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THE FUNDAMENTAL CONCEPTS OF SPINOZA'S METAPHYSICS IN THE LIGHT OF GEOMETRICAL LOGIC

I.

Geometry was for Spinoza the paragon of all precision, objectivity and rationality; the logic of geometrical proof according to him best expressed the laws of nature, "naturae leges et regulae, secundum quas omnia fiunt et ex unis formis in alias mutantur" (Ethics, III, preface). Hence his predilection for the geometrical methods of exposition and proof also in the domain of philosophy, which has primarily to reveal the principles of this order, regularity and rationality of the world. But geometry remains in yet another, much closer and more profound, union with Spinoza's philosophy. For, according to him, extension is one of the attributes of God, and it is geometry which investigates the modes of this attribute. Should we therefore succeed in revealing the order which reigns in this extensive domain, determine its objective principles and arrange them according to their generality or some other criterion so that it would be possible to pass from them to the singular modes of this domain, then such a philosophical, purely qualitative and genetic, geometry would have been considered by Spinoza as a really potent method in philosophy - one which would not only be a means of exposition and proof, but also one making for actual progress in our metaphysical knowledge. Since then, in line with his proposition of the identity of the order which reigns in all attributes (Ethics, II, 7), we should cognize not only order existing in the spatial domain but, in general, the order and organization of the world - its universal structure; we could then venture to make a closer examination of the relation which binds substance with its singular modes. For that matter, similar hopes would be aroused by science, which could in exact manner present to us the structure of the second of the attributes of substance known to us - the attribute of thought. It would have to be a logic as exact as mathematics, and at the same time a structural (architectonical) and genetic logic. But in Spinoza's times there was no such philosophical geometry nor any such logic; still less was there any science which would bind the two together - which would constitut their union.

Such a science, however, now exists. This is the geometry of logic

or geometrical logic (topologic), which presents algebraic logic in spatial form. We are indebted to Leibniz for the concept and first attempts to develop algebraic logic; its systematic form was, however, given us by Boole as late as the middle of the 19th century. And here the thought arises: since non-spatial sets of two or three numerical elements find faithful depiction in the spatial formations of Descartes's analytical geometry; since the relations and propositions of ordinary quantitative algebra can likewise be depicted spatially - all this would appear to support the idea that the elements and relations of qualitative algebra (i.e., algebraic logic) can also be depicted spatially, geometrically, in qualitative geometrical representation, hence in projective geometry - the geometry of position. This supposition has been fully confirmed and we have for more than ten years past possessed a logic which has revealed the relations and structure concealed deep in algebraic logic, and presented them intuitively and spatially[x]. We receive an image of the logical world by introducing a system of logico-geometrical co-ordinates analogous to those of Descartes in analytical geometry, with the centre of the co-ordinates at 0, which here represents the minimal concept of algebraic logic, the comprehension being the minimum; besides utilizing the correspondence which exists between the operations of algebraic logic and of projective geometry. Namely, in logic, we have two dual operations, addition and multiplication, and similarly, in projective geometry two likewise dual operations: section and projection. The point, as the intersection (union) of two straight lines, corresponds in geometrical logic to the logical sum (or totality) of two logical elements; whilst the straight line as the common substrate of two points corresponds dually to the logical product (or community) of two logical elements. Basing ourselves on these premisses we receive the following bi-elemental geometrical scheme of algebraic logic (the simple elements a and b, and their negations a and b :[xx]

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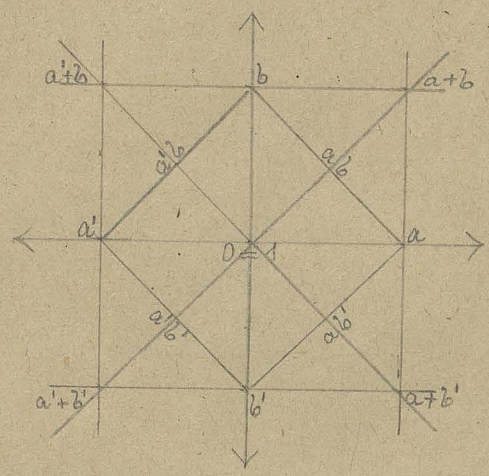
[x] Cf. Benedykt Bornstein. La logique géométrique et sa portée philosophique. Bibliotheca Universitatis Liberae Polonae, No. 20, Warsaw, 1928.

[xx] We restrict ourselves here to bi-elemental logic: for the purposes of the present paper this suffices and we can quite disregard the more complicated tri-elemental (tridimensional) logic.



The elements, relations and operations of bi-elemental algebraic logic find their geometrical correspondences here, so that the propositions of this logic are given us by intuition and can be read off this diagram direct. Logic, spatialized in such wise, acquires a marked architectonic, structural and figurative character.

We must in this connexion, first of all realize the exceptionally important fact that logical structures revealed by geometrical logic are simultaneously geometrical or spatial structures. It turns out here that two domains which are so toto genere diverse - the domain of non-spatial thought and that of spatial formations - possess the same construction, that they present the same relations and structures, that the same categories of elements, relations and operations are analogously represented in these so different spheres. The connexion with Spinoza's philosophy is obvious: the fact of geometrical logic scientifically confirms the validity of proposition No. 7 in Part II of Spinoza's Ethics, so important for his system, affirming that "ordo et connexio idearum idem est, ac ordo et connexio rerum," where "res", as follows from the Scholium to this proposition, signifies "the mode of extension".

