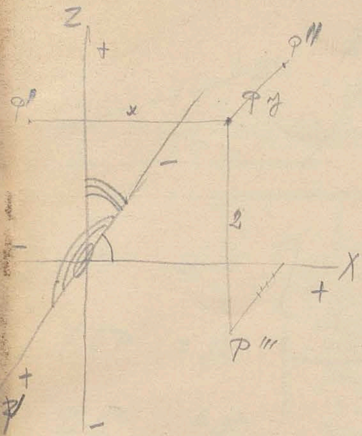
I. V. S. 1892/93

Dr. Emil Weyr

Analytische Geometrie des Raumes

M. Smoluchowski

17/10



P' P'' P''' = orthog. Proj.

[illegible]

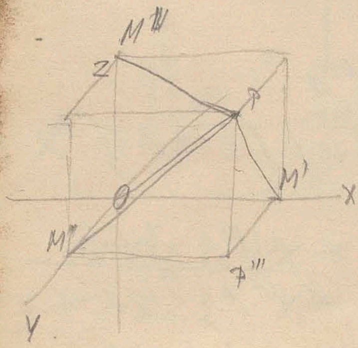
[illegible]

8 Octanten [illegible]

+ x	+ y	+ z	[illegible] X Y
+	−	+	
−	+	+	
−	−	+	
+	+	−	[illegible]
+	−	−	
−	+	−	
−	−	−	

$x = a$
$y = b$
$z = c$ } [illegible]

[illegible] constr. [illegible]:



$OM' = P'P = x$

etc.

[illegible] $OP = \sqrt{\ } = r$

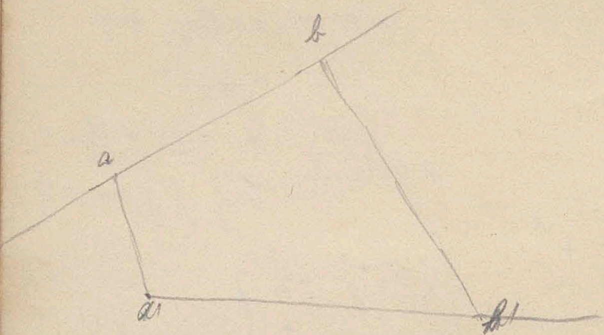
$PM' \perp X$ [illegible]

$PM'' \perp Y$

$PM''' \perp Z$

Projection einer Geraden auf eine andere

Proj. [illegible]



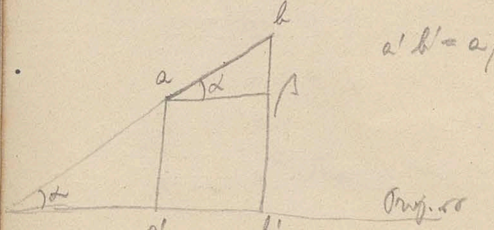
x = Proj. r [illegible] X-Ax

y = Proj. r [illegible] Y -

z = " - Z -

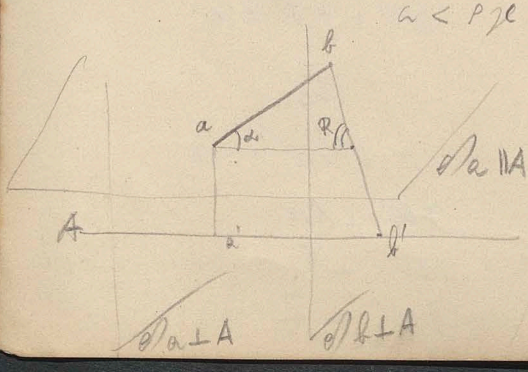
BH.

$\overline{a'b'} = \overline{ab} \cos\alpha$ [illegible]



$a'b' = a\beta = ab \cos\alpha$

[illegible]

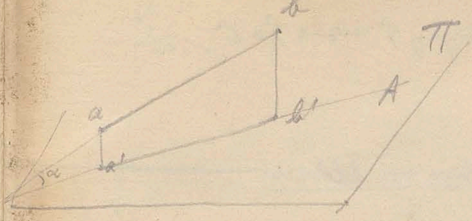


[illegible] ∥ A [illegible]

$a'b' = ab \cos\alpha$

∡ [illegible]

[illegible]



[illegible] Π [illegible]

$\text{proj.}\ \overline{ab} = \overline{ab}\cos\alpha$

$\angle X\,OP = \alpha$
$\angle Y\,OP = \beta$
$\angle Z\,OP = \gamma$
} [illegible]

$x = r\cos\alpha$
$y = r\cos\beta$
$z = r\cos\gamma$

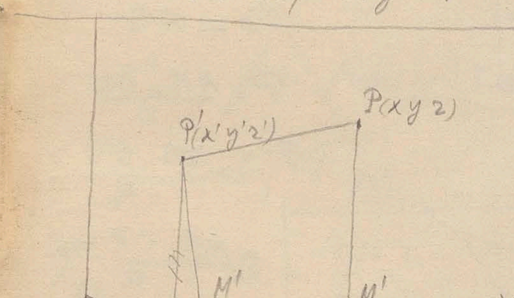
$r^2 = OM'^2 + M'P^2 = OM'^2 + P'''P^2 + M'P'''^2$

$= OM'^2 + OM'''^2 + OM''^2$

$r^2 = x^2 + y^2 + z^2$

$r^2 = r^2[\cos^2\alpha + \cos^2\beta + \cos^2\gamma]$

$\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$



$\overline{P'P} = e$
$PM' \perp X$
$PM'_1 \perp X$
$OM' = x$
$OM'_1 = x'$
$\overline{M'_1M'} = x - x'$
$= \text{Proj.}\ P'P$

$x - x' = \text{Proj.}\ e$ [illegible] X [illegible]
$y - y' = \text{Proj.}\ e$ Y
$z - z' =$ Z

$\alpha, \beta, \gamma = \angle$ [illegible]

$x - x' = e \cos\alpha$

$y - y' = e \cos\beta$

$z - z' = e \cos\gamma$

$$(x-x')^2 + (y-y')^2 + (z-z')^2 = e^2$$

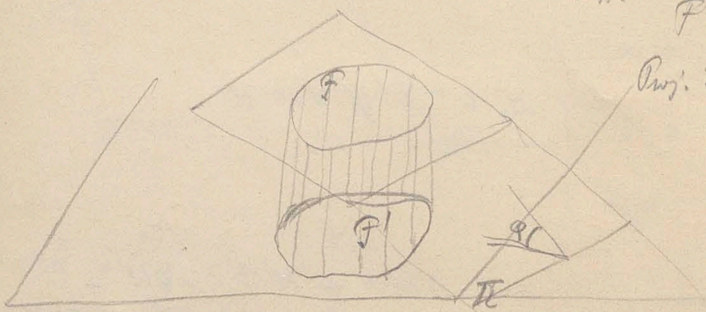
Projection von Flächen

[illegible]

BH:

$F' = F \cos\varphi$

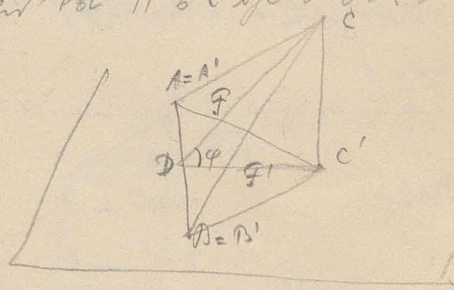
$\text{Proj. } F = F \cos\varphi$



[illegible]

[illegible] || Proj. [illegible]

[illegible] π [illegible]



$CD \perp AB$

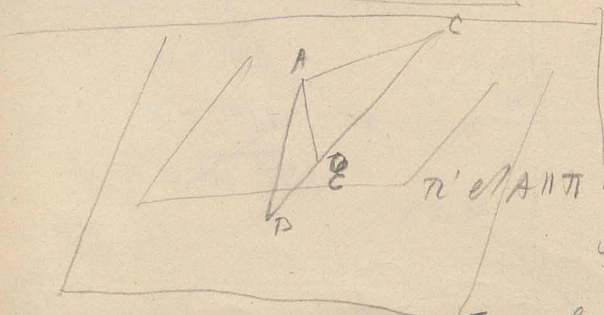
$C'D \perp AB$

$F = \frac{AB \cdot CD}{2}$

$F' = \frac{AB \cdot C'D}{2}$

$= \frac{AB \cdot CD \cos\varphi}{2}$

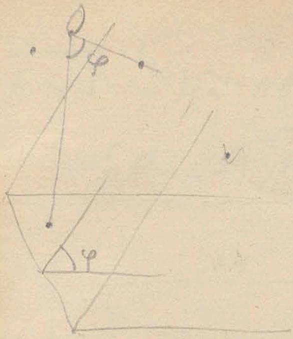
$= F \cos\varphi$



$\pi' \ni A \parallel \pi$

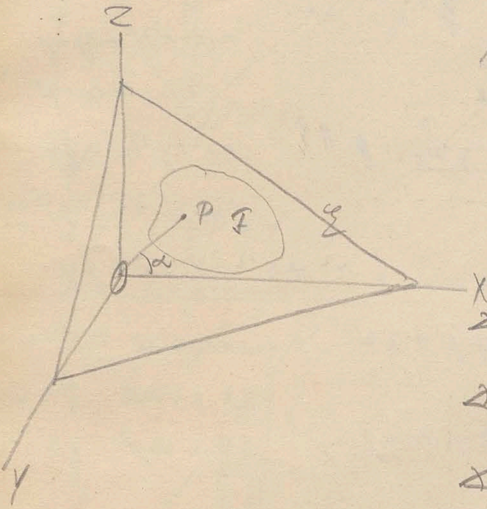
[illegible]

[illegible] Σ



[illegible]

∢ [illegible] 2 [illegible]



[illegible]

[illegible]

$OP \perp$ Eb. $\mathfrak{E}$ ~~XYZ~~

$\sphericalangle \alpha = \sphericalangle\ \mathfrak{E},\ \cancel{Y}OZ$

$\sphericalangle \beta = \mathfrak{E}\widehat{\ XOZ}$

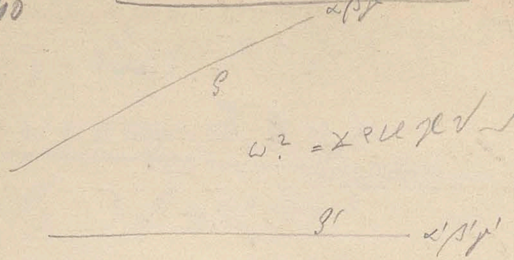
$\sphericalangle \gamma = \mathfrak{E}\widehat{\ XOY}$

[illegible] 3 C. [illegible]

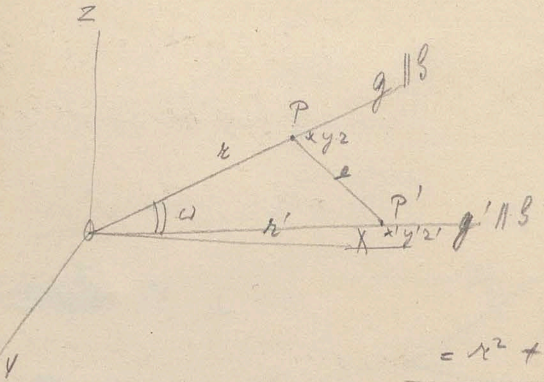
$F_{xy},\ F_{xz},\ F_{yz}$

$$\left.\begin{aligned} F_{yz} &= F \cos\alpha \\ F_{xz} &= F \cos\beta \\ F_{xy} &= F \cos\gamma \end{aligned}\right\}\quad (F_{yz})^2 + (F_{xz})^2 + (F_{xy})^2 = F^2$$

Winkel zweier Geraden



$\omega^2 =$ [illegible]



$$e^2 = r^2 + r'^2 - 2rr'\cos\omega$$
$$= (x-x')^2 + (y-y')^2 + (z-z')^2$$
$$= x^2+y^2+z^2 - 2xx' - 2yy' - 2zz' + x'^2 + y'^2 + z'^2$$
$$= r^2 + r'^2 - 2(xx' + yy' + zz')$$

$$rr'\cos\omega = xx' + yy' + zz'$$

$x = r\cos\alpha$	$x' = r'\cos\alpha'$
$y = r\cos\beta$	$y' = r'\cos\beta'$
$z = r\cos\gamma$	$z' = r'\cos\gamma'$

$$\cos\omega = \cos\alpha\cos\alpha' + \cos\beta\cos\beta' + \cos\gamma\cos\gamma'$$

[illegible] $r = r' = 1$ [illegible]

[illegible]:

$g' \perp g \qquad \cos\alpha\cos\alpha' + \cos\beta\cos\beta' + \cos\gamma\cos\gamma' = 0$

$$\cos\omega = \cos\alpha\cos\alpha' + \cos\beta\cos\beta' + \cos\gamma\cos\gamma'$$

$$\sin^2\omega = 1 - [\cos\alpha\cos\alpha' + \cos\beta\cos\beta' + \cos\gamma\cos\gamma']^2$$

$$= [\cos^2\alpha + \cos^2\beta + \cos^2\gamma]\,[\cos^2\alpha' + \cos^2\beta' + \cos^2\gamma'] -$$

$$- [\cos\alpha\cos\alpha' + \cos\beta\cos\beta' + \cos\gamma\cos\gamma']^2$$

$$= [\cos\alpha\cos\beta' - \cos\beta\cos\alpha']^2 + [\cos\beta\cos\gamma' - \cos\gamma\cos\beta']^2 +$$

