g \int z\,dx = \alpha$ oczywiście! ~~[illegible]~~ równowaga sił

metoda Wilhelmy : płytki [illegible] pionowe   zwykłe ważenie ciężaru



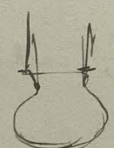
błędy areometrów

Adhäsionsplatten :



tutaj opór napięcia pow. na brzegu jemu ciężar hydrostat. na powierzchnię

Krople (Quincke)

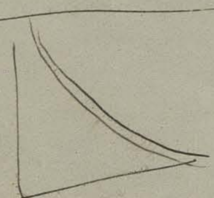


$p = 2\pi r \alpha \Delta$ podobnie jak przy metodzie bańki?

nie dokładnie bo nie wiemy czy ciężar kropli oderwanej rzeczywiście ~~[illegible]~~ równa się ciężarowi kropli oderwanej

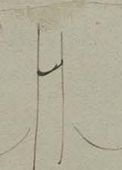
dokładniej na tworze z ∠ ośmiostnie

[znany błąd jeśli krople się zbytko tworzą od takiej przy powolnym —



$$z = \left(\frac{1}{R_1} + \frac{1}{R_2}\right)\frac{\alpha}{\rho g} = \frac{2}{r}\frac{\alpha}{\rho g} = \frac{2c}{x}\frac{\alpha}{\rho g}$$

płytki równoległe

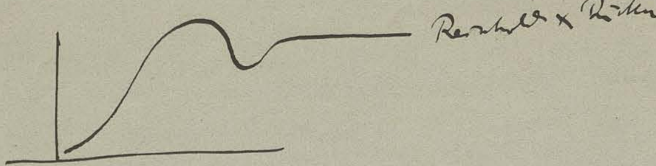


wysokość $= \frac{1}{2}$ rurki

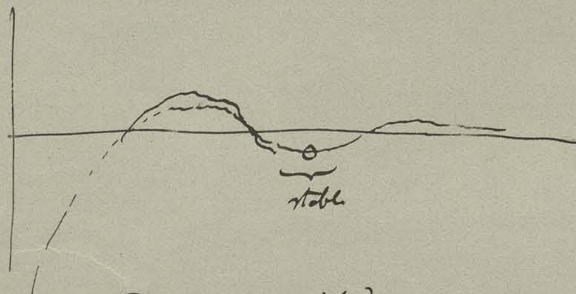
siła ściskająca jest

und Rotationen in ruhig ist

$$= \frac{2\alpha}{r} + \frac{\partial a}{\partial r} - \frac{e^2}{8\pi r^4}$$



Isobian Alkohol



$$\left(\frac{e}{r^2} + \frac{V}{d}\right)^2$$

Projecte u. Pläne

127

optische [illegible] ; [illegible] Refl. [illegible] etc. [illegible]

[illegible]

[illegible]

Interfer. [illegible] — — — | [illegible] refl. [illegible] absorb. [illegible]

[illegible] Newton [illegible]

[illegible]

illopt. pol. [illegible] etc.

[illegible] Refl. [illegible]

Pol. [illegible]

Analogie [illegible]

[illegible] Fresnel [illegible]

[illegible] Kirchhoff. [illegible] Poisson

Wüllner p. 406

Interf.

[illegible] Medium [illegible]

Saccharimeter : direkte [illegible] — Gummi [illegible]

[illegible]

Schlieren [illegible]

[illegible] diff. Refl. [illegible]

[illegible]

[illegible] Newton [illegible]

[illegible]

[illegible] Newton [illegible]

[illegible]

[illegible] Ampl. [illegible]

[illegible] Interf. [illegible]

[illegible]

[illegible] 2 [illegible]

[illegible] Fresnel [illegible] Äthers [illegible] Neum. [illegible]

Piezometer: relative Vol. [illegible]

[illegible] spec. [illegible]

Zerrungs Invarianten [illegible] zu Bezugsebene [illegible]

[illegible] ?

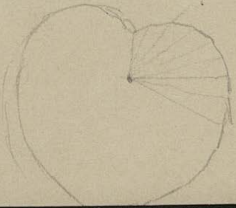
[illegible] $\frac{p}{p_0} = \left(\frac{\rho}{\rho_0}\right)^k$? [illegible] [illegible]

[illegible] 2 Medien [illegible] k, [illegible]

Refl. [illegible] ? [illegible] Konst. [illegible]

[illegible] Refl. [illegible] ?

[illegible]



[illegible] Vol. [illegible] !

[illegible] contr. [illegible]

[illegible]

Apparat [illegible] : [illegible]

[illegible]

[illegible]

[illegible] Elastic.-Coeff. [illegible]

[illegible] Coeff.; [illegible] Elast. [illegible]

[illegible]

[illegible]

Compr.-Coeff., [illegible]

dynamischer [illegible] statischer El. Coeff.

[illegible] conform [illegible] Elast. etc. 168 f Subst 2' = a2

[[illegible]

[illegible] coeff ?

[transversal [illegible] horiz. [illegible]

[illegible] ? [[illegible]]

[illegible]

~~[illegible]: [[illegible], [illegible]]~~

~~[illegible] [[illegible], AR, [illegible], [illegible]]~~

[illegible], [illegible], Ind. [illegible], [illegible],

[illegible]

Photometer [[illegible]]
[illegible]

indue. [illegible]

[illegible]

Meteorol.: [illegible]

Meteorol. Ballonstatt

[illegible]

th. [illegible], elast. [illegible]

[illegible] 20 × 01

Max-Min. Therm.

[illegible] Phantom, Saccharin etc. [illegible]

[illegible]

Capac. [illegible]

[illegible]

[illegible]

Cap. Const. elc. [Ostw. 78]

[illegible]

[illegible]

[illegible] = ----

[illegible]

[illegible]

[illegible]

[illegible] 14 ele[illegible]

[illegible] elektr. [illegible]

[illegible]

[illegible]

131

[illegible]

[illegible] Rendiloj.

[illegible] 2 [illegible] elektr. [illegible]

[illegible] elektr. [illegible] Coeff.

[illegible]

[illegible] Dalton [illegible]

(Ostw. p. 422) [illegible] Henry [illegible]

[illegible]

[illegible]

Capill. + constr. [illegible]

[illegible] Vol. [illegible] ?

[illegible]

[illegible] ?

[illegible] Refl. [illegible]

[illegible]

[illegible] Refl.

[illegible] Refl. [illegible]

[illegible] contin. [illegible] ?

[illegible]

[illegible] ?

[illegible]

Pt [illegible]

Pd [illegible]

[illegible]

[illegible]

[illegible]

[illegible] Aggr. [illegible]

[illegible] ? [illegible]

[illegible] Polar. [illegible]

[illegible] ?

[illegible]

[illegible]

[illegible]

Photometer: Selendraht mit ~~Draht~~ Tang.-bus. 132

Electr.messer: Thermoelem. Erwärmung resp. Erkältung

Electr. Seifenblasen

Spiegelablesung bei Drehwage

Influenzmasch. mittelst

1/. Quecksilber

2/. Kugel von [illegible] Rad. z. B. Kautschukballon

Bahnen d. Cyclonen [illegible]

[illegible] !!!

Schall beim Durchfliegen eines Körpers durch die Luft.

Berechnung d. Widerstand. d. Mittels aus d. kinet. Gas-Theorie

Reibung von Flüssigk. an fest. Körpern

Analogie des Huyghens'schen Principes [Optik] auf Elektr. [illegible]

~~[illegible]~~

[illegible]

[illegible]

[illegible] Electr. [illegible]

[illegible] Atmosph.

[illegible]

[illegible]

[illegible] parall. [illegible]

[illegible]

[illegible] anod. [illegible]

[illegible]

[illegible] elast. [illegible]

[illegible] Redoms [illegible]

[illegible] Res. [illegible]

[illegible] temp. ...

[illegible]

[illegible]

[illegible] Elektr.

[illegible] Cl Br J, spectra } [illegible] temp.

[illegible] etc. } [illegible]

[illegible]

Entropie [illegible]

Elastiz., Reibung, [illegible]

[illegible]

[illegible] ~~Hydrodyn.~~ [illegible] Elektr. [illegible]

[illegible]

[illegible] 133

[illegible] Adhäs. [illegible] Th. [illegible]

[illegible]

[illegible] [[illegible] etc.] [illegible]

[illegible]

[illegible] [[illegible]]

[illegible] [[illegible]]; math. [illegible]!

[illegible]

[illegible]

[illegible]

Demonstr. [illegible]

[illegible]

[illegible]

[illegible] Urst. App. [illegible]

Kater'sches Revers. Pendel [illegible]; [illegible]?

Continuität der Lufthülle [illegible]

[illegible]

[illegible]

[illegible]

$\frac{d^n f(x)}{dx^n}$ [illegible] $f_c(n)$; [illegible]

[illegible]

$\frac{d^n f(x)}{dx^n} = \varphi(x)$ [illegible]?

[illegible] ??

[illegible] [[illegible]

[illegible]] [illegible]

[illegible] $f_c(n)$

[illegible]

[illegible] [=1, 2, ———]

[illegible]

[illegible] Alkohol [illegible]

[illegible]

[illegible] Analogon [illegible]

[illegible]

[illegible] Constanten K [illegible] Capill.?

Capillare [illegible]

[illegible]

[illegible] Torsions [illegible]

[illegible]

[illegible]

[illegible] max. [illegible]

[illegible]

[illegible]

Capill. [illegible]

[illegible]

[illegible]

[illegible]

[illegible] Concessi [illegible]

[illegible] Torsions [illegible]

[illegible]

[illegible] spec. [illegible]

Capill. [illegible]

[illegible]

elast. [illegible]

[illegible]

[illegible]

[illegible] ; Interf.

[illegible]

Interfer. [illegible] Intens.

[illegible] Med. [illegible]

[illegible] = [illegible] elect.

[illegible]

[illegible]

[illegible] cond.

[illegible]

matter [illegible] :

D. [illegible]

[illegible] ;

[[illegible] Lampe [illegible]]. [illegible]

[illegible]

[illegible] temp. [illegible]

[illegible] ; [illegible]

[illegible]

Coordinaten [illegible] variiren

[illegible] Hydrodyn.

statische [illegible] = Result. [illegible] Ver. ... Stileanal. :

A Treatise on the Application of generalised Coordinates to the Kinetics of a Material [illegible]. H. W. Watson and Burbury

Oxford Clarendon Press 1879

Experimentum crucis d. Newton

[illegible]

[illegible]

[illegible]

[illegible]

Refl. [illegible]

Refl. [illegible]

[illegible]

[illegible] contin. [illegible]

[illegible] refl. [illegible] [[illegible] Refl. [illegible]]

Reflex. d. [illegible]

[illegible]

Refl. d. [illegible]

[illegible]

[illegible] ?

[illegible] Phosph. [illegible]

Kubische Determ. [illegible] ?

Phosphor [illegible] ?

Diffusion [illegible] ?

[illegible] etc. [illegible]

[illegible] Regul.,

[illegible] Pendel etc. [illegible] etc.

[illegible]

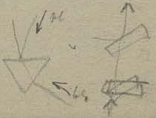
[illegible] rotirende [illegible] ; etc.

[illegible] (a | b) (b | a) vibrir

[illegible] Spectrum

[illegible] ?

[illegible]



136

93/53

II 7

137

Niewiadome

różne

oraz

Miscellanea

(np. historję Katedr nauk. [illegible])

p. 281 – 345   Z le [illegible]   Prz. do do [illegible] Krak. 138

561 str.   [illegible] po pauzie 600 str. U.K.

[illegible] prz CH. T. N. K.

Ref. Kołłątaja 1780

Stan. Zatorski Biskup krak. zamierzał fundować katedrę mat. i fizyki [illegible]

wykład Marcina Świątkowskiego prof. Univ. w połowie 1746 – 1750

dla wykształcenia mat. i fiz.

tenże wykładał mat. ale wnet zginął Krak.

J. Sierak. w Ks. [illegible] [illegible]

[illegible] stan [illegible]

Kołłątaj powołał 1778 Ks. Rydzańskiego do wykładu fizyki

spis 1784 40 narzędzi mechanicznych i fizycznych

dopiero [illegible] zdawanych gabinetu [illegible]

---

1). Ks. Andrzej Trzciński Dr fil. 1771 wykładał ~~1778~~ – 1782

1783 – 1804 prof. fizyki +1823

spór z kolegą Jaśkiewiczem 1789 który mu zarzucał nieuctwo [illegible]

ale pozostał na katedrze do 1804

został kanonikiem

2). Radwański Feliks Dr Fil 1775 1780 Prof. mechaniki

– 1804/5

znowu 1820/1 – 1823/4 +1826

3). Antoni Gloisner 1805 potem do Lwowa

ze Lwowa

4). Zermanteck Jan 1805/6 (zaprowadził austryackiego planu nauk) – 1809

(przeszedł Krak. do księstwa Warszawsk.) wykładał po [illegible]

5). 1810 1811 po [illegible]: Jan Chrystian Hoffmann

urodz. 1768 w Schlettau [illegible]

6). pazdz. - maja 1812 - 1813 Joachim Markiewicz (ze Lwowa) [illegible]
zastępca

7). Roman Markiewicz 1813 po polsku - 1837/8 + 1841
Doktor 1814 fizyka
mechaniki 1833/4 - 1836/7

wspom. w R. T. N. Kr.

8). 1838/9 zastęp. Józef Podolski + 1850

9). 1839 - Stefan Ludwik Kuczyński ur. Lwów 1811
Doktor 1835

~~[illegible]~~
Gabinet fizyczny
1784 [illegible] 9 [illegible] przyrządów, itd. (bardzo [illegible] + [illegible]
[illegible]
[illegible]

(1805 inwentarz przy oddaniu [illegible] ([illegible]) 82 numerów
[illegible] J. Śniadecki [illegible]

[illegible] 1796 - 1805 zostawił [illegible] przy [illegible]
[illegible] do Gabinetu

Gabinet mechaniczny 1784 inwentarz 24 machin
w ośmiu latach 10,400 złp.

1803/4 dawny kollegium fizyczne i moralne
w [illegible] i [illegible]

1805 oddano gabinet mechan. do fizycznego [illegible] przeniesiono do koll. [illegible]
Inwentarz roczna dotacja 500 złp. 1807 mechanik [illegible]
[illegible] w 1817/8 800 złp.

1809 przyłączono do Ks. Warszawskiego
Inwentarz 1814 ~~[illegible]~~

1813 przeniesiony do kol. fizycznego Markiewicz 1818 mechanik
Tomasz Taborski

dotacja 1000 złp. od 1815/6   w ostatnich latach 22.000

469 narzędzi i sprzętów

240

także z gabinetu chemicznego

i mechanicznego

Katedra mechaniki zwinięta 1833 // powstał zakład techniczny w Krakowie 1834

adjunkt od 1828 D. Podolecki

1850/1 Nowy plan naukowy Instytut. Wydz. fil. dotychczas szkoła przygotowawcza odtąd sam.

dotacja 1000 złp = 250 złr. wal. austr.

od 1858/9 1200 złp

od 1860 mechanik Jan Niemotz z Wiednia

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C. K. UNIWERSYTETU JAGIELLOŃSKIEGO

Kraków ....
Gołębia 13.

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Physikalische Zeitschrift

Gewünscht wird:

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~~8. 9~~

(14.) (5)

~~13. 20~~

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2		20
4		1, 26 b
6		17
13		18, 21/22

141

Zorin

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2). [illegible] perpher [illegible] Z...

3). Zbiór zagadn. i metod psychologii. Poradn. samokszt. 1909

4). [illegible] Disp. [illegible] (Z. s. Zadub.) [illegible] X (1908) Wied. mat. 13 (1909)

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8). O dysp. światła w parach metali R A U (1910)

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(Z. s. Zakrz.) Bull. 1910

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Ann. d. Ph. 38 (1912)

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$C_7H_6O_2 + KClO_3 =$

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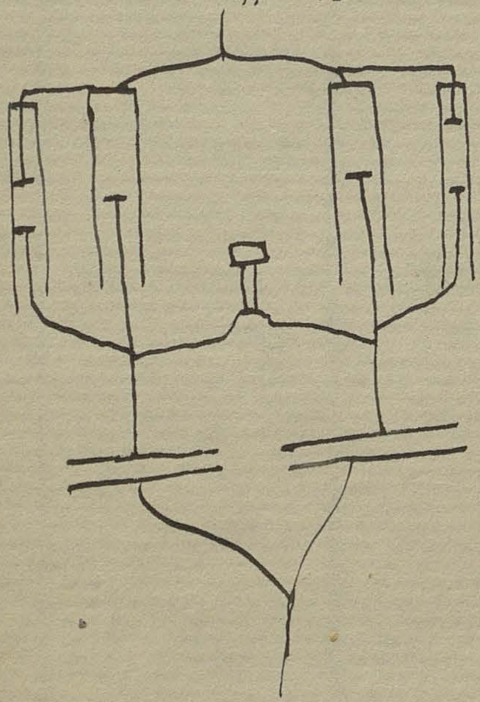
$$C_2 = C_{12} + C.$$

$$2C = C_{11} - C_{12}$$



~~$a_2^2\mu_2 = (v_1x_1 - u_1y_1) - a_1^2\mu_1 + (\mu_2 - \mu_1)\frac{c_1^2 - a_1^2 - a_2^2}{2}$~~

$$\mu_2 a_2^2 = \frac{v_1x_1 - u_1y_1 - a_1^2\mu_1 + (\mu_2-\mu_1)\frac{c_1^2 - a_1^2}{2}}{\mu_2 + \frac{\mu_2-\mu_1}{2}}\mu_2$$

$$a_2^2\mu_2 = \frac{U}{\mu_1+\mu_2} - a_1^2\mu_1$$

$$(v_1x_1 - u_1y_1)\mu_2 - a_1^2\mu_1\mu_2 + (\mu_2-\mu_1)\mu_2\frac{c_1^2-a_1^2}{2} = \frac{3\mu_2-\mu_1}{2}\left[\frac{U}{\mu_1+\mu_2} - a_1^2\mu_1\right]$$

$$\left[v_1x_1 - u_1y_1 + \frac{(\mu_2-\mu_1)c_1^2}{2}\right]\mu_2 - \frac{3\mu_2-\mu_1}{2}\frac{U}{\mu_1+\mu_2} = a_1^2\left[\mu_1\mu_2 + (\mu_2-\mu_1)\frac{\mu_2}{2} - \frac{3\mu_2-\mu_1}{2}\mu_1\right]$$

$$\frac{2\mu_1\mu_2 + \mu_2^2 - \mu_1\mu_2 - 3\mu_1\mu_2 + \mu_1^2}{2}$$

$$\frac{\mu_1^2 + \mu_2^2 - 2\mu_1\mu_2}{2}$$

$$= a_1^2\frac{(\mu_1-\mu_2)^2}{2}$$

$$a_1^2\mu_1 - a_2^2\mu_2 = \frac{-U}{\mu_1+\mu_2} + \frac{4\mu_1}{(\mu_1-\mu_2)^2}\left\{-\frac{3\mu_2-\mu_1}{2}\frac{U}{\mu_1+\mu_2} + \mu_2\left[v_1x_1 - u_1y_1 + \frac{\mu_2-\mu_1}{2}c_1^2\right]\right\}$$

$$= \frac{-U}{\mu_1+\mu_2}\left\{1 - \frac{2\mu_1(3\mu_2-\mu_1)}{(\mu_1-\mu_2)^2}\right\}$$

$$x = f(t, R_0, a_0, \alpha_0 \ldots)$$

$$x = f_{R_0=0}(t) + R_0 \left(\frac{\partial f}{\partial R}\right)_{R_0=0} + \ldots$$

Ale ~~[illegible]~~ (jeśli $\frac{\partial x}{\partial t}$ nieciągłe dla $t=0$, co tutaj ma miejsce jeśli pole R powstaje natychmiastowo w momencie $t=0$.)