Let us now examine the image of the logical world from another standpoint - from the point of view of the number of elements visible in it; new connexions between geometrical logic and Spinoza's system then become apparent. We are struck by the small number of elements to which has been reduced the infinite multiplicity of concepts on the one hand, and of points and straight planes on the other. We of course have before us a categorial condensation of the world of thought and of the world of space, in which countless elements are reduced to a few categories which represent them and under which they fall.

It is the idea of such a categorial condensation which we find in Spinoza's concept of "series rerum fixarum aeternarumque" - a concept of great significance not only from the metaphysical standpoint but also from that of the theory of cognition. Spinoza states that when we seek the essence of singular variable things (existing in time) and the laws which govern them,we cannot but return to these "fixed and eternal things". He affirms: "Imo haec mutabilia singularia adeo intime atque essentialiter (ut sic dicam) ab iis pendent, ut sine iis nec esse nec concepi possent". Just before this passage he stated: "Haec (i.e., intima essentia rerum) vero tantum est petenda a fixis atque aeternis rebus, et simul a legibus in iis rebus, tanquam in eius veris codicibus inscriptis, secundum quas omnia singularia et fiunt et ordinantur".x) (Tractatus de intellectus emendatione, § 101).

But is this "series of fixed and eternal things" really according to Spinoza a condensation and as if general,categorial essence of a countless multitude of singular things existing in time? Of this there can be no douht. Spinoza states: "Haec fixa et aeterna, quamvis sint singularia tamen ab eorum ubique praesintiam ac latissimam potentiam erunt nobis tanquam universalia sive genera definitionum rerum singularium mutabilium et causae proximae omnium rerum".xx) The whole cognitive value of this "series of fixed and eternal things" is primarily based on this that insetad of the countless multitude of singular things it provides us with a concise conspectus of them. Spinoza states: "Seriem enim rerum singularium mutabilium impossibile foret humanae imbecillitati assequi,quum propter earum omnem numerum superantem multitudinem, tum propter..."xxx) It is just this reason,inter alia, that Spinoza introduces the concept of a series of causes and actual beings which are simultaneously fixed, eternal and categorially restricted in their number. Hence we really have in the above representation of elements and laws of categorial geometrical logic a realization of Spinoza's concept of a "series rerum fixarum aeternarumque" and of "leges saecundum quas omnia singularia et fiunt et ordinantur". It is in these fixed and eternal things that the singular modes of the world participate as in their categorial (not individualxxxx)) essences and the laws, inscribed in

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x) These laws inscribed in the domain of fixed and eternal things are claimed to be, according to the above, universal laws. We thus have as it were the inception of the concept of some scientiae universalis in the Platonic or Leibnizian style.

xx) l.c.par.101.

xxx) l.c.par.100.

xxxx) The examination of a series of individual essences would not help the restrictedness of the human mind in any way, for this would be just as impossible, "owing to their magnitude surpassing all number", as the examination of a series of the singular things corresponding to them in time.

these things as in their real codes, flow in such wise from eternity to the world of duration.

This series of fixed and eternal things is to bind one, single substance with the multitude of its individual modes, to present a series of stages leading from the most général things to the singular ones; it is to indicate the degrees by which the substance and its attributes develop and differentiate themselves. It so happens that geometrical logic is above all a genetic logic, a logic of developments and differentiations, and covers the range from the most general concepts (i.e., of minimal comprehension) to ever more specified and determinate ones. The element 0 - the logical minimum, a comprehension which is minimally determinate - develops into the more determinate elements a and a′; each of these elements is determinated further, when a yields the specifications a + b and a + b′ and a′ the specifications a′ + b and a′ + b′ (cf. the figure above). Within the range of bi-elemental logic, these four specifications represent the most determinate, the most concrete, categorial elements, if we for the moment disregard the element 1 (the logical maximum), which will be discussed on another page.

But we shall not make a closer examination here of these members of a series of eternal things which already ~~xxxxxx~~ enters a finite domain (a, a + b, etc.). True we may suppose that Spinoza's concept of fixed and eternal things also embraces essences which are finite (although not individual but general or categorial); but there can be no doubt that the leading part in his case is played by infinite elements, primarily by the so-called infinite modes (modi infiniti), although nothing forbids us to join to these eternal and infinite things of Spinoza's substance and attributes,[x] elements which above all possess the features of immutability, eternity and infinity.

These infinite elements will now be examined by us. We have a whole series of these elements in Spinoza's system: infinite substance, infinite attributes, infinite modes and these modes are of two kinds; in addition these modes also appear in polar form, e.g. as motion and rest. If then, geometrical logic (or logical geometry) is really in so close a relation to Spinoza's metaphysics, as we affirm and as we have striven to demonstrate, then we should also find in it a whole graduation of infinite elements. And in actual fact, we do find them. In this connexion it is necessary to add that this series of infinite elements is clearly discernable only in geometrical logic: they are not graduated in ordina-

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[x] Elisabeth Schmitt. Die unendlichen Modi bei Spinoza, in Zeitschrift für Philosophie und philosophische Kritik, vol. 140 (1910), pp.64-65, and Zur Problematik der unendlichen Modi, in Chronicon Spinozanum, vol. II, 1922.

ry algebraic logic and in principle appear only as a single zero (the logical minimum) and as a single unity (the logical maximum). Both these elements are limitary, i.e. non-finite: zero can be conside red as a sub-finite element, and unity, representing the totality of opposite elements $(1 = a + a')$ - as a trans-finite element (in logic often known as the "universe of discourse"). As stated above, we have a whole graduation of these zeros and unities in geometrical logic. Leaving the question of unity aside for the moment, we shall now deal with the series of zeros in bi-dimensional geometrical logic.