$+[\cos\gamma\cos\alpha' - \cos\alpha\cos\gamma']^2$

f. Identität [illegible]:

$$[a^2+b^2+c^2][a'^2+b'^2+c'^2] - [aa'+bb'+cc']^2 \equiv$$

$$[ab'-a'b]^2 + [bc'-b'c]^2 + [ca'-c'a]^2$$

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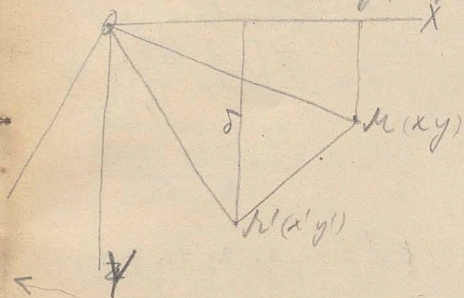
f. [illegible] $\alpha \sim [\text{Proj.}]^2$:



$$\delta = \frac{y+y'}{2}(x-x') + \frac{x'y'}{2} - \frac{xy}{2}$$

$$= \frac{1}{2}\left[xy + xy' - x'y - x'y' + x'y' - xy\right]$$

$$= \frac{1}{2}\left[xy' - x'y\right]$$

$OP' = 1$ $\quad OP = 1$

$x = \cos\alpha$ $\quad x' = \cos\alpha'$

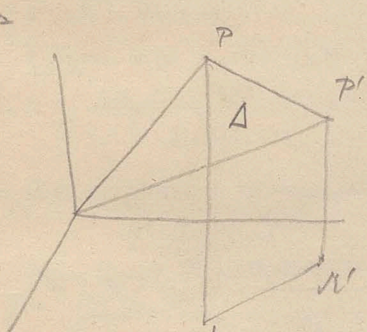
$y = \cos\beta$ $\quad y' = \cos\beta'$

$\cos\alpha\cos\beta' - \cos\beta\cos\alpha' = 2\,\delta$

$\Delta = \frac{1}{2}\sin\omega$

$\Sigma\, e^2\, e$ Proj. $= [\sin\omega]^2$

s. f.:



$\frac{1}{2}\cos\alpha\cos\beta' - \cos\beta\cos\alpha'$

$\frac{1}{2}\cos\beta\cos\gamma' - \text{— —}$ $\quad\Big\}$ Proj. e Δ

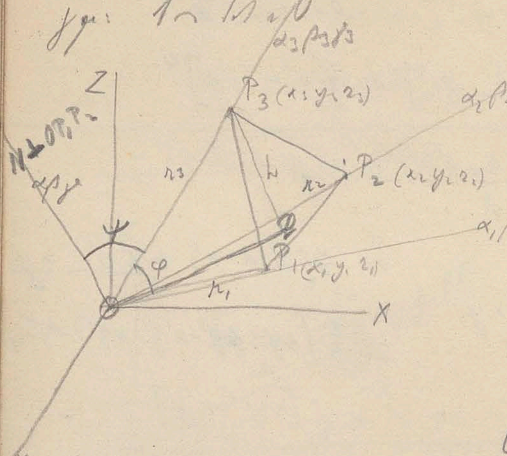
$\frac{1}{2}\cos\gamma\cos\alpha'$ ———

$$\Delta^2 = \frac{1}{4}\left\{[\quad]^2 + [\quad]^2 + [\quad]^2\right\} = \frac{1}{4}\sin^2\omega$$

etc. [illegible]

Rauminh. d. Tetraed. [illegible]

[illegible]



[illegible]

$\measuredangle P_3 O \mathfrak{q} = \varphi \qquad \measuredangle P_1 O P_2 = \omega$

$$V = \mathfrak{F}[O P_1 P_2] \cdot \frac{h}{3}$$

$$= r_1 r_2 \frac{\sin\omega \; h}{6}$$

$$6V = r_1 r_2 r_3 \sin\varphi \sin\omega$$

[illegible] $r_1 r_2 r_3$ [illegible]

[illegible]

$\sin^2\omega =$

$$[\cos\alpha_1 \cos\beta_2 - \cos\beta_1 \cos\alpha_2]^2 + [\cos\beta_1 \cos\gamma_2 - \cos\gamma_1 \cos\beta_2]^2 +$$
$$+ [\cos\gamma_1 \cos\alpha_2 - \cos\alpha_1 \cos\gamma_2]^2$$

$N \perp OP_1 P_2$

$N \perp O\mathfrak{q}$

$\sin\varphi = \cos\psi$

$$6V = r_1 r_2 r_3 \sin\omega \cos\psi$$

$$\cos\psi = \cos\alpha \cos\alpha_3 + \cos\beta \cos\beta_3 + \cos\gamma \cos\gamma_3$$

[illegible] $\alpha \beta \gamma$ [illegible]

$ON \perp OP_1: \quad \cos\alpha\cos\alpha_1 + \cos\beta\cos\beta_1 + \cos\gamma\cos\gamma_1 = 0$

$ON \perp OP_2: \quad \cos\alpha\cos\alpha_2 + \cos\beta\cos\beta_2 + \cos\gamma\cos\gamma_2 = 0$

$\begin{vmatrix} \cos\alpha_2 & \cos\beta_2 \\ \cos\alpha_1 & \cos\beta_1 \end{vmatrix}$

$\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$

$$\cos\beta\,[\cos\beta_1\cos\alpha_2 - \cos\beta_2\cos\alpha_1] + \cos\gamma\,[\cos\gamma_1\cos\alpha_2 - \cos\gamma_2\cos\alpha_1] = 0$$

$$\frac{\cos\beta}{\cos\gamma_1\cos\gamma_2 - \cos\alpha_2\cos\gamma_1} = \frac{\cos\gamma}{\cos\beta_1\cos\alpha_2 - \cos\beta_2\cos\alpha_1}$$

$$\frac{\cos\beta}{\cos\gamma_1\cos\alpha_2-\cos\alpha_1\cos\gamma_2}=\frac{\cos\gamma}{\cos\alpha_1\cos\beta_2-\cos\beta_1\cos\alpha_2}$$

$$\cos\alpha[\cos\alpha_1\cos\beta_2-\cos\alpha_2\cos\beta_1]+\cos\gamma\,[\cos\gamma_1\cos\beta_2-\cos\gamma_2\cos\beta_1]=0$$

$$\frac{\cos\alpha}{\cos\beta_1\cos\gamma_2-\cos\beta_2\cos\gamma_1}=\frac{\cos\gamma}{\cos\alpha_1\cos\beta_2-\cos\alpha_2\cos\beta_1}$$

$$\frac{\cos\alpha}{\cos\beta_1\cos\gamma_2-\cos\beta_2\cos\gamma_1}=\frac{\cos\beta}{\cos\gamma_1\cos\alpha_2-\cos\gamma_2\cos\alpha_1}=\frac{\cos\gamma}{\cos\alpha_1\cos\beta_2-\cos\alpha_2\cos\beta_1}=\lambda$$

$$\cos\alpha=\lambda\,[\cos\beta_1\cos\gamma_2-\cos\beta_2\cos\gamma_1]$$
$$\cos\beta=\lambda\,[\cos\gamma_1\cos\alpha_2-\cos\gamma_2\cos\alpha_1]$$
$$\cos\gamma=\lambda\,[\cos\alpha_1\cos\beta_2-\cos\alpha_2\cos\beta_1]$$

$$1=\lambda^2\sin^2\omega \qquad \lambda=\frac{1}{\pm\sin\omega}$$

$$\begin{cases}\cos\alpha=\dfrac{\cos\beta_1\cos\gamma_2-\cos\beta_2\cos\gamma_1}{\pm\sin\omega}\\[2ex]\cos\beta=\dfrac{\cos\gamma_1\cos\alpha_1-\cos\gamma_2\cos\alpha_1}{\pm\sin\omega}\\[2ex]\cos\gamma=\dfrac{\cos\alpha_1\cos\beta_2-\cos\alpha_2\cos\beta_1}{\pm\sin\omega}\end{cases}$$

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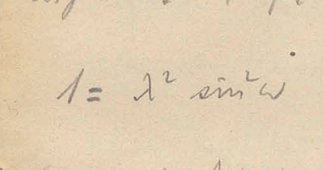


$$\cos\alpha=\frac{\cos\beta_1\cos\gamma_2-\cos\beta_2\cos\gamma_1}{\sin\omega}$$

$$\cos\beta=\frac{\cos\gamma_1\cos\alpha_2-\cos\gamma_2\cos\alpha_1}{\sin\omega}$$

$$\cos\gamma=\frac{\cos\alpha_1\cos\beta_2-\cos\alpha_2\cos\beta_1}{\sin\omega}$$

$$6V = r_1 r_2 r_3 \sin\omega \cos\psi$$

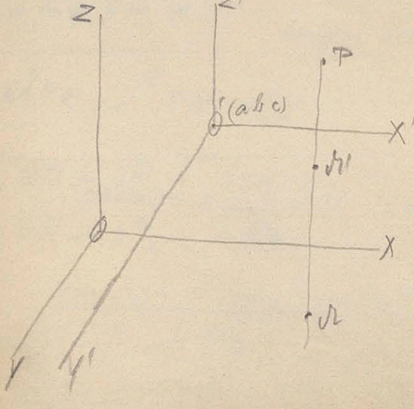
$$\cos\psi = \cos\alpha \cos\alpha_3 + \cos\beta \cos\beta_3 + \cos\gamma \cos\gamma_3$$

$$\sin\omega \cos\psi = \cos\alpha_3(\cos\beta_1 \cos\gamma_2 - \cos\beta_2 \cos\gamma_1) +$$
$$+ \cos\beta_3(\cos\gamma_1 \cos\alpha_2 - \cos\gamma_2 \cos\alpha_1) +$$
$$+ \cos\gamma_3(\cos\alpha_1 \cos\beta_2 - \cos\alpha_2 \cos\beta_1)$$

$$6V = r_3 \cos\alpha_3 (r_1 \cos\beta_1 \, r_2 \cos\gamma_2 - r_2 \cos\beta_2 \, r_1 \cos\gamma_1) +$$
$$+ \text{— — —}$$
$$= x_3(y_1 z_2 - y_2 z_1) + y_3(z_1 x_2 - z_2 x_1) +$$
$$+ z_3(x_1 y_2 - x_2 y_1)$$

$$= \begin{vmatrix} x_1 & y_1 & z_1 \\ x_2 & y_2 & z_2 \\ x_3 & y_3 & z_3 \end{vmatrix}$$

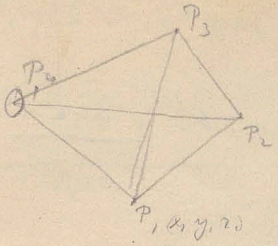
$$6V = z_1(x_2 y_3 - x_3 y_2) + z_2(x_3 y_1 - x_1 y_3) + z_3(\quad)$$



C. - Transform.

I - Tr.

$$\begin{cases} z = z' + c \\ x = x' + a \\ y = y' + b \end{cases}$$



[illegible]

$$\begin{cases} x_1 = x'_1 + x_4 \\ y_1 = y'_1 + y_4 \\ z_1 = z'_1 + z_4 \end{cases}$$

$$\begin{cases} x_2 = x'_2 + x_4 \\ x \;\text{———} \\ \text{———} \end{cases} \qquad \begin{cases} x_3 = x'_3 + x_4 \\ \text{———} \\ \text{———} \end{cases}$$

$$6V = z'_1 (x'_2 y'_3 - \;\; —) + \text{— —}$$

$$= (z_1 - z_4)\left[(x_2 - x_4)(y_3 - y_4) - (x_3 - x_4)(y_2 - y_4)\right] +$$
$$+ (z_2 - z_4)\left[(x_3 - x_4)(y_1 - y_4) - (x_1 - x_4)(y_3 - y_4)\right] +$$
$$+ (z_3 - z_4)\left[(x_1 - x_4)(y_2 - y_4) - (x_2 - x_4)(y_1 - y_4)\right]$$

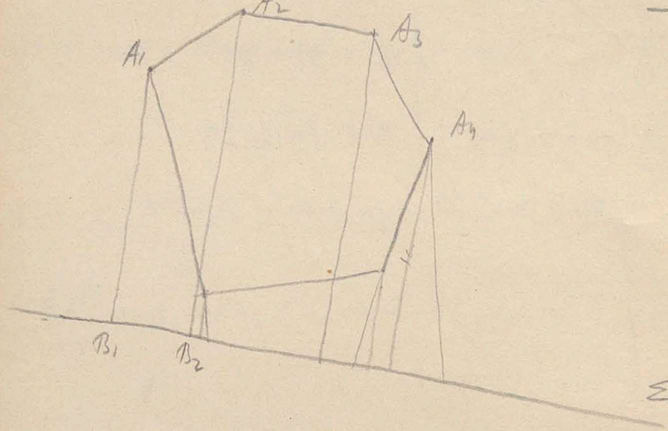
$$= \begin{vmatrix} 1 & x_1 & y_1 & z_1 \\ 1 & x_2 & y_2 & z_2 \\ 1 & x_3 & y_3 & z_3 \\ 1 & x_4 & y_4 & z_4 \end{vmatrix}$$

[illegible] : e Tetr. = 0

$6V = 0$, $V = 0$ $\qquad \Delta = 0$

[illegible] Polyg. [illegible] Proj. [illegible] Σ [illegible] Proj. [illegible] $=0$.