$f_{R_0=0}(t)$ nie będzie równem $f\left(\begin{matrix} t=0 \\ R_0=0 \end{matrix}\right)$

Według Voigta stadyum II (i III) jest określone

za czasu $t=0$:

$$x_0 = a\cos\varepsilon = x_1 \qquad u_1 = -a\alpha\sin\varepsilon + \frac{eR\tau}{2m\omega}\,a\alpha\cos\varepsilon$$

$$y_0 = a\sin\varepsilon = y_1 \qquad v_1 = a\alpha\cos\varepsilon + \frac{eR\tau}{2m\omega}\left(\omega + a\alpha\sin\varepsilon\right)$$

co musi być identyczne z początkowemi wartościami (27):

$$x_1 = a_1\cos\alpha_1 + a_2\cos\alpha_2 = a\cos\varepsilon$$

$$y_1 = a_1\sin\alpha_1 - a_2\sin\alpha_2 = a\sin\varepsilon$$

$$u_1 = -a_1 p_1\sin\alpha_1 - a_2 p_2\sin\alpha_2 = -a\alpha\sin\varepsilon + \frac{eR\tau}{2m\omega}\,a\alpha\cos\varepsilon$$

$$v_1 = a_1 p_1\cos\alpha_1 - a_2 p_2\cos\alpha_2 = a\alpha\cos\varepsilon + \frac{eR\tau}{2m\omega}\left(\omega + a\alpha\sin\varepsilon\right)$$

z tych obliczyć niewiadome $a_1\, a_2\, \alpha_1\, \alpha_2$ oprowadzając się na pamiętny potęgi względem R

$$p_1 = p\left[\left(1+\left(\frac{\Pi}{p}\right)^2\right)^{\frac{1}{2}} - \frac{\Pi}{p}\right] = ~~p\left[1+\frac{1}{2}\left(\frac{\Pi}{p}\right)^2\right]~~ = \alpha\left(1-\frac{eR}{2m\omega\alpha}\right)$$

$$p_2 = \qquad = \alpha\left(1+\frac{eR}{2m\omega\alpha}\right)$$

~~$-a_1(\sin\alpha_1 ...) - a_2\sin\alpha_2 + \frac{a_1 eR}{2m\omega\alpha}\,p\sin\alpha_1 - \frac{a_2 eR}{2m\omega\alpha}\,p\sin\alpha_2 = -a\sin\varepsilon + \frac{eR\tau}{2m\omega}\,a\cos\varepsilon$~~

Moment pierwotny (Voigt 30): $\quad p_1 a_1^2 - p_2 a_2^2 = \alpha\left[(a_1^2 - a_2^2) - \frac{eR}{2m\omega\alpha}\left(a_1^2 + a_2^2\right)\right]$

Moment początkowy : $\quad \alpha a^2$

~~St~~ Stawiając u Voigta (23) : ~~X~~ (uwzględniając porostającą siłę stałą Y)

$$m\frac{d^2y}{dt^2}+ky = \frac{eR}{\omega}\left(-\frac{dx}{dt}+1\right)$$

$$y = a_1\sin(p_1 t+\alpha_1) - a_2\sin(p_2 t+\alpha_2) + \frac{eR}{k\omega}$$

(30) pozostaje niezmienione

$$\left.\begin{aligned} x_1 &= a_1\cos\alpha_1 + a_2\cos\alpha_2 \\ y_1 &= a_1\sin\alpha_1 - a_2\sin\alpha_2 + \left(\frac{eR}{k\omega}\right)\end{aligned}\right\}\bar{x}$$

$$\left.\begin{aligned} -u_1 &= a_1 p_1\sin\alpha_1 + a_2 p_2\sin\alpha_2 \\ v_1 &= a_1 p_1\cos\alpha_1 - a_2 p_2\cos\alpha_2\end{aligned}\right\} \quad \begin{aligned} u_1^2+v_1^2 &= a_1^2 p_1^2 + a_2^2 p_2^2 + 2a_1a_2p_1p_2(\sin\alpha_1\sin\alpha_2 - \cos\alpha_1\cos\alpha_2) \\ W_1^2 &= \end{aligned}$$

$$c_1^2 = x_1^2+y_1^2 = a_1^2+a_2^2+2a_1a_2(\cos\alpha_1\cos\alpha_2 - \sin\alpha_1\sin\alpha_2)$$

$$W_1^2 + c_1^2 p_1p_2 = a_1^2p_1^2 + a_2^2p_2^2 + a_1^2p_1p_2 + a_2^2p_1p_2$$

$$= (a_1^2p_1 + a_2^2p_2)(p_1+p_2)$$

$$v_1x_1 = a_1^2p_1\cos\alpha_1^2 - a_2^2p_2\cos\alpha_2^2 + a_1a_2\cos\alpha_1\cos\alpha_2(p_1-p_2)$$

$$u_1y_1 = -a_1^2p_1\sin\alpha_1^2 + a_2^2p_2\sin\alpha_2^2 + a_1a_2\sin\alpha_1\sin\alpha_2(p_1-p_2)$$

$$v_1x_1 - u_1y_1 =$$

$$= a_1^2p_1 + a_2^2p_2 + a_1a_2(p_1-p_2)(\cos\alpha_1\cos\alpha_2 - \sin\alpha_1\sin\alpha_2)$$

$$= a_1^2p_1 + a_2^2p_2 + (p_1-p_2)\frac{(c_1^2-a_1^2-a_2^2)}{2}$$

Vorft:

148

$$\int dt \left[\begin{array}{l} m\frac{d^2x}{dt^2} + kx = \frac{eR}{\omega}\frac{dy}{dt} \\ m\frac{d^2y}{dt^2} + ky = -\frac{eR}{\omega}\frac{dx}{dt} + eY \end{array} \right.$$

$$m(u_1 - u_0) = \frac{e v_1}{\omega} \int R\,dt$$

$$m(v_1 - v_0) = -\frac{e u_1}{\omega} \int R\,dt + e\int Y\,dt$$

$$\frac{M_1 - M_0}{e} = -\frac{eR}{4m^2\omega p^2}\left(m\,W_0^2 - k\,c_0^2\right) = -\frac{eR}{4m^2\omega\,\frac{k}{m}}\left[m(u_0^2+v_0^2) - k(x_0^2+y_0^2)\right]$$

$$= \frac{eR}{4m\omega}\left[x_0^2+y_0^2 - \frac{m}{k}(u_0^2+v_0^2)\right]$$

~~$v_1 = v_0 + \Pi\tau(\omega - u_0)$ | $x_1 = x_0$~~

~~$u_1 = u_0 + \Pi\tau\, v_0$ | $y_1 = y_0$~~

$$(v_1 x_1 - u_1 y_1) - (v_0 x_0 - u_0 y_0) = \Pi\tau\,(\omega x_0 - u_0 x_0 + v_0 y_0)$$

$$x = a_1\cos(pt+\alpha_1) + a_2\cos(pt+\alpha_2) + \Pi\left[a_1\sin(pt+\alpha_1) - a_2\sin(pt+\alpha_2)\right]$$

$$y = a_1\sin(pt+\alpha_1) + a_2\sin(pt+\alpha_2) + \Pi\left[-a_1\cos(pt+\alpha_1) + a_2\cos(pt+\alpha_2)\right]$$

$$u = -p a_1\sin(pt+\alpha_1) - a_2 p\sin(pt+\alpha_2) + \Pi p\left[a_1\cos(pt+\alpha_1) - a_2\cos(pt+\alpha_2)\right]$$

$$v = p a_1\cos(pt+\alpha_1) + a_2 p\cos(pt+\alpha_2) + \Pi p\left[a_1\sin(pt+\alpha_1) + a_2\sin(pt+\alpha_2)\right]$$

Resistance of Ellipsoid:

$$R = 16\pi\mu u \frac{1}{\int_0^\infty \frac{d\lambda}{\Delta} + a^2\int_0^\infty \frac{d\lambda}{(a^2+\lambda)\Delta}} \qquad \Delta = (a^2+\lambda)\sqrt{c^2+\lambda}$$

~~$$\frac{1}{\alpha - c^3}\left[\frac{2\alpha - c^3}{\sqrt{\frac{\alpha}{c} - c^2}}\left[\frac{\pi}{2} - \arccos\sqrt{1-\left(\frac{c}{a}\right)^2}\right] - \frac{c^4}{a^2}\right]$$~~

$$\frac{1}{a^2-c^2}\left[\frac{2a^2-c^2}{\sqrt{a^2-c^2}} \arccos\sqrt{1-\left(\frac{c}{a}\right)^2} - \frac{c^3}{a^2}\right] \quad \Bigg| \quad \frac{1}{c^2-a^2}\left[\frac{c^2-2a^2}{2\sqrt{c^2-a^2}} \lg\frac{c+\sqrt{c^2-\ldots}}{c-\sqrt{}} + \frac{c^3}{a^2}\right]$$

$$ac^2 = \alpha \qquad \frac{c}{a} = 1-\delta$$

$$c^3 = \alpha(1-\delta)$$

$$c = \sqrt[3]{\alpha}\,(1-\delta)^{1/3}$$

$$a = \sqrt[3]{\alpha}\,(1-\delta)^{-2/3}$$

$$\frac{1}{a}\left\{-\frac{1}{1-(1-\delta)^2} + \frac{2-(1-\delta)^2}{\left[1-(1-\delta)^2\right]^{3/2}} \lg\frac{1+\sqrt{2\delta\ldots}}{1-\sqrt{\ldots}}\right\}$$

$$\lg\frac{1+x}{1-x} = 2\left(x + \frac{x^3}{3} + \frac{x^5}{5}\right)$$

$$-1 + (1+2\delta-\delta^2)\left(1 + \frac{2\delta-\delta^2}{3} + \frac{(2\delta-\delta^2)^2}{5}\right)$$

$$-1 + 1 + 2\delta - \delta^2 + \frac{2\delta}{3} + \frac{4\delta^2}{3} - \frac{\delta^2}{3} + \frac{4\delta^2}{5}$$

$$\frac{8\delta}{3} + \cancel{\frac{-15+20-5}{15}} + \frac{4\delta^2}{5}$$

$$N = \frac{\left(1-\frac{2\delta}{3}\right)}{1-\frac{\delta}{2}}\left(1+\frac{3}{10}\delta\right)$$

$$1 - \frac{2}{3}\delta + \frac{\delta}{2} + \frac{3}{10}\delta$$

$$\frac{-20+15+9}{30}$$

$\frac{c}{a} = 1+\delta \qquad a = \sqrt[3]{\alpha}\,(1+\delta)^{-2/3}$

$$N = \frac{(1+\delta)^{2/3}}{\sqrt[3]{\alpha}\,(2\delta+\delta^2)}\left[1 + \frac{(1+\delta)^2 - 2}{(2\delta+\delta^2)^{3/2}}\left[\frac{\pi}{2} - \arccos\sqrt{1 - \frac{1}{(1+\delta)^2}}\right]\right.$$

$$\arcsin\sqrt{1-\frac{1}{(1+\delta)^2}}$$

$$1 + (2\delta+\delta^2-1)\left(\frac{1}{1+\delta} + \frac{2\delta+\delta^2}{6\,(1+\delta)^3} + \frac{3}{2\cdot4\cdot5}\,4\delta^2\right) \qquad \frac{\sqrt{2\delta+\delta^2}}{1+\delta}$$

$$1-\delta+\delta^2 + \frac{2\delta+\delta^2}{6}(1-3\delta) + \frac{3}{10}\delta^2$$

$$1-\delta+\delta^2+\frac{\delta}{3}+\frac{\delta^2}{6}-\delta^2+\frac{3\delta^2}{10} \qquad \frac{+5+9}{30} \quad \frac{14}{30}$$

$$1-\frac{2\delta}{3}+\frac{7\delta^2}{15}$$

$$1 - 1 + \frac{2\delta}{3} - \frac{7\delta^2}{15} + 2\delta - \frac{4\delta^2}{3} + \delta^2$$

$$\frac{8\delta}{3} + \frac{15-20-7}{15} = \frac{8\delta}{3} - \frac{4}{5}\delta^2 = \frac{8}{3}\delta\left(1-\frac{49}{80}\delta\right)$$

$$\left(1+\frac{2\delta}{3}\right)\left(1-\frac{\delta}{2}\right)\left(1-\frac{49}{80}\right) \qquad \frac{160-120-147}{240} \qquad \frac{-107}{240}$$

$$\left(1+\frac{2\delta}{3}\right)\left(1-\frac{\delta}{2}\right)\left(1-\frac{3}{10}\right)$$

$$\frac{+20-15-9}{30}$$

Resistance of Ellipsoid:

150

$$F = 16\pi\mu U \frac{1}{\int_0^\infty \frac{d\lambda}{\Delta} + a^2 \int_0^\infty \frac{d\lambda}{(a^2+\lambda)\Delta}} \qquad \Delta = \sqrt{(a^2+\lambda)(b^2+\lambda)(c^2+\lambda)}$$

for axial symetry $b = c$

$$F = 16\pi\mu U \frac{1}{\int_0^\infty \frac{d\lambda}{(c^2+\lambda)\sqrt{a^2+\lambda}} + a^2 \int_0^\infty \frac{d\lambda}{(c^2+\lambda)(a^2+\lambda)\sqrt{a^2+\lambda}}} = \frac{16\pi\mu U}{N}$$

By putting

$$= \left(\frac{1}{x^2} - \frac{1}{x^2+c^2-a^2}\right)\frac{1}{c^2-a^2}$$

$$\frac{\partial}{\partial a}\int_0^\infty \frac{d\lambda}{(c^2+\lambda)\sqrt{a^2+\lambda}} = -\int_0^\infty \frac{a\,d\lambda}{(c^2+\lambda)\sqrt{a^2+\lambda}^3}$$

$$N = \left[1 - a\frac{\partial}{\partial a}\right]\int_0^\infty \frac{d\lambda}{(c^2+\lambda)\sqrt{a^2+\lambda}} = 2\left[1 - a\frac{\partial}{\partial a}\right]\int_a^\infty \frac{dx}{x^2+c^2-a^2}$$

I). $c < a$

$$\int_a^\infty \frac{dx}{x^2-(a^2-c^2)} = \frac{1}{\sqrt{a^2-c^2}}\log\frac{a+\sqrt{a^2-c^2}}{a-\sqrt{a^2-c^2}}$$

$$\frac{\partial}{\partial a} \text{ - - - -} = \frac{-a}{\sqrt{a^2-c^2}^3}\log\frac{a+\sqrt{\ }}{a-\sqrt{\ }} + \frac{1}{\sqrt{a^2-c^2}}$$

Broglie Sur les obsv. d. mouv. brown. dans l. gaz à basse pression CR. 154 p 112 1912 151

agitation tend à s'accentuer à partir de 70^{mm} – 1^{mm}

~~fumées de phosphore~~ (mesure de Δ pas encore faite

vaseline phosphorée fume ; fumée, ultramicrosk. (ca 50 $\mu\mu$)

CR 1909 p 1315 Broglie Tome VI Valeur de Phys. Payne

Phys. Rev. 1911 Fletcher

Duclaux Absorption de gaz par les corps poreux ! p 1217

Probabilité de condensation ref 152 p. 1776

Unrim

Fournard p 1152 polarisation sur la mé. d. Lormoss

Poincaré p 1103 rapp. th. de l'act. & Quanta ...

Brillouin Polymorphisme et rotation moléculaire CR p 240

rotation optique que les deux formes polym. sont constituées des mêmes molec. arrangées

d'une façon différente

Leduc Pression interne dans les gaz et attractions moléculaires p 179

$$\pi = \theta \frac{\partial p}{\partial \theta} - p = p(\beta\theta - 1)$$

à ~~pression~~ temp. cste $\pi \sim \frac{m^2}{v^2}$

Brillouin Surface de glissement, Généralisation de la th. de Helmholtz p. 43

Bouchon Sur un effet de déplacement ... métal ... électrostatique ...

153 p 47

Einleitung: Keine Spezial-Untersuchung, sondern Beleuchten von verschiedenen Seiten;

Keine Theorien, bloß Experim. Sammeln von Material

Leitfähigkeit ~~d. Paraffin u. Glas~~ von Dielectricis. Entdeckg. von B. & H., B. & S., J. J. Th.,

[illegible] B. & B. S. H. / Voller Erfolg, dagegen Röntgen;

Erste Anordnung: Messing Kugel bedeckt mit Paraffin etc.

Scheinbare Entladg. aber nicht bis 0, nachträgl., weil weniger etc.

Erklärbar durch Condensatorwirkg.

Zweite Anordnung: Metal innerhalb Alumin. Cylinder.

Kein merklicher Effect bis 3 V

Dritte Anordng.: zwei Condensatoren; bis 2400 V.

Analog mit Glas.

Leitfähigkeit von Gasen Für gewöhnlich ausgezeichnete Isolatoren; Leitungsvermögen wächst bei Verdünnung, aber immer eine gewisse EMF erforderlich um Strom einzuleiten.

Ausnahmen: Flammen, erhitzte Gase, Phosphor, Ultraviol. Licht, Röntg. Str., Ur.

I). J.J. Thomson's Gesetz betreffs Abhäng. von Potentialdiff. bestätigt

Ebenso für Uranium (Beattie und Becquerel) (Doch abhängig von Dimension d. Apparats)

Analoges scheint auch für Flammen zu gelten und für UVL, vielleicht auch für Entladung in Geissler Röhren?

Contrast mit metallischer Leitung, electrolytische Leitg. mit Polarisation, Funkenentladg.

J.J. Thomson's Erklärungs Versuch aus Dissociation; Sonstige Theorien (Convections theorie und Righi's Zerstäubungs Theorie für UVL scheinen da zu versagen)

II). ~~Electrolyt.~~ Erzeugung von Electricität, electrolytisches Zers.

für Röntg. Str., (Minchin, Perrin, Murray; dagegen Dewit & H.)

UVL (Righi) aber abhängig von Distanz! daher Contact Potential —

Ur — CuO – Al = 2 V (größer als mittelst H_2O)

Rauchgase

Specielle Eigenthümlichkeiten (Meinung dass nur − Elektr. wird durch UVL)
Elektrisch. Körper?)

a). <u>Unipolarität</u> bei Flammen?; UVL; dagegen keine bei RS und Ur;
(Analogie in negativen Metallen in Geissler Röhren)

b). bei UVL großer Einfluss der <u>Natur d. Metalls etc</u>; bei RS und Ur nur geringer

c). Strahlung zwischen Metallflächen, <u>ohne dieselben zu berühren</u>

Starker Effect bei RS (Perrin's Kraftröhren Satz), sogar Luft die fortgeblasen wird, behält ihre Leitfähigkeit für einige Zeit

Letzteres auch bei Ur

Auch bei UVL schwache Wirkung

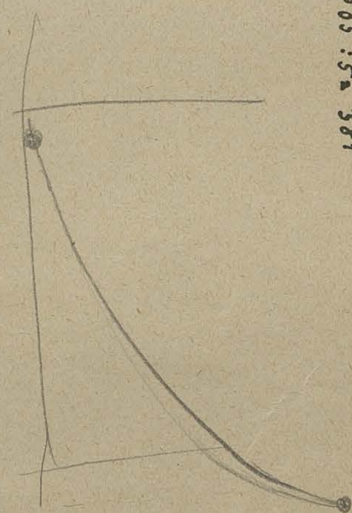
(Zerstreuungs Th. somit jedenfalls bei RS und Ur nicht anwendbar)

In verschiedenen Gasen

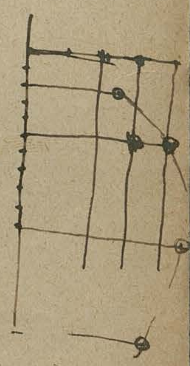
Bei verschiedenen Drucken

86·67 0·5547
508 7059
35·87 0·8488
620 8·6 2
0·7060
870 2·5
703
2·17 || 7032
7060
0·5
3
3365 6730
1644 9138
5009 5868
2·8
50·8 0
1·2
13·2
62·8 1·0
64·0 1·2 1·1
3171
3861
7032
0·198 0·9
0·2047
0170
7059
3111
15
0·146
82
0·228 1·0
0·260 1·1
090
9542 9084
1644 9138
1186 8222
1314
664
0·198
6·7
30
0·1
0·922 0·22
94·87 6442
508 7059
44·07 9383
0·8685
0·878 — 2·5
0823 2·4
3802 7604
1644 9138
5446 6742
10
55
0·248
3504
4723
8227

2041 4082 2336
1644 9138 2099
3685 3220 4435 1·6
74·7
508
23·9
3784
7059
6725
4704
404 1·5
2304 4608
1644 9138
3948 3746
485 1·7
15
4704
4435 1·6
2482
2369
485
1·67
146
415
= 0·3



517
2
= 258
1207:3 = 402
2242:5 = 448
1905:5 = 381



$$\frac{\partial p}{\partial x} = \frac{\mu}{r}\cdot\frac{\partial}{\partial r}\left(r\frac{\partial u}{\partial r}\right) = \frac{p_1 - p_2}{l}$$

$$u = \frac{p_1 - p_2}{4\mu l}(R^2 - r^2)$$

$$V = \frac{R^4\pi}{8\mu}\,\frac{p_1 - p_2}{l}$$

~~$u = \frac{1}{\mu}\left[-\frac{r^2}{6}\frac{\partial}{\partial x}\left(\frac{Ax}{r^3}\right) - \frac{2}{3}\ \frac{\partial}{\partial x}\right.$~~

$$u = -\frac{3}{4}\frac{ca}{r^3}\left(1 - \frac{a^2}{r^2}\right)x^2 + c\left(1 - \frac{3}{4}\frac{a}{r} - \frac{1}{4}\frac{a^3}{r^3}\right)$$

$$v = -\frac{3}{4}\frac{ca}{r^3}\left(1 - \frac{a^2}{r^2}\right)xy$$

$$w = -\frac{3}{4}\frac{ca}{r^3}\left(1 - \frac{a^2}{r^2}\right)xz$$

$$p = p_0 - \frac{3}{2}\mu\frac{c\,a\,x}{r^3}$$

~~$\Delta^2[\varphi(x)\cdot x] = x^2\Delta^2\varphi + 4x\frac{\partial\varphi}{\partial x} + 2\varphi$~~

$$\Delta^2[\varphi\cdot xy] =$$

$$\Delta^2\cdot\left(\frac{xy}{r^3}\right) \qquad \Delta^2\left(\frac{x^2}{r^3}\right) \qquad \Delta^2\left(\frac{1}{r^3}\right)$$

$$\left(\frac{xy}{r^5}\right) \qquad \frac{x^2}{r^5}$$

~~$\frac{\partial}{\partial x}\left(\frac{x^m y}{r^3}\right) = \frac{y}{r^3}$~~

$$\frac{\partial}{\partial x}\left(\frac{x^m y^n}{r^p}\right) = m\frac{x^{m-1}y^n}{r^p} - p\frac{x^{m+1}y^n}{r^{p+2}}$$

$$\frac{p\,z}{r^{p+2}}\ x^m y^n$$

$$\frac{\partial^2}{\partial x^2} = m(m-1)\frac{x^{m-2}y^n}{r^p} - [p(m+1) + pm]\frac{x^m y^n}{r^{p+2}} + p(p+2)\frac{x^{m+2}y^n}{r^{p+4}}$$

$$n(n-1)\frac{x^m y^{n-2}}{r^p} - p(2n+1)\frac{x^m y^n}{r^{p+2}} + p(p+2)\frac{x^m y^{n+2}}{r^{p+4}}$$

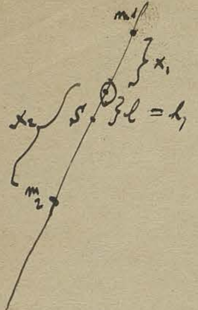
$$-p\frac{x^m y^n}{r^{p+2}} + p(p+2)\frac{x^m y^n z^2}{r^{p+4}}$$

$$\Delta^2\left(\frac{x^m y^n}{r^p}\right) = p\left[(p+2) - (2m+1) - (2n+1) - 1\right]\frac{x^m y^n}{r^{p+2}} + m(m-1)\frac{x^{m-2}y^n}{r^p} + n(n-1)\frac{x^m y^{n-2}}{r^p}$$

$$p[p - 2m - 2n - 1]$$

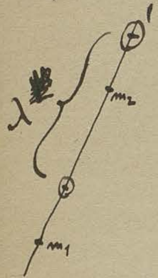
154

$$l = \frac{m_2 x_2 - m_1 x_1}{m_1 + m_2}$$



$$T = 2\pi \sqrt{\frac{K}{Mgl}} = 2\pi \sqrt{\frac{m_1 x_1^2 + m_2 x_2^2}{(m_1+m_2)\, g \, \frac{m_2 x_2 - m_1 x_1}{(m_1+m_2)}}}$$

$$= \frac{2\pi}{\sqrt{g}} \sqrt{\frac{m_1 x_1^2 + m_2 x_2^2}{m_2 x_2 - m_1 x_1}} = 2\pi \sqrt{\frac{\lambda_1}{g}}$$



$$\lambda_1 = \frac{m_1 x_1^2 + m_2 x_2^2}{m_2 x_2 - m_1 x_1}$$

$$\lambda_2 = \frac{m_1 (\lambda + x_1)^2 + m_2 (\lambda - x_2)^2}{m_1(\lambda + x_1) + m_2 (\lambda - x_2)}$$