First of all, the axes of co-ordinates are zero-straight lines. The horizontal axis is the substrate of the opposite points a and a', hence that which these elements have in common; but elements which are opposite (or negative) to each other are marked by their community being a minimal one, i.e. 0: in other words, their logical product (a, a'), in which the community of the elements is expressed, is 0. We thus have $0 = aa'$, and this 0 is depicted geometrically in the form of the horizontal axis of co-ordinates which joins (multiplicatively, i.e. by community) the polar points a and a'. Further the same holds good in the case of the second axis of co-ordinates - the vertical one: it is also a common substrate of the opposite points b and b' and is ~~thereforelikewise~~ therefore likewise 0 (here $0 = bb'$). In order to distinguish between these two zeros, we shall designate the first one $0_{aa'}$ and the second one $0_{bb'}$. This is not all, however. These axes unite (straight lines on a plane, as we know, unite additively) at the point 0, the origin of the co-ordinates - so that we have here a third zero - a more determinate and more concrete one than the previous two, since it depicts the logical sum $(0 = 0_{aa'} + 0_{bb'})$ x). We shall designate it for the sake of brevity as $0_{aa'} + 0_{bb'}$. Finally we have a fourth zero - a more general and still less determinate one than the zero-axes of the co-ordinates: this zero is the common substrate of these axes - the plane in which they rest - the plane of the co-ordinates. This is $0 = 0_{aa'} \times 0_{bb'}$, or, more briefly, $0_{aa'.bb'}$. We thus have four zero elements; the like number of unity elements dual to these zero ones will be examined later (cf. p. 17).

We have seen in the field of geometrical logic what far-reaching differentiation of infinite elements there is, and this at once suggests the thought that it is not unconnected with the series of infinite elements in Spinoza's system of metaphysics. We shall therefore more closely examine the question of these infinite elements of Spinoza's and shall ascertain to what extent his architectonical concept of the supreme principles of the universe agrees with their architectonics, which give us categorial geometrical logic, or otherwise: categorial geometry

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x) In the algebra of logic, the logical sum of zeros - as also their product - likewise yields zero.

- the depiction of categorial algebraic logic.

II.

Let us first examine the nature of the infinite modes of the first genus. As is known, Spinoza, in his "Tractatus brevis de Deo" introduced as mediate members between natura naturans (substance and attributes) and natura naturata particularis (finite particular things) none but members of a single kind (natura naturata generalis). Later, in his Letters and his Ethics, he added to them mediate elements of another kind-infinite modes of the second genus. As regards infinite modes of the first genus, for the attribute of extension they are, according to Spinoza: motion and rest - ancient Pythagorean-Platonic principles, which also occupy a prominent place in the metaphysics and physics of Descartes. Constant, eternal and infinite, they are according to Spinoza, the first direct product of substance as far as the attribute of extension goes, and it is only thanks to them that this extension can be differentiated, that it can reveal itself in the form of countless multitudes of singular things. For things differ from each other just in respect of motion and rest, speed and slowness: "supponimus... quamcumque rem corpoream particularem nihil aliud esse nisi certam motus quietisque proportionem." And Spinoza lays special stress on this that only collaboration of motion and rest can condition the genesis of a world of individualized finite modes, that this requires the collaboration of these polar factors, that one of them does not suffice, and "nisi in extensione aluid non esset, quam sive motus sive quies sola, nulla in illa res particularis ostendi vel esse posset". (Tractatus brexi brevis de Deo, Corollary II). If, however, an exactly defined, individual proportion of motion and rest is that which causes bodies to differ from each other, then motion in general and rest in general - motion and rest still undifferentiated, only implying infinite possibilities of differentiation - hence an infinite mode proper of the first genus will be that which is common to all things - that which is implied in the concept of every thing and in rhiphcof of which all things are in agreement with each other. Lemma 2 of the second part of Ethics (between the 13th and the 14th propositions), affirming that "omnia corpora in quibusdam conveniunt", is based on the following: "In his enim omnia corpora conveniunt quod unitus eiusdemque attributi conceptum involvunt; deinde quod iam tardius, iam celerius, et absolute iam moveri, iam quiescere possunt."

We see here that Spinoza fundamentally draws nearer to each and relates the attribute of extension to its direct infinite mode; that

the attribute of extension as also motion and rest express that whereby all things agree with each other, that which they have in common, that which their concept implies. Taking the matter logically, both the attribute of extension and its infinite mode of the first genus will hence correspond to the logical minimal comprehensions, those which are most general and which are implied in every concept; in other words, this will be the logical zero (0). Here, the attribute of extension will be a more primary zero than motion in general and rest in general, since the concepts of motion and rest already imply the concept of extension, which is common to both. If we bear in mind that the community of two elements is expressed in algebraic logic by their product, we shall then be in a position to state that the attribute of extension is the logical product of motion in general and of rest in general (motion in general x rest in general). In addition, we must further remember that Spinoza stressed the point that it is only a union of motion and rest which constitutes an infinite mode of the first genus; that this mode is therefore expressed as the logical sum of motion in general and of rest in general (motion in general + rest in general), i.e., as a totality, in which motion in general and rest in general will be only dependent, limitary moments. In such wise, a logical analysis of the concepts of the attribute of extension and of its infinite mode of the first genus givesus the following graduation of these concepts:

$0_1 + 0_2$ (rest and motion in general)

0_1 (rest in general); 0_2 (motion in general)

$0_1 \times 0_2$ (extension in general).