[illegible]



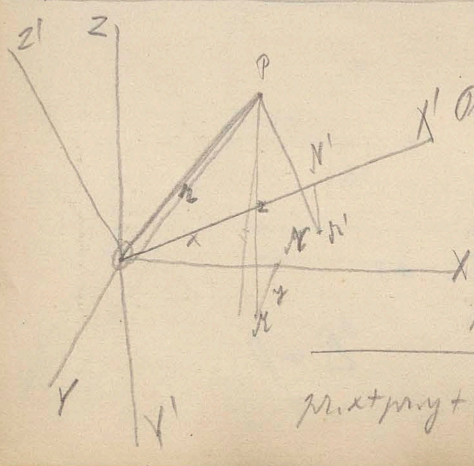
$B_1 B_2 =$ Proj. $A_1 A_2$; — $B_2 B_3$ — $B_3 B_4$

$B_1 B_2 + B_2 B_3 + B_3 B_4 + B$ — — $B_{n-1} B_n + B_n B_1 = 0$

Proj. $A_1 A_2 +$ Proj. $A_2 A_3 +$ — — Proj. $A_n A_1 = 0$

$B_1 B_n = B_1 B_2 + B_2 B_3 +$ — — $B_{n-1} B_n$

Proj. $A_1 A_n =$ Proj. $A_1 A_2 +$ Proj. $A_2 A_3 +$ — Proj. $A_{n-1} A_n$



Proj. [illegible] Σ

Proj. $OP =$ Proj. $ON +$ Proj. NM + Proj. MP

proj. $r =$ pr. $x +$ pr. $y +$ pr. z

proj. $r =$ pr. $x' +$ pr. $y' +$ pr. z'

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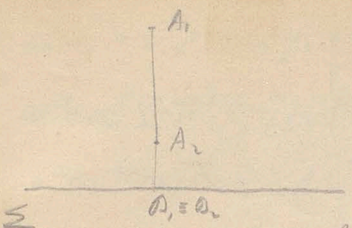
pr. $x +$ pr. $y +$ pr. $z =$ pr. $x' +$ pr. $y' +$ pr. z'



$A_1 A_2 \perp \Sigma$

Proj. $A_1 A_2 = 0$

1er gr C.

$\Sigma \perp OZ\ OY$ ∴ at as proj. $x = \mu_1 x' + \mu_2 y' + \mu_3 z'$

$\Sigma \perp OX\ OZ$ — — — $y =$ — — —

$\Sigma \perp OX\ OY$ — $z =$ — —

∴ 6 9 p/ C. 8 as lin fn. e ~s fe.

$x = a_1 x' + b_1 y' + c_1 z'$ ∴ ^{e} p/ C. homog. lin fn. e

$y = a_2 x' + b_2 y' + c_2 z'$

$z =$

Rectan. C. :

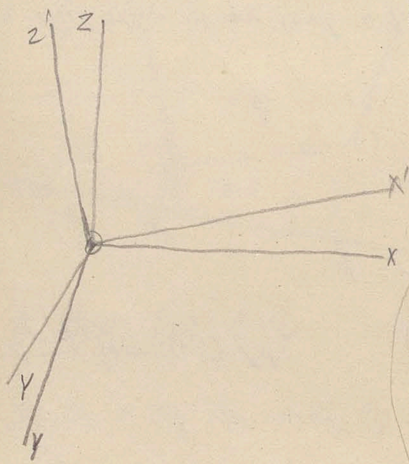
$\measuredangle\ X'\ \phi X = \alpha_1\ \beta_1\ \gamma_1$

$\measuredangle\ Y'\ \phi X$

$\measuredangle\ Z' = \alpha_3\ \beta_3\ \gamma_3$

$$\left.\begin{aligned} \Sigma = X\sigma: &\quad x = x'\cos\alpha_1 + y'\cos\alpha_2 + z'\cos\alpha_3 \\ \Sigma = Y\sigma: &\quad y = x'\cos\beta_1 + y'\cos\beta_2 + z'\cos\beta_3 \\ \Sigma = Z\sigma: &\quad z = x'\cos\gamma_1 + y'\cos\gamma_2 + z'\cos\gamma_3 \end{aligned}\right\}$$

$$\left.\begin{aligned} x &= x'\cos\alpha_1 + y'\cos\alpha_2 + z'\cos\alpha_3 \\ y &= x'\cos\beta_1 + y'\cos\beta_2 + z'\cos\beta_3 \\ z &= x'\cos\gamma_1 + y'\cos\gamma_2 + z'\cos\gamma_3 \end{aligned}\right\}$$ [illegible] $\perp$ [illegible]



[illegible] $\perp$ [illegible]:

$\cos\alpha_1\cos\alpha_2 + \cos\beta_1\cos\beta_2 + \cos\gamma_1\cos\gamma_2 = \cos X'OY'$

$\cos\alpha_2\cos\alpha_3 + \cos\beta_2\cos\beta_3 + \cos\gamma_2\cos\gamma_3 = \cos Y'OZ'$

$\cos\alpha_3\cos\alpha_1 + \cos\beta_3\cos\beta_1 + \cos\gamma_3\cos\gamma_1 = \cos Z'OX'$

$x \,\Big|\, \cos\alpha_1, \cos\beta_1, \cos\gamma_1$

[illegible] $X' \perp Y' \perp Z'$ | $=0$

[illegible] $=0$ | $=0$ $=0$

$$\begin{aligned} x' &= x\cos\alpha_1 + y\cos\beta_1 + z\cos\gamma_1 \\ y' &= x\cos\alpha_2 + y\cos\beta_2 + z\cos\gamma_2 \\ z' &= x\cos\alpha_3 + y\cos\beta_3 + z\cos\gamma_3 \end{aligned} \;\Bigg|\; \begin{aligned} &\cos\alpha_1 \\ &\cos\beta_1 \\ &\cos\gamma_1 \end{aligned}$$

[illegible]

$$x\cos\alpha_1 + y\cos\beta_1 + z\cos\gamma_1 = x'(\cos^2\alpha_1 + \quad) + \left.\begin{aligned} &+ y'(\cos\alpha_2\cos\alpha_1 \quad) + \\ &+ z'(\cos\alpha_3\cos\alpha_1 \quad) \end{aligned}\right\} = 0$$

$$\left.\begin{aligned}\cos^2\alpha_1+\cos^2\beta_1+\cos^2\gamma_1&=1\\ \cos^2\alpha_2+\cos^2\beta_2+\cos^2\gamma_2&=1\\ \cos^2\alpha_3+\cos^2\beta_3+\cos^2\gamma_3&=1\end{aligned}\right\}$$

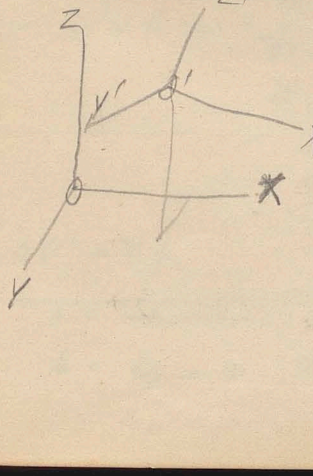
$$\left.\begin{aligned}\cos\alpha_1\cos\alpha_2+\cos\beta_1\cos\beta_2+\cos\gamma_1\cos\gamma_2&=0\\ \cos\alpha_1\cos\alpha_3+\cos\beta_1\cos\beta_3+\cos\gamma_1\cos\gamma_3&=0\\ \cos\alpha_2\cos\alpha_3+\cos\beta_2\cos\beta_3+\cos\gamma_2\cos\gamma_3&=0\end{aligned}\right\}$$ [illegible] $\perp$

[illegible]:

$$\left.\begin{aligned}\cos^2\alpha_1+\cos^2\alpha_2+\cos^2\alpha_3&=1\\ \cos^2\beta_1+\cos^2\beta_2+\cos^2\beta_3&=1\\ \cos^2\gamma_1+\cos^2\gamma_2+\cos^2\gamma_3&=1\end{aligned}\right\}$$

$$\left.\begin{aligned}\cos\alpha_1\cos\beta_1+\cos\alpha_2\cos\beta_2+\cos\alpha_3\cos\beta_3&=0\\ \cos\beta_1\cos\gamma_1+\cos\beta_2\cos\gamma_2+\cos\beta_3\cos\gamma_3&=0\\ \cos\gamma_1\cos\alpha_1+\cos\gamma_2\cos\alpha_2+\cos\gamma_3\cos\alpha_3&=0\end{aligned}\right\}$$ [illegible] $\perp$ [illegible]

[illegible] $\perp$ [illegible]



[illegible] abc

[illegible]

X	$\alpha_1\ \alpha_2\ \alpha_3$
Y	$\beta_1\ \beta_2\ \beta_3$
Z	$\gamma_1\ \gamma_2\ \gamma_3$

[illegible]

$$\left.\begin{aligned} x &= a + x'\cos\alpha_1 + y'\cos\alpha_2 + z'\cos\alpha_3 \\ y &= b + x'\cos\beta_1 + y'\cos\beta_2 + z'\cos\beta_3 \\ z &= c + x'\cos\gamma_1 + y'\cos\gamma_2 + z'\cos\gamma_3 \end{aligned}\right\}$$ [illegible] $\perp$

[illegible] lin. fc ([illegible] AC) [illegible]

[illegible] homog. fc.

[illegible]

[illegible] lin. fc [illegible] $\perp$ [illegible]

[illegible] Transf. [illegible]

[illegible]

$x = a$ [illegible] a ‖ [illegible] YZ [illegible] | [illegible]

[illegible]

[illegible]

[illegible] $x = a$, $y = b$ [illegible] ‖ Z [illegible]

$\left.\begin{aligned} x &= a \\ y &= b \\ z &= c \end{aligned}\right\}$ [illegible] 3 [illegible]

[illegible] $x = a$ ‖ OZ [illegible]

[illegible] Transf. d. Coord.

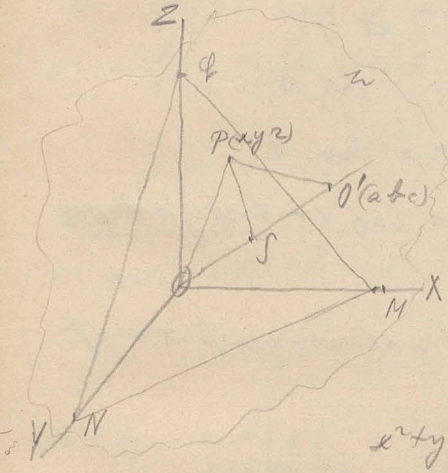
$x = x' \cos\alpha_1 + y' \cos\alpha_2 + z' \cos\alpha_3$

$a = x' \cos\alpha_1 + y' \cos\alpha_2 + z' \cos\alpha_3$ [illegible]

[illegible]

[illegible]

[illegible]



[illegible] : [illegible] Perp. etc.

$OS \perp$ Ebene

$SO' = OS$

$OP = O'P$

$OP^2 = O'P^2$

$x^2 + y^2 + z^2 = (x-a)^2 + (y-b)^2 + (z-c)^2$

$ax + by + cz = \frac{a^2 + b^2 + c^2}{2}$ [illegible]

[illegible] : [illegible] $OS = p$

$\left.\begin{matrix} \measuredangle \alpha, \\ \beta, \\ \gamma \end{matrix}\right\} \measuredangle$ [illegible] perp. [illegible] $\left\{\begin{matrix} X \\ Y \\ Z \end{matrix}\right\}$ [illegible]

$OO' = 2p$

$a = 2p \cos\alpha$

$b = 2p \cos\beta$

$c = 2p \cos\gamma$

$a^2 + b^2 + c^2 = 4p^2$

$2p \cos\alpha . x + 2p \cos\beta . y + 2p \cos\gamma . z = \frac{4p^2}{2}$

$x \cos\alpha + y \cos\beta + z \cos\gamma = p$

$x \cos\alpha + y \cos\beta + z \cos\gamma - p = 0$ [illegible] Normal-Form

[illegible]

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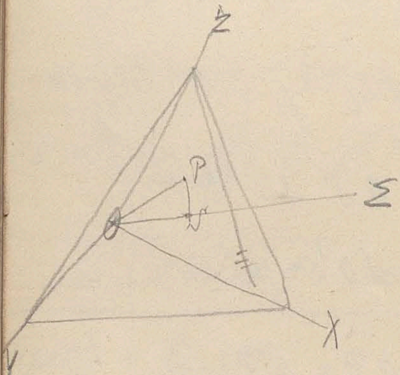
$x \cos\alpha + y \cos\beta + z \cos\gamma - p = 0$

[illegible] CS. [illegible] ∡



OS ⊥

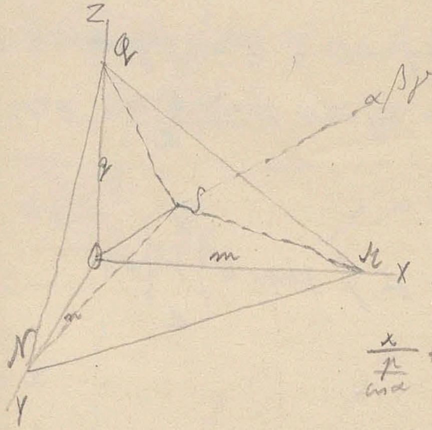
OSP ∡ 90°

OS = Proj. OP

p = Proj. r

p = Proj. x + Proj. y + Proj. z

$= x \cos\alpha + y \cos\beta + z \cos\gamma$



$p = m \cos\alpha$

$= n \cos\beta$

$= q \cos\gamma$

$x \cos\alpha + y \cos\beta + z \cos\gamma - p = 0$

$$\frac{x}{\frac{p}{\cos\alpha}} + \frac{y}{\frac{p}{\cos\beta}} + \frac{z}{\frac{p}{\cos\gamma}} = 1$$

[illegible] $\frac{x}{m} + \frac{y}{n} + \frac{z}{q} = 1$

BA. $Ax + By + Cz + D = 0$ [illegible]

BW.

$$\frac{x}{-\frac{D}{A}} + \frac{y}{-\frac{D}{B}} + \frac{z}{-\frac{D}{C}} = 1 \qquad \begin{cases} m = -\frac{D}{A} \\ n = -\frac{D}{B} \\ q = -\frac{D}{C} \end{cases}$$

[illegible]

[illegible]:

$$z = -\frac{A}{C}x - \frac{B}{C}y - \frac{D}{C} \qquad -\frac{A}{C} = a \quad -\frac{B}{C} = b \quad -\frac{D}{C} = c$$

$$z = ax + by + c$$

[illegible]