Wähle $\lambda_2 = \lambda$

$$m_1 \lambda^2 + m_1 \lambda x_1 + m_2 \lambda^2 - m_2 \lambda x_2 = m_1 \lambda^2 + 2 m_1 \lambda x_1 + m_1 x_1^2 + m_2 \lambda^2 - 2 m_2 \lambda x_2 + m_2 x_2^2$$

$$m_1 x_1^2 + m_1 \lambda x_1 + m_2 x_2^2 - m_2 \lambda x_2 = 0$$

$$\lambda = \frac{m_2 x_2^2 + m_1 x_1^2}{m_2 x_2 - m_1 x_1} = \lambda_1$$

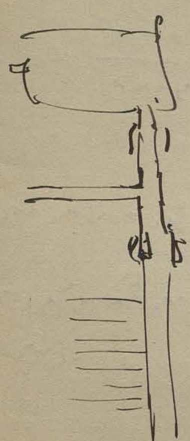
$$\lambda_1 - \lambda_2 = \left[\frac{m_1 x_1^2 + m_2 x_2^2}{m_2 x_2 - m_1 x_1}\right] - \frac{m_1(\lambda + x_1)^2 + m_2(\lambda - x_2)^2}{m_1(\lambda + x_1) + m_2(\lambda - x_2)}$$

$$= m_1^2 x_1^2 \lambda + m_1^2 x_1^3 + m_1 m_2 x_1^2 \lambda + m_1 m_2 x_1 x_2^2 + m_1 m_2 \lambda x_1^2 - m_1 m_2 x_1^2 x_2 + m_1 m_2 \lambda x_2^2 + m_1 m_2 x_1 x_2^2 + m_2^2 \lambda x_2^2 - m_2^2 x_2^3 - m_1 m_2 x_2 (\lambda + x_1)^2 - m_2^2 x_2 (\lambda - x_2)^2 + m_1^2 x_1 (\lambda + x_1)^2 - m_1 m_2 x_1 (\lambda - x_2)^2$$

$$\lambda - l = \frac{(m_2 x_2^2 + m_1 x_1^2)(m_1 + m_2) - (m_2 x_2 - m_1 x_1)^2}{(m_2 x_2 - m_1 x_1)(m_1 + m_2)}$$

$$= m_1 m_2 x_2^2 + m_1^2 x_1^2 + m_2^2 x_2^2 + m_1 m_2 x_1^2 - m_2^2 x_2^2 - m_1^2 x_1^2 + 2 m_1 m_2 x_1 x_2$$

$$= m_1 m_2 (x_1 + x_2)^2$$



Jüdi

$\lambda_2 < \lambda$

$\frac{m_2 x_2^2 + m_1 x_1^2}{m_1 x_1 - m_2 x_2} + \lambda < \ell$

$-\lambda_1$

$\lambda < \lambda_1$

$m_1 + m_2$

$$\lambda - \ell = \frac{m_1 m_2 (x_1+x_2)^2}{(m_2 x_2 - m_1 x_1)(m_1+m_2)}$$

$$\frac{\lambda - \ell}{\ell} = \frac{m_1 m_2 (x_1+x_2)^2}{(m_1+m_2)^2}$$

Jüdi

$$\lambda_2 = \lambda (1+\delta)$$

$$\frac{m_1(\lambda+x_1)^2 + m_2(\lambda-x_2)^2}{m_1(\lambda+x_1) + m_2(\lambda-x_2)} = \lambda(1+\delta)$$

$$m_1(\lambda^2 + 2\lambda x_1 + x_1^2) + m_2(\lambda^2 - 2\lambda x_2 + x_2^2) = m_1\lambda^2 + m_1 \lambda x_1 + m_2 \lambda^2 - m_2 \lambda x_2$$
$$+ \delta\left[m_1(\lambda+x_1) + m_2(\lambda-x_2)\right]\lambda$$

$$-\lambda + \underbrace{\frac{m_2 x_2^2 + m_1 x_1^2}{m_2 x_2 - m_1 x_1}}_{\lambda_1} = \frac{\delta\left[m_1(\lambda+x_1) + m_2(\lambda-x_2)\right]\lambda}{m_2 x_2 - m_1 x_1}$$

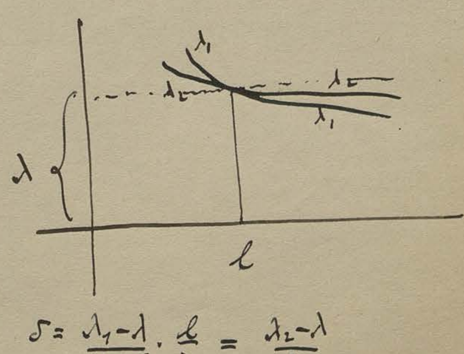
$$\lambda_1 - \lambda = \frac{\lambda\delta\left[m_1(\lambda+x_1) + m_2(\lambda-x_2)\right]}{m_2 x_2 - m_1 x_1} = \lambda\delta\left\{\lambda\frac{(m_1+m_2)}{m_2 x_2 - m_1 x_1} - 1\right\}$$

$$= \lambda\delta\left\{\frac{\lambda}{\ell} - 1\right\}$$

$$\lambda_1 - \lambda = (\lambda_2 - \lambda)\left\{\frac{\lambda}{\ell} - 1\right\}$$

$$= (\lambda_2 - \lambda)\left\{\frac{\lambda - \ell}{\ell}\right\}$$

$$= \left[\lambda_2 - \lambda\right] \frac{m_1 m_2 (x_1+x_2)^2}{\ell\,(m_1+m_2)^2}$$



$$\delta = \frac{\lambda_1 - \lambda}{\lambda - \ell} \cdot \frac{\ell}{\lambda} = \frac{\lambda_2 - \lambda}{\lambda}$$

$$\xi_1 = x_1(1+\delta)$$

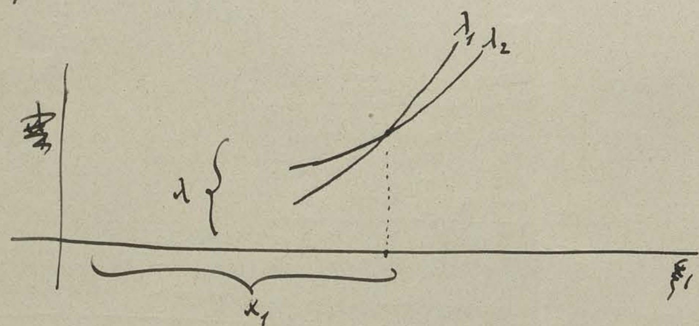
$$\lambda_1 = \lambda + \frac{2m_1 x_1^2 \delta}{m_2 x_2 - m_1 x_1} + \frac{m_1 x_1 \delta}{m_2 x_2 - m_1 x_1}\lambda$$

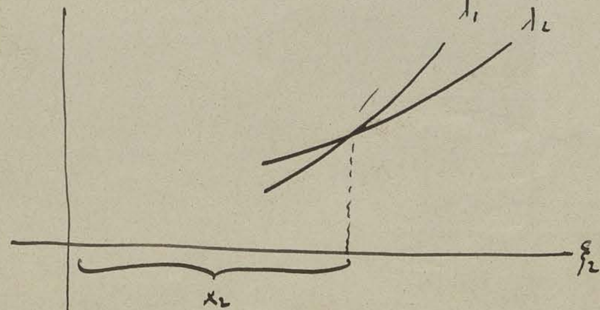
$$= \lambda + \delta \frac{m_1 x_1}{m_2 x_2 - m_1 x_1}\left[2x_1 + \lambda\right]$$

$$\lambda_2 = \frac{m_1(\lambda + x_1 + x_1\delta)^2 + m_2(\lambda - x_2)^2}{m_1(\lambda + x_1 + x_1\delta) + m_2(\lambda - x_2)} = \lambda + \frac{2m_1 x_1(\lambda + x_1)\delta}{m_1(\lambda + x_1) + m_2(\lambda - x_2)} - \frac{m_1 x_1 \delta}{m_1(\lambda + x_1) + m_2(\lambda - x_2)}\lambda$$

$$= \lambda + \delta \frac{m_1 x_1}{m_1(\lambda + x_1) + m_2(\lambda - x_2)}\Big[\underbrace{2(\lambda + x_1) - \lambda}_{(\lambda + 2x_1)}\Big]$$

ξ_1 zunehmen, x_2 konst





x_1 dane

x_2 zmienne $= \xi_2$

~~$x_2 = x$~~ $\xi_2 = x_2(1+\delta)$

$x_1 + h = .$

~~$\lambda_1 = \lambda(1+\varepsilon)$~~

~~$\dfrac{m_2 x_2^2 + m_1 x_1^2}{m_2 x_2 - m_1 x_1} = \lambda(1+\varepsilon)$~~

$$\lambda_1 = \frac{m_1 x_1^2 + m_2 x_2^2 (1+\delta)^2}{m_2 x_2 (1+\delta) - m_1 x_1} = \lambda + \delta \left\{ \frac{2 m_2 x_2^2}{m_2 x_2 - m_1 x_1} - \frac{m_2 x_2}{m_2 x_2 - m_1 x_1} \lambda \right\}$$

$$\frac{m_2 x_2}{m_2 x_2 - m_1 x_1} \left\{ 2 x_2 - \lambda \right\}$$

~~$\dfrac{m_2 x_2^2 - 2 m_1 x_1 x_2 - m_1 x_1^2}{m_2 x_2 - m_1 x_1}$~~

$\xi_2 = x_2(1+\delta)$

$$\lambda_1 = \lambda + \delta . \frac{m_2 x_2}{m_2 x_2 - m_1 x_1} \left[2 x_2 - \lambda \right]$$

$$\lambda_2 = \frac{m_1 (\lambda + x_1)^2 + m_2 (\lambda - x_2 - x_2 \delta)^2}{m_1 (\lambda + x_1) + m_2 (\lambda - x_2 - x_2 \delta)} = \lambda - \frac{2 m_2 (\lambda - x_2) x_2 \delta}{m_1 (\lambda + x_1) + m_2 (\lambda - x_2)} + \frac{m_2 x_2 \delta}{m_1 (\lambda + x_1) + m_2 (\lambda - x_2)} \lambda$$

$$= \lambda - \frac{m_2 x_2 \delta \left[\lambda - 2 x_2 \right]}{m_1 (\lambda + x_1) + m_2 (\lambda - x_2)}$$

$$\lambda_2 = \lambda + \delta \frac{m_2 x_2 \left[2 x_2 - \lambda \right]}{m_1 (\lambda + x_1) + m_2 (\lambda - x_2)}$$

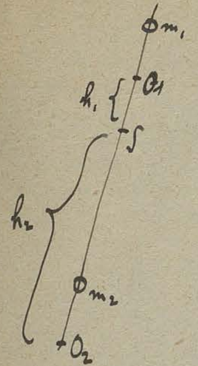
$$\frac{1}{m_2 x_2 - m_1 x_1} \pm \frac{1}{m_1 (\lambda + x_1) + m_2 (\lambda - x_2)} = \frac{m_1 \lambda + m_1 x_1 + m_2 \lambda - m_2 x_2 \pm \left[m_2 x_2 - m_1 x_1 \right]}{(\quad)(\quad)}$$

$$\lambda_1 + \lambda_2 = 2\lambda + \delta\, m_2 x_2 (2 x_2 - \lambda) \frac{(m_1 + m_2) \lambda}{(m_2 x_2 - m_1 x_1) \left[m_1 (\lambda + x_1) + m_2 (\lambda - x_2) \right]}$$

$$\lambda_1 - \lambda_2 = \delta\, m_2 x_2 (2 x_2 - \lambda) \frac{m_1 \lambda + 2 m_1 x_1 + m_2 \lambda - 2 m_2 x_2}{(\quad)\left[\quad\right]}$$

$$2\lambda = (\lambda_1 + \lambda_2) - (\lambda_1 - \lambda_2) \frac{(m_1 + m_2) \lambda}{(m_1 + m_2) \lambda + 2 (m_1 x_1 - m_2 x_2)} = (\lambda_1 + \lambda_2) - (\lambda_1 - \lambda_2) \frac{\lambda}{\lambda - 2 l}$$

157



$$T_1 = 2\pi\sqrt{\frac{K}{Mgh_1}} = 2\pi\sqrt{\frac{K_0 + h_1^2 M}{Mgh_1}} = \frac{2\pi}{\sqrt{g}}$$

$$\tau = 2\pi\sqrt{\frac{l}{g}}$$

$$g\frac{T_1^2}{4\pi^2} = \frac{K_0}{M}\frac{1}{h_1} + h_1$$

$$g\frac{T_2^2}{4\pi^2} = \frac{K_0}{M}\frac{1}{h_2} + h_2$$

$$\frac{T_1^2}{\tau^2} l = \frac{k^2}{h_1} + h_1$$

$$\frac{T_2^2}{\tau^2} l = \frac{k^2}{h_2} + h_2 = \frac{k^2}{l-h_1} + l - h_1$$

$$\frac{g}{4\pi^2}\left[h_1 T_1^2 - h_2 T_2^2\right] = \left[h_1^2 - h_2^2\right] = \frac{[h_1+h_2]}{\tau^2}\left[h_1 T_1^2 - h_2 T_2^2\right]$$

$$\frac{g}{4\pi^2}\tau^2 = h_1 + h_2 = l$$

$$h_1 + h_2 = l$$

$$h_2 = l - h_1$$

$$g\frac{\tau^2}{4\pi^2} = h_1 + h_2$$

~~$l(2h_1 - l) = \frac{g}{4\pi^2}\left[h_1 T_1^2 - (l - h_1) T_2^2\right]$~~

$$h_1 - h_2 = \frac{1}{\tau^2}\left[h_1 T_1^2 - h_2 T_2^2\right]$$

$x_1)^2$

~~$h_1\{2l - \frac{g}{4\pi^2}(T_1^2 + T_2^2)\} = l^2 - g\frac{l}{4\pi^2}T_2^2$~~

~~$2h_1 - l = \frac{1}{\tau^2}\left[h_1 T_1^2 - (l-h_1) T_2^2\right]$~~

~~$h_1\left\{2l - \frac{l}{\tau^2}(T_1^2 + T_2^2)\right\} = l^2\left\{1 - \frac{T_2^2}{\tau^2}\right\}$~~

~~$h_1\left\{2 - \frac{1}{\tau^2}[T_1^2 + T_2^2]\right\} = l\left\{1 - \frac{T_2^2}{\tau^2}\right\}$~~

$$\tau^2 = \frac{h_1 T_1^2 - h_2 T_2^2}{h_1 - h_2}$$

$$h_1\left[1 - \frac{T_1^2}{\tau^2}\right] = [l - h_1]\left[1 - \frac{T_2^2}{\tau^2}\right]$$

$$\frac{T_1^2 + T_2^2}{2} + \frac{h_1 T_1^2 - h_2 T_2^2}{h_1 - h_2} - \frac{T_1^2 h_1}{2} + h_2\frac{T_1^2}{2} - \frac{T_2^2 h_1}{2} + \frac{h_2 T_2^2}{2}$$

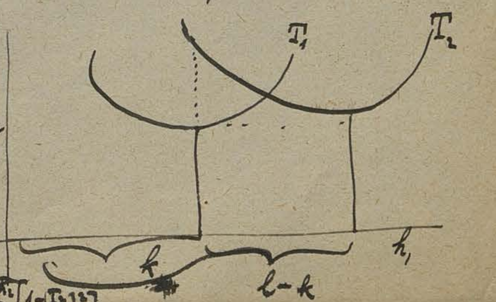
~~$\left(\frac{T_2}{\tau}\right)^2 = 1 - \frac{h_1}{l-h_1}\left[1 - \left(\frac{T_1}{\tau}\right)^2\right] = \frac{l + h_1\left(\frac{T_1}{\tau}\right)^2 - 2h_1}{l - h_1}$~~ $= \frac{T_1^2}{2}(h_2 + h_1) + \frac{T_2^2}{2}(h_1 + h_2)$

~~$\tau^2 = T_1^2 + \frac{h_2 (T_2^2 + T_1^2)}{h_1 - h_2} = T_1^2 + \frac{h_2}{h_1 - h_2}(T_1^2 - T_2^2)$~~

$h_1 T_1^2 + (h_2 - h_1) T_1^2$

$$\tau^2 = \frac{T_1^2 + T_2^2}{2} + \frac{1}{2}\frac{h_1 + h_2}{h_1 - h_2}\left[T_1^2 - T_2^2\right]$$

~~$\left[1 - \left(\frac{T_2}{\tau}\right)^2\right]\left[1 - \frac{h_2}{h_1}\right] = \frac{T_1^2 + T_2^2}{2} + \frac{1}{2}\frac{h_1 + h_2}{\tau}\left[1 - [illegible]\right]$~~



$$T_1 = \tau(1+\delta)$$

$$T_2^2 = \frac{-\tau^2(h_1-h_2) + h_1 T_1^2}{h_2}$$

$$= \frac{-\tau^2 h_1 + \tau^2 h_2 + h_1\tau^2 + 2h_1\tau^2\delta}{h_2}$$

$$= \tau^2\left(1 + \frac{2h_1}{h_2}\delta\right)$$

$$\frac{m_1x_1^2 + m_2x_2^2}{m_2x_2 - m_1x_1} = \lambda$$

$$m_2x_2^2 - m_2x_2\lambda = -m_1x_1\lambda - m_1x_1^2$$

$$x_2^2 - x_2\lambda = -\frac{m_1}{m_2}(x_1\lambda + x_1^2)$$

$$x_2 = \frac{\lambda}{2} \pm \sqrt{\frac{\lambda^2}{4} - \frac{m_1}{m_2}(x_1\lambda_1 + x_1^2)}$$

motive o ile

$$x_2 = \frac{\lambda}{2} - \sqrt{\frac{\lambda^2}{4} - \frac{m_1}{m_2}(x_1\lambda_1 + x_1^2)}$$

$$\frac{\lambda^2}{4} > \frac{m_1}{m_2}(x_1\lambda_1 + x_1^2)$$

$$\left(\frac{\lambda}{2} - \frac{m_1}{m_2}x_1\right)^2 > x_1^2\left(\frac{m_1}{m_2} + \frac{m_1^2}{m_2^2}\right)$$

$$\frac{\lambda}{2} - \frac{m_1}{m_2}x_1 > x_1\sqrt{\frac{m_1}{m_2} + \left(\frac{m_1}{m_2}\right)^2}$$

$$\frac{\lambda}{2} > x_1\left\{\frac{m_1}{m_2} + \sqrt{\frac{m_1}{m_2} + \left(\frac{m_1}{m_2}\right)^2}\right\}$$

$$> x_1\frac{m_1}{m_2}\left[1 + \sqrt{1 + \frac{m_2}{m_1}}\right]$$

$$m_2(\lambda - x_2)^2 - m_2\lambda(\lambda - x_2) = m_1\lambda(\lambda + x_1) - (\lambda + x_1)^2$$

$$\lambda - x_2 = \frac{\lambda}{2} \pm \sqrt{\frac{\lambda^2}{4} + \frac{m_1}{m_2}\Big[\underbrace{\lambda^2 + \lambda x_1 - (\lambda + x_1)^2}_{-\lambda x_1 - x_1^2}\Big]}$$

$$\left[\frac{\lambda}{2} \pm \sqrt{\frac{\lambda^2}{4} - \frac{m_1}{m_2}(x_1\lambda_1 + x_1^2)}\right] > \frac{m_1}{m_2}x_1$$

$$(+)\ \sqrt{\quad} > \frac{m_1}{m_2}x_1 - \frac{\lambda}{2}$$

$$\frac{\lambda^2}{4} - \frac{m_1}{m_2}(x_1\lambda + x_1^2) > \frac{\lambda^2}{4} - \lambda\frac{m_1}{m_2}x_1 + \left(\frac{m_1}{m_2}\right.$$

$$(-)\ \frac{\lambda}{2} - \frac{m_1}{m_2}x_1 > \sqrt{\frac{\lambda^2}{4} - \frac{m_1}{m_2}(x_1\lambda + x_1^2)}$$

$$\frac{\lambda^2}{4} - \lambda\frac{m_1}{m_2}x_1 + \left(\frac{m_1x_1}{m_2}\right)^2 > \frac{\lambda^2}{4} - \lambda\frac{m_1}{m_2}x_1 - \frac{m_1}{m_2}x_1^2$$

wenn man wählt $h_2 = 2h_1$ so ist

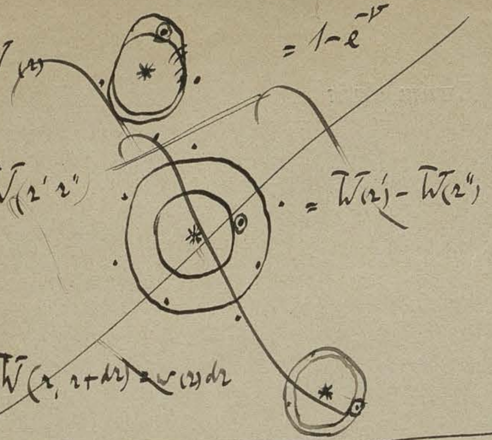
$$\tau^2 = \frac{h_1 T_1^2}{h_1} = T_1^2$$

$\int_{-\infty}^{\infty} e^{-\frac{h}{2}s^2} \, ds \sqrt{\frac{h}{2}} = \sqrt{\pi}$

$W_{(r)} = 1 - e^{-\nu}$

$W_{(r' r'')} = W(r') - W(r'')$

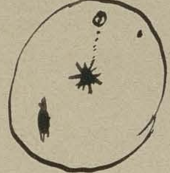
$W(r, r+dr) = w(r)\,dr$



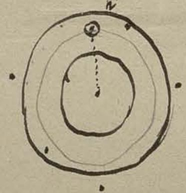
$W(r) = 1 - e^{-\nu}$ = [illegible] e 1, 2, 3 [illegible]

$= \sum \nu e^{-\nu} + \nu^2 \frac{e^{-\nu}}{2} + \ldots$

= [illegible] e wenigstens 1 [illegible]

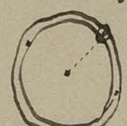


$W_{(r' r'')}$ $= W(r') \; W(r'')$ ~~[illegible] $= W(r') \, (1 - W(r''))$~~



$W(r, r+dr) = w(r)\,dr = \frac{\partial W}{\partial r} dr = e^{-\nu} \frac{d\nu}{dr} dr = 4\pi r^2 \cdot e^{-\nu} dr \; n$

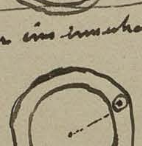
$W(r+dr)[1 - W(r)] =$



Wahrsch. dass nur eins innerhalb r_1 aber gar keins innerhalb r_2:

~~$\frac{4\pi}{3} n r^3 e^{-\frac{4\pi}{3}[(r+dr)^3 - r^3] n}$~~

$= \frac{4\pi}{3} n [(r+dr)^3 - r^3] \, e^{-\frac{4\pi n}{3} r^3} = 4\pi r^2 n \, dr \, e^{-\frac{4\pi n r^3}{3}}$



Für Wahrsch. dass nur eins zwischen $r_1 r_2$ vorkommt, kann man nicht sagen: $W r_1 = W(r_1, r_2) + W(r_2)$

sondern $W(r_1) = \frac{4\pi}{3}n(r_1^3 - r_2^3)\, e^{-\frac{4\pi}{3}n(r_1^3 - r_2^3)} \cdot e^{-\frac{4\pi}{3} n r_2^3}$ + $\frac{4\pi}{3} n r_2^3 \, e^{-\frac{4\pi}{3} n r_2^3}$

$= \frac{4\pi}{3} n r_1^3 \, e^{-\frac{4\pi}{3} n r_1^3}$ stimmt

POLSKIE
TOWARZYSTWO PRZYRODNIKÓW
IM. KOPERNIKA.