If we now recall the series of zeros in bi-dimensional geometrical logic - if we realize their spatial representation,mentioned above - we shall simultaneously receive a spatial image of the four infinite elements of Spinoza's metaphysics examined above. The attribute of extension (extension in general) - the substrate of the physical world - will be represented on our diagram as a plane of a system of co-ordinates, the lowest zero,implied in all the modes of this attribute, primarily in its infinite mode: motion in general (0_2) and rest in general (0_1). This attribute of extension is that which motion and rest have in common with each other ($0_1 \times 0_2$) - the common substrate of 0_1 and 0_2, which are represented in our diagram in the form of the zero axes of co-ordinates $0_{aa'}$ and $0_{bb'}$. These zero axes of co-ordinates join additively at the origin of the logico-geometrical co-ordinates ($0_{aa'} + 0_{bb'}$), which in such wise becomes the depiction of the infinite mode of extension of the first genus in its totality, in the unity of its moments. - We have here before us the attribute of extension in the form

of a still absolutely undifferentiated plane of co-ordinates, and the first formation, its first differentiation, the direct mode of this extension, i.e. the infinite mode of the first genus (motion and rest in general) in the shape of a system of co-ordinates: the origin of the co-ordinates and of the axes implied in it as its determining moments. All the ordinary points (beyond the system of co-ordinates) lying on this basic plane are determinate and particularized by the exactly defined proportion of motion and rest appropriate to each of them; in other words, by the definite co-ordinates of motion and rest - these are the finite modes of the attribute of extension. But one point of this plane occupies quite a specific and pre-eminent place amongst all the others; this is the origin of the co-ordinates - the infinite mode of the attribute of extension, motion and rest in general. It represents a combination of the co-ordinates of motion and rest, not in their determinate form (Type a + b), but in a general, indeterminate form (aa' + bb') implying none the less the possibility of an infinite multitude of finite modes, i.e., points which have exactly defined co-ordinates of motion and rest.x)

We say: the possibility of finite modes; but in what manner can this possibility be realized in actuality? This query must arise inevitably. So far, we have only the attribute of extension and its first, sole differentiation in the form of an infinite mode; we have, in other words, a plane of co-ordinates and a system of co-ordinates, but, apart from the origin of co-ordinates, not a single actual point on the plane - not a single finite mode. It is not enough in this case to introduce two moments (motion and rest) into the infinite mode: it does not suffice actually to develop the multitude of finite modes. True, it is essential to introduce this polar pair in order to receive any point in general on the plane; but this is not enough when we seek to actualize the multitude of points having exactly defined co-ordinates. Something else is necessary here, yet another factor is required - one which would introduce a differentiation of the co-ordinates and help them to issue from their indeterminate state (motion and rest in general) and endow them with defined finite values (a, a', b, b' and their derivatives). It can be supposed that Spinoza realized these difficulties and understood that the introduction of a single infinite mode (even a polar complicated one) does not suffice to develop the multitude of finite modes from the attributes. Probably for this reason, inter alia, he later introduced (in his

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x) But if this infinite multitude of finite modes is taken categorially and not multitudinously, it will be expressed in the form of four points, characteristic of the quarters of the plane. These will be the points a+b, a+b', a'+b, a'+b'.

Letters and Ethics) a second infinite mode - one of the second genus. We shall now make a closer examination of its relation to the infinite mode of the first genus.

However we comprehend the role of the infinite mode of the second genus in Spinoza's system, we can be sure of one thing: of its integral nature. "Facile concipiemus, totam naturam unum esse individuum, cuius partes, hoc est omnia corpora infinitis modis variant absque ulla totius individui mutatione", Spinoza writes (Ethics II, lemma 7, Scholium). This integrity of nature, this infinite individual in its totality, is called "facies totius universi" by Spinoza (in Letter 64); and it is this "face of the whole world" which he states to be the infinite mode of the second genus. Berelears earlier, in 1665, he had written to Oldenburg (Letter 32): "omne corpus quatenus certo modo modificatum existit, ut partem totius universi considerari debere, cum suo toto convenire et cum reliquis cohaerere." And in the face of the infinite power which the universe has, "eius partes infinitis modis moderantur et infinitas variationes pati coguntur," but "servata semper in omnibus simul, hoc est in toto universo, eadem ratione motus ad quietem." Hence, the face of the universe as a totality remains eternally the same and immutable, in spite of the changes and modifications of parts. And it is this immutable face of the universe which isthe infinite mode of the second genus - the infinite totality of the modes of motion in its immutable relation to the infinite totality of the modes of rest. It therefore represents the average of all the states of motion and rest, as some constant of the universe which governs the phenomena of the universe. Naturally, although it might seem superfluous to add, this average of motion and rest - this infinite mode of the second genus - precedes Nature in the multitude of its forms; it is its source and ~~xpxixxi~~ a priori law which makes it possible and realizes it - in no wise does it derive from it a posteriori. For Spinoza could never have haveed that the aggregate of finite and transient modes could condition the infinite and eternal mode, and not the converse.