[illegible]

$$ax + by - z + c = 0$$

$$\frac{x}{\frac{c}{a}} + \frac{y}{\frac{c}{b}} - \frac{z}{c} + 1 = 0$$

[illegible] N. [illegible]?

$$Ax + By + Cz + D = 0 \mid \lambda$$

$$\lambda Ax + \lambda By + \lambda Cz + \lambda D = 0$$

$$x\cos\alpha + y\cos\beta + z\cos\gamma - p = 0$$

$$\lambda A = \cos\alpha \quad \lambda B = \cos\beta \quad \lambda C = \cos\gamma \quad \lambda D = -p$$

$$\lambda^2[A^2 + B^2 + C^2] = 1$$

$$\lambda = \frac{1}{\pm\sqrt{A^2 + B^2 + C^2}}$$

$$\cos\alpha = \frac{A}{\pm\sqrt{A^2+B^2+C^2}}$$

$$\cos\beta = \frac{B}{\pm\sqrt{A^2+B^2+C^2}}$$

$$\cos\gamma = \frac{C}{\pm\sqrt{A^2+B^2+C^2}}$$

$$p = -\frac{D}{\pm\sqrt{A^2+B^2+C^2}}$$

p [illegible] $\gtrless 0$ [illegible]

[illegible]

$\gtrless 0$ [illegible]

[illegible]

---

3 [illegible] Const. [illegible] 3 [illegible]

[illegible] 3 [illegible]

$P_i\,(x_i\; y_i\; z_i)$

$$\left.\begin{array}{l} Ax_1 + By_1 + Cz_1 = 0 \\ Ax_2 + By_2 + Cz_2 = 0 \\ Ax_3 + By_3 + Cz_3 = 0 \end{array}\right\} \quad \begin{array}{l} A = \\ B = \\ C = \end{array}$$

[illegible]

$Ax + By + Cz + D = 0$ $\qquad m = -\frac{D}{A}$ $\quad n = -\frac{D}{B}$ $\quad q = -\frac{D}{C}$

$0.x + By + Cz + D = 0$ [[illegible] x / $=\infty$]

$By + Cz + D = 0 \qquad m = \infty$ [illegible] ∞ [illegible]

[illegible] $\parallel X$ [illegible]

$Ax + Cz + D = 0 \quad \parallel OY$

$Ax + By + D = 0 \quad \parallel Z$

$D = 0 \quad Ax + By + Cz = 0 \qquad m = 0,\; n = 0,\; q = 0$ [illegible] 0

$A=0$, $B=0$

$m=\infty$
$n=\infty$

$Cz+D=0$ ([illegible])

$z=-\frac{D}{C}=q$ [illegible] $\parallel XOY$

[illegible] $D=0$ $z=0$ $=XOY$

$Ax+D=0$ $\parallel YOZ$

$By+D=0$ $\parallel XOZ$

$A=0$, $B=0$, $C=0$ $D \gtrless 0$

$0x+0y+0z+D=0$

$D=0$ poradoxe [illegible] [illegible]

$m=\infty$
$n=\infty$
$q=\infty$

[illegible]

$Ax+By+Cz+D=0$ $|\lambda$

$m=-\frac{D}{A}$

$\lambda Ax+\lambda By+\lambda Cz+\lambda D=0$

$n=$

$m=-\frac{\lambda D}{\lambda A}$ [illegible]

$q=$

[illegible] | [illegible]

$D=0$ $D'=0$

$\lambda D=0$

$\lambda=\frac{D'}{D}$ $D'=0$ [illegible]

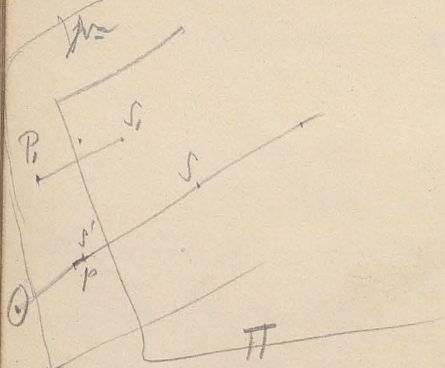
$1=0$

Länge des Perpendikels auf eine Ebene

[illegible]

P_1 [illegible]

31/10 $x\cos\alpha + y\cos\beta + z\cos\gamma - p = 0$ [illegible]



Π_1 [illegible] $P_1 \parallel \Pi$

$OS = p,\ OS' = p'$

$S'S = P_1S_1$

$P_1S_1 = OS - OS'$

$= p - p'$

Π_1 — $x\cos\alpha + y\cos\beta + z\cos\gamma - p' = 0$

$x_1\cos\alpha + y_1\cos\beta + z_1\cos\gamma - p' = 0$

$p' = x_1\cos\alpha + y_1\cos\beta + z_1\cos\gamma$

$P_1S_1 = p - (x_1\cos\alpha + y_1\cos\beta + z_1\cos\gamma)$

$= -[x_1\cos\alpha + y_1\cos\beta + z_1\cos\gamma - p]$

[illegible] N. F. [illegible]

[illegible]

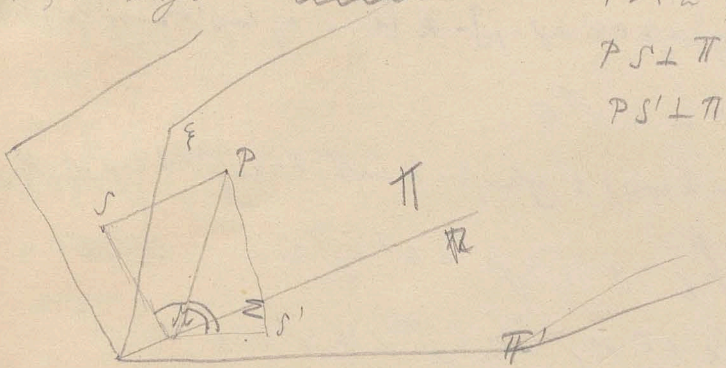
[illegible] N. F. [illegible]

$Ax + By + Cz + D = 0$

$\lambda = \frac{1}{\pm\sqrt{A^2+B^2+C^2}}$ — $\frac{Ax+By+Cz+D}{\pm\sqrt{A^2+B^2+C^2}}$

$$P_1 S_1 = -\frac{Ax_1 + By_1 + Cz_1 + D}{\pm\sqrt{A^2+B^2+C^2}}$$

[illegible] **Ebenen-Büschel**



P [illegible] ξ

$PS \perp \Pi$

$PS' \perp \Pi'$

$$\frac{PS}{PS'} = k = \text{constant} = \frac{\sin \measuredangle}{\sin \measuredangle}$$

$\measuredangle\ SMP = \measuredangle\ \Pi\xi$

[illegible]

[illegible]

BW:

[illegible] Π: $x\cos\alpha + y\cos\beta + z\cos\gamma - p = 0$

Π': $x\cos\alpha' + y\cos\beta' + z\cos\gamma' - p' = 0$

[illegible] k [illegible] constant [illegible]

$P(x'y'z')$ $\quad PS = -[x'\cos\alpha + y'\cos\beta + z'\cos\gamma - p]$

$PS' = -[x'\cos\alpha' + y'\cos\beta' + z'\cos\gamma' - p']$

$$k = \frac{x'\cos\alpha + y'\cos\beta + z'\cos\gamma - p}{x'\cos\alpha' + y'\cos\beta' + z'\cos\gamma' - p'}$$

[illegible] k [illegible] x' [illegible] Indices

$$k = \frac{x \cos\alpha + y \cos\beta + z \cos\gamma - p}{x \cos\alpha' + y \cos\beta' + z \cos\gamma' - p'}$$

$$[x \cos\alpha + y \cos\beta + z \cos\gamma - p] - k\,[x \cos\alpha' + y \cos\beta' + z \cos\gamma' - p'] = 0$$

[illegible]:

$$x(\cos\alpha - k\cos\alpha') + y(\cos\beta - k\cos\beta') + z(\cos\gamma - k\cos\gamma') - (p - kp') = 0$$

[illegible] ... [illegible]

[illegible]

[illegible] $\equiv 0$ ...

[illegible] $k =$ [illegible]

Abgekürzte Bezeichnungsweise

π : $x \cos\alpha + y \cos\beta + z \cos\gamma - p = 0$ $\qquad x \cos\alpha + y \cos\beta + z \cos\gamma - p \equiv \pi$

$\pi = 0$

[$-\pi =$ [illegible]]

$$x \cos\alpha' + y \cos\beta' + z \cos\gamma' - p' \equiv \pi'$$

ξ : $\pi - k\pi' = 0$

$$k = \frac{\sin \pi\xi}{\sin \pi'\xi}$$

[illegible] ξ harm. conj. Ebene:

η : $\pi + k\pi' = 0$

$\pi = 0$

$\pi' = 0$

ξ : $\pi - k\pi' = 0$

η : $\pi - l\pi' = 0$

[illegible] $= \frac{k}{l}$

$$(\pi\,\pi'\,\xi\,\eta) = \frac{k}{l}$$

Doppelverhältnis

$\pi - k\pi' = 0$ $\qquad \frac{k}{\lambda} = $ [illegible]

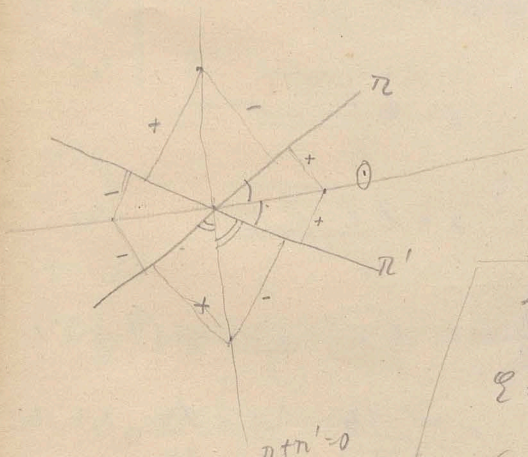
$\pi - \lambda\pi' = 0$

$\left.\begin{array}{l}\pi = 0 \\ \pi' = 0 \\ \pi + k\pi' = 0 \\ \pi - k\pi' = 0\end{array}\right\}$ ~~[illegible]~~ [illegible]

$k = \pm 1$ $\qquad \left.\begin{array}{l}\pi = 0 \\ \pi' = 0 \\ \pi + \pi' = 0 \\ \pi - \pi' = 0\end{array}\right\}$ [illegible]

$k = -1$ [illegible]

$k = +1$ [illegible]



Allgem. Form.

$Ax + By + Cz + D = 0$

$\mathcal{E} \equiv Ax + By + Cz + D$

$\mathcal{E} = 0$

$$\frac{Ax + By + Cz + D}{\sqrt{A^2 + B^2 + C^2}} = 0$$

$$\pi \equiv \frac{Ax + By + Cz + D}{\sqrt{}} \equiv \frac{\mathcal{E}}{\sqrt{A^2 + B^2 + C^2}} \qquad \mathcal{E} = \pi\sqrt{A^2 + B^2 + C^2}$$

$\mathcal{E}=0$ $\qquad n = \dfrac{\mathcal{E}}{\sqrt{A^2+B^2+C^2}}$ $\qquad n' = \dfrac{\mathcal{E}'}{\sqrt{A'^2+B'^2+C'^2}}$

$\mathcal{E}'=0$

$n=0$

$n'=0$ $\qquad n - k n' = 0$

$$\frac{\mathcal{E}}{\sqrt{A^2+B^2+C^2}} - k\frac{\mathcal{E}'}{\sqrt{A'^2+B'^2+C'^2}} = 0$$

$$\mathcal{E} - k\sqrt{\frac{A^2+B^2+C^2}{A'^2+B'^2+C'^2}}\;\mathcal{E}' = 0$$

$$\mathcal{E} - \kappa\;\mathcal{E}' = 0$$

$$\kappa = k\sqrt{\frac{A^2+B^2+C^2}{A'^2+B'^2+C'^2}}$$

[illegible]

[illegible]

$\mathcal{E}=0$

$\mathcal{E}'=0$ $\qquad \kappa = k\dfrac{\sqrt{}}{\sqrt{}}$

$\mathcal{E}-\kappa\,\mathcal{E}'=0$ — k

$\mathcal{E}-\lambda\,\mathcal{E}'=0$ — l $\qquad \lambda = l\dfrac{\sqrt{}}{\sqrt{}}$

$$\frac{\kappa}{\lambda} = \frac{k}{l} = \text{[illegible]} = (1\,2\,3\,4)$$

$$= \frac{\sin 13}{\sin 23} : \frac{\sin 14}{\sin 24}$$

$\mathcal{E}=0$

$\mathcal{E}'=0$

$\mathcal{E}-\kappa\,\mathcal{E}'=0$

$\mathcal{E}+\kappa\,\mathcal{E}'=0$

} 4 harm. Eb.

$\mathfrak{E} - \kappa \mathfrak{E}' = 0$

[illegible]

$F(xyz) = 0 \qquad F = 0$

$F' = 0 \qquad$ [illegible]

$F - \kappa F' = 0$

~~[illegible]~~ [illegible]:

$mF + nF' = 0 \qquad \kappa = -\frac{n}{m}$

$F = 0$	m
$F' = 0$	n
$F'' = 0$	p

[illegible]

$mF + nF' + pF'' = 0$ ← [illegible]

$\mathfrak{E} = 0$

$\mathfrak{E}' = 0 \qquad m\mathfrak{E} + n\mathfrak{E}' + p\mathfrak{E}'' = 0$ [illegible]

$\mathfrak{E}'' = 0 \qquad$ 3 [illegible]

[illegible] Coeff. [illegible]

$\mathfrak{E} \equiv Ax + By + Cz + D = 0$

$\mathfrak{E}' \equiv A'x + B'y + C'z + D' = 0$

$\mathfrak{E}'' \equiv A''x + B''y + C''z + D'' = 0$

$m(Ax + \text{———}) + n(\quad) + p(\quad) = 0$

$(mA + nA' + pA'')x + (mB + nB' + pB'')y + (mC + nC' + pC'')z +$

$+ (mD + nD' + pD'') = 0$

[illegible] $mA + nA' + pA'' = 0$ [illegible] ident. $= 0$ [illegible]

etc.