Lwów dnia ....

$$e^{-nk_1} - e^{-nk_2} \geq n(k_2-k_1)\, e^{-nk_2}$$

$$e^{-n(k_1-k_2)} \geq n(k_2-k_1) + 1$$

$$e^{n(k_2-k_1)} > 1 + n(k_2-k_1)$$

$$e^{-nk_1}$$

$$\frac{d(e^{-nk_1})}{dn} dn$$

$$e^{-nk}$$

160

$$\lim_{\sigma=0} \psi = -\frac{a^2\alpha \sin^2\theta}{2}\Big[\Big\{\cos(\sigma t + \ldots) + \frac{3}{2\beta a}(\cos\sigma t + \sin\sigma t) + \frac{3}{2\beta^2 a^2}\sin\sigma t\Big\}\frac{a}{r}$$

$$-\Big\{\frac{3}{2\beta a}\big[\cos(\sigma t - \beta(r-a)) + \sin(\sigma t - \beta(r-a))\big] - \frac{3}{2\beta^2 a^2}\sin(\sigma t - \beta(r-a))\Big\}\Big\{1 - \beta(r-a) + \frac{\beta^2 (r-a)^2}{2}\Big\}\Big]$$

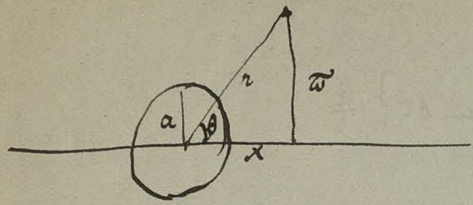
$$\beta = \sqrt{\frac{\sigma\rho}{2\mu}}$$

$$= -\frac{3\alpha \sin^2\theta}{4\beta^2}\Big[\Big\{\sin\sigma t + \beta a(\cos\sigma t + \sin\sigma t) + \frac{2\beta^2 a^2}{3}\cos\sigma t\Big\}\frac{a}{r}$$

$$-\underbrace{\Big\{\sin(\sigma t - \beta(r-a)) + \beta a\cos(\sigma t - \beta(r-a))\Big\}}\Big\{1 - \beta(r-a) + \beta^2\frac{(r-a)^2}{2}\Big\}\Big]$$

$$\sin\sigma t \cos\beta(r-a) - \cos\sigma t \sin\beta(r-a)$$

$$+\cos\sigma t \cos\beta(r-a) + \sin\sigma t \sin\beta(r-a)$$



$$u = -\frac{1}{\varpi}\frac{\partial\psi}{\partial\varpi} \qquad V = \frac{1}{\varpi}\frac{\partial\psi}{\partial x}$$

$$U = \alpha\cos(\sigma t+\varepsilon)$$

$$\beta = \sqrt{\frac{\sigma\rho}{2\mu}}$$

$$\psi = -\frac{1}{2}\alpha a^2\sin^2\theta\Big[\Big\{\Big(1+\frac{3}{2\beta a}\Big)\cos(\sigma t+\varepsilon) + \frac{3}{2\beta a}\Big(1+\frac{1}{\beta a}\Big)\sin(\sigma t+\varepsilon)\Big\}\frac{a}{r}$$

$$-\frac{3}{2\beta a}\Big\{\cos[\sigma t-\beta(r-a)+\varepsilon] + \Big(1+\frac{1}{\beta r}\Big)\sin[\sigma t-\beta(r-a)+\varepsilon]\Big\}e^{-\beta(r-a)}\Big]$$

$$= \psi_1 + \psi_2$$

$$\Phi_1 = -4\mu\Big[\frac{\partial v}{\partial y}\frac{\partial w}{\partial z} - \frac{\partial v}{\partial z}\frac{\partial w}{\partial y} + \frac{\partial w}{\partial z}\frac{\partial u}{\partial x} - \frac{\partial w}{\partial x}\frac{\partial u}{\partial z} + \frac{\partial u}{\partial x}\frac{\partial v}{\partial y} - \frac{\partial u}{\partial y}\frac{\partial v}{\partial x}\Big] = \overset{1=}{\text{(irrotational)}}$$

$$= -4\mu\Big[\frac{\partial^2\varphi}{\partial y^2}\frac{\partial^2\varphi}{\partial z^2} - \Big(\frac{\partial^2\varphi}{\partial y\partial z}\Big)^2 + \frac{\partial^2\varphi}{\partial z^2}\frac{\partial^2\varphi}{\partial x^2} - \Big(\frac{\partial^2\varphi}{\partial x\partial z}\Big)^2 + \frac{\partial^2\varphi}{\partial x^2}\frac{\partial^2\varphi}{\partial y^2} - \Big(\frac{\partial^2\varphi}{\partial x\partial y}\Big)^2\Big] =$$

$$= 4\mu\Big[\Big(\frac{\partial^2\varphi}{\partial x^2}\Big)^2 + \Big(\frac{\partial^2\varphi}{\partial y^2}\Big)^2 + \Big(\frac{\partial^2\varphi}{\partial z^2}\Big)^2 + 2\Big(\frac{\partial^2\varphi}{\partial y\partial z}\Big)^2 + 2\Big(\frac{\partial^2\varphi}{\partial x\partial z}\Big)^2 + 2\Big(\frac{\partial^2\varphi}{\partial x\partial y}\Big)^2\Big]$$

$$v = V\frac{y}{\varpi} = V\frac{y}{\sqrt{y^2+z^2}} = \frac{y}{y^2+z^2}\frac{\partial\psi}{\partial x}$$

$$w = V\frac{z}{\sqrt{y^2+z^2}} = \frac{z}{y^2+z^2}\frac{\partial\psi}{\partial x}$$

hence $\varphi = \boxed{-A}\,\frac{x}{r^3}$

$$\frac{\partial\varphi}{\partial x} = \frac{1}{r^3} - \frac{3x^2}{r^5}$$

$$\frac{\partial\varphi}{\partial y} = -\frac{3xy}{r^5}$$

$$\frac{\partial\varphi}{\partial z} = -\frac{3xz}{r^5}$$

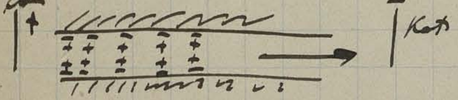
$$\frac{\partial^2\varphi}{\partial x^2}$$

$$\Phi_1 = 4\mu\Big[\Big[-\frac{9x}{r^5} + \frac{15x^3}{r^7}\Big]^2 + \Big[-\frac{3x}{r^5} + \frac{15xy^2}{r^7}\Big]^2 + \Big[-\frac{3x}{r^5} + \frac{15xz^2}{r^7}\Big]^2 + 2\Big(\frac{15xyz}{r^7}\Big)^2 + 2\Big(-\frac{3z}{r^5} + \frac{15x^2z}{r^7}\Big)^2$$

$$Vol = \frac{K(\varphi_i - \varphi_0)}{4\pi} \frac{J\sigma}{\mu}$$

		J
$-SiO_2 + H_2O$ (Sol)		> 4·7 . 10^1 Vol.
Sb_2O_5		> 6·4 "
$WO_3 . H_2O$		8·0 "
$SnO_2 + H_2O$		11. 10^5
SnO_2		8. "
$TiO_2 + H_2O$		1 "
Fe_2O_3		7·3 10^{-11} NaOH
$Fe(OH)_3$		5·6
CuO		4·6
$Cu(OH)_2$		< 2·9
ZnO		2·8
$Zn(OH)_2$		1·3
$Ni(OH)_2$		1·5 . 10^{-13}
$Mg(OH)_2$		< 6·6 "
MgO		1·5 . 10^{-14}

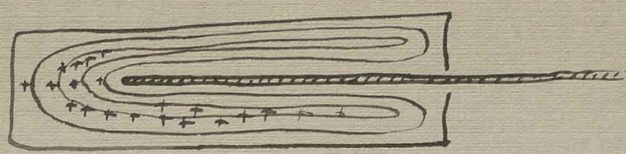
$$v = A - k \lg p$$

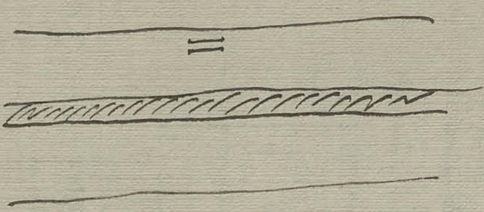


H_2WO_4		v = +2·66
+ HCl	0·006	+ 2·81
	0·045	+ 1·12
	0·207	+ 0·28
	0·8	0·0
	(norm)	

ZnO	(norm)	v = −0·31
NaOH	0·0002	−0·22
	0·0003	+ 0·16
	0·0004	+ 0·24

$Mg(OH)_2$				
	0·0006	v = −0·64	0·015	+ 0·65
	0·005	− 0·23	0·012	+ 0·99
	0·01	− 0·05	+ 0·009	+ 0·32
	0·011	− 0·10		
	0·02	+ 0·90		





$$-\frac{\partial}{\partial r}\left(r\frac{\partial V}{\partial r}\right) + \frac{r q}{\omega\varepsilon} = 0$$

$$d\left(r\frac{dV}{dr}\right) = r\frac{q}{\omega\varepsilon}$$

$$r\frac{dV}{dr} = \frac{r^2}{2}\frac{q}{\omega\varepsilon} + \alpha$$

$$V = \frac{r^2}{4}\left(\frac{q}{\omega\varepsilon}\right) + \alpha \lg r + \beta$$

$$V_1 = \frac{r_1^2}{4}\gamma + \alpha \lg r_1 + \beta$$

$$V_2 = \frac{r_2^2}{4}\gamma + \alpha \lg r_2 + \beta$$

$$V_1 - V_2 = \frac{r_1^2 - r_2^2}{4}\gamma + \alpha \lg \frac{r_1}{r_2}$$

q = liczba jonów pro sek. wytworzonych

$\varepsilon\omega$ = prędkość jonu w polu $\frac{1\,\text{Volt}}{1\,\text{cm}}$

$$V - V_2 = \frac{(r^2 - r_2^2)}{4}\gamma + \alpha \lg \frac{r}{r_2}$$

$$= \frac{r^2 - r_2^2}{4}\gamma + \left\{(V_1 - V_2) - \frac{r_1^2 - r_2^2}{4}\gamma\right\}\frac{\lg r - \lg r_2}{\lg r_1 - \lg r_2}$$

$$= (V_1 - V_2)\frac{\lg r - \lg r_2}{\lg r_1 - \lg r_2} +$$

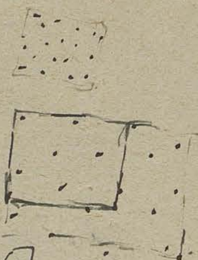
$$\omega\varepsilon\left(r\frac{\partial V}{\partial r}\right)_1 = \omega\varepsilon\left[\frac{r_1^2}{2}\frac{q}{\omega\varepsilon} + \alpha\right] = \frac{r_1^2}{2}q + \alpha\,\omega\varepsilon$$

$$= \frac{r_1^2 q}{2} + \frac{V_1 - V_2 - \frac{r_1^2 - r_2^2}{4}\gamma}{(\lg r_1 - \lg r_2)}\,\omega\varepsilon$$

163

$$e^{-\frac{v}{2}\delta^2 \cdot \frac{v_0^2}{R\theta_0}\left(\frac{\partial p}{\partial v}\right)_0}$$

$$e^{-v\frac{\delta^2}{2} - \frac{\Delta U}{\frac{R\theta}{2}}}$$



$$pv = R\theta - \frac{1}{2}\sum r f(r)$$

$$\frac{mu^2}{2} = \frac{mc^2}{6} = \frac{R\theta}{2}$$

$$v\frac{\partial p}{\partial v} + p = -\frac{1}{2}\frac{\partial}{\partial v}\left[\sum r f(r)\right]_{n=const}$$

$$v^2\frac{\partial p}{\partial v} + pv = -\frac{1}{2} v \frac{\partial}{\partial v}\left[\sum r f(r)\right]$$

$$\sum\sum r f(r) = \sum\iint r f(r)\, e^{-hU(x, y, \dots)} dx_1 \dots$$

$$\sum(xX + yY + zZ)\, e^{-hU(x, y, z)} dx\, dy\, dz$$

$$\frac{v^2\frac{\partial p}{\partial v}}{R\theta} + \frac{pv}{R\theta} = = \frac{\frac{1}{2} v \frac{\partial}{\partial v}\left(\sum r f\right)}{R\theta}$$

$$\frac{v^2\left(\frac{\partial p}{\partial v}\right)}{R\theta} = \frac{\frac{1}{2}\overbrace{\sum\sum r f(r)}^{V} - v\frac{\partial}{\partial v}\sum r f(r)_{n=const}}{R\theta} - 1$$

$$V - \frac{v}{\cancel{2}}\frac{\partial}{\partial v}V$$

$$\cancel{= \frac{1}{v}\frac{\partial}{\partial v}(vV)}$$

$$p\,\mathrm{vol} = R\theta - \overbrace{\left(\sum\sum r f(r)\right)}^{V}$$

$$p_0\,\mathrm{vol} = R\theta = V_0$$

$$v^2\frac{V dv - v\, dV}{v^2 dv}$$

$$v\left(\frac{V}{v} - \frac{1}{v}\frac{\partial V}{\partial v}\right) = v\frac{\partial}{\partial v}\left(\frac{V}{v}\right)$$

$$\frac{\partial p}{\partial v}\cdot \mathrm{vol} = -\frac{\partial V_{\mathrm{vol}}}{\partial v}\Big|_{\mathrm{vol}=const}$$

$$= -v^2\frac{d}{dv}\left(\frac{V}{v}\right)$$

$$\frac{\Delta V_{n=const}}{\partial v} = \frac{\Delta V}{\partial v}_{\mathrm{vol}=const} + V\frac{\partial}{\partial n}\Delta n$$

$$= -\frac{1}{\rho^2}\frac{d(V\rho)}{d\left(\frac{1}{\rho}\right)} = = -\frac{d(V\rho)}{\rho}$$

$$\int(p - p_0)\,dv = R\theta\left[\log\frac{v}{v_0} - \frac{v - v_0}{v_0}\right] -$$

$$\sum r\,(\dots) - \sum r_0 f(r_0)\Delta r$$

$$\int_1^2 r\frac{\partial U}{\partial r}dr = (Ur)\Big|_1^2 - \int_1^2 U\,dr$$

$$- r_1\left(\frac{\partial U}{\partial r}\right)_1 (r_2 - r_1)$$

$$\sum\sum\int_1^2 r\frac{\partial U}{\partial r}dr - r_1\left(\frac{\partial U}{\partial r}\right)_1 (r_2 - r_1)$$

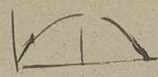
$$U\Delta r + r\Delta U - U\Delta r - r\underbrace{\frac{\partial U}{\partial r}\Delta r}_{\Delta U} = 0$$

$$\frac{\partial(Ur)}{\partial r}\Delta r + \frac{\partial^2(Ur)}{\partial r^2}\Delta r^2 - -$$

$$\Delta r^2\cdot\left[2\frac{\partial U}{\partial r} + r\frac{\partial^2 U}{\partial r^2}\right]$$

$\boxed{t = 10 \text{ sek.}}$

1). $c = 50 \frac{m}{sek.}$ $t = 5$



$\frac{100}{2} 10 = 500 \text{ m.}$

$p = -\frac{\partial F}{\partial v}$

$p_0 = -\left(\frac{\partial F}{\partial v}\right)_0$

$\int (p - p_0)\, dv = F - F_0 - \left(\frac{\partial F}{\partial v_0}\right)(v - v_0)$

$U = F - \theta \frac{\partial F}{\partial \theta}$

$\frac{\partial U}{\partial v} = \frac{\partial F}{\partial v} - \theta \frac{\partial^2 F}{\partial \theta \partial v} = p - \theta \frac{\partial p}{\partial \theta}$

$\delta U = \frac{U_{-1} + U_{+1} - 2U_0}{2} = \frac{\partial^2 U}{\partial v^2} \delta v^2$

$\frac{\partial^2 U}{\partial v^2} = \frac{\partial p}{\partial v} - \theta \frac{\partial}{\partial \theta}\left(\frac{\partial p}{\partial v}\right)$ (1 ??)

$p + \frac{a}{v^2} = \frac{R\theta}{\;}\left[\frac{1}{v} + \frac{b}{v^2} + \frac{5}{8}\frac{b^2}{v^3}\right]$

$\frac{\partial p}{\partial v} = -\frac{2a}{v^3} - R\theta\left[\frac{1}{v^2} + \frac{2b}{v^3} + \frac{15}{8}\frac{b^2}{v^4}\right]$

$-\frac{v^2}{R\theta}\left(\frac{\partial p}{\partial v}\right) = 1 + \frac{2b}{v} + \frac{15}{8}\frac{b^2}{v^2} + \frac{2a}{v.R\theta}$

$\frac{v}{2}\delta^2\left[\frac{v_0^2}{R\theta_0}\left(\frac{\partial p}{\partial v}\right)_0\right]$

$b = 4\left[\frac{4}{3}\pi\left(\frac{\sigma}{2}\right)^3\right]\nu = \frac{2}{3}\pi\sigma^3\nu$

$2b = \alpha v$ $= \frac{\alpha v}{2}$

$(2.10^{-5})^3 . 3.10^{19}$

$v = 24.10^5$

$\sqrt{v} = 5.10^2$

$\frac{1}{500}$

$h = \frac{3N}{2L} = \frac{3N}{2N\frac{mc^2}{2}} = \frac{3}{mc^2}$

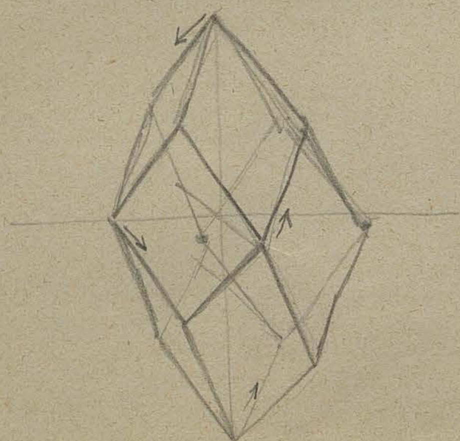
$\frac{mnc^2}{3} = R\theta$

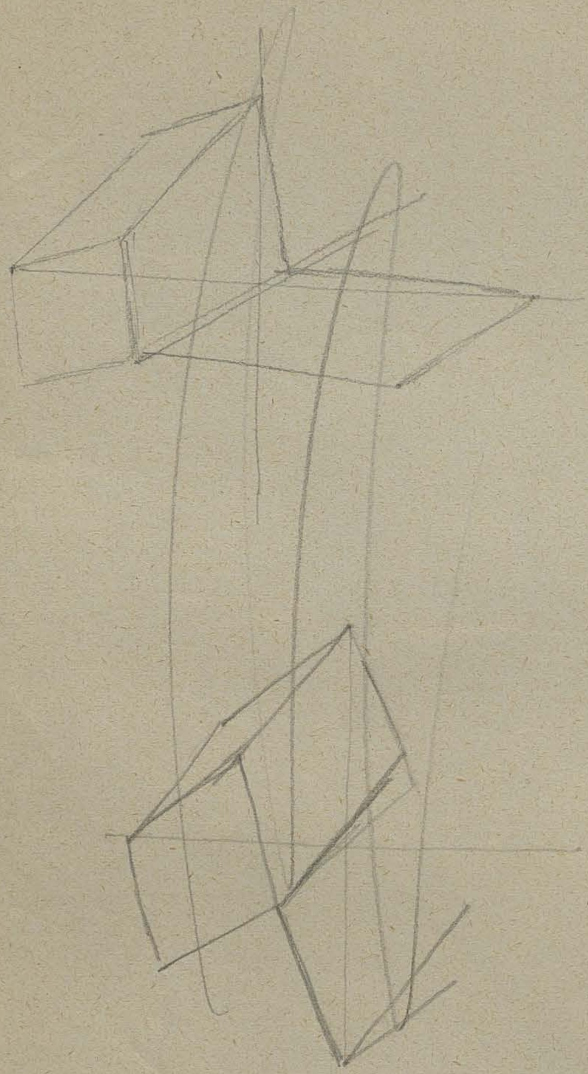
$\frac{3}{mc^2} = \frac{n}{R\theta} = \frac{n\mu}{H\theta} = \frac{nm}{mR\theta}$

$h = \frac{N}{H\theta}$

$\sqrt{3.10^{19}}$

$= \frac{10^{-9}}{\sqrt{30.}} = \frac{1}{5.5} = 2.10^{-10}$





165

Dlaczego tylko 100 km asejsmicznej, bo ciśnienie [górotwórcze] tylko w tej

2. Sztywność wnętrza Ziemi Rudzki   Kalabria 1905   7 km

S. Francisco 1906   16-32

. Helmert, Hayford Anglicy, fizycy, ponieważ który już opływ geologom [illegible]

głębsze warstwy plastyczne

sprawdzić ciało stałe, bo inaczej nie byłoby poprzecznych, ale

skorupa spoczywa na głębszych warstwach o wyższym stopniu plastyczności

Teorya izostazyi   równowaga hydrostatyczna gór lżejszych

przyczynek Rudzkiego :   Norwegia 280 km – ślady morza podniesienie epoki lodowej

dowody pod lód   933 m

7 km

ale nie masa płynna jak woda tylko plastyczna; proces trwający dłuższy! Ruch 3-40,000 lat

Kanada, Chicago, Ameryka

Wiek Ziemi

Kelvin   Rudzki cylinder   139 - 486 mil lat

Ker   40 - 100   całkiem przemawia

Strutt : ~~He~~		Holmes U → Pb	
olejowe	8	węgl.	340
eocen	31	sylur	430
wapień węgl.	150	arch.	1025 - 1640
arch.	710		

Dotychczas tylko [illegible] teorie z zakresu [illegible]
fale wodne Helmholtz

dzieło { ~~[illegible]~~ Fizyka Ziemi
meteorologia

Geofizyka : { zjawiska przyrody
{ dążność do ścisłości rozumowania | właściwość ujęcia tej nauki pomoca geologii ; studiuje matematyk i fiz.-tec.

geofizyka stwarza jej...