We thus come to the conclusion that the infinite mode of the second genus presents itself logically and categorially in quite different fashion than the infinite mode of the first genus. The latter possesses, as we have seen, a general character, one of community, whilst the former has a marked and indubitable character of totality. Issuing from finite modes, we can combine them in twofold manner: taking them distributively, we can unite them in their community, and taking them collectively, we can unite them in their totality. And this dual method of combining individuals was comprehended by Spinoza as early as his Tractatus

brevis de Deo (cf. Dialogue II, second discourse of Theophilus), when he compared and distinguished the concepts of totality and generality.[x] We now encounter these two concepts in their metaphysical realization, as infinite modes of extension of the first and the second genus: on the one hand we have motion in general and rest in general; on the other, totality of motion and totality of rest in their immutable relation.

Let us now examine the logical and geometrical structure of these two categorial concepts.

In algebraic logic we arrive at general ~~byxthex~~ concepts by the operation of multiplication; combining two elements multiplicatively, we receive their logical product - a concept which is more general than the conceptions joined in this way. If we unite multiplicatively two concepts polar to each other (e.g., a and a'), we secure the logical product in the shape of the logical 0 - the concept of the most general comprehension. In mathematical logic we have yet another operation - that of addition. In the logical sum, we unite the elements added, we join them into a totality and not into a community and generality as in the product. The correlation which arises in mathematical logic between multiplication and addition - between the logical product and the logical sum, between generality and totality - is called duality, and the operations and elements in question are known as dual ones. Now, to the logical product of the polar elements a and a' (a.a') or to the logical 0 corresponds dually the logical sum of these opposite and complementary elements (a + a'), i.e. the logical maximum, "the universe of discourse".[xx] This is, apart from 0, the second non-finite element in mathematical logic - in this case, the trans-finite one. In accordance with the principle of duality, we shall be able to examine a series of these logical unities, a number of trans-finite elements, which are dual with respect to the sub-finite, zero elements. To attain this, however, we must turn to geometrical logic - to the geometry of logic - for aid.

The reader is reminded in the first place that in projective geometry the principle of the duality of operations and of elements is likewise binding, analogously to the principle of logical duality. We have two dual geometrical operations: section and projection (union), as also two dual elements (on a plane): the straight lines intersecting each other, yield a point as an outcome of this union (corresponding to the logical sum) whilst two points joined to each other yield a straight line as their common substrate (corresponding to the logical product). If we now revert to our basic image of a logical plane, we shall easily

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[x] Cf. p. 24.

[xx] Cf. p. 8.

locate there three logical unities, dual with respect to the three logical zeros: $0_{aa'}$, $0_{bb'}$, and $0_{aa'+bb'}$.x) These three logical zeros have their geometrical correspondences: the horizontal axis of co-ordinates, the vertical axis, and the point of origin of the co-ordinatesxx). We shall first of all find the dual geometrical correspondence for the horizontal axis. This axis represents the common substrate of the points $\underline{a}$ and $\underline{a}'$ (their product: $\underline{aa'}$). If we now take the elements dual with respect to the points $\underline{a}$ and $\underline{a}'$ (the straight lines $\underline{a}$ and $\underline{a}'$, perpendicular with regard to the horizontal axis at the points $\underline{a}$ and $\underline{a}'$), then the point ($\underline{a}$ + $\underline{a}'$) at which these lines intersect gives us an element dual with respect to the axis under examination. As the straight lines $\underline{a}$ and $\underline{a}'$ are parallel, the point of their intersection will lie at infinity upon the vertical axis. In such wise, we have secured the first of the trans-finite elements, $1_{a+a'} = a + a'$, dual to $0_{aa'} = aa'$. In just the same way we can arrive at the second trans-finite (total) element, dual with respect to the vertical axis $0_{bb'} = bb'$; this will be the point at infinity situate on the horizontal axis $1_{b+b'} = b + b'$. Finally, the third unity will be found to be the element dual with respect to the origin of the co-ordinates, to the point $0_{aa'+bb'} = 0_{aa'+bb'}$, in which the axes $0_{aa'}$ and $0_{bb'}$ are joined. The element dual to it will therefore represent the straight line joining the points dual with respect to the two axes, i.e., the points at infinity: $1_{a+a'}$ and $1_{b+b'}$. This straight line will be the "straight line at infinity", expressed algebraically as $1_{a+a'} \times 1_{b+b'}$, or, in short as: $1_{(a+a')(b+b')}$.

In such manner, we have arrived at three topological unity elements, dual with respect to the three topological zero elements; the straight line at infinity with the two points situate upon it, dual with respect to the origin of the co-ordinates with the two axes implied in it. And just as the infinite mode of extension of the first genus is a point-origin of the co-ordinates, and the axes of the co-ordinates determining it are only its dependent moments, so does the infinite mode of the second genus appear in the form of a straight line at infinity, and the points at infinity determining it are only its dependent moments. The point at infinity $1_{a+a'}$ is an infinite totality of the modes of the motion of the universe; the point at infinity $1_{b+b'}$ is an infinite totality of the modes of its rest; whilst the infinite mode of the second genus the face of the whole universe, $1_{a+a'} \times 1_{b+b'}$, represents some function of the above-mentioned two totalities, not their sum, but their logical product - as if their arithmetical mean, their average, the average of the motion and the rest of all the things in the universe around which the countless particular values of these states oscillate.

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x) We shall not for the moment examine the fourth unity (corresponding to the fourth zero.
xx)Cf. page 8,9.