[illegible]

$m\varphi + n\varphi' + p\varphi'' \equiv 0$ [illegible]

BW. $\left.\begin{array}{l}\varphi = 0 \\ \varphi' = 0\end{array}\right\}$ [illegible] $m\varphi + n\varphi' = 0$ [illegible]

$m\varphi + n\varphi' + p\varphi'' = 0$ [illegible]

[illegible] $p\varphi'' = 0$

$\varphi'' = 0$ [illegible]

[illegible] φ', φ [illegible]

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$F_1 = 0$	m
$F_2 = 0$	n
$F_3 = 0$	p

$mF_1 + nF_2 + pF_3 \equiv 0$ [illegible]

[illegible]: $F_3 = \dfrac{-mF_1 - nF_2}{p}$

[illegible] $F_3 = 0$ [illegible] $\dfrac{-mF_1 - nF_2}{p} = 0$

$m'F_1 + n'F_2 = 0$ [illegible]

$F_1 = 0$	[illegible]	m
$F_2 = 0$		n
$F_3 = 0$		p
$F_4 = 0$	[illegible]	q

[illegible] $F_1 m + F_2 n + F_3 p + F_4 q \equiv 0$ [illegible]

$m F_1 + n F_2 + p F_3 + q F_4 \equiv 0$

$P \begin{cases} F_1 \\ F_2 \\ F_3 \end{cases}$ [illegible] $m F_1 + n F_2 + p F_3 = 0$

[illegible]

[illegible] $q F_4 = 0$

$F_4 = 0$ [illegible]

$F_1 = 0$ } m

$F_2 = 0$ } n

$F_3 = 0$ } p

~~$m F_1 + n F_2 + p F_3 = 0$~~

[illegible] F_4 [illegible]:

[illegible]: $m F_1 + n F_2 + \ldots$

$$F_4 = \frac{-m F_1 - n F_2 - p F_3}{q}$$

$$\frac{-m F_1 - n F_2 - p F_3}{q} = 0$$

$m' F_1 + n' F_2 + p' F_3 = 0$ [illegible]

[illegible]

$\varphi_1 = 0$

$\varphi_2 = 0$

$m \varphi_1 + n \varphi_2 = 0$ [illegible]

$\varphi_1 = 0$

$\varphi_2 = 0$

$\varphi_3 = 0$

$m \varphi_1 + n \varphi_2 + p \varphi_3 = 0$ [illegible]

[illegible] $m \varphi_1 + n \varphi_2 + p \varphi_3 \equiv 0$ [illegible]

$\varphi_1 = 0$

$\varphi_2 = 0$

$\varphi_3 = 0$

$\varphi_4 = 0$

$m \varphi_1 + n \varphi_2 + p \varphi_3 + q \varphi_4 \equiv 0$ [illegible]

[illegible]

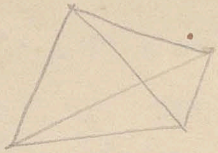
[illegible]

$\mathfrak{L}_1 = 0$

$\mathfrak{L}_2 = 0$

$\mathfrak{L}_3 = 0$

$\mathfrak{L}_4 = 0$



[illegible] Tetraeder

$m\,\mathfrak{L}_1 + n\,\mathfrak{L}_2 + p\,\mathfrak{L}_3 + q\,\mathfrak{L}_4 = 0$ [illegible] $\mathfrak{L}$

[illegible]

$\mathfrak{L}_i = A_i x + B_i y + C_i z + D_i$

$\mathfrak{L} = A x + B y + C z + D = 0$

$$m\,\mathfrak{L}_1 + n\,\mathfrak{L}_2 + p\,\mathfrak{L}_3 + q\,\mathfrak{L}_4 \equiv (m A_1 + n A_2 + p A_3 + q A_4)x + (m B_1 + n B_2 + p B_3 + q B_4)y + (m C_1 + n C_2 + p C_3 + q C_4)z + m D_1 + n D_2 + p D_3 + q D_4 = 0$$

[illegible] $m\,n\,p\,q$ [illegible]

[illegible] Coeff. [illegible]

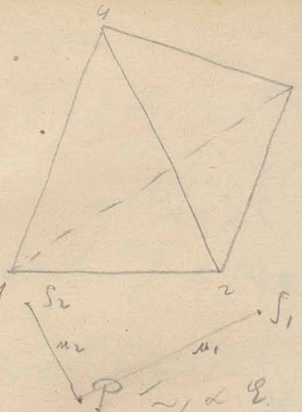
[illegible]

$m A_1 + n A_2 + p A_3 + q A_4 = A$ | $m =$

$m B_1 + n B_2 + p B_3 + q B_4 = B$ | $n =$

$m C_1 + n C_2 + p C_3 + q C_4 = C$ | $p =$

$m D_1 + n D_2 + p D_3 + q D_4 = D$ | $q =$



$$Q_1 = 0;\quad Q_2 = 0;\quad Q_3 = 0;\quad Q_4 = 0$$

$$m\,Q_1 + n\,Q_2 + p\,Q_3 + q\,Q_4 = 0$$

$$PS_1 \perp Q_1 = 234 \qquad = u_1$$

$$PS_2 \perp Q_2 \qquad = u_2$$

$$PS_i \perp Q_i \qquad = u_i$$

$Q_i = 0$ N. F. :

$$\frac{Q_i}{\sqrt{A_i^2 + B_i^2 + C_i^2}} = 0$$

$$u_i = -\frac{Q_i}{\sqrt{A_i^2 + B_i^2 + C_i^2}} = \lambda_i Q_i$$

$$\lambda_i = -\frac{1}{\sqrt{}}$$

$$u_1 = \lambda_1 Q_1$$
$$u_2 = \lambda_2 Q_2$$
$$u_3 = \lambda_3 Q_3$$
$$u_4 = \lambda_4 Q_4$$

$$\frac{m u_1}{\lambda_1} + \frac{n u_2}{\lambda_2} + \frac{p u_3}{\lambda_3} + \frac{q u_4}{\lambda_4} = 0$$

$$\frac{m}{\lambda_1} = a_1 \qquad \frac{n}{\lambda_2} = a_2$$

$$a_1 u_1 + a_2 u_2 + a_3 u_3 + a_4 u_4 = 0 \leftarrow$$

[illegible]

[illegible]

[homogenen = tetraedrischen Coordi[illegible]]

$f_1 \quad f_2 \quad f_3 \quad f_4$ [illegible]

$$\frac{1}{3}(f_1 u_1 + f_2 u_2 + f_3 u_3 + f_4 u_4) = Vol = V$$

$$f_1 u_1 + f_2 u_2 + f_3 u_3 + f_4 u_4 = 3V$$

[illegible]

[illegible] homog.

$$\alpha_1 u_1 + \alpha_2 u_2 + \alpha_3 u_3 + \alpha_4 u_4 + \beta.1 = 0$$ [illegible] homog. [illegible]

$$1 = \frac{f_1 u_1 + f_2 u_2 + f_3 u_3 + f_4 u_4}{3V}$$

$$u_1 \underbrace{\left[\alpha_1 + \frac{\beta f_1}{3V}\right]}_{a_1} + u_2 \left[\alpha_2 + \frac{\beta f_2}{3V}\right] + \dots = 0$$

$$a_1 u_1 + a_2 u_2 + a_3 u_3 + a_4 u_4 = 0$$

$$Ax + By + Cz + D = 0$$ [illegible] Parallel [illegible] homog. [illegible]

$$x = \frac{\xi}{\omega};\ y = \frac{\eta}{\omega};\ z = \frac{\zeta}{\omega}$$ [illegible]

$$A\frac{\xi}{\omega} + B\frac{\eta}{\omega} + C\frac{\zeta}{\omega} + D = 0$$ [illegible]

$$A\xi + B\eta + C\zeta + D\omega = 0$$

$\xi\ \eta\ \zeta$ — [illegible] = ~~[illegible]~~

= Hesse'schen

homogenen Coordinaten

$P(x y z)$

$P(\xi\ \eta\ \zeta\ \omega)$ [illegible] ∞ [illegible]

$$P(\lambda\xi, \lambda\eta, \lambda\zeta, \lambda\omega) = P(\xi, \eta, \zeta, \omega)$$

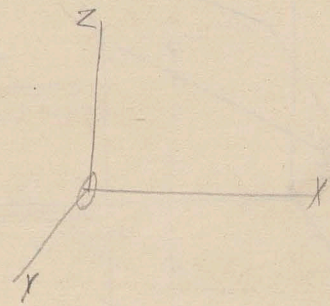
$\omega = 1$ [illegible]

[illegible] Fund. Tetraeder

$\xi = 0$ [illegible] YOZ

$\eta = 0$ " ZOX

$\zeta = 0$ " XOY



$\omega = 0$ [illegible] ∞ [illegible]

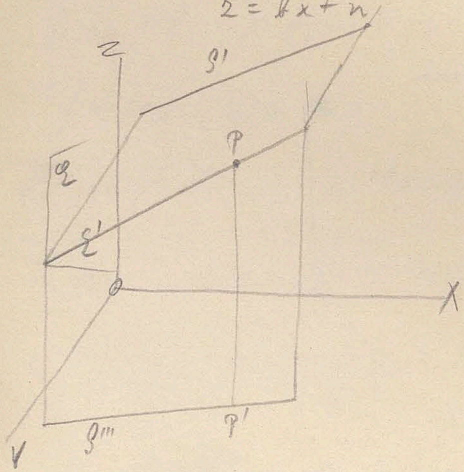
[illegible] $\omega = 0$ [illegible] $x = y = z = \infty$

Gleichung der Geraden

$$\left.\begin{aligned} Q &: Ax + By + Cz + D = 0 \\ Q' &: A'x + B'y + C'z + D' = 0 \end{aligned}\right\}$$

$$y = ax + m$$

$$z = bx + n$$



$$Q = 0 \quad Q' = 0$$

$$Q - \lambda Q' = 0$$

$$[Ax + By + Cz + D] - \lambda [A'x + B'y + C'z + D'] = 0$$

$$C - \lambda C' = 0$$

$$\lambda = \frac{C}{C'}$$

$$(A - \lambda A')x + (B - \lambda B')y + (D - \lambda D') = 0$$

$$y = -\frac{A - \lambda A'}{B - \lambda B'}x - \frac{D - \lambda D'}{B - \lambda B'} = ax + m \qquad \lambda = \frac{C}{C'}$$

$$z = bx + n \qquad \lambda = \frac{B}{B'}$$

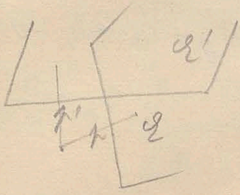
$$z = cy + p \qquad \lambda = \frac{A}{A'}$$

$y = ax + m$	$\parallel Z$		$g''' =$ proj. g ;	XY
$z = bx + n$	$\parallel Y$		g'	XZ
$z = cy + p$	$\parallel X$		g''	YZ

Winkel zweier Ebenen

$$\mathfrak{E} \equiv Ax + By + Cz + D = 0$$

$$\mathfrak{E}' \equiv A'x + B'y + C'z + D' = 0$$



$n - \alpha\beta\gamma$

$n' - \alpha'\beta'\gamma'$

$$\cos\alpha = \frac{A}{\sqrt{A^2+B^2+C^2}} \qquad \cos\beta = \frac{B}{\sqrt{A^2+B^2+C^2}}$$

$$\cos\alpha' = \frac{A'}{\sqrt{A'^2+B'^2+C'^2}} \qquad \cos\beta' = \frac{B'}{\sqrt{A'^2+B'^2+C'^2}}$$

$$\cos\gamma = \frac{C}{\sqrt{A^2+B^2+C^2}} \qquad \cos\gamma' = \frac{C'}{\sqrt{A'^2+B'^2+C'^2}}$$

$$\cos\varepsilon = \cos\alpha\cos\alpha' + \cos\beta\cos\beta' + \cos\gamma\cos\gamma'$$

$$= \frac{AA' + BB' + CC'}{\sqrt{(A^2+B^2+C^2)}\sqrt{A'^2+B'^2+C'^2}}$$

$\mathfrak{E}' \perp \mathfrak{E} \qquad \cos\varepsilon = 0 \qquad AA' + BB' + CC' = 0$

$\mathfrak{E}'' \parallel \mathfrak{E} \qquad \cos\alpha' = \pm\cos\alpha \quad \cos\beta' = \pm\cos\beta \quad \cos\gamma' = \pm\cos\gamma$

$$\frac{\cos\alpha'}{\cos\alpha} = \frac{\cos\beta'}{\cos\beta} = \frac{\cos\gamma'}{\cos\gamma} = \pm 1$$

$$\frac{A'\sqrt{A^2+B^2+C^2}}{A\sqrt{A'^2+B'^2+C'^2}} = \frac{B'}{B}\frac{\sqrt{A^2+B^2+C^2}}{\sqrt{A'^2+B'^2+C'^2}} = \frac{C'}{C}\frac{\sqrt{A^2+B^2+C^2}}{\sqrt{A'^2+B'^2+C'^2}}$$

$$\frac{A'}{A} = \frac{B'}{B} = \frac{C'}{C} = \pm\frac{\sqrt{A'^2+B'^2+C'^2}}{\sqrt{A^2+B^2+C^2}} = K$$

$A' = \kappa A \quad B' = \kappa B \quad C' = \kappa C$

$A' : A = B' : B = C' : C$

$\mathcal{E} - Ax + By + Cz + D$

$\mathcal{E}' \parallel \mathcal{E} \quad \kappa(Ax + By + Cz) + D' = 0 \quad Ax + By + Cz + D_1 = 0$