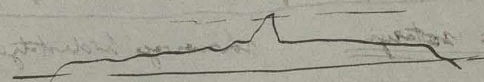
Słowo o epoce lodowej

charakteryzuje właściwości wobec tej nauki, wszystko tylko w tem co daje się wyrazić matematycznie

Dziwna dlaczego trudno dać pojęcie nie fachowcom

ograniczę się do ogólnikowego poglądu na wielkie zagadnienie

1). wnętrze i kształt kuli ziemskiej

na pozór rzecz prosta. Giewont 1900 m – co to znaczy? 

Geoida, obliczenie teoretyczne

Helmert metoda kondensacji, Rudzki inny sposób

związek z ~~[illegible]~~ rozkładem ciężkości, z anomaliami

2). Seismologia

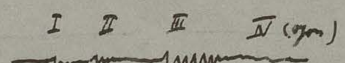
najwłaściwszy instrument do poznania wnętrza ziemi, tak jak prześwietlenie

30 – 40.000 trzęsień rocznie, 100 stacji, tylko obserwowanie!

fakty znane : gęstość, [illegible] sprężystość objętości, fale objętości, podłużne

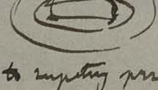
ciało stałe izotropowe, 2 [illegible] 2 fale oraz fale Rayleigh – Lamb wzdłuż powierzchni (jak fale wody)

analiza sejsmogramów : 3 rodzaje fal



I	II	III
7 km	4	
12	7	3-4

Badania Rudzkiego : 1). skały skorupy anizotropowe (Gneis, Łupki) wskutek ściśnięcia jednostronnego

3 fale  analogia z podw. załamaniem

~~[illegible]~~ Stanowiłoby to rzeczywisty przewrót, rewolucję, gdyby cała kula ziemska złożona

Tymczasem (tylko 100 – 120 km. , 6370 km. równikowa
Rudzki sam!)

odwrotnego pierwiastka z gęstości ((proporcyonalny do N), podczas gdy szybkość opadania zostaje niezmienioną, a szybkość prądów związanych z różnicami temperatury (muszą się zmniejszyć) w stosunku gęstości.

Zauważę też, że obserwacye kondensacyi pary na jonach (Wilson, Thomson etc.) i związane z tem zjawiska elektryczne (mogą tu ciekawego materyału dostarczyć. a co

Unoszenie się proch!

---

§15). Znacznie większe trudności przedstawia zastosowanie tych rozumowań do cieczy, ponieważ wogóle mało jeszcze wiemy o mechanizmie ich ruchów cząsteczkowych. Do obliczenia liczby uderzeń n nie możemy użyć (wzoru z teoryi gazów (16) gdyż ważnym jest tylko dla gazu rozrzedzonego. Można w każdym razie stawić

$$n = 4\pi R^2 \nu \tag{19}$$

a zatem

$$\Delta = \frac{2\sqrt{2}}{3\sqrt{\pi}} \frac{c}{R\sqrt{\nu}} \tag{20}$$

gdzie ν oznacza ilość uderzeń na $1 cm^2$ powierzchni, ale pytanie (znów) jak oznaczyć ν. Podamy kilka sposobów mniej lub więcej (nie)ścisłych, które jednak choć co do rzędu tej wielkości ν mogą dać wskazówki.

a). Według Van der Waalsa istnieje w cieczy wewnętrzne ciśnienie $p_i = a\rho^2$ które obliczyć można z (ciepła parowania wewnętrznego) (t.j. z ciepła parowania pozostającego po odciągnięciu pracy zewnętrznej) na podstawie wzoru: $L_i = a\rho$

Owo ciśnienie (wywierane na powłokę wewnętrzną) jest równoważone przez uderzenia drobin, z których każde przeciętnie reprezentuje moment ruchu $2mc\int_0^{\frac{\pi}{2}}\sin\theta\cos\theta\,d\theta = mc$; zatem wynika równanie:

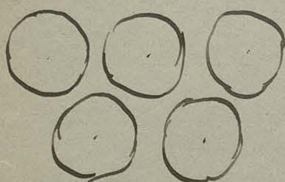
$$\nu = \frac{p_i}{mc} = \frac{L_i\rho}{mc} \tag{21}$$

~~Podstawiwszy B w λ) otrzymujemy:~~

~~$$\Lambda = \frac{2\sqrt{2}}{3\sqrt{\pi}} \frac{c^{3/2}}{R} \sqrt{\frac{m}{p_i}}$$~~

b). Spółczynnik lepkości ~~z punktu widzenia teoryi kinetycznej~~ jest to ilość ruchu udzieloną na sekundę płaszczyźnie nieruchomej XZ przez drobiny o nią uderzające, jeżeli spad prędkości $\frac{\partial u}{\partial y}$ (równoległej do X) ~~wynosi~~ równa się jedności.

Otrzymamy ten spółczynnik mnożąc ilość uderzeń przez przeciętną ich ilość ruchu. Gdyby drobiny były punktami, wtedy (te które) pod kątem θ ~~się poruszające~~ ~~pochodzi~~ ~~ostatnio drogę swobodną~~ uderzają, pochodziłyby z warstwy odległej o $\lambda \cos\theta$ od płaszczyzny XZ, jeżeli λ jest ich drogą swobodną, i posiadałyby ilość ruchu $\lambda \cos\theta$.



Z powodu rozmiarów drobin trzeba jednak dodać do λ ~~podwójny promień~~ średnicę $2r$ dla uderzeń prostych, a pewien ułamek tej wielkości dla skośnych uderzeń. Ponieważ chodzi nam tylko o rząd wielkości, nie wdamy się w dokładne obliczenie tego ułamka, tylko stawiamy $\lambda + 2r$ na miejsce λ. Zatem mamy

$$N m (\lambda + 2r) \int^{\frac{\pi}{2}} \cos\theta \sin\theta \, d\theta = \mu$$

Rozmaite względy wskazują ~~Ponieważ musimy przyjąć~~ że w cieczach przestrzeń wolna dla ruchu jest małym ułamkiem ~~całej~~ przestrzeni zajętej, więc ~~pominąć~~ pominiemy λ w porównaniu z $2r$ i otrzymamy:

$$N = \frac{\mu}{r m} \qquad (22)$$

c). ~~W poprzedniej rozprawie wyprowadziliśmy~~ Wzór dla spółczynnika dyfuzyi $D = \frac{\lambda c}{3}$ (patrz poprzednią rozprawę) pod pewnemi założeniami także dla cieczy będzie przybliżonym ważnym. ~~Otrzymać~~ Spółczynnik dyfuzyi wody przez wodę wprawdzie nie

$$+2\left(-\frac{3y}{r^5}+\frac{15x^2y}{r^7}\right)^2\Big]$$

$$= 4\mu\left(\frac{3x}{r^5}\right)^2\left\{\left(3-5\frac{x^2}{r^2}\right)^2+\left(1-5\frac{y^2}{r^2}\right)^2+\left(1-\frac{5z^2}{r^2}\right)^2+2\left(\frac{5yz}{r^2}\right)^2\right\}+2\cdot\frac{9z^2}{r^{10}}\left(1-\frac{5x^2}{r^2}\right)^2$$

$$+2\cdot\frac{9y^2}{r^{10}}\left(1-\frac{5x^2}{r^2}\right)^2\Big]$$

$$\left[1-10\frac{y^2+z^2}{r^2}+25\frac{(y^2+z^2)^2}{r^4}\right]$$

$$=\left(\frac{3x}{r^5}\right)^2\left\{\left[3-5\frac{x^2}{r^2}\right]^2+\left[1-10\left(1-\frac{x^2}{r^2}\right)+25\left(1-\frac{x^2}{r^2}\right)^2\right]\right\}+18\left(1-\frac{5x^2}{r^2}\right)^2\left(1-\frac{x^2}{r^2}\right)\frac{1}{r^8}$$

$$1-10\frac{x^2}{r^2}+25\frac{x^4}{r^4}$$

$$9-30\frac{x^2}{r^2}+25\frac{x^4}{r^4}+1-10+10\frac{x^2}{r^2}+25-50\frac{x^2}{r^2}+25\frac{x^4}{r^4}\Big]$$

$$-\frac{x^2}{r^2}+10 \quad -25$$

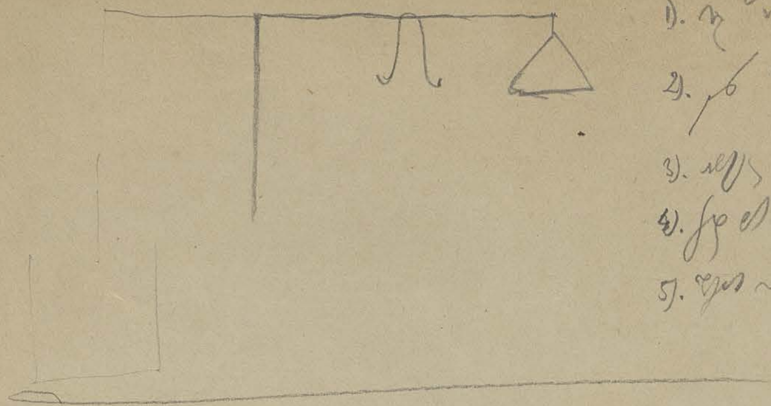
$$=\frac{9}{r^8}\left\{\frac{x^2}{r^2}\left[25-70\frac{x^2}{r^2}+50\frac{x^4}{r^4}\right]+2\left[1-11\frac{x^2}{r^2}+35\frac{x^4}{r^4}-25\frac{x^6}{r^6}\right]\right\}$$

$$=\frac{9}{r^8}\left\{2+3\frac{x^2}{r^2}\right\}$$

$$\int \cos^2\vartheta\sin\vartheta\,d\vartheta = -\frac{\cos^3}{3}\Big|_0^\pi = \frac{2}{3}$$

$$\Phi_1 = \frac{36\mu}{r^8}\left[2+3\frac{x^2}{r^2}\right]A^2 \qquad A=\frac{\alpha a^3}{2}\left[\left(1+\frac{3}{2\beta a}\right)\cos\dots+\frac{3}{2\beta a}\left(1+\frac{1}{\beta a}\right)\sin\dots\right]$$

$$\Sigma\Phi_1=\iint_{\vartheta=0}^{\pi} 2\pi r^2\sin\vartheta\,d\vartheta\,dr\;\Phi_1 = 2\pi\cdot 36\mu A^2\cdot\left(\frac{1}{5a^5}-\frac{1}{5r^5}\right)\left[4+3\cdot\frac{2}{3}\right]=\frac{2\cdot36\cdot6\cdot\pi\mu A^2}{5\cdot a^5}$$



1). [illegible]

2). [illegible]

3). [illegible]

4). [illegible]

5). [illegible]

[illegible] : 1). [illegible] [illegible] — [illegible] : —

2). dist. [illegible] — —

3). [illegible] resp. [illegible] + —

[illegible]

[illegible] Thermoelement [illegible]

[illegible] poröse [illegible]

$$T = \frac{\mu}{2}\left[\dot y_1^2 + \dot y_2^2 + \dots \dot y_n^2 + \dot y_{n+1}^2 + \dot y_{n+2}^2\right]$$

$$U = \frac{T_1}{2a}\left[(y_2-y_1)^2+(y_3-y_2)^2+\dots (y_n-y_{n+1})^2+(y_{n+2}-y_{n+1})^2\right] \qquad y_1 = y_{n+2} = 0$$

~~$\mathcal{D} y_1 + A y_2 + \mathcal{D} y_3 = 0$~~

~~$\mathcal{D} y_2 + A y_3 + \mathcal{D} y_4 = 0$~~

$\vdots$

~~$\mu \ddot y_1 = +\frac{T_1}{a}(y_2-y_1)$~~

$$\mu \ddot y_2 = -\frac{T_1}{a}\left[(y_2-y_1)-(y_3-y_2)\right]$$

$$\mu \ddot y_2 + \frac{2T_1}{a} y_2 = \frac{T_1}{a}(y_1+y_3)$$

$$y_m = \sum_{s=1}^{n} P_s \sin\frac{(m-1)s\pi}{n+1} \cos(n_s t - \varepsilon_s)$$

$$n_s = 2\sqrt{\frac{T_1}{\mu a}} \sin\frac{s\pi}{2(n+1)} \qquad s = 1 \dots n$$

$s=1$: $\quad y_m = P_1 \sin\frac{m-1}{n+1}\pi \cos\dots$

$y_1 = 0$

$\left.\begin{aligned} y_2 &= P_1 \sin\frac{\pi}{n+1} \\ y_3 &= P_1 \sin\frac{2\pi}{n+1} \\ y_4 &= P_1 \sin\frac{3\pi}{n+1} \\ &\vdots \end{aligned}\right\} \cos\dots$

$y_2 = \sin\frac{2\pi}{n+1}$

$y_3 = \sin\frac{4\pi}{n+1}$

$y_4 = \sin\frac{6\pi}{n+1}$

$y_2 = \sin\frac{3\pi}{n+1}$

$y_3 = \sin\frac{6\pi}{n+1}$

$y_4 = \sin\frac{9\pi}{n+1}$

$q_1 = \alpha_{11} y_1 + \alpha_{12} y_2 + \alpha_{13} y_3 + \dots$

$q_2 = \alpha_{21} y_1 + \alpha_{22} y_2 + \alpha_{23} y_3 \dots$

$q_3 \dots$

$y_1 = \beta_{11} q_1 + \beta_{12} q_2 + \dots$

$y_2 = \beta_{21} q_1 + \beta_{22} q_2 + \dots$

$\vdots$

$$q_s = \frac{2}{n+1} \sum_{r=1}^{n} y_r \sin\frac{r s \pi}{l}$$

#

$$\begin{vmatrix} c & 1 & 0 & 0 & 0 \\ 1 & c & 1 & 0 & 0 \\ 0 & 1 & c & 1 & 0 \\ 0 & 0 & 1 & c & 1 & 0 \end{vmatrix} = 0$$

$$\begin{cases} \varphi_1 = 0 \\ \varphi_{n+1} = 0 \end{cases}$$

$\varphi_r = P \sin(r-1)\beta \cos(nt-\varepsilon)$ $\quad (n+1)\beta = s\pi$

$B \sin(r-2)\beta + A \sin(r-1)\beta + B \sin r\beta = 0$

$B[\sin(r-2)\beta + \sin r\beta] + A \sin(r-1)\beta = 0$

$2\sin(r-1)\beta \cos\beta$

$2B\cos\beta + A = 0$

$$\frac{A}{B} = -2 + \frac{\mu a n^2}{T} = -2\cos\beta$$

$$n = 2 \sin\frac{\beta}{2}\sqrt{\frac{T}{\mu a}}$$

$$y_r = \sum_{s=1}^{m} P_s \sin\frac{(r-1)s\pi}{m+1} \cos(n_s t - \varepsilon) \qquad = \sum \alpha q$$

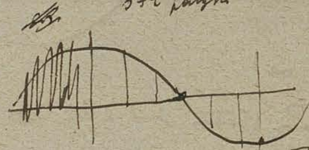
$$n_s = 2\sqrt{\frac{T}{\mu a}} \sin\frac{s\pi}{2(m+1)}$$

$$\sum_{s=1}^{m} \sin^2\frac{s r a \pi}{l} = \frac{m+1}{2}$$

$$\sum_{n=1}^{m} \sin\frac{s n a \pi}{l} \sin\frac{t n a \pi}{l} = 0$$

$$\cos(s+t)\frac{n a \pi}{l} - \cos(s-t)\frac{n a \pi}{l}$$

$s+t$ [illegible]



$$q_s = \sum_{r=1}^{m} y_r \sin\frac{(r-1)s\pi}{m+1}$$

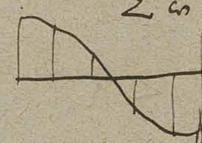
$$\dot q_s = \sum_{r=1}^{n} \dot y_r \sin\frac{(r-1)s\pi}{m+1}$$

$$\sum \dot q_s^2 = \sum_{s=1}^{m} \left[\dot y_2 \sin\frac{s\pi}{m+1} + \dot y_3 \sin\frac{2s\pi}{m+1} + \dot y_4 \sin\frac{3s\pi}{m+1} + \cdots\right]^2 \qquad \frac{1}{2}\left[1 - \cos\frac{2 s r \pi}{m+1}\right]$$

$$= \dot y_2^2 \sum_{s=1}^{m} \sin^2\frac{s\pi}{m+1} + \dot y_3^2 \sum_{s=1}^{m} \sin^2\frac{2s\pi}{m+1} + \cdots$$

$$+ \dot y_2 \dot y_3 \sum \sin\frac{s\pi}{m+1} \sin\frac{2s\pi}{m+1}$$

$\sum \cos$



$$e^{i\frac{s\pi}{m+1}} = \cos\frac{s\pi}{m+1} + i \sin\cdots$$

$$x + x^2 + x^3 + \cdots + x^n = S$$

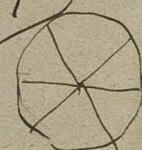
$$\frac{S}{x} = S - x^n + 1$$

$$S = Sx - x^{n+1} + x$$

$$S = \frac{x - x^{n+1}}{1 - x}$$

$\sum$

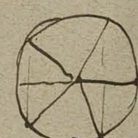
$$\left(e^{\frac{2i\pi}{m+1}}\right)^2 + \left(e^{\frac{2i\pi}{m+1}}\right)^2 + \left(e^{\frac{3i\pi}{m+1}}\right)^2 + \cdots \left(e^{\frac{m i\pi}{m+1}}\right)^2 = \frac{e^{\frac{2i\pi}{m+1}} - e^{(i\pi)^2}}{1 - e^{\frac{2i\pi}{m+1}}} = \text{[illegible]} \quad 1$$



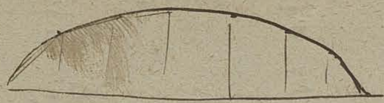
$$y_r = \sum_{s=1}^{m} q_s \sin\frac{(r-1)s\pi}{m+1}$$

$$y_{r+1} - y_r = \sum_{s=1}^{n} q_s \cos\frac{(2r-1)s\pi}{2(m+1)} \sin\frac{s\pi}{2(m+1)}$$

$$V = \frac{T}{2a}\left\{\sum_{s=1}^{m} y^2 - 2(y_1 y_2 + y_2 y_3 + \cdots)\right\}$$

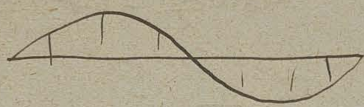


$s=1$



$\sin\frac{(m-1)\pi}{n+1}$

$s=2$



$\sin\frac{n-1}{n+1}2\pi$

$$y_x = \sum_{s=1}^{\infty} A_s \sin s\frac{\pi x}{l} \cos n_s t$$

$$n_s = \frac{s\pi}{l}\sqrt{\frac{T}{\rho}}$$

$$\dot{y}_x = \sum A_s n_s \sin\frac{s\pi x}{l} \sin n_s t$$

$$\int \sum\left[A_s n_s \sin\frac{s\pi x}{l}\right]^2 =$$

$$Y_1 = A_1 \sin\frac{\pi}{n+1} + A_2 \sin\frac{2\pi}{n+1} + \ldots$$

$$Y_2 = \ldots$$

$$A_s = \frac{2}{n+1}\sum_{r=1}^{m} y_r \sin\left(r\frac{s\pi}{n+1}\right)$$

$a r = x$

$a = dx$

$$A_s = \frac{2}{l}\int_0^l y_x \sin\left(\frac{s\pi x}{l}\right) dx$$

$$\left(x+\frac{1}{x}\right)^8 = \binom{8}{0}x^8 + \binom{8}{1}x^6 + \binom{8}{2}x^4 + \binom{8}{3}x^2 + \binom{8}{4}x^0 + \binom{8}{3}\frac{1}{x^2} + \binom{8}{2}\frac{1}{x^4} + \binom{8}{1}\frac{1}{x^6} + \binom{8}{0}\frac{1}{x^8}$$

$$\left(x+\frac{1}{x}\right)^8\left(x-\frac{1}{x}\right) = \binom{8}{0}x^9 + \left[\binom{8}{1}-\binom{8}{0}\right]x^7 + \left[\binom{8}{2}-\binom{8}{1}\right]x^5 + -$$

$$\frac{1}{x}\left(x+\frac{1}{x}\right)^9 = \binom{9}{0}x^8 + \binom{9}{1}x^6 + \binom{9}{2}x^4 + \binom{9}{3}x^2 + \binom{9}{4}x^0 + \binom{9}{4}\frac{1}{x^2} + \binom{9}{3}\frac{1}{x^4} + \binom{9}{2}\frac{1}{x^6} + \binom{9}{1}\frac{1}{x^8} + \binom{9}{0}\frac{1}{x^{10}}$$

$$\frac{d}{dx}\left[\frac{1}{x}\left(x+\frac{1}{x}\right)^n\right] = 8\binom{9}{0}x^7 + \dots \qquad -10$$