We thus now already have between the attribute of extension and its particular finite modes, two (and not only one) infinitie modes with the moments which determine them. This circumstance is of decisive significance for the realization of the singular modes of this domain. For we have not only motion and rest in general - an infinite mode which is absolutely void in its generality, in spite of its implication of countless possibilities for the particularization of the modes of motion and rest - but also the totality (still undifferentiated) of these particular values, the plenitude of the modes of motion, and the plenitude of the modes of rest which can determine and actualize the empty places, the possibilities of determinate states of motion and rest implied in the infinite mode of the first genus and in its moments (axes). United and intermingled with each other in $1_{a+a'}$, the elements a and a' can now develop themselves on the plane in the shape of the straight lines a and a', which in their turn determine and actualize the categorial points a and a' on the horizontal axis. Similarly with the totality $1_{b+b'}$, and the vertical axis. By the reciprocal action of unity, as a plenitude of particular (though non-divided) values for motion and rest, and of the zero elements, the void but implying infinite possibilities, infinite "place" for these values, a reciprocal differentiation of these infinite elements may take place: parts may issue from the totality, i.e., straight lines from points at infinity, and the dual appearance of points on the axes of co-ordinates (and on the straight lines parallel to them). In such wise (see diagram), the finite modes of the plane may appear upon it in the shape of categorial points, and this categorial image of creation, this image of "constant and eternal things", is as if the very first model of multitudinous creation: eternal creation of particular essences and creation of individuals in time.

The philosophical, qualitative analysis of extension and its modes led Spinoza to determine two principles for this domain - two infinities - two infinite modes - and the constitution of each is marked by two polar moments. We shall not diverge from the lines traced out by Spinoza, and can only separate more precisely that which is differentiated by him, when we perceive generality in the essence of the infinite mode of the first genus and the category of totality in the essence of the second infinite mode. But we transcend Spinoza, when in the concluding passage - by the aid of geometrical logic - we define more clearly the collaboration of these two infinite principles, cognized as the dual topological principles: the point of origin (and two axes) of the co-ordinates, and the straight line (and two points) at infinity. It is now possible to perceive more clearly the connexion between finite and infinite modes,

to see thir genesis from infinity, from eternal things ("a modo quo ab aeternitate fluunt", Letter 12); we see how they arise on the substrates of the axes and of the lines parallel to them caused by the universe-creating rays proceeding from infinity. We can follow this methodically and are able to give perhaps a more determinate reply to the ever-recurring, stubborn query of Tschirnhaus aimed at Spinoza: In what way can multitude and diversity of things de deduced a priori from extension? (Letters 59 and 82). Six months before his death, Spinoza was pondering this problem and in Letter 83, dated the 15th of July 1676, he wrote to Tschirnhaus: "Sed de his forsan aliquando, si vita suppetit, clarius tecum agam. Nam huc usque nihil de his ordine disponere mihi licuit."

III.

We shall now leave this fundamental matter of the individualization of finite things by means of infinite modes and make an attempt to elucidate the nature of these modes from another aspect. We know that the infinite mode of the first genus is expressed geometrically as the point of origin of the co-ordinates, and the infinite mode of the second genus - facies totius universi - as a straight line at infinity. This straight line at infinite distance can also be taken as the circumference of a circle, since such a circle (having an infinitely great radius) will have a curvature equivalent to 0, i.e., it will really be a straight line. In such wise, the complex of both of Spinoza's infinite modes finds its geometrical depiction in the form of a circle whose centre is at the origin of the co-ordinates and its circumference at infinity. This origin of the co-ordinates as the poorest, zero element - will be implied in all the elements ($0 < a$), whilst the circumference of the circle at infinity (or, the straight line at infinity) as a unitary element - a maximal, all-embracing one ($a < 1$), will be implied in nothing and will be present in none of the elements. We thus ascertain that the complex of both of Spinoza's infinite modes is represented geometrically as a circle, "the centre of which is everywhere, and the circumference of which is nowhere."

We know the geometrical symbol of such a circle or sphere. Through the course of millenia, beginning with Plato, in the teachings of the Neo-Platonists and of Dyonysius the Areopagite, through the mysticism of the Middle Ages and that of Nicolas of Cusa, through the philosophy of the early and late Renaissance pre-eminently represented by Ficino and Giordano Bruno, this symbol penetrates into the philosophy of Spinoza's times, and plays a great role in Pascal's and Leibnitz's philosophy[x]. The

x) Cf. the basic work on this concept by Dietrich Mahnke: Unendliche Sphäre und Allmittelpunkt. Beiträge zur Genealogie der mathematischen Mystik. Halle, 1937.

philosophers used this symbol in the most diverse interpretations and applications in order to draw nearer to a cognition either of God himself or of the soul of the universe, or of the totality of Nature. We do not find it in Spinoza in any distinct formulation, but the analysis of the concept of infinite modes effected above has led us to their geometrical image, which is nothing else but just that "circulus infinitus, cuius centrum ubique, circumferentia vero nusquam." The implication which we perceive here does not refer to the supreme concept of Spinoza's, that of God or substance, but only to the infinite modes of this substance, to the possibility and totality of motion and rest or will and reason, provided we likewise strictly parallelly consider the attribute of thought[x]; this duplication refers not to God himself, but to his primaeval creation of the still undifferentiated universe, or of the soul or idea of such universe ("natura naturata generalis",and,it must be added, "natura naturata totalis").

Let us now examine more closely the reciprocal relation of these two infinite modes of Spinoza's in connexion with the nature of their geometrical symbol.