$D_1 = \frac{D'}{\kappa}$

$\mathcal{E} — Ax + By + Cz + D = 0$

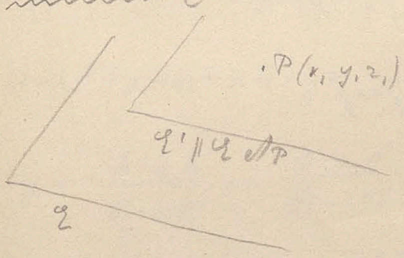
$\mathcal{E}' \parallel \mathcal{E} — Ax + By + Cz + D_1 = 0$

$D - D_1 = 0 \qquad D \gtrless D'$

∞ / ℬ ℊ ∠ $\mathcal{E} : \mathcal{E}'$

$\mathcal{E} — Ax + By + Cz + D = 0 \quad \mathcal{E}' — Ax_1 + By_1 + Cz_1 + D_1 = 0$

$\mathcal{E}' \parallel \mathcal{E}$ ℬ P



$D_1 = -Ax_1 + By_1 + Cz_1$

$Ax + By + Cz - (Ax_1 + By_1 + Cz_1) = 0$

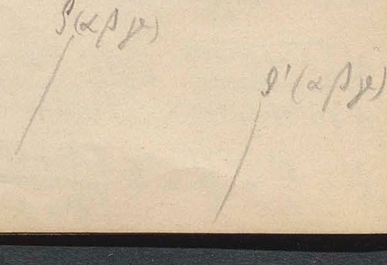
$A(x - x_1) + B(y - y_1) + C(z - z_1) = 0$

$P \equiv 0 \qquad x_1 = y_1 = z_1 = 0$

$Ax + By + Cz = 0$

ℬ $O \parallel \mathcal{E}$

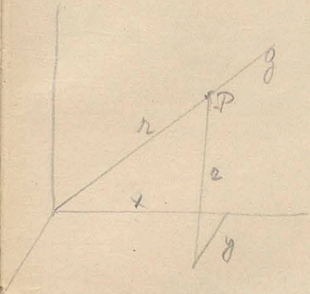
ℊ ∧ ℛ :



$S\begin{cases} Ax + By + Cz + D = 0 \\ A'x + B'y + C'z + D' = 0 \end{cases}$

$S' \parallel S \begin{cases} Ax + By + Cz + D = 0 \\ A'x + B'y + C'z + D_1' = 0 \end{cases}$

$$S\left\{\begin{array}{l} Ax+By+Cz=0 \\ ' \qquad\qquad =0 \end{array}\right| \begin{array}{l} A' \\ B' \end{array} \qquad g \parallel \mathfrak{S} \text{ durch } O$$



$x = r\cos\alpha$

$y = r\cos\beta$

$z = r\cos\gamma$

$$x(AB'-A'B)+z(CB'-C'B)=0$$

$$\frac{x}{z} = \frac{BC'-B'C}{AB'-A'B} \qquad \frac{y}{z} = \frac{AC'-A'C}{BA'-B'A} = \frac{A'C-AC'}{AB'-A'B}$$

$$x:y:z = BC'-B'C : CA'-C'A : AB'-A'B$$

$$\cos\alpha = \lambda(BC'-B'C) \qquad 1 = \lambda^2\left[(AB'-A'B)^2+(\ )^2+(\ )^2\right]$$

$\cos\beta$

$\cos\gamma =$

$$\lambda = \frac{1}{\sqrt{(AB'-A'B)^2+}}$$

$$\cos\alpha = \frac{BC'-B'C}{\sqrt{}}$$

$$\cos\beta = \frac{CA'-C'A}{\sqrt{}}$$

$$\cos\gamma = \frac{AB'-A'B}{\sqrt{}}$$

14/11

$$\left.\begin{array}{l} y = ax + m \\ z = bx + n \end{array}\right\} g$$

$$\frac{a}{\sqrt{1+a^2+b^2}} \qquad \frac{b}{\sqrt{1+a^2+b^2}} \qquad \frac{1}{\sqrt{1+a^2+b^2}}$$

2 [illegible]

$$\left.\begin{array}{l} y = a'x + m' \\ z = b'x + n' \end{array}\right\} g'$$

$$\frac{a'}{\sqrt{1+a'^2+}}$$

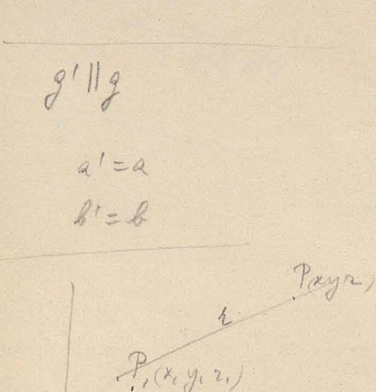
$$\cos w = \frac{aa' + bb' + 1}{\sqrt{(1+a^2+b^2)(1+a'^2+b'^2)}} = \cos \measuredangle$$ [illegible]

$aa' + bb' + 1 = 0$ [illegible] $g' \perp g$

$g' \parallel g$

$a' = a$

$b' = b$



$$x - x_1 = r\cos\alpha$$
$$y - y_1 = r\cos\beta$$
$$z - z_1 = r\cos\gamma$$

$$\frac{x-x_1}{\cos\alpha} = \frac{y-y_1}{\cos\beta} = \frac{z-z_1}{\cos\gamma} = r$$

[illegible]

$$\frac{x-x_1}{A} = \frac{y-y_1}{B} = \frac{z-z_1}{C}$$

$$\frac{x-x_1}{\lambda A} = \frac{y-y_1}{\lambda B} = \frac{z-z_1}{\lambda C}$$

λ [illegible]:

$$\lambda A = \cos\alpha$$
$$\lambda B = \cos\beta$$
$$\lambda C = \cos\gamma$$

$$\lambda = \frac{1}{\sqrt{A^2+B^2+C^2}}$$

$$\frac{x-x_1}{\cos\alpha} = \frac{y-y_1}{\cos\beta} = \frac{z-z_1}{\cos\gamma}$$

$g \qquad \dfrac{x-x_1}{A} = \dfrac{y-y_1}{B} = \dfrac{z-z_1}{C}$

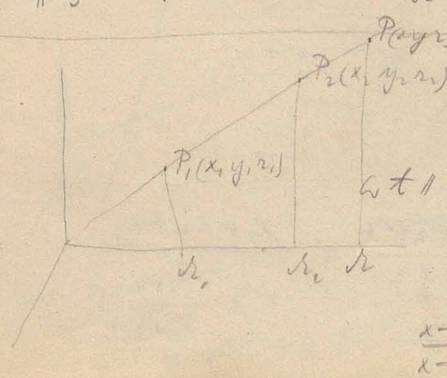
$g' \qquad \dfrac{x-x'}{A'} = \dfrac{y-y'}{B'} = \dfrac{z-z'}{C'}$

$$\frac{AA'+BB'+CC'}{\sqrt{(A^2+B^2+C^2)(A'^2+B'^2+C'^2)}} = \cos\measuredangle$$

$$\sin^2\omega = \frac{(AB'-A'B)^2+(BC'-B'C)^2+(CA'-C'A)^2}{\sqrt{(A^2+B^2+C^2)}\;\sqrt{(A'^2+B'^2+C'^2)}}$$

$g' \perp g \quad : \quad AA'+BB'+CC'=0$

$g \parallel g' \quad : \quad A : A' = B : B' = C : C'$



$$\frac{P_1P}{P_2P} = t \quad = [illegible]$$

$\omega\, t \parallel \; : \; \ell\,(\ldots)$

$$\frac{M_1M}{M_2M} = \frac{P_1P}{P_2P} = t$$

$$\frac{x-x_1}{x-x_2} = t$$

$$\frac{y-y_1}{y-y_2} = t \qquad \frac{z-z_1}{z-z_2} = t$$

$$\frac{x-x_1}{x-x_2} = \frac{y-y_1}{y-y_2} = \frac{z-z_1}{z-z_2}$$ = [illegible]

$$x - x_1 = tx - tx_2$$

$$x(1-t) = x_1 - tx_2$$

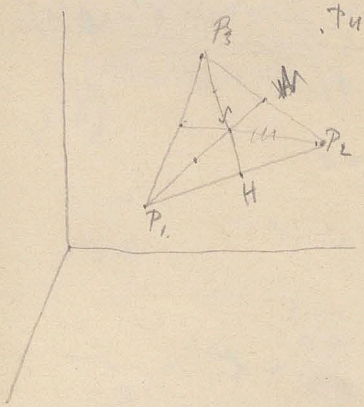
$$\begin{cases} x = \dfrac{x_1 - tx_2}{1-t} \\[2ex] y = \dfrac{y_1 - ty_2}{1-t} \\[2ex] z = \dfrac{z_1 - tz_2}{1-t} \end{cases}$$

2B. 2d) H $t=-1$

$$x = \frac{x_1+x_2}{2}$$

$$y = \frac{y_1+y_2}{2}$$

$$z = \frac{z_1+z_2}{2}$$



$AS = \frac{1}{3} HP_3$

$S(xyz) = ?$

H: $\dfrac{x_1+x_2}{2}$

$\dfrac{y_1+y_2}{2}$

$\dfrac{z_1+z_2}{2}$

$t = -\frac{1}{2}$

$$x = \frac{x_1 - tx_2}{1-t}$$

$$x = \frac{\frac{x_1+x_2}{2} - \left(-\frac{1}{2}\right)x_3}{1-\left(-\frac{1}{2}\right)} = \frac{x_1+x_2+x_3}{3}$$

$$y = \frac{y_1+y_2+y_3}{3} \qquad z = \frac{z_1+z_2+z_3}{3}$$

[illegible] p_4 : Tetraeder [illegible]

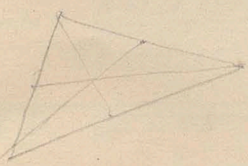
$$\delta\Sigma = \frac{1}{4}\,\delta P_4 \qquad t = -\frac{1}{3}$$

$$\Sigma:\; x = \frac{\frac{x_1+x_2+x_3}{3} - \left(-\frac{1}{3}\right)x_4}{1-\left(-\frac{1}{3}\right)} = \frac{x_1+x_2+x_3+x_4}{4}$$

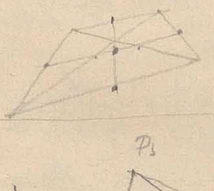
$$= \frac{\frac{x_1+x_2}{2} + \frac{x_3+x_4}{2}}{2}$$

$$y = \frac{y_1+y_2+y_3+y_4}{4}$$

$$z = \frac{z_1+z_2+z_3+z_4}{4}$$

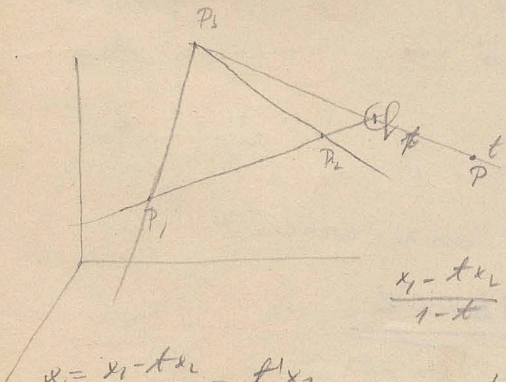


[illegible]



[illegible]

[illegible]



[illegible]:

$$\frac{P_1 Q}{P_2 Q} = t$$

$$\frac{Q P}{P_3 P} = t' \quad \text{[illegible]}$$

$$\frac{x_1 - t x_2}{1-t} \qquad \frac{y_1 - t y_2}{1-t} \qquad \frac{z_1 - t z_2}{1-t}$$

$$x = \frac{\frac{x_1 - t x_2}{1-t} - t' x_3}{1-t'} = \frac{x_1 - t x_2 - t'(1-t)x_3}{(1-t)(1-t')}$$

$$\frac{1}{(1-t)(1-t')} = m_1$$

$$\frac{-t}{1-t'} = m_2$$

$$\frac{-t'}{1-t} = m_3$$

$$\left.\begin{aligned} x &= m_1 x_1 + m_2 x_2 + m_3 x_3 \\ y &= m_1 y_1 + m_2 y_2 + m_3 y_3 \\ z &= m_1 z_1 + m_2 z_2 + m_3 z_3 \end{aligned}\right\}$$ [illegible]

$$m_1 + m_2 + m_3 = 1$$

16/11 [illegible] $\angle\omega$

$$Ax + By + Cz + D = 0 \quad \} \alpha\beta\gamma$$

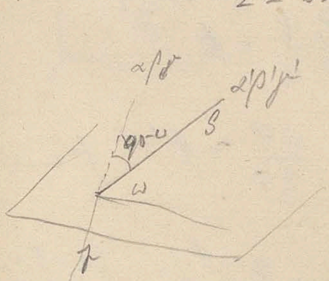
$$\left.\begin{aligned} y &= ax + m \\ z &= bx + n \end{aligned}\right\} \alpha'\beta'\gamma'$$

$$\cos\alpha = \frac{A}{\sqrt{A^2+B^2+C^2}}$$

$$\cos\alpha' = \frac{1}{\sqrt{1+a^2+b^2}}$$

$$\cos\beta' = \frac{a}{\sqrt{}}$$

$$\cos\gamma' = \frac{b}{\sqrt{}}$$



$$\sin\omega = \cos\alpha\cos\alpha' +$$

$$\sin\omega = \frac{A + Ba + Cb}{\sqrt{A^2+B^2+C^2}\sqrt{1+a^2+b^2}}$$

ω g ‖ ~ sin = 0

$A\,a + B\,b + C\,c = 0$

g ⊥ E : $\cos\alpha' = \pm\cos\alpha$

$\cos\beta' = \pm\cos\beta$

$\cos\gamma' = \pm\cos\gamma$

$$\frac{A}{a} = \frac{B}{b} = \frac{C}{c} = \frac{\sqrt{}}{\sqrt{}}$$

ω ∠ g =

g $$\frac{x-x'}{A'} = \frac{y-y'}{B'} = \frac{z-z'}{C'}$$

$$\cos\alpha' = \frac{A'}{\sqrt{A'^2+B'^2+C'^2}}$$

$$\sin\omega = \frac{AA' + BB' + CC'}{\sqrt{A^2+B^2+C^2}\sqrt{A'^2+B'^2+C'^2}}$$

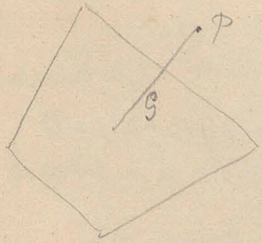
g ‖ E $\sin\omega = 0$

$AA' + BB' + CC' = 0$

g ⊥ E $$\frac{A'}{A} = \frac{B'}{B} = \frac{C'}{C}$$

E ~ ‖, ~ ⊥ P', $\left\{\begin{matrix} x' \\ y' \\ z' \end{matrix}\right.$; l ∠ s Bez. = ?