$n^2 - (n-1)^2 = 2n - 1$

II

$$1^2(S_1 - S_2) + 2^2(S_2 - S_3) + 3^2(S_3 - S_4) + \cdots\ 8^2(S_8 - S_9) + 9^2 S_9$$

$$= S_1 + (2^2 - 1^2) S_2 + (3^2 - 2^2) S_3 + \cdots\cdots + (9^2 - 8^2) S_9$$

$$= 2S_1 + 2.2.\ S_2 + 3.2\ S_3 + \cdots + 9.2.\ S_9$$

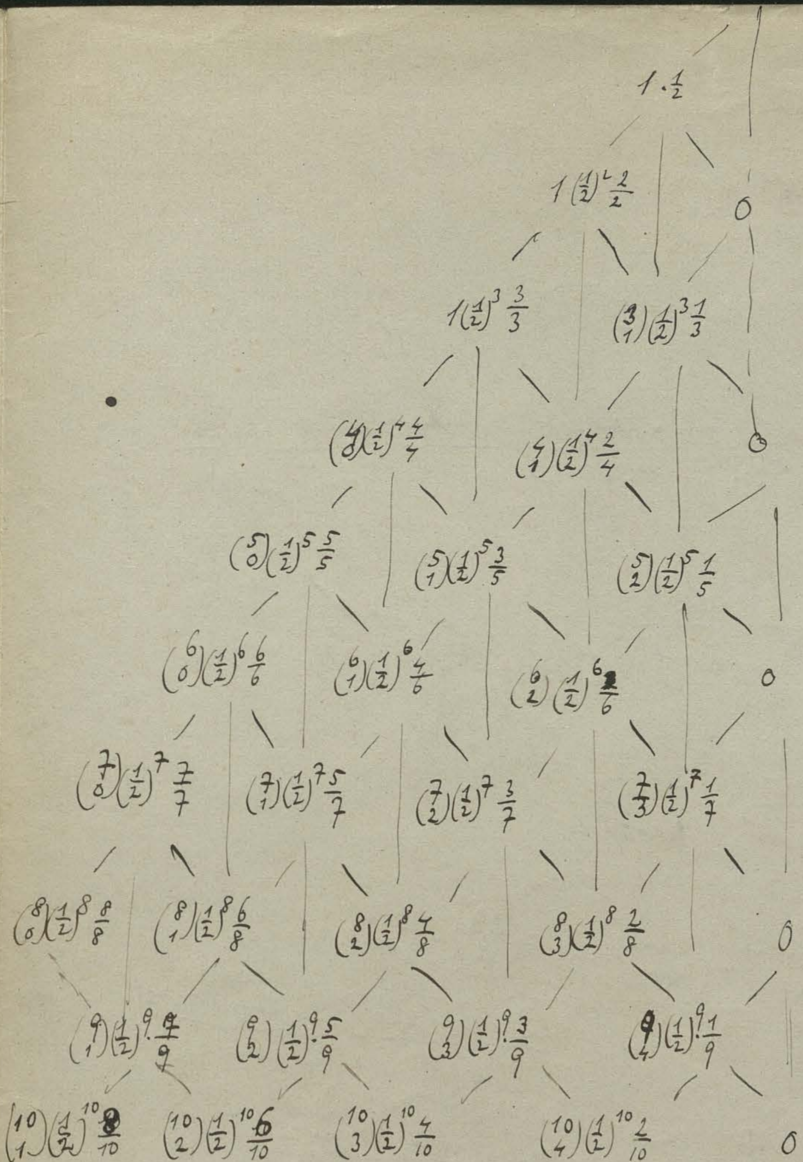
$$-S_1 \quad - S_2 \quad - S_3 \quad\quad - S_9$$

$$\overline{E_m^2} = 2\Big[\underbrace{1.S_1 + 2S_2 + 3S_3 + \cdots\ 9 S_9}_{\frac{1}{2}\cdot 9}\Big] - \Big[S_1 + S_2 + S_3 \cdots S_9\Big]$$

$$\boxed{\overline{E_m^2} = m - \overline{E_m}}$$

$\binom{c}{2}$

$\frac{1}{1}\ \frac{2}{1} \cdot 1$



2

3

4

$$\frac{1}{2^n}\left\{\sin n\varphi + \frac{n-2}{n}\binom{n}{1}\sin(n-2)\varphi + \frac{n-4}{n}\binom{n}{2}\sin(n-4)\varphi + \cdots\cdots \quad \text{u.}\ \frac{\sin\varphi}{\sin\varphi}\right\} = \frac{1}{2}\sin\varphi\cos^{n-1}\varphi$$

$$\sum_{n=1}^{m}\sum_{\mu=n}^{m} a_{n\mu}\sin n\varphi = \frac{1}{2}\sin\varphi\sum_{n=1}^{m}\cos^{n-1}\varphi$$

$$1+\cos\varphi+\cos^2\varphi+\cdots\cos^{n-1}\varphi = \frac{1-\cos^m\varphi}{1-\cos\varphi}$$

$$\int_0^{\frac{\pi}{2}} \sin^{2a} x \cos 2bx \, dx = 0 \qquad a < b$$

$$\int_0^{\frac{\pi}{2}} \sin^{2a} x \cos 2bx \, dx = \frac{(-1)^b}{2^{2a+1}} \pi \binom{2a}{a-b} \qquad a > b$$

für $b=0$:

$$\int_0^{\frac{\pi}{2}} \sin^{2a} x \, dx = \frac{\pi}{2^{2a+1}} \binom{2a}{a} = \frac{\pi}{2^{2a+1}} \frac{2a(2a-1)(2a-2)\cdots(a+1)}{1.\,2.\,3.\,\cdots a} = \frac{\pi}{2^{2a+1}} \frac{2a!}{(a!)^2}$$

$a = k-1$

$$\sum A = \sum_{k=1}^{k=m} \binom{m}{k} (-1)^{k+1} \frac{2^{k-1}}{2^{2k+1}} \cdot \frac{(2k-2)!}{((k-1)!)^2}$$

$$+ \sum \binom{m}{k} (-1)^{k-1} \frac{2^{k-1}(-1)}{2^{2k+1}} \binom{2k-2}{k-2}$$

$$= \sum \binom{m}{k} (-1)^{k-1} \frac{1}{2^{k+2}} \left[\binom{2k-2}{k-1} - \binom{2k-2}{k-2} \right]$$

$$= \sum_{k=1}^{k=m} (-1)^{k-1} \binom{m}{k} \frac{1}{2^{k+2}} \binom{2k-2}{k-1} \frac{1}{k}$$

$$\binom{2a}{a} - \binom{2a}{a-1} =$$

$$\frac{2a(2a-1)\cdots(a+1)}{a!} - \frac{2a(2a-1)\cdots(a+2)}{a-1!}$$

$$= \frac{2a(2a-1)\cdots[(a+1)-a]}{a!}$$

$$= \frac{2a(2a-1)\cdots(a+2)}{a!} = \binom{2a}{a-1} \frac{1}{a}$$

$$= \binom{2a}{a} \frac{1}{a+1}$$

$$\left(x+\frac{1}{x}\right)^n = x^n + \binom{n}{1}x^{n-2} + \binom{n}{2}x^{n-4} + \cdots + \binom{n}{n-1}\frac{1}{x^{n-2}} + \binom{n}{n}\frac{1}{x^n}$$

$$x\frac{\partial}{\partial x} \quad = n x^n + (n-2)\binom{n}{1}x^{n-2} + (n-4)\binom{n}{2}x^{n-4} + \cdots - (n-2)\binom{n}{n-1}\frac{1}{x^{n-2}} - n\binom{n}{n}\frac{1}{x^n}$$

$$= n\left(x+\frac{1}{x}\right)^{n-1}\left(x-\frac{1}{x}\right)$$

$x = e^{i\varphi}$

$$n(2\cos\varphi)^{n-1}(2i\sin\varphi) = n(2i\sin n\varphi) + (n-2)\binom{n}{1}2i\sin(n-2)\varphi + \cdots$$

$$2^{n-1} n \cos^{n-1}\varphi \sin\varphi = n\sin n\varphi + (n-2)\binom{n}{1}\sin(n-2)\varphi + (n-4)\binom{n}{2}\sin(n-4)\varphi + \cdots$$

$$\Big|\frac{1}{n\,2^n}$$

$$\frac{1}{2}\sin\varphi\cos^{n-1}\varphi = \frac{1}{2^n}\left[\sin n\varphi + \frac{n-2}{n}\binom{n}{1}\sin(n-2)\varphi + \frac{n-4}{n}\binom{n}{2}\sin(n-4)\varphi + \cdots\right]$$

~~$\left(x-\frac{1}{x}\right)^n = x^n - \binom{n}{1}x^{n-2} + \binom{n}{2}x^{n-4} + \cdots + \binom{n}{\frac{n}{2}}x^0 + \cdots - \binom{n}{n-1}\frac{1}{x^{n-2}} + \binom{n}{n}\frac{1}{x^n}$~~ (n grande)

~~$n = 2k+1$~~

~~$= x^n - \binom{n}{1}x^{n-2} + \binom{n}{2}x^{n-4} + \binom{n}{\frac{n}{2}-1}x - \binom{n}{\frac{n}{2}+1}\frac{1}{x} + \binom{n}{n-1}\frac{1}{x^{n-2}} - \binom{n}{n}\frac{1}{x^n}$~~

~~$n\left(x-\frac{1}{x}\right)^{n-1}\left(x+\frac{1}{x}\right) = n x^n - (n-2)\binom{n}{1}x^{n-2} + (n-4)\binom{n}{2}x^{n-4} - \cdots + (n-2)\binom{n}{n-1}\frac{1}{x^{n-2}} - n\binom{n}{n}\frac{1}{x^n}$~~

~~$= n x^n - \cdots - (n-2)\binom{n}{n-1}\frac{1}{x^{n-2}} + n\binom{n}{n}\frac{1}{x^n}$~~

~~$x = e^{i\varphi}$~~

$$= \sum_{n\ \text{const}} a_n \sin n\varphi \qquad A_m \sin m\varphi + A_{m-1}\sin(m-1)\varphi + \cdots$$

$$\frac{1}{2}\sin\varphi\sum_{n=1}^{m}\cos^{n-1}\varphi = \sum\sum a_n \sin n\varphi =$$

$$A_k = \frac{2}{\pi}\int_{0}^{+\pi}\left[\frac{1}{2}\sin\varphi\sum_{n=1}^{m}\cos^{n-1}\varphi\right]\sin k\varphi\, d\varphi$$

$$\sum_{k=1}^{m} A_k = \frac{2}{\pi}\int_{0}^{+\pi}\frac{1}{2}\sin\varphi\sum_{n=1}^{m}\cos^{n-1}\varphi\sum_{k=1}^{m}\sin k\varphi\, d\varphi$$

174

$$\frac{\cos\varphi}{\sin^2\varphi}\left[\cos\varphi - \cos^n 2\varphi \cdot \cos\varphi - \cos(2m+1)\varphi - \cos(2m+1)\varphi \cos^n 2\varphi\right] d\varphi$$

~~1 + cos 2φ / 2~~ 2

10	9	8	7	6	5	4	3	2	1	0
		1	1	1	1	1	1	1	1	
				1		6		15		20
			1		7		21		35	
		1		8		28		56		70
	1		9		36		84		126	
1		10		45		120		210		252

$\frac{1}{n 2^n}$ Σ

$6 \cdot \frac{1}{2^6}$ || $\frac{6 \cdot 6}{6 \cdot 2^6}$ || $\frac{15 \cdot 2}{6 \cdot 2^6}$

$\frac{1}{64}$ $\frac{1}{16}$ $\frac{5}{64}$

$$n + (n-2)\cdot\frac{n(n-1)}{1.2} + (n-4)\frac{n(n-1)(n-2)(n-3)}{1.2.3.4} + (n-6)\frac{n(n-1)(n-2)(n-3)(n-4)(n-5)}{1.2.3.4.5.6}$$

$$+ (n-4)\frac{n(n-1)\cdots(n-7)}{8!} + (n-2)\frac{n\cdots(n-10)}{10!} + n$$

$$n + (n-2)\frac{n}{1} + (n-4)\frac{n(n-1)}{1.2} + (n-6)\frac{n(n-1)(n-2)}{1.2.3} + (n-4)\frac{n(n-1)(n-2)(n-3)}{1.2.3.4} + (n-2)\frac{n\cdots(n-4)}{1\cdots5} + n\frac{n\cdots(n-5)}{1\cdots6}$$

$$\left(x+\frac{1}{x}\right)^n = x^n + \binom{n}{1}x^{n-2} + \binom{n}{2}x^{n-4} + \cdots \binom{n}{\frac{n}{2}}x^0 + \cdots + \frac{1}{x^n}$$

$$\sum_1^m \frac{n}{2^n}\left[\frac{1}{n} + \binom{n+2}{1}\frac{1}{n+2}\frac{1}{2^2} + \binom{n+4}{2}\frac{1}{n+4}\frac{1}{2^4} + \cdots \binom{m}{m-\cdot}\frac{1}{m}\frac{1}{2^m}\right]$$

$$\frac{1}{n} + \frac{n+2}{1}\frac{1}{n+2}\frac{1}{2^2} + \frac{(n+4)(n+3)}{1.2}\frac{1}{n+4}\frac{1}{2^4} + \frac{(n+6)(n+5)(n+4)}{1.2.3}\frac{1}{n+6}\frac{1}{2^6} + \cdots$$

$$A_n = \sum_{\mu=n}^{m} a_{n\mu}$$

$$\sin\alpha \cos\beta = \frac{\sin(\alpha+\beta) + \sin(\alpha-\beta)}{2}$$

$$\sum_{n=1}^{m} A_n \sin n\varphi = \frac{1}{2}\sin\varphi \frac{1-\cos^m\varphi}{1-\cos\varphi}$$

$$\sin m\psi \cos\psi = \frac{\sin(m+1)\psi}{2} + \frac{\sin(m-1)\psi}{2}$$

$$A_n = \frac{2}{\pi}\int_0^{\pi} \frac{1}{2}\sin\varphi \sin n\varphi \frac{[1-\cos^m\varphi]}{1-\cos\varphi}\, d\varphi$$

~~$\sin\alpha \sin\beta$~~

$$\sin\alpha \sin\beta = \frac{\cos(\alpha-\beta) - \cos(\alpha+\beta)}{2}$$

$$\sum_{n=1}^{m} \sin n\varphi = \sin\varphi + \sin\varphi + \cdots \sin m\varphi = \frac{\sin\frac{m\varphi}{2}\sin\frac{m+1}{2}\varphi}{\sin\frac{\varphi}{2}}$$

$$\sum_{n=1}^{m} A_n = \frac{1}{\pi}\int_0^{\pi} \frac{\sin\varphi \sin\frac{m\varphi}{2}\sin\frac{(m+1)\varphi}{2}\,[1-\cos^m\varphi]}{\sin\frac{\varphi}{2}\,[1-\cos\varphi]}\, d\varphi =$$

$$= \frac{1}{\pi}\int_0^{\pi} \frac{\cos\frac{\varphi}{2}\sin\frac{m\varphi}{2}\sin\frac{(m+1)\varphi}{2}}{\sin^2\frac{\varphi}{2}}\,[1-\cos^m\varphi]\, d\varphi$$

$$\frac{\varphi}{2} = \psi \qquad d\varphi = 2d\psi$$

$$= \frac{2}{\pi}\int_0^{\frac{\pi}{2}} \frac{\cos\psi \sin m\psi \sin(m+1)\psi}{\sin^2\psi}\,[1-\cos^m 2\psi]\, d\psi$$

$$= \frac{2}{\pi}\int_0^{\frac{\pi}{2}} \left[\frac{\sin^2(m+1)\psi}{2\sin^2\psi} + \frac{\sin(m+1)\psi \sin(m-1)\psi}{2\sin^2\psi}\right][1-\cos^m 2\psi]\, d\psi$$

$$= \frac{1}{\pi}\int_0^{\frac{\pi}{2}} \underbrace{\frac{1-\cos^m 2\psi}{\sin^2\psi}}\left\{\frac{1-\cos 2(m+1)\psi}{2} + \frac{\cos 2\psi - \cos 2m\psi}{2}\right\} d\psi$$

$$\frac{1-(1-2\sin^2\psi)^m}{\sin^2\psi} = \left\{\binom{m}{1}\cdot 2 - \binom{m}{2}\cdot 2^2\sin^2\psi + \binom{m}{3}\cdot 2^3\sin^4\psi - \binom{m}{4}\cdot 2^4\sin^6\psi + \cdots\right\}$$

$$= \frac{1}{\pi}\int_0^{\frac{\pi}{2}} \left\{\binom{m}{1} - \binom{m}{2}2\sin^2\psi + \binom{m}{3}2^2\sin^4\psi + \binom{m}{4}2^3\sin^6\psi + \cdots + (-1)^{m}\binom{m}{m}2^{m-1}\sin^{2m-2}\psi\right\}\left\{1+\cos 2\psi - \cos 2m\psi - \cos 2(m+1)\psi\right\} d\psi$$

~~$x^k + x^{k-1} + x^{k-2} + \cdots + \frac{1}{x^{k-1}} + \frac{1}{x^k}$~~ $= \sum_1^k \sin k\varphi$

~~$= \frac{1+x+x^2+\cdots+x^{2k}}{x^k} = \frac{1+x^{2k+1}}{(1-x)x^k}$~~

$\sin\alpha \sin\beta = \frac{\cos(\alpha-\beta) - \cos(\alpha+\beta)}{2}$

$$\sum_1^m \sin k\varphi = \frac{\sin\frac{m\varphi}{2}\sin\frac{m+1}{2}\varphi}{\sin\frac{\varphi}{2}} = \frac{\cos\frac{\varphi}{2} - \cos\left(m\varphi + \frac{\varphi}{2}\right)}{\sin\frac{\varphi}{2}}$$

$$\sum_1^m \cos^{n-1}\varphi = \frac{1-\cos^n\varphi}{1-\cos\varphi}$$

$$\sum_1 A_k = \frac{1}{\pi}\int_0^\pi 2\sin\frac{\varphi}{2}\,\frac{\sin\frac{m\varphi}{2}\sin\frac{m+1}{2}\varphi}{\sin\frac{\varphi}{2}}\,\frac{(1-\cos^n\varphi)}{2\sin^2\frac{\varphi}{2}}\cos\frac{\varphi}{2}\,d\varphi$$

$$\leq \frac{2}{\pi}\int_0^{\frac{\pi}{2}} \frac{\sin m\varphi \sin(m+1)\varphi\,[1-\cos^n 2\varphi]\cos\varphi}{\sin^2\varphi}\,d\varphi$$

$$= \frac{1}{\pi}\int \frac{[-\cos(2m+1)\varphi + \cos\varphi][1-\cos^n 2\varphi]\cos\varphi}{\sin^2\varphi}\,d\varphi$$

$$\int \sin\varphi\,\frac{[1-\cos^n\varphi]}{1-\cos\varphi}\,\frac{\cos\frac{\varphi}{2} - \cos\left(m\varphi+\frac{\varphi}{2}\right)}{2\sin\frac{\varphi}{2}}$$

$$\int \frac{1-\cos^n\varphi}{1-\cos\varphi}\left[\cos^2\frac{\varphi}{2} - \cos\frac{\varphi}{2}\cos\left(m\varphi+\frac{\varphi}{2}\right)\right]d\varphi$$

$$\left[\frac{1+\cos\varphi}{2} - \frac{\cos(m+1)\varphi + \cos m\varphi}{2}\right]d\varphi$$

$= \frac{1}{2}\int$

$$\frac{1}{1.2.3}\left\{ \frac{\cancel{7}.6.5}{2^7}.1 + \frac{\cancel{8}.7.6}{2^8}.2 + \frac{\cancel{9}.8.7}{2^9}.3 + \frac{\cancel{10}.9.8}{2^{10}}.4 \right.$$

$$\frac{1}{1.2.3.4}\left\{ \frac{\cancel{9}.8.7.6}{2^9}.1 + \frac{\cancel{10}.9.8.7}{2^{10}}.2 + \frac{\cancel{11}.10.9.8}{2^{11}}.3 + \frac{\cancel{12}.11.10.9}{2^{12}}.4 + 1 \right.$$

$$\frac{d}{dy}\frac{d}{dx}\frac{d}{dx}\frac{d}{dx}\left\{ \left(\frac{x}{2}\right)^8 y + \left(\frac{x}{2}\right)^9 y^2 + \left(\frac{x}{2}\right)^{10} y^3 + \left(\frac{x}{2}\right)^{11} y^4 \right.$$

$$\int \frac{\cos px}{\cos^2 x}\,dx = 2^{p-1}\pi \qquad \int \frac{\cos 2ax \cos px}{\cos^p x}\,dx = (-1)^a\, 2^{p-2}\pi$$

P – As – Sb – x – Bi
31 75 120 165 210

} Jedes Glied ist genau des arithm. Mittel aus den daneben steh.

Ähnlich auch bei

Mg Zn Cd x Hg
24 65 112 156 200

x – Cu – Ag – x – Au
18? 63 108 153 197

x Al? – Ge – Sn – x – Pb
27 72 117 162 206

x – Cr – Mo – x – W
8 52 96 140 184

S – Se T
32 79 126

Ca – Sr – Ba
40 87 137

Cl Br J
35'4 79'7 126'5

Sc Y? La? – Yb?
44 89 138 173?

Al?? Ga In ~~x~~ Th
27 70 113 158 204 ??

? ?

C? Co Rh x Ir
12 58 104 149 193

?

Li – Na – K – Rb – Cs
7 23 39 85 133

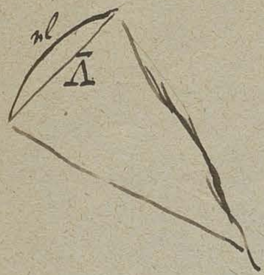
Schmelztemp.
180° 95'6 62'5 38'5 26'5
Rothgl. 720 270 Siedep.

x V Nb x Ta
8 – 51 94 138 182

Wenn das als Kreisbogen aufgefasst wird.