It may be charged at this juncture that, although Spinoza undoubtedly distinguished and differentiated two types of infinite modes,he did not separate them in such radical fashion from each other that they could be reciprocally opposed as the inextensible centre of a circle and its infinitely great circumference, as a minimum and a maximum. Such a charge would not, however, be justified in spite of all appearances to the contrary. For, this symbol is by no means intended to contrast these two infinities - the minimal one and the maximal one; on the contrary, it has to express their union and inseparability in the essence of the circle; it has to show their profound agreement and convergence; it has to be the expression of the co-incidence of these contrarities: in the inextensible contre of the circle the infinity of its circumference is already implicitly contained; and the infinite, limitary circumference which embraces all space, itself already extends beyond it and is hence already inextensible. Applying this symbol, pholosophers have considered that the distinction and even the conceptional separation (with the corresponding spatialone) of the elements of a thing do not yet signify its definitive dismemberment. For, upon closer analysis it may appear that these distinct and provisionally separate elements are its convergent

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x) We shall not occupy ourselves more closely with the infinite modes of this second attribute here. In concord with Proposition 7 of Ethics, II, they should represent elements strictly analogous to the infinite modes of extension. As we know, however, this parallellism was not maintained by Spinoza.

eiusdem generis, totum autem ex diversis conjunctis individuis

moments, such as do not impair its basic unity. Hence the close connexion and union which undoubtedly exists (according to Spinoza) between its two infinite modes can quite well be maintained when they are geometrically depicted in the form of a circle thus comprehended with infinite circumference. Moreover, this symbol may even express their complete convergency and unity, differentiating them only as two moments (generality and totality) of one and the same thing - as two objectively based points of view respecting the same thing. And Spinoza actually had a distinct tendency to smooth out the difference between generality and totality, to represent it as one which should disappear upon a closer examination of the matter. Let us recall to mind the passage in Tractatus Brevis, in which the idea of totality and generality are compared and distinguished:x) "Adde, quod totum ens rationis tantum sit nec a generali differat, nisi in eo, quod generale fiat ex diversis non conjunctis individuis." Even if we postulate that the duality of Spinoza's infinite modes is only provisional and that it will in the final resort be reduced to unity, then, even so, this ancient geometrical symbol (in which minimum and maximum coincide) could be a faithful depiction of the dialectical nature of these two qualitative infinities.

Let us, however, examine this matter from the systematic point of view, from the standpoint of geometrical logic. We have found there the two infinities in question: the origin of the co-ordinates ($O_{aa'} + O_{bb'}$), which can be considered as the centre of a circle having its circumference at infinity, and this circumference itself ($1_{a+a'} \times 1_{b+b'}$). But have we any ground for accepting this logico-geometrical formation as a dialectical one, in which the centre of the circle, the zero point and the infinite, unitarian circumference woud coincide within each other? Can geometrical logic confirm such comprehension of this circle? In actual fact, geometrical logic, being a metaphysical logic and a metaphysical geometry in its limitary elements (zero and unity), fully confirms this dialectical convergence of the two infinities.

w polskim rękopisie jest $O_{a+a'} + O_{b+b'}$

Let us now revert to our diagram. We have O' as $O_{aa'} + O_{bb'}$ as the union of the chief logico-geometrical co-ordinates at the origin of the co-ordinates. But if this origin be examined from another point of view, viz., as the union of the slanting axes, we receive, surprisingly enough, something quite different. One of these slanting axes is the straight line joining the points (a+b) and (a'+b'), i.e., the straight line which is a product of these points, hence (a+b)(a'+b') or aa'+ab'+a'b+bb' or 0+ab'+a'b+0 = ab'+a'b. The other slanting axes is the straight line

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x) See above, page 15.

$(a'+b)(a+b')$, or $aa'+a'b'+ab+bb'$, or $ab+a'b'$. The point of their intersection (or union) - the origin of the co-ordinates - is then found to be: $ab'+a'b+ab+a'b'$, and this expression is, as we know, equal to unity. Therefore the same point is 0 from one standpoint, and 1 from another as we have just demonstrated. The minimum and the maximum actualy do coincide in this case. The same, dually, can be demonstrated for the straight line (circle) at infinity: as determined by points at infinity on the main axes, it is, as we know, unity: $1_{a+a'} \times 1_{b+b'}$; but if we regard it as determined by points on the slanting axes, it then turns out to be 0. The minimum and the maximum are again found to coincide. We therefore have for the centre and the circumference of the infinite circle: 0 = 1, or more strictly speaking: $0_{aa'} + 0_{bb'} = 1_{a+a'} \times 1_{b+b'}$.

This is so undoubtedly, but how can this dialectical coincidence be understood without the help of algebraic-geometrical logic? Does it not perhaps run counter to the principles of common sense? The answer is in the negative. For we have to do here with a convergence not of the lowest zero with the highest unity, but of the highest zero with the lowest unity. Let us recall to mind that both zeros and unities are marked by graduation. Thus, the origin of the co-ordinates, as we already know, with respect to comprehension, is the highest zero - it is the additive union of the axial zeros; and, dually, the straight line (circle) at infinity is the lowest unity - the multiplication union of the unitarian points. The origin of the co-ordinates is the totality of the zeros, the circle at infinity - the community or generality of unities. And here we have the cause of the co-incidence of such a zero with such a unity ($0_{aa'}+0_{bb'} = 1_{a+a'} \times 1_{b+b'}$). In the series of zeros and unities we come across such an element in which they converge as at their common limit, as the degrees of heat and frost converge on a thermometric scale at zero.