$$\frac{x-x'}{A'} = \frac{y-y'}{B'} = \frac{z-z'}{C'}$$



$$\frac{A'}{A} = \frac{B'}{B} = \frac{C'}{C} = \lambda$$

$A' = \lambda A$ etc.

$$\frac{x-x'}{\lambda A} = \frac{y-y'}{\lambda B} = \frac{z-z'}{\lambda C}$$

$$\frac{x-x'}{A} = \frac{y-y'}{B} = \frac{z-z'}{C}$$

[illegible] $\left\{ \begin{matrix} S \\ S' \end{matrix} \right.$ [illegible] $P' \left\{ \begin{matrix} x' \\ y' \\ z' \end{matrix} \right.$ [illegible]

S —— $y = ax + m$ $\qquad$ S' —— $y = a'x + m'$

$\qquad z = bx + n$ $\qquad\qquad z = b'x + n'$

P' —— $x'y'z'$

E — $Ax + By + Cz + D = 0$

$$Ax' + By' + Cz' + D = 0$$

$$A(x-x') + B(y-y') + C(z-z') = 0$$

$$A + Ba + Cb = 0$$

$$A + Ba' + Cb' = 0$$

$$B(a-a') + C(b-b') = 0$$

$$B : C = b'-b = a-a'$$

~~A [illegible]~~

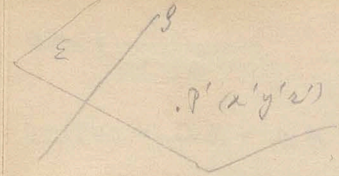
$$A(b'-b) + B(ab'-a'b) = 0$$

$$A : B = a'b - ab' : b'-b$$

$$A : B : C = a'b - ab' : \sout{b'-ab} : a-a'$$

$$(x-x')(a'b-ab') + (b'-b)(y-y') + (a-a')(z-z') = 0$$

[illegible] !!!



ε [illegible] P' [illegible] ⊥ g

$$A(x-x')+B(y-y')+C(z-z')=0$$

$$y=ax+m$$

$$z=bx+n$$

$$\frac{A}{1}=\frac{B}{a}=\frac{C}{b}$$

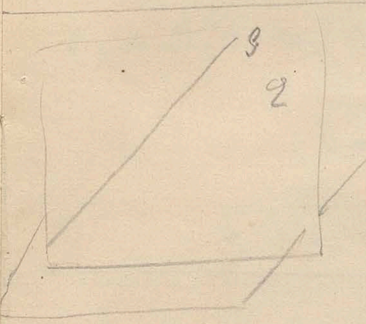
$$x-x'+a(y-y')+b(z-z')=0$$

[illegible] ∥:

$$\frac{x-x_1}{A'}=\frac{y-y_1}{B'}=\frac{z-z_1}{C'}$$

$$\frac{A}{A'}=\frac{B}{B'}=\frac{C}{C'}$$

ε: --- $A'(x-x')+B'(y-y')+C'(z-z')=0$



[illegible] ∥

[illegible] ⊥ [illegible]

ε — $Ax+By+Cz+D=0$

g — $\begin{cases} A_1x+B_1y+C_1z+D_1=0 \\ A_2x+B_2y+C_2z+D_2=0 \end{cases}$

$$(A_1-\lambda A_2)x+(B_1-\lambda B_2)y+(C_1-\lambda C_2)z+D_1-\lambda D_2=0$$

[illegible]

$A(A_1-\lambda A_2)+B(B_1-\lambda B_2)+C(C_1-\lambda C_2)=0$ [illegible] ⊥ [illegible]

$$\lambda=\frac{AA_1+BB_1+CC_1}{AA_2+BB_2+CC_2}$$

$$Ax + By + Cz + D = \frac{AA_1 + BB_1 + CC_1}{AA_2 + BB_2 + CC_2}\left[\quad\right] = 0$$

$$\varepsilon_1 (AA_2 + BB_2 + CC_2) - \varepsilon_2 (AA_1 + BB_1 + CC_1) = 0$$

$$\frac{\varepsilon_1}{AA_1 + BB_1 + CC_1} - \frac{\varepsilon_2}{AA_2 + BB_2 + CC_2} = 0$$

[illegible]:

$y = ax + m$ $\qquad ax - y$

$z = bx + n$

$$ax - y + m - \lambda(bx - z - n) = 0$$

$$(a - \lambda b)x - y + \lambda z + m - \lambda n = 0$$

$$A(a - \lambda b) - B + C\lambda = 0$$

$$\lambda = \frac{Aa - B}{Ab - C}$$

$$ax - y + m - \frac{Aa - B}{Ab - C}(bx - z - n) = 0$$

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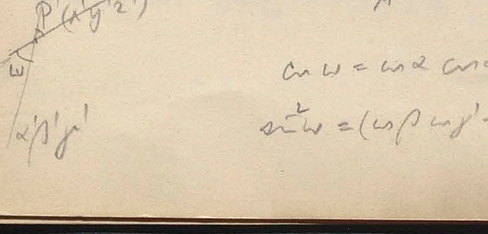
sn. porthy. proj. [illegible]



[illegible] (P₁ [illegible] δ

$$\frac{x - x'}{A} = \frac{y - y'}{B} = \frac{z - z'}{C}$$

$\cos\omega = \cos\alpha\cos\alpha' + $ ——

$\sin^2\omega = (\cos\beta\cos\gamma' - \cos\gamma\cos\beta')^2 + (\quad)^2 + (\quad)^2$

$= 0$ $\cos\alpha = \dfrac{A}{\sqrt{A^2+B^2+C^2}}$ $\cos\alpha' = \dfrac{x_1 - x'}{r}$ $r^2 = (x_1 - x')^2 + (\ )^2 + (\ )^2$

$) = 0$ $\cos\beta = \text{———}$ $\cos\beta' = \dfrac{y_1 - y'}{r}$

$$\sin^2 w = \frac{[B(z_1 - z') - C(y_1 - y')]^2 + [\qquad]^2 + [\qquad]^2}{r^2(A^2+B^2+C^2)}$$

$d = r \sin w$

$$d^2 = [B(z_1 - z') - C(y_1 - y')]^2 + [C(x_1 - x') - A(z - z')]^2 + [A \qquad]^2$$

19/11 $Ax + By + Cz + D = 0$

$\left.\begin{array}{l} y = ax + m \\ z = bx + n \end{array}\right\} S$ } $\mathcal{P}_l$ S_{le}

$$Ax + B(ax+m) + C(bx+n) + D = 0$$

$$x = -\frac{Bm + Cn + D}{A + Ba + Cb}$$

$$y = -a\,\text{———————} + m$$

$$z = -b\,\text{———————} + n$$

$S \parallel \mathcal{E}$ ω $\mathcal{P}_l$ ∞ [illegible]

$A + Ba + Ca = 0$

ω S [illegible] $\mathcal{E}$? ω ∞ / [illegible] $x = \dfrac{0}{0}$

$)^2$

$Bm + Cn + D = 0$

$A + Ba + Cb = 0$

$Ax + By + Cz + D = 0$

$$g = \begin{cases} A'x + B'y + C'z + D' = 0 \\ A''x + B''y + C''z + d'' = 0 \end{cases}$$ [illegible]

$x = 0$

$y = 0$

$z = 0$

[illegible]:
$$A(B'C'' - B''C') + B(C'A'' - C''A') + C(A''C'' - A''C')$$

$g \parallel E$ [illegible] $= 0$.

[illegible]:
$$g \begin{cases} y = ax + m \\ z = bx + n \end{cases} \qquad g' \begin{cases} y = a'x + m' \\ z = b'x + n' \end{cases}$$

[illegible]

$$0 = (a - a')x + m - m' \quad \Big| \; b - b'$$
$$0 = (b - b')x + n - n' \quad \Big| \; a - a'$$

$$(m - m')(b - b') - (n - n')(a - a') = 0$$

$$\frac{a - a'}{m - m'} = \frac{b - b'}{n - n'}$$

Gleichung einer Ebenen welche durch 2 Geradegeht

$$g - \begin{cases} E_1 = 0 \\ E_2 = 0 \end{cases} \qquad g' - \begin{cases} E_3 = 0 \\ E_4 = 0 \end{cases}$$

[illegible] g, g' [illegible]:

[illegible] 4 [illegible]

$k_1 E_1 + k_2 E_2 + k_3 E_3 + k_4 E_4 \equiv 0$

BH. $k_1 E_1 + k_2 E_2 = 0$ [illegible] $g\,g'$

[illegible] | [illegible] $k_3 E_3 + k_4 E_4 = 0$ [illegible]

[illegible]

$E_1 \equiv y - ax - m$

$E_2 \equiv z - bx - n$

$E_3 \equiv y - a'x - m'$

$E_4 \equiv z - b'x - n'$

$E_3 - E_1 \equiv (a-a')x + m - m'$ | $b-b'$

$E_4 - E_2 \equiv (b-b')x + n - n'$ | $a-a'$

$(b-b')(E_3 - E_1) - (a-a')(E_4 - E_2) \equiv 0$

[illegible] $g\,g'$:

$(b-b')\,E_1 - (a-a')\,E_2 = 0$

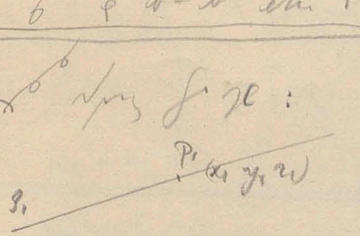
[illegible]: $(b-b')\,E_3 - (a-a')\,E_4 = 0$

$= (b-b')(y - ax - m) - (a-a')(z - bx - n) = 0$

$= (b-b')(y - a'x - m') - (a-a')(z - b'x - n') = 0$

[illegible] $b - b'$ [illegible] $m - m'$ [illegible] prop.

[illegible]:



$g_1 \quad \frac{x - x_1}{\cos\alpha_1} = \frac{y - y_1}{\cos\beta_1} = \frac{z - z_1}{\cos\gamma_1}$

$g_2 \quad \frac{x - x_2}{\cos\alpha_2} = \frac{y - y_2}{\cos\beta_2} = \frac{z - z_2}{\cos\gamma_2}$

$\perp S_1 - z$ $Q_1 \parallel S_2$ [illegible] $z =$ [illegible]

$\perp S_2 - z$ $Q_2 \parallel S_1$

$= K$

$\angle 0$ [illegible] p_1, p_2 — $\lambda\mu\nu$

$p_1 - p_2 = K$

$\angle S S_1 = \theta$

$$\cos\lambda = \frac{\cos\beta_1\cos\gamma_2 - \cos\gamma_1\cos\beta_1}{\sin\theta}$$

$\cos\mu =$

$\cos\nu =$

Q_1 — $(x-x_1)\cos\lambda + (y-y_1)\cos\mu + (z-z_1)\cos\nu = 0$

Q_2 — $(x-x_2)\cos\lambda + (y-y_2)\cos\mu + (z-z_2)\cos\nu = 0$

$p_1 = x_1\cos\lambda + y_1\cos\mu + z_1\cos\nu$

$p_2 = x_2\cos\lambda + y_2\cos\mu + z_2\cos\nu$

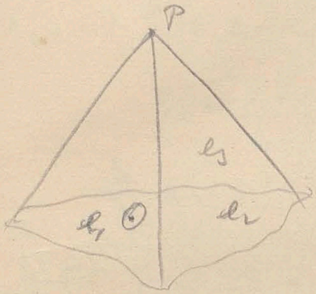
$k = (x_1-x_2)\cos\lambda + (y_1-y_2)\cos\mu + (z_1-z_2)\cos\nu$

$$k = \frac{\left\{\begin{array}{l}(x_1-x_2)[\cos\beta_1\cos\gamma_2 - \cos\gamma_1\cos\beta_2] + \\ + (y_1-y_2)[\cos\gamma_1\cos\alpha_2 - \cos\alpha_1\cos\gamma_2] + \\ + (z_1-z_2)[\cos\alpha_1\cos\beta_2 - \cos\beta_1\cos\alpha_2]\end{array}\right\}}{\left\{\sqrt{(\cos\beta_1\cos\gamma_2 - \cos\gamma_1\cos\beta_2)^2 + (\cos\gamma_1\cos\alpha_2 - \cos\alpha_1\cos\gamma_2)^2 + (\quad)^2}\right\}}$$

[illegible] !!!