178

Formel (11):



$$\frac{\bar{\Lambda}}{nl} = 1 - \frac{n\delta}{6} = \frac{2a \sin \frac{\varphi}{2}}{a\varphi} = 1 - \frac{(\frac{\varphi}{2})^2}{2 \cdot 3}$$

$$\left(\frac{\varphi}{2}\right)^2 = n\delta$$

$$\varphi = 2\delta\sqrt{n}$$

Vergl. Rieger Pr. R. S. 83 p. 499

Rayleigh Theory of Sound 2 ed. 1894 p. 39

$$\Lambda = c\sqrt{\frac{2M}{S}}$$

$$M\frac{d^2x}{dt^2} = -S\frac{dx}{dt}$$

$$V = Ce^{-\frac{t}{M/S}}$$

~~$S = \frac{2\pi}{3}R^2$~~ mNc

$$= c\sqrt{\frac{2M}{\frac{2}{3}mn}} = c\sqrt{\frac{2m}{\frac{2mn}{3}}} = c\sqrt{\frac{3}{n}}$$

$$= NR^2\pi c \cdot \frac{2m}{3M}$$

$$S = \frac{2\pi}{3}R^2\rho c = \frac{2\pi}{3}R^2 N m c$$

$$S = 6\pi R\mu$$

$$S = \frac{2}{3}\frac{\pi m c R^2 N}{M}$$

[illegible]

$$\frac{8}{3}4R^2 N\frac{\sqrt{\pi M m}}{m+M} \neq \frac{32}{3}R^2 N\sqrt{\frac{\pi m}{M}}$$

[illegible]

I.: Red. [illegible]

[illegible] RV [illegible] v $\frac{c}{n} = \lambda$

~~$\sigma = a \sin ct - x$~~

$$\sigma = a \sin 2\pi n \left(t - \frac{x}{c}\right) = a \sin 2\pi \left(n t - \frac{x}{\lambda}\right)$$ } [illegible]

$$\sigma' = a \sin 2\pi n \frac{c}{c-v}\left(t - \ldots\right) = a \sin 2\pi n \left(\frac{c t}{c-v} - \frac{x}{c}\right)$$

[illegible] en [illegible] c? $\lambda' = \frac{c-v}{n}$

II.: Red. [illegible]

$$\sigma' = a \sin 2\pi n \left(\frac{c t}{c-v} - \frac{x}{c-v}\right)$$

III.: [illegible]; Red. [illegible]

$$\sigma' = a \sin 2\pi n \left(t - \frac{x}{c-v}\right)$$

fest	bewegl.	[illegible]	[illegible]
M A E	E MA	+	
M E A	A ME	+	+
E A M	M EA		+

$$\frac{b\rho r_1^2\pi + h\sigma r_1^2\pi + Bt\sigma r_1^2\pi + A\sigma v_0}{A\sigma r_1^2\pi} = M$$

$$\frac{[760\rho + h\sigma] v_0 \frac{T}{T_0} - v_0 [b\rho + h\sigma + Bt\sigma]}{A\sigma r_1^2\pi} = N$$

$$x_1^2 + Mx_1 = N$$

$$x_1 = -\frac{M}{2} \pm \sqrt{\frac{M^2}{4} + N}$$

$$\frac{\partial x_1}{\partial t} = -\frac{1}{2}\frac{\partial M}{\partial t} \pm \frac{1}{2}\frac{1}{\sqrt{\frac{M^2}{4}+N}}\left[\frac{M}{2}\frac{\partial M}{\partial t} + \frac{\partial N}{\partial t}\right]$$

$$= 0$$

$$\frac{\partial M}{\partial t} \mp \frac{\frac{M}{2}\frac{\partial M}{\partial t} + \frac{\partial N}{\partial t}}{\sqrt{\frac{M^2}{4}+N}} = 0$$

$$\sqrt{\frac{M^2}{4}+N} = \pm\left[\frac{M}{2} + \overbrace{\frac{\frac{\partial N}{\partial t}}{\frac{\partial M}{\partial t}}}^{q}\right]$$

$$\frac{M^2}{4} + N = \frac{M^2}{4} + Mq + q^2$$

$$q = \frac{\frac{[760\rho + h\sigma] v_0}{T_0} - v_0 \frac{h\alpha}{r_2^2\pi}\sigma}{\frac{h\alpha}{r_2^2\pi}\frac{r_1^2+r_2^2}{r_2^2}} =$$

$$q = v_0(r_1^2+r_2^2)\left[\frac{760\rho + h\sigma}{T_0 h\alpha}\pi - \frac{\sigma}{r_2^2}\right]$$

$$\frac{[\overbrace{760\rho + h\sigma}^{c}]\frac{T}{T_0} - [b\rho + h\sigma + Bt\sigma]}{} =$$

$$= \frac{b\rho r_1^2\pi + h\sigma r_1^2\pi + Bt\sigma r_1^2\pi + A\sigma v_0}{} (r_1^2 + r_2^2)\left[\frac{760\rho + h\sigma}{T_0 h\alpha}\pi - \frac{\sigma}{r_2^2}\right]$$

$$+ (\quad q \quad)^2 \frac{A\sigma r_1^2\pi}{v_0}$$

A)

$$c\frac{t}{T_0} - Bt\sigma + Bt\sigma r_1^2\pi\,(r_1^2+r_2^2)\left[\frac{c\pi}{T_0 h\alpha}x - \frac{\sigma}{r_2^2}\right]$$

$$= t\left[\frac{c}{T_0} - \frac{h\alpha\sigma}{r_2^2\pi} + \frac{h\alpha\sigma r_1^2}{r_2^2}(r_1^2+r_2^2)\left(\frac{c\pi}{T_0 h\alpha} - \frac{\sigma}{r_2^2}\right)\right]$$

$$= t\left[\frac{c}{T_0} - \frac{h\alpha\sigma}{r_2^2\pi}\right]\left[1 + \frac{h\alpha\sigma r_1^2}{r_2^2}(r_1^2+r_2^2)\frac{\pi}{h\alpha}\right]$$

I Lösung: $\frac{c}{T_0} - \frac{h\alpha\sigma}{r_2^2\pi} = 0 \qquad q = 0$

II " : $\frac{\sigma r_1^2}{r_2^2}(r_1^2+r_2^2)\pi + 1 = 0$

B) $-b\rho = b\rho r_1^2\pi\,(r_1^2+r_2^2)\left[\frac{c\pi}{T_0 h\alpha} - \frac{\sigma}{r_2^2}\right]$

$$b\rho\left[r_1^2\pi(r_1^2+r_2^2)\left(\frac{c\pi}{T_0 h\alpha} - \frac{\sigma}{r_2^2}\right) + 1\right] = 0$$

$N=0$

$x_1 = -M$

$N=0$:

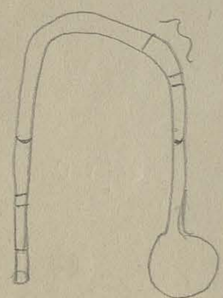
$c\frac{T}{T_0} - b\rho - h\sigma - \mathfrak{B}t\sigma = 0$

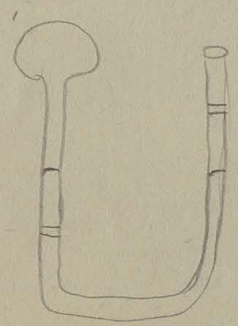
$\mathfrak{B} = \frac{M\alpha}{r_1^2\pi}$

~~$\left(\frac{c}{T_0} - \frac{M\alpha\sigma}{r_1^2\pi}\right)t + \frac{273c}{T_0} - b\rho - h\sigma = 0$~~

$$x_1 = -\frac{b\rho}{A\sigma} + \frac{h + \mathfrak{B}t}{A} + \frac{v_0}{r_1^2\pi}$$

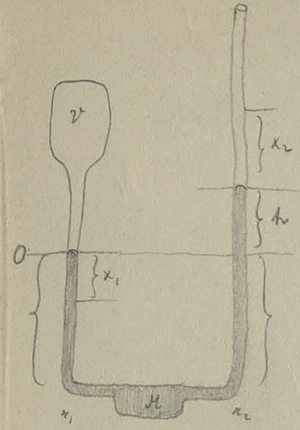
$$= -b\frac{r_2^2}{r_1^2+r_2^2}\frac{\rho}{\sigma} + \underbrace{\frac{h r_2^2}{r_1^2+r_2^2} + \frac{v_0}{r_1^2\pi}}_{?} + \frac{M\alpha}{(r_1^2+r_2^2)\pi}t$$





$A_g - A_q$

$A_g + A_q$



$$pv = RT$$

$$p_0 v_0 = RT_0$$

$$pv = p_0 v_0 \frac{T}{T_0}$$

$$p_0 = 760 + h\frac{\sigma}{\sigma_q}$$

$$p = 760 - \underbrace{\beta}_{b + [h + x_1 + x_2]\frac{\sigma}{\sigma_q}}$$

$$= b + \left[h + x_1 + x_1\left(\frac{r_1}{r_2}\right)^2 + \frac{M\alpha t}{r_2^2\pi}\right]\frac{\sigma}{\sigma_q}$$

$$M - x_1 r_1^2\pi + x_2 r_2^2\pi = M(1+\alpha t)$$

$$x_2 = \frac{M\alpha t + x_1 r_1^2\pi}{r_2^2\pi}$$

$$v = v_0 + x_1 r_1^2\pi$$

$$\left\{b + \left[h + x_1\frac{r_1^2+r_2^2}{r_2^2} + \frac{M\alpha t}{r_2^2\pi}\right]\frac{\sigma}{\sigma_q}\right\}\left\{v_0 + x_1 r_1^2\pi\right\} =$$

$$= \left[760 + h\frac{\sigma}{\sigma_q}\right] v_0 \frac{273+t}{273+t_0}$$

$\sigma_q = \rho$

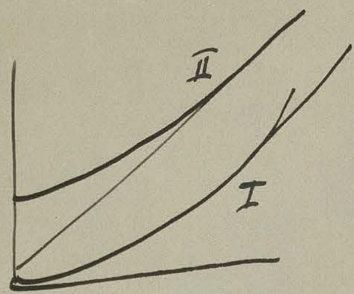
$x_1 = ?$ $\quad \frac{r_1^2+r_2^2}{r_2^2} = A \quad \frac{M\alpha}{r_2^2\pi} = \mathfrak{B}$

$$\left[b\sigma_q + h\sigma + A\sigma x_1 + \mathfrak{B}t\sigma\right]\left[v_0 + x_1 r_1^2\pi\right] = \left[760\sigma_q + h\sigma\right]v_0\frac{T}{T_0}$$

$$A\sigma x_1^2 r_1^2\pi + x_1\left[b\rho r_1^2\pi + h\sigma r_1^2\pi + \mathfrak{B}t\sigma r_1^2\pi + A\sigma v_0\right] =$$

$$= \left[760\rho + h\sigma\right]v_0\frac{T}{T_0} - v_0\left[b\rho + h\sigma + \mathfrak{B}t\sigma\right]$$

~~$\frac{h\nu}{\frac{h\nu}{kT} + \left(\frac{h\nu}{kT}\right)^2} = kT\left(1 + \frac{h\nu}{2kT}\right)$~~



I). $E = \dfrac{h\nu}{e^{\frac{h\nu}{kT}} - 1}$

II). $E = \uparrow + \dfrac{h\nu}{2}$

I). $\lim_{T=\infty} E = kT - \dfrac{h\nu}{2}$

II). $= kT$

$\sigma = \dfrac{C}{\lambda^5} \dfrac{1}{e^{\frac{hc}{k\lambda\theta}} - 1}$

Rayleigh $\dfrac{C k \theta}{\lambda^4}$

181e

[illegible] 3 dimension [illegible] 2 [illegible]

$$\int_0^y e^{-\beta y^2}dy\left[\int_0^y e^{-\alpha x^2}dx\right]^{n-1} = \left[\int_0^y e^{-\alpha x^2}dx\right]^{n-1}\int_0^y e^{-\beta y^2}dy - \int dy\,(n-1)\,e^{-\alpha y^2}\left[\int_0^y e^{-\alpha x^2}dx\right]^{n-2}\cdot\int_0^y e^{-\beta y^2}dy$$

$$= \int_0^y e^{-\alpha y^2}\cdot e^{-(\beta-\alpha)y^2}dy\,/\ldots = \frac{1}{n}\left[\frac{1}{2}\sqrt{\frac{\pi}{\alpha}}\right]^n\cdot e^{-(\beta-\alpha)y^2}\Big|_0^\infty + \int 2(\beta-\alpha)y\,e^{-(\beta-\alpha)y^2}\left[\int_0^y\quad\right]^n dy$$

$$2\nu P + 2\nu^2 P + \nu^2 P^2(2-1+2+2-1)$$

$$+\nu^2 P^3\left(\frac{1}{2}+\frac{1}{2}\right.$$

$$m^2\binom{n}{m} = \frac{n(n-1)(n-2)\cdots(n-m+1)}{m!}m^2 =$$

$$(n-m)^2\cdots \qquad (m)^2$$

$n-m+1\ldots$

$2n-m$

$-m$

$n+i=-m$

$i=n+m$

183

$$(1)^2\left[\frac{b_1}{1!}\frac{b}{0!} + \frac{b^2}{2!}\frac{b^1}{1!} + \frac{b^3}{3!}\frac{b^2}{2!} + \frac{b^4}{4!}\frac{b^3}{3!} + \frac{b^5}{5!}\frac{b^4}{4!}\right.$$

$$0\left[\frac{b^0}{0!} + \frac{b}{1!}\frac{b^1}{1!} + \frac{b^2}{2!}\frac{b^2}{2!} + \frac{b^3}{3!}\frac{b^3}{3!} + \right.$$

$$+(1)^2\left[\frac{b^1}{1!}\frac{b^0}{0!} + \frac{b^2}{2!}\frac{b^1}{1!} + \frac{b^3}{3!}\frac{b^2}{2!} + \frac{b^4}{4!}\frac{b^3}{3!}\right.$$

$$+(2)^2\left[\frac{b^2}{2!}\frac{b^0}{0!} + \frac{b^3}{3!}\frac{b^1}{1!} + \frac{b^4}{4!}\frac{b^2}{2!} + \cdots\right.$$

$$+(3)^2\left[\frac{b^3}{3!}\frac{b^0}{0!} + \frac{b^4}{4!}\frac{b^1}{1!} + \frac{b^5}{5!}\frac{b^2}{2!} + \cdots\right.$$

$$\sum_{m=1}^{\infty} m\frac{b^m}{m!} = be^b$$

$$= \sum_{m=0}^{\infty} \frac{b^m}{m!}\left[0\cdot\frac{b^m}{m!} + (1)^2\frac{b^{m+1}}{m+1!} + (2)^2\frac{b^{m+2}}{m+2!} + \right.$$

$$= \sum_{m=0}^{\infty} \frac{b^m}{m!}\left[m^2\frac{b^0}{0!} + (m-1)^2\frac{b^1}{1!} + (m-2)^2\frac{b^2}{2!} + (m-3)^2\frac{b^3}{3!} + \cdots + (2)^2\frac{b^{m-2}}{m-2!} + (1)^2\frac{b^{m-1}}{m-1!} + 0b^m\right]$$

$$+ (1)^2\frac{b^m}{m!} + (2)^2\frac{b^{m+1}}{m+1!} + \cdots$$

$$+ \sum_{m=1}^{\infty}\frac{b^m}{m!}\left[m^2\frac{b^0}{0!} + (m+1)\frac{b^1}{1!} + (m+2)^2\frac{b^2}{2!} + (m+3)^2\frac{b^3}{3!} + \cdots\right]$$

$$x^m e^{\alpha x} = x^m + \alpha\frac{x^{m+1}}{1!} + \alpha^2\frac{x^{k+2}}{2!} + \alpha^3\frac{x^{k+3}}{3!} + \cdots$$

$$x\frac{d}{dx}(x^m e^{\alpha x}) = m x^{m} + (m+1)\frac{\alpha x^{m+1}}{1!} + (m+2)\frac{\alpha^2 x^{m+2}}{2!} + \cdots$$

$$\frac{d}{dx}\left[x\frac{d}{dx}(x^m e^{\alpha x})\right]_{x=1} = m^2 + (m+1)^2\frac{\alpha}{1!} + (m+2)^2\frac{\alpha^2}{2!} + \cdots = \frac{d}{dx}\left[x.(mx^{m-1}e^{\alpha x} + \alpha x^m e^{\alpha x})\right]$$

$$= m^2 e^{\alpha} + \alpha(m+1)e^{\alpha}$$

$$e^b\left[b^2 - 2bm + b + m^2 + \Big\|\, + m^2 + b(m+1)\right]$$

$$= e^b\left[b^2 - bm + 2b + 2m^2\right] \Big\|\Big\| e^b\left[(b^2+2b)e^b - b.be^b + 2(b+b^2)e^b\right]$$

$$x_0^2\left(1+e^{-2\beta t}\right) + \xi^2\left(1-e^{-2\beta t}\right) - 2x_0^2 e^{-\beta t}$$

$$= x_0^2\left(1-e^{-\beta t}\right)^2 + \xi^2\left(1-e^{-2\beta t}\right)$$

185

1241	1562	18368	98677	02118	00945	73799	2356. 164
2803	0097	86082	86082	86082	86082	86082	
2900	1050	05450	84759	88201	87027	59881	
1850	1022	11337	07040	2900	1850	0828	354
0828	0547	1241	2803	07621	07418	07970	
0281		2375	2873	2138	1108	0431	25.

0901	1474	16850	69723	86629	01284	83059
2375	0498	86082	86082	86082	86082	86082
2873	0735	02932	55805	72711	87366	69141
2138	1030	07614		2873	2138	1108
1108	0677	1070	2375	05335	07476	04914
0431		0901		2340	1390	0617
		1971	2736			

171 34 15 — 22

3216 643 27 20 — 414

278 556 250 — 359

15. 129 645 — 1935

575 115. 52 — 742

32. 24 — 64 128 — 77

129.4 — 516

69 138

138 552 — 193

29.69 174 261 — 20

86 43 — 13

29. 32

21. 32 — 63 4

83.69 — 498 747 — 572

325 32 9 — 36

32 32 365 76 4 — 405

205 20 2 — 23

77 77 — 854

22 2

$P = \frac{2\cdot 25}{3\cdot 10} =$

$P = 0\cdot 7258$

$P^2 = 0\cdot 5268$

$Pv = 1\cdot 1250$

35218
49136
86082
19033
05115

$P^2(n-v)^2 - P(n-v) - P^2 n + 2Pn$

$\Delta_n^2 = $ ~~$P(n+v)[1+P(n-v)] - nP^2$~~

$= P^2(n-v)^2 - (n-v)P - nP^2 + 2nP$

$= \left[P(n-v) - \frac{1}{2}\right]^2 - \frac{1}{4} - n(P-1)^2 + n$

$= \left[P(n-v) - \frac{1}{2}\right]^2 + n\left[1-(P-1)^2\right] - \frac{1}{4}$

0·92481

05115
09691
14806

− 0·14062

0·2742
0·87614
0·07519

1·6250
21085
42170

+2·6406
− 25
− 2·3906
+

− 1·6250
+ 7258
0·8992

95386
90772
+0·80858
92481
1·7334
− 25
1·4834

− 1·6250
+ 1·4516
0·1734
23905
47810

0·03007
184962
1·8797
− 25
1·6297

27774
1·6250
0·5524
74225
48450

0·30514
2·77443
3·0796
− 25
2·8296

2·9032
1·6250
1·2782
10660
21320

1·6338
3·6992
5·3330
− 25
5·083

3·6290
1·6250
2·0040
40160
46240
8·640
− 25
8·390

0·43429 . 1125
4343
869
217
0·48858
0·51142
0·3247

51142
05115
56257
3652

51142
10230
61372
4109
2054

51142
15345
66487
4622
07703

51142
20460
71602
5200
1300
02167

51142
25575
76717
07918
68799
·004875

51142
86082
37224
86082
23306
09388

4875
914
3961 . 7258
95470
81552

59780
86082
45862
4875
− 002875
0020

51142
30690
81832
− 85733
96099
·0009141

$\frac{v^n e^{-v}}{n!}$ $v = 1.55$

186

	2.04118		
·19033	·23151	·1704	
38066	42184	2641	
57099	61217	4094	
76132	80250	6346	
		1058	
95165	99283	9837	
14198	18316	1525	128
33231	37349	2363	

109.9	108.7
170.4	168.4
132.1	130.6
68.2	67.4
26.45	26.15
8.20	8.105
1.81	1.79
0.35	

164
164
8
1812

43429
217145
21715
·67315
·32685 −1
2·71433
2·04118

$\frac{6}{518} = 1.16\%$

1099
12
108.7

1321
15
130.6

170.4
17.04
17
2
168.4

3247
260
23
35.3

3652
272
216
39.48

68.2
68
7

820
95

2645
26
[illegible]

2194
54
4
22.42

7609
761
8.37

218
11
7
2.36

488.109
4392
0.532

181

67.4
1348
265
8.35

1348
539
4
18.9

2356
1414
188
9
39.67

3541
2125
283
14
59.6

2492
1495
195
10
41.9

1684
168
33
3
18.88

1685
2211
295
15
62.06

168
8
16

1348
6066
19.5

674
539
34
12.47

5392
135
54
55.8

171
513
10
22.3

3216
965
19
42.0

278
834
16
36.3

1306
653
19.6

575
1725
34
7.51

1306
914
2.22

0.5

1.9

0·3652
0·3247
0·0405 · 7258
29032
363
0·029395
3247
0·3541

·17102

3652
20545
·15975 · 7258
16 · ·725
4350
− ·11600
+ 3652
0·2492

20545
07703
·12842
10864
86082
96946
−0·09321
+ 20545
0·11224

07703
2167
·05536
74320
86082
60402
−0·04018
+ 07703
0·03685

2167
487
·01680
22531
86082
08613
0·02167
−0·01219
+0·00948

2356
3541
2492
1122
03685
00948
200

1185
1049
1370
07535
02737
00748

07072
86082
93454

87708
86082
73790

·02078
86082
88160

43727
86082
29809

13672
86082
99754

0·08601
2356
·3216

1122
·0547
·0575

3541
0·07614
·2780

03685
·019865
·01698

2492
·099435
·1498

51142

87390
86082
73472

948
·00543
·00405

1710
3216
2780
1498
0575
0170

1506
0436
1282
0923
0405

17782
86082
03864
+ ·1093
1710
·2803

63949
86082
50031
3216
03165
2900

10789
86082
96871
2780
09305
1850

96520
86082
82602
1498
06[illegible]
0828

60746
86082
46828
0575
0294
0281

$(1+x)^{1/2} = 1+\frac{x}{2} - \frac{x^2}{8}$

187

$$P^2 + \frac{n+\nu}{(n-\nu)^2 - n}\, P = \frac{1}{(n-\nu)^2 - n}$$

$$P = -\frac{n+\nu}{2[(n-\nu)^2-n]} \pm \sqrt{\frac{1}{[(n-\nu)^2-n]} + \frac{(n+\nu)^2}{4[\quad]^2}}$$

$$= \frac{-(n+\nu) \pm \sqrt{4[(n-\nu)^2-n] + (n+\nu)^2}}{2[(n-\nu)^2-n]}$$

Falls $n \ll (n-\nu)^2$

$1+\delta \ll \nu\delta^2$

$1 \gg \delta^2 \gg \frac{1}{\nu}$

$$P = \frac{-(n+\nu) \pm \sqrt{4(n-\nu)^2 + (n+\nu)^2}}{2(n-\nu)^2} = \frac{-(2+\delta) + \sqrt{4\delta^2 + (2+\delta)^2}}{2\nu\delta^2}$$

$$= \frac{-(2+\delta) + \sqrt{4+4\delta+5\delta^2}}{2\nu\delta^2}$$

$$= \frac{-(2+\delta) + 2\left[1+\delta+\frac{5}{4}\delta^2\right]^{1/2}}{2\nu\delta^2}$$

$$= \frac{1+\frac{\delta}{2}+\frac{5}{8}\delta^2 - \frac{\delta^2}{8} - 1 - \frac{\delta}{2}}{\nu\delta^2} = \frac{1}{2\nu}$$

$$P = \frac{-(n+\nu) + (n+\nu)\left[1+\frac{4[(n-\nu)^2-n]}{(n+\nu)^2}\right]^{1/2}}{2[(n-\nu)^2-n]} = \frac{(n+\nu)\left\{\frac{2[(n-\nu)^2-n]}{(n+\nu)^2} - 2\frac{[(n-\nu)^2-n]^2}{(n+\nu)^4}\right\}}{2[(n-\nu)^2-n]}$$

$$= \frac{1}{n+\nu} - \frac{(n-\nu)^2-n}{(n+\nu)^3} \cdots$$

$$= \frac{1}{n+\nu}\left\{1 - \frac{(n-\nu)^2-n}{(n+\nu)^2} \cdots\right\}$$

Falls $\frac{(n-\nu)^2-n}{(n+\nu)^2} \ll 1$ immer erfüllt!