It then turns out that from the standpoint of geometrical logic the division of Spinoza's infinite modes - the depiction of which is the origin of the co-ordinates and the circle at infinity - is by no means final; that they may actually be comprehended as being most straitly joined with each other, even converging into each other and being equivalent reciprocally - as if two aspects of one and the same thing, just as Spinoza probably supposed.

x x

x

[tu następuje 1½ strony do przetłumaczenia]

Powstaje teraz pytanie: jaka to jedność - z punktu widzenia logiki geometrycznej - odpowiadać winna atrybutowi rozciągłości, najniższemu zeru, zobrazowanemu w dwuwymiarowej logice geometrycznej przez samą nieskończoną płaszczyznę współrzędnych. Rozwinięcia tego zera odpoznaliśmy w systemie Spinozy jako ruch w ogóle (zerowa oś pozioma), spoczynek w ogóle (zerowa oś pionowa) i ich zjednoczenie (zerowy środek współrzędnych). Wiemy, że elementem dualnym względem osi poziomej będzie punkt w nieskończoności na osi pionowej (jedność $1_{a+a'}$ - całość spoczynków); elementem dualnym względem osi pionowej będzie punkt w nieskończoności na osi poziomej (jedność $1_{b+b'}$ - całość ruchów), zaś elementem dualnym względem środka współrzędnych (zerowego objawu nieskończonego) będzie prosta w nieskończoności (jednościowy objaw nieskończony). Zachodzi teraz pytanie, jaka to będzie ta czwarta jedność odpowiadać mająca dwoiście czwartemu zeru, samej płaszczyźnie współrzędnych, tj. atrybutowi rozciągłości? Biorąc rzecz algebraicznie, będzie to punkt $1_{a+a'} + 1_{b+b'}$ (wobec tego że płaszczyzna współrzędnych wyraża się jako $0_{aa'} \times 0_{bb'}$). Geometrycznie będzie to punkt, wychodzący już poza płaszczyznę współrzędnych i znajdujący się w nieskończoności na trzeciej osi (w trzecim wymiarze). Ontologicznie - będzie to całość spoczynków $(1_{a+a'})$ i ruchów $(1_{b+b'})$[x], pełnia treści światowej. I ta właśnie najwyższa jedność wyrażałaby substancję Spinozy, o ile się tu przejawia w atrybucie rozciągłości. Tej jedności substancjalnej odpowiadałoby dualnie najniższe zero, najmniej określony substrat przestrzenny, atrybut przestrzeni (przy dwóch wymiarach - płaszczyzna współrzędnych, przy trzech - przestrzeń trójwymiarowa).[xx]

Oczywiście, i ten najmniej określony substrat bytowy zawarty jest (i przezwyciężony) w substancji, albowiem wszystko jest w niej zawarte, a więc i to najniższe zero bytowe $(0 < 1)$. Świat jednak cały, ogół elementów poszczególnych, a przede wszystkim zasada immanentna tego świata $(0=1)$ powstać może tylko z współdziałania substancji i substratu, zasad ostatecznych bytu, powstać może wtedy, gdy ten najmniej określony substrat, ta "natura chaotyczna", wyłonił się z substancji i stał się "miejscem", odbiornikiem jej kształtujących, prawdziwie boskich czynności. To jest warunkiem powstania immanentnej zasady świata, która u Spinozy występuje jako dwu-

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x) Trzeba tę całość spoczynków i ruchów $(1_{a+a'} + 1_{b+b'})$ odróżnić od $1_{a+a'} \times 1_{b+b'}$ (objaw nieskończony drugiego rodzaju), czyli od przeciętnej ruchu i spoczynku wszystkich ciał w świecie.

xx) Tak się sprawa przedstawia, o ile bierzemy pod uwagę jeden tylko atrybut substancji - rozciągłość. Co dotyczy drugiego atrybutu - myśli, to wiemy, że struktura jego jest analogiczna do struktury rozciągłości, co już głosił Spinoza i co zostało potwierdzone przez fakt logiki geometrycznej (por. str. 3). W ten sposób obraz rozwinięcia atrybutu myśli (obraz przestrzeni myślnej) pokrywałby się całkowicie z obrazem architektonicznym, jaki otrzymaliśmy dla atrybutu rozciągłości.

stronny modus infinitus i - jak widzimy - jest zbiegiem obniżonej najwyższej substancjalnej jedności i sublimowanego najniższego substratowego zera (0 = 1) - por. str. 17.

Bogiem-Naturą (Deus sive Natura) jest więc nie substancja Spinozy (najwyższa jedność),w której natura jest jeszcze przezwyciężona,lecz modus infinitus,najniższa jedność zrównoważona z najwyższym zerem: Bóg, przejawiony już substratowo,przejawiony w naturze lub odwrotnie a równoważnie: natura już nie chaotyczna,lecz uporządkowana.

Tak oto widzimy,jak logika (ontologia) geometryczna w swej matematyczno-jakościowej analizie świata myśli i rozciągłości przenika metodycznie do głębin metafizycznych,w które Spinoza był wpatrzony,i oświetla jego system,ujawniając leżącą u jego podstawy architektonikę,która w zasadzie okazuje się architektoniką prawdziwą,wymaga jednak na swych szczytach pewnych ważnych korektur.

1939 r.