~~$\frac{\nu\delta^2-(1+\delta)}{(2+\delta)^2} \ll 1$~~ ~~$\nu\delta^2 - 1 - \delta \ll \nu\delta^2 + 4\nu + 4\nu\delta$~~

$n^2+\nu^2-2n\nu-n \ll n^2+\nu^2+2n\nu$

aber Entwicklung erfordert eigentlich $\ll \frac{1}{4}$

also

$\nu\delta^2 - 1 - \delta \ll \frac{\nu\delta^2}{4} + \nu + \nu\delta$

$\frac{3}{4}\nu\delta^2 - 1 - \delta \ll \nu + \nu\delta$

$-1-\delta \ll \nu + \nu\delta - \frac{3}{4}\nu\delta^2$

Jedenfalls gültig falls $\delta < 1$

188

$\frac{26{\cdot}15}{2{\cdot}35}$ $\quad$ $\frac{237\,.\,262}{474}$ 142 5 6·21 $\quad$ $\frac{287\,.\,262}{574}$ 172 6 7·52 $\quad$ $\frac{214\,.\,261}{428}$ 128 2 5·58 $\quad$ 2615 2615 21 2·8973 $\quad$ 1·1

0·5 $\quad$ 1·6 $\quad$ 2·2 $\quad$ 1·9 $\quad$ 1·1 $\quad$ 0·4

---

1) $\left[1+\frac{b^2}{1!}+\frac{b^4}{1\,2\,.\,(2.3)}+\frac{b^6}{3!\;2.3.4}+\cdots\right]\frac{b}{1}$

$+\left[1+\frac{b^2}{1!}+\right.$

1). $1\left\{\frac{b}{1!}+\frac{b^3}{1!\,2!}+\frac{b^5}{2!\,3!}+\frac{b^7}{3!\,4!}+\cdots\right\}+$

2). $+4\left\{\frac{b^2}{2!}+\frac{b^4}{1!\,3!}+\frac{b^6}{2!\,4!}+\frac{b^8}{3!\,5!}+\cdots\right\}+$

$+9\left\{\frac{b^3}{3!}+\frac{b^5}{1!\,4!}+\frac{b^7}{2!\,5!}+\right.$

$+16\left\{\frac{b^4}{4!}+\frac{b^6}{1!\,5!}+\cdots\right.$

$\frac{(b)^n}{n!}\left[1+\frac{b^2}{1!}+\frac{b^4}{2!(n+1)(n+2)}\right.$

$=\sum_{k=-n}^{\infty}k^2\sum_{n=0}^{\infty}\frac{b^{k+n}}{k+n!}\;\frac{b^n}{m!}=\sum\;\sum\frac{b^m}{m!}\;b^n$

$=(-n)^2\sum_{m=0}^{\infty}\left(\frac{b^{m+n}}{m+n!}\;\frac{b^m}{m!}\right)+(-n+1)^2\sum\left(\frac{b^{m+n-1}}{m+n-1!}\;\frac{b^m}{m!}\right)+\cdots\;0\cdot\sum\frac{b^m}{m!}\frac{b^m}{m!}+\cdots$

$=\frac{b^0}{0!}\left[(-n)^2\sum_{m=0}^{\infty}\frac{b^{m+n}}{m+n!}+(-n+1)\sum\frac{b^{m+n-1}}{m+n-1!}+\cdots\right.$

$+\frac{b^1}{1!}\left[(-n)^2\sum\right.$

$$+(2)^2\left[\frac{b^2}{2!}\frac{b^0}{0!}+\frac{b^3}{3!}\frac{b^1}{1!}+\frac{b^4}{4!}\frac{b^2}{2!}+\frac{b^5}{5!}\frac{b^3}{3!}+\frac{b^6}{6!}\frac{b^4}{4!}+\cdots\right.$$

$$+1\left[\frac{b^1}{1!}\frac{b^0}{0!}+\frac{b^2}{2!}\frac{b^1}{1!}+\frac{b^3}{3!}\frac{b^2}{2!}+\frac{b^4}{4!}\frac{b^3}{3!}+\frac{b^5}{5!}\frac{b^4}{4!}+\cdots\right.$$

$$0\left[\frac{b^0}{0!}\frac{b^0}{0!}+\frac{b^1}{1!}\frac{b^1}{1!}+\frac{b^2}{2!}\frac{b^2}{2!}+\frac{b^3}{3!}\frac{b^3}{3!}+\frac{b^4}{4!}\frac{b^4}{4!}+\cdots\right.$$

$$+1\left[\frac{b^1}{1!}\frac{b^0}{0!}+\frac{b^2}{2!}\frac{b^1}{1!}+\frac{b^3}{3!}\frac{b^2}{2!}+\frac{b^4}{4!}\frac{b^3}{3!}+\frac{b^5}{5!}\frac{b^4}{4!}+\cdots\right.$$

$$+(2)^2\left[\frac{b^2}{2!}\frac{b^0}{0!}+\frac{b^3}{3!}\frac{b^1}{1!}+\frac{b^4}{4!}\frac{b^2}{2!}+\right.$$

$$+(3)^2\left[\frac{b^3}{3!}\frac{b^0}{0!}+\frac{b^4}{4!}\frac{b^1}{1!}+\right.$$

$$+(4)^2\left[\frac{b^4}{4!}\frac{b^0}{0!}+\right.$$

Hilla i Innesa dają wyniki najzupełniej zgodne i tem samem umacniają podstawy poprzednich obliczeń.

$$v^2 = 2\frac{p}{\rho}$$

$$\frac{v^2\rho}{2} - p$$

$$\sigma = \frac{10^6}{9.10^{11}}$$

$$\eta = 0{\cdot}02$$

$$K\Delta\varphi = \frac{4}{300}$$

$$E = \frac{4}{4\pi}\cdot\frac{P}{9.10^5}\,\frac{1}{0{\cdot}02}$$

pro Atmosph.: $P = 10^6$

$$E = \frac{1}{9\pi}\cdot 50.10 = 20 \text{ Volt}$$

$$\Delta P = \frac{2\,E\,K\Delta\varphi}{R^2\pi} = \frac{2(K\Delta\varphi)^2}{4\pi^2}\,\frac{P\sigma}{\eta}\,\frac{1}{R^2}$$

$R = 0{\cdot}1$ mm

$$\frac{\Delta P}{P} = \left(\frac{K\Delta\varphi}{\pi}\right)^2 \frac{1}{2}\,\frac{\sigma}{\eta}\,\frac{1}{R^2} = \frac{1}{2}\left(\frac{4}{1000}\right)^2 \frac{10^{-5}}{9}\,\frac{1}{0{\cdot}02}\,\frac{1}{10^{-4}}$$

$$= \frac{16}{2.9}\,\frac{50.10^{-5}}{10^{6-4}} = 5\,\frac{.10^{-4}}{10^2} = 5.10^{-6}\,!$$

$\cos\varepsilon = \cos\alpha_1 \cos\alpha_2 + \cos\beta_1 \cos\beta_2 + \cos\gamma_1 \cos\gamma_2$ $\qquad = (\cos\alpha_1 \cos\beta_2 - \cos\alpha_2 \cos\beta_1)^2 + (\quad)^2 + (\quad)^2$

$\sin\varepsilon = \sqrt{1-(\quad)^2} = \cos^2\alpha_1 \cos^2\alpha_2 + \cos^2\alpha_1 \cos^2\beta_2 + \cos^2\alpha_1 \cos^2\gamma_2 + - \left\{ \cos^2\alpha_1 \cos^2\alpha_2 + \cos^2\beta_1 \cos^2\beta_2 + \right.$

$\cos\ldots = \qquad \cos^2\beta_1 \cos^2\alpha_2 + \cos^2\beta_1 \cos^2\beta_2 + \cos^2\beta_1 \cos^2\gamma_2 \qquad 2\cos\alpha_1 \cos\beta_1 \cos\alpha_2 \cos\beta_2 -$

$\cos^2\gamma_1 \cos^2\alpha_2 \ldots$

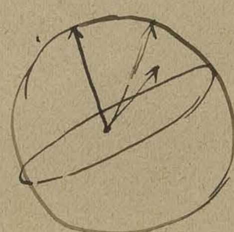
$\cos\alpha_1 \cos\lambda + \cos\beta_1 \cos\mu + \cos\gamma_1 \cos\nu = 0 \qquad (\cos\alpha_1 \cos\gamma_2 - \cos\alpha_2 \cos\gamma_1)\cos\lambda + (\cos\beta_1 \cos\gamma_2 - \cos\beta_2 \cos\gamma_1)\cos\mu = 0$

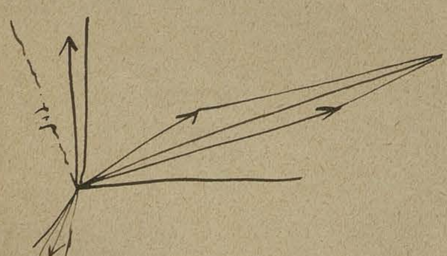
$\cos\alpha_2 \cos\lambda + \cos\beta_2 \cos\mu + \cos\gamma_2 \cos\nu = 0$

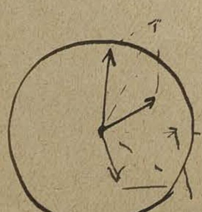
$\cos^2\lambda + \cos^2\mu + \cos^2\nu = 1$

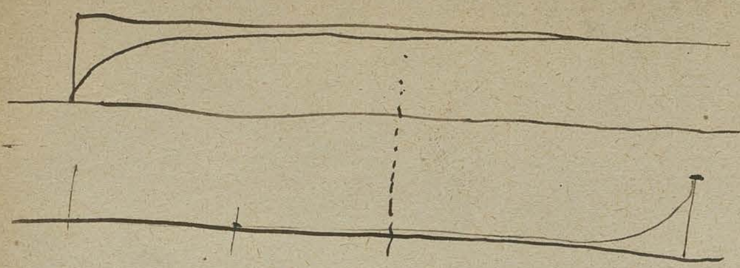
$$\frac{\cos\lambda}{\cos\beta_1 \cos\gamma_2 - \cos\beta_2 \cos\gamma_1} = \frac{\cos\mu}{\cos\gamma_1 \cos\alpha_2 - \cos\gamma_2 \cos\alpha_1} = \frac{\cos\nu}{\quad} = K$$

$$\cos\lambda = \frac{\cos\beta_1 \cos\gamma_2 - \cos\beta_2 \cos\gamma_1}{\sqrt{(\quad)}}$$



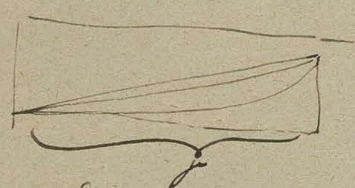






$$\sin\left(n\pi - \frac{n\pi\xi}{c}\right)$$

$$= (-1)^{n+1} \sin\frac{n\pi\xi}{c}$$



$$u = \gamma\left\{\frac{x}{c} + \frac{2}{\pi}\sum \frac{(-1)^n}{n} e^{-a^2\left(\frac{n\pi}{c}\right)^2 t} \sin\frac{n\pi x}{c}\right\}$$

$\theta = \gamma - u$

~~$x = c - \xi$~~

$\xi = c - x$

$\frac{x}{c} = 1 - \frac{\xi}{c}$

$$\theta - \gamma = \gamma\left\{1 - \frac{\xi}{c} + \frac{2}{\pi}\sum \frac{(-1)^n}{n} e^{-a^2\left(\frac{n\pi}{c}\right)^2 t} \underbrace{\sin n\pi\left(1 - \frac{\xi}{c}\right)}_{\sin n\pi\frac{\xi}{c}}\right\}$$

~~$$\theta = \gamma\left\{1 - \frac{\xi}{c} + \frac{2}{\pi}\sum \frac{(-1)^n}{n} e^{-a^2\left(\frac{n\pi}{c}\right)^2 t} \sin n\pi\frac{\xi}{c}\right\}$$~~

$$\theta = \gamma\left\{2 - \frac{\xi}{c} + \frac{2}{\pi}\sum \frac{(-1)^{2n+1}}{n} e^{-a^2\left(\frac{n\pi}{c}\right)^2 t} \sin\frac{n\pi\xi}{c}\right\}$$

$$= \gamma\left\{2 - \frac{\xi}{c} - \frac{2}{\pi}\sum \frac{1}{n} e^{-a^2\left(\frac{n\pi}{c}\right)^2 t} \sin\frac{n\pi\xi}{c}\right\}$$

~~$$\sum \frac{\sin n\alpha}{n} = \frac{\pi - \alpha}{2} \quad 1 - \frac{\xi}{c} - \frac{x}{\pi} \ldots$$~~

$$\frac{d\varrho}{dt} = \frac{c_B \, s \, v}{e^{\frac{vh}{k}} - 1} \qquad v = \alpha \frac{d\varrho}{dt}$$

$$c_B = \frac{m - \varrho}{W}$$

$$\frac{d\varrho}{dt}\left[\frac{vh}{k} + \frac{1}{2}\left(\frac{vh}{k}\right)^2\right] = c_B \, s \, v$$

$$\frac{d\varrho}{dt}\left[1 + \frac{1}{2}\frac{\alpha h}{k}\frac{d\varrho}{dt}\right] = \frac{m-\varrho}{W}\,\frac{s\,k}{h}$$

$$\left(\frac{d\varrho}{dt}\right)^2 + 2\frac{k}{\alpha h}\frac{d\varrho}{dt} = \frac{m-\varrho}{W}\,\frac{2s}{\alpha}\left(\frac{k}{h}\right)^2$$

$$\frac{d\varrho}{dt} = -\frac{k}{\alpha h} \pm \sqrt{\frac{m-\varrho}{W}\,\frac{2s}{\alpha}\left(\frac{k}{h}\right)^2 + \left(\frac{k}{\alpha h}\right)^2}$$

$$= -\frac{k}{\alpha h}\left[1 + \left(1 + \frac{m-\varrho}{W}\,2s\alpha\right)^{\frac{1}{2}}\right]$$

$$\frac{d\varrho}{dt} = -\frac{k}{\alpha h}\left[\frac{m-\varrho}{W}\,2s\alpha - \frac{1}{8}\left(\frac{m-\varrho}{W}\,2s\alpha\right)^2\right]$$

$$\theta = \gamma\left\{1 - \frac{\xi}{c} - \frac{2}{\pi}\sum \frac{1}{n} e^{-a^2\left(\frac{n\pi}{c}\right)^2 t} \sin\frac{n\pi\xi}{c}\right\}$$

$t = 0 \qquad \theta = 0$

$\xi = 0 \qquad \theta = \gamma$

$\xi = c \qquad \theta = 0$

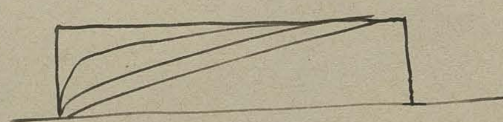
$$\theta = \gamma\left\{\frac{\xi}{c} + \frac{2}{\pi}\sum \frac{1}{n} e^{-a^2\left(\frac{n\pi}{c}\right)^2 t} \sin\frac{n\pi\xi}{c}\right\}$$

$$\sum \frac{\sin n\alpha}{n} = \frac{\pi - \alpha}{2}$$

$$\frac{\xi}{c} + \frac{2}{\pi}\,\frac{\pi - \frac{\pi\xi}{c}}{2} = 1$$

$t = 0 \qquad \theta = \gamma$

$\xi = 0 \qquad \theta = 0$



$\xi = c \qquad \theta = \gamma$

$$\left.\frac{\partial\theta}{\partial\xi}\right|_0 = \frac{\gamma}{c}\left\{1 + 2\sum_{n=1}^{\infty} e^{-a^2\left(\frac{n\pi}{c}\right)^2 t}\right\}$$

wyszukuję czas kiedy $\frac{2}{c} e^{-\frac{a^2\pi^2}{c^2}t}$ spadnie do wartości ε

$$e^{-\frac{a^2\pi^2}{c^2}t} = \frac{c}{2}\varepsilon$$

$$\frac{\partial c}{\partial t} = a^2\frac{\partial^2 c}{\partial x^2}$$

$$-\frac{\pi^2 a^2}{c^2}t = \lg\frac{c\varepsilon}{2}$$

$a^2 = K = 0{\cdot}89$

$c = 3{\cdot}65$

$$t = \frac{c^2}{\pi^2 a^2}\lg\frac{2}{c\varepsilon}$$

$$= \frac{c^2}{\pi^2 a^2}\left[\lg\frac{2}{c} - \lg\varepsilon\right]$$

Wł. Umiński [illegible]

Pogadanki naukowe, Pioruny i Błyskawice. Warszawa Arct 108 str. 1897.

Zbiór różnych opowiadań o piorunach, po części faktów prawdziwych, po części wprost śmiesznie dziecinnych bajek i podane zupełnie bezkrytycznie i bez zrozumienia naukowego, za to wszystko silnie zabarwione sensacyjnością i [illegible] ilustrowane okropnie marnemi ilustracyami.

[illegible]
(Takie książki [illegible] więcej szkody przynoszą umysłom niedojrzałym [illegible] pożytku.

...

M. Jamrógiewicz. Maciej Brzoza, zbiór nakł. Tow. Naucz. Szk. Wyż. 1906 str 34

Książka do czytania (dla chłopców w wieku szkolnym) napisana poprawnie (lepiej niż przeciętnie) i zachęcająca swą treścią do zajęć fizycznych i zręcznościowych; [illegible] Sądzimy że i tu niektóre szczegóły (np. możnaby powiedzieć że niektóre z opisanych robót (np. (twaróg z mąk bezowych)) są za trudne do wykonania w praktyce i może lepiej byłoby ograniczyć się na nieco skromniejszych, ale łatwiejszych do wykonania wypadkach. Ale bardzo wysoko cenimy intencyę autora, (które powinny niezawodnie wydać) [illegible] (tę książkę [illegible] nie [illegible] (polecamy młodzieży szkolnej))

[illegible]

$$\int_0^\infty f(y)\,[f(y)]^{n-1}\,dy = \frac{[f(y)]^n}{n}\Big|_0^\infty$$

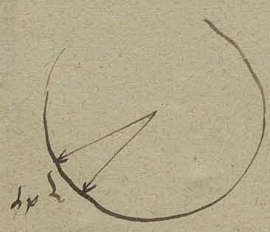
$$\int_0^\infty e^{-\alpha y^2}\,dy\left[\int_0^y e^{-\alpha x^2}\,dx\right]^{n-1} = \frac{1}{n}\left[\int_0^\infty e^{-\alpha x^2}\,dx\right]^n = \frac{1}{n}\left[\frac{1}{2}\sqrt{\frac{\pi}{\alpha}}\right]^n$$

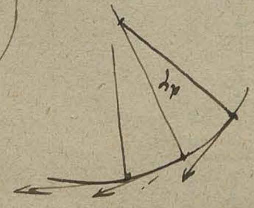
$\frac{\partial}{\partial\alpha}$

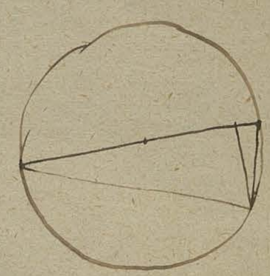
$$-\int_0^\infty y^2 e^{-\alpha y^2}\,dy.\left[\int_0^y e^{-\alpha x^2}\,dx\right]^{n-1} - \int_0^\infty e^{-\alpha y^2}\,dy.\,(n-1)\left[\int_0^y e^{-\alpha x^2}\,dx\right]^{n-2}.\int_0^y x^2 e^{-\alpha x^2}\,dx$$

$$\int_0^y x^2 e^{-\alpha x^2}\,dx = -\frac{y.\,e^{-\alpha y^2}}{2\alpha} + \frac{1}{2\alpha}\int_0^y e^{-\alpha x^2}\,dx$$

$$\int e^{-\alpha y^2}\,dy$$







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