$e_1 = 0$	k_1
$e_2 = 0$	k_2
$e_3 = 0$	k_3

$$k_1 e_1 + k_2 e_2 + k_3 e_3 \equiv 0$$ [illegible]



[illegible]

$e_1 = 0$
$e_2 = 0$
$e_3 = 0$ } [illegible]

O [illegible] [illegible]

$e_1 - e_2 = 0$
$e_2 - e_3 = 0$ } [illegible]
$e_3 - e_1 = 0$

$k_1 = 1$
$k_2 = 1$
$k_3 = 1$

$$(e_1 - e_2) + (\quad) + (\quad) \equiv 0$$ [illegible]

[illegible] Rotation [illegible]

$e_1 - e_2 = 0$
$e_2 + e_3 = 0$ } [illegible]
$e_3 + e_1 = 0$

} [illegible] | 1, 1, −1

[illegible] Rotation [illegible]

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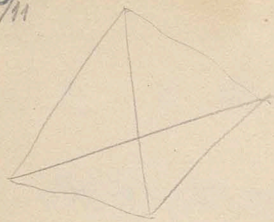

[illegible] [illegible] [illegible] [illegible] [illegible] [illegible

[illegible]]

[illegible] !!!

I. $\Pi_1 = 0$

II. $\Pi_2 = 0$

III. $\Pi_1 - k\,\Pi_2 = 0$ | IV. $\Pi_1 - l\,\Pi_2 = 0$

$$(1234) = \frac{k}{l}$$

$$= \frac{\sin 13}{\sin 23} : \frac{\sin 14}{\sin 24}$$

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1). $\varphi_1 = 0$

2). $\varphi_2 = 0$

3). $\varphi_1 - k\,\varphi_2 = 0$ — k $\qquad k = \kappa c$

4). $\varphi_1 - \lambda\,\varphi_2 = 0$ — l $\qquad l = \lambda c$

$$\frac{k}{l} = \frac{\kappa}{\lambda}$$

$$(1234) = \frac{\kappa}{\lambda}$$

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4 [illegible] :

$S \begin{cases} \varphi_1 \\ \varphi_2 \end{cases}$

1). $\varphi_1 - \lambda_1\,\varphi_2 = 0$

2). $\varphi_1 - \lambda_2\,\varphi_2 = 0$ $\qquad (1234) = ?$

3). $\varphi_1 - \lambda_3\,\varphi_2 = 0$

4). $\varphi_1 - \lambda_4\,\varphi_2 = 0$

$\varphi_1 - \lambda_1 \varphi_2 \equiv l_1$

$\varphi_1 - \lambda_2 \varphi_2 \equiv l_2$

$l_1 = 0$

$l_2 = 0$

$(\lambda_2 - \lambda_1)\, \varphi_2 \equiv l_1 - l_2$

$$\varphi_2 \equiv \frac{l_1 - l_2}{\lambda_2 - \lambda_1} \qquad \varphi_1 \equiv l_1 + \lambda_1 \frac{l_1 - l_2}{\lambda_2 - \lambda_1} = \frac{l_1 \lambda_2 - l_2 \lambda_1}{\lambda_2 - \lambda_1}$$

3). $\dfrac{\lambda_2 l_1 - \lambda_1 l_2}{\lambda_2 - \lambda_1} - \lambda_3 \dfrac{l_1 - l_2}{\lambda_2 - \lambda_1} = 0$

4). $\dfrac{\lambda_2 l_1 - \lambda_1 l_2}{\lambda_2 - \lambda_1} - \lambda_4 \dfrac{l_1 - l_2}{\lambda_2 - \lambda_1} = 0$

3). $l_1 [\lambda_2 - \lambda_3] - l_2 [\lambda_1 - \lambda_3] = 0$

$$l_1 - l_2 \frac{\lambda_1 - \lambda_3}{\lambda_2 - \lambda_3} = 0$$

4). $$l_1 - l_2 \frac{\lambda_1 - \lambda_4}{\lambda_2 - \lambda_4} = 0$$

$$(1234) = \frac{\lambda_1 - \lambda_3}{\lambda_2 - \lambda_3} : \frac{\lambda_1 - \lambda_4}{\lambda_2 - \lambda_4}$$

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a harmon. lje 6

$\dfrac{\lambda_1 - \lambda_3}{\lambda_2 - \lambda_3} : \dfrac{\lambda_1 - \lambda_4}{\lambda_2 - \lambda_4} = -1 \qquad \dfrac{\lambda_1 - \lambda_3}{\lambda_2 - \lambda_3} + \dfrac{\lambda_1 - \lambda_4}{\lambda_2 - \lambda_4} = 0$

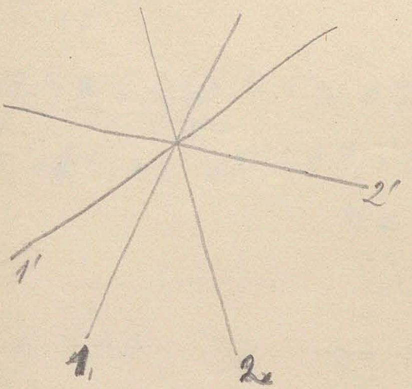
$\lambda_1 \lambda_2 \text{ —— } = 0$

$2\lambda_1 \lambda_2 + 2\lambda_3 \lambda_4 - [\lambda_1 + \lambda_2][\lambda_3 + \lambda_4] = 0$

[illegible] 12 [illegible] 34:

$$\lambda_1\lambda_2 + \frac{1}{2}(\lambda_1+\lambda_2)(\lambda_3+\lambda_4) + \lambda_3\lambda_4 = 0$$

[illegible] 2 [illegible] 8 [illegible]



$1: \quad \varphi_1 - \lambda_1\varphi_2 = 0$

$2 \quad \varphi_1 - \lambda_2\varphi_2 = 0$

$1': \quad \varphi_1 - \lambda_1'\varphi_2 = 0$

$2': \quad \varphi_1 - \lambda_2'\varphi_2 = 0$

[illegible]:

[illegible] 12 [illegible] 1'2' [illegible]

$3 \quad \varphi_1 - \lambda_3\varphi_2 = 0$

$4 \quad \varphi_1 - \lambda_4\varphi_2 = 0$

$(1234) = -1$

$(1'2'34) = -1$

$$\left.\begin{aligned}\lambda_1\lambda_2 - \tfrac{1}{2}(\lambda_1+\lambda_2)(\lambda_3+\lambda_4) + \lambda_3\lambda_4 &= 0\\ \lambda_1'\lambda_2' - \tfrac{1}{2}(\lambda_1'+\lambda_2')(\lambda_3+\lambda_4) + \lambda_3\lambda_4 &= 0\end{aligned}\right\}\quad \begin{aligned}\lambda_3\lambda_4 &= \text{—}\\ \lambda_3+\lambda_4 &= \text{— —}\end{aligned}$$

$$\lambda^2 - p\lambda + q = 0$$

[illegible] 3 [illegible]

[illegible]

[illegible] ham. [illegible]

[illegible] – Involution [illegible]:

[illegible] 2 [illegible]:

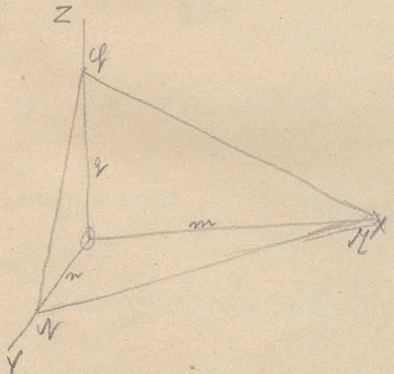
$$\left.\begin{aligned} a_1 - \lambda_1'' a_2 &= 0 \\ a_1 - \lambda_2'' a_2 &= 0 \end{aligned}\right\}$$

$$\lambda_1'' \lambda_2'' - \tfrac{1}{2}(\lambda_1'' + \lambda_2'')(\lambda_3 + \lambda_4) + \lambda_3 \lambda_4 = 0$$

[illegible] $\left\{\begin{matrix} \lambda_3 \lambda_4 \\ \lambda_3 + \lambda_4 \end{matrix}\right\}$ elim. !!!

[illegible]

[illegible]



$$\frac{x}{m} + \frac{y}{n} + \frac{z}{q} - 1 = 0$$

$$-\frac{1}{m} = u$$

$$-\frac{1}{n} = v$$

$$-\frac{1}{q} = w$$

$$-ux - vy - wz - 1 = 0$$

$$ux + vy + wz + 1 = 0$$

[illegible]

[illegible]

[illegible] $[x\,y\,z] = $ [illegible]

[illegible] $=$ [illegible]

$ux + vy + wz + 1 = 0$ [illegible] $u\,v\,w$ [illegible] $x\,y\,z$ [illegible]

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$Au + Bv + Cw + D = 0$

$\frac{A}{D}u + \frac{B}{D}v + \frac{C}{D}w + 1 = 0$

$a = \frac{A}{D} \quad b = \frac{B}{D} \quad c = \frac{C}{D}$

$au + bv + cw + 1 = 0$ [illegible] C, abc

$Au + Bv + Cw + D = 0$ [illegible]

$ux + vy + wz + 1 = 0$

$w = -\frac{Au + Bv + D}{C}$

$ux + vy - \frac{Au + Bv + D}{C}z + 1 = 0$

$u(Cx - Az) + v(Cy - Bz) + (C - Dz) = 0$ [illegible]

$$\left.\begin{aligned} Cx - Az &= 0 \\ Cy - Bz &= 0 \\ C - Dz &= 0 \end{aligned}\right\}$$ [illegible]

$Au + Bv + Cw + D = 0$ [illegible]

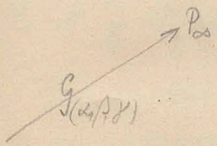
$ux + vy + wz + 1 = 0$ Normal " "

$Au + Bv + Cw = 0$ $\boxed{D = 0}$ [illegible]

$x = \frac{A}{D}$; $y = \frac{B}{D}$ $z = \frac{C}{D}$

$x = y = z = \infty$ $D = 0$

$x : y : z = A : B : C$ [illegible] P_∞



$\cos\alpha : \cos\beta : \cos\gamma = A : B : C$

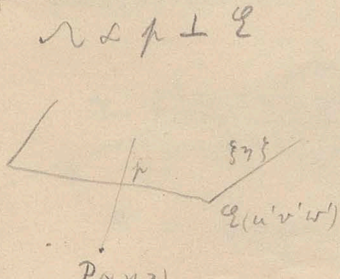
$\cos\alpha = \frac{A}{\sqrt{A^2 + B^2 + C^2}}$ - - - - -

$D \gtrless 0$ $D = 0$

$Au + Bv + Cw + D = 0$ $A = B = C = 0$

$x = y = z = 0$ [illegible] C.

[illegible] $p \perp \mathfrak{Q}$



P — $xu + yv + zw + 1 = 0$

$\mathfrak{Q}$ — $u'\xi + v'\eta + w'\zeta + 1 = 0$

$\mathfrak{Q}$ N.F. $\frac{u'\xi + v'\eta + w'\zeta + 1}{-\sqrt{u'^2 + v'^2 + w'^2}} = 0$

$$p = -\left\{ \frac{u'x + v'y + w'z + 1}{-\sqrt{u'^2 + v'^2 + w'^2}} \right\} = \frac{u'x + v'y + w'z + 1}{\sqrt{u'^2 + v'^2 + w'^2}}$$

[illegible]:

P — $Au + Bv + Cw + D = 0$

P — $\frac{A}{D} + \frac{B}{D} v + \frac{C}{D} w + 1 = 0$

$$p = \frac{\frac{A}{D} u' + \frac{B}{D} v' + \frac{C}{D} w' + 1}{\sqrt{u'^2 + v'^2 + w'^2}} = \frac{Au' + Bv' + Cw' + D}{D\sqrt{u'^2 + v'^2 + w'^2}}$$

$$p = \frac{Au + Bv + Cw + D}{D\sqrt{u^2 + v^2 + w^2}}$$ [illegible] α

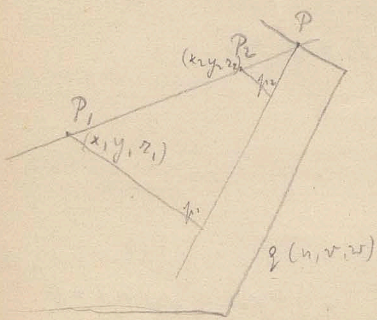
P — $Au + Bv + Cw + D = 0$ [illegible] $(u\,v\,w)$.

[illegible]

$\frac{P_1 P}{P_2 P} = t$ P ? — $P_1 \equiv x_1 u + y_1 v + z_1 w + 1 = 0$

[illegible] $P_2 \equiv x_2 u + y_2 v + z_2 w + 1 = 0$

$t_x = \frac{p_1}{p_2}$



$$p_1 = \frac{x_1 u + y_1 v + z_1 w + 1}{\sqrt{u^2 + v^2 + w^2}}$$

$$p_2 = \frac{x_2 u + y_2 v + z_2 w + 1}{\sqrt{u^2 + v^2 + w^2}}$$

$$\frac{x_1 u + y_1 v + z_1 w + 1}{x_2 u + y_2 v + z_2 w + 1} = t$$

$(x_1 u + y_1 v + z_1 w + 1) - t(x_2 u + y_2 v + z_2 w + 1) = 0$ — P

$P_1 = 0$

$P_2 = 0$ P — $P_1 - t P_2 = 0$

[illegible]:

$$\frac{P_1}{D_1}=0 \qquad \frac{P_2}{D_2}=0 \qquad \frac{P_1}{D_1}-t\frac{P_2}{D_2}=0 \qquad P_1 - t\frac{D_1}{D_2}P_2=0$$

$$P_1=0 \qquad P_2=0$$

$\alpha_1 P_1 \mp \alpha_2 P_2 = 0$ [illegible] $\overline{P_1P_2}$

$$P_1-\frac{\alpha_2}{\alpha_1}P_2=0 \qquad \frac{\alpha_2}{\alpha_1}=t\cdot\frac{D_1}{D_2}$$

$$P_1 - t\frac{D_1}{D_2}P_2=0 \qquad t=\frac{\alpha_2 D_2}{\alpha_1 D_1}$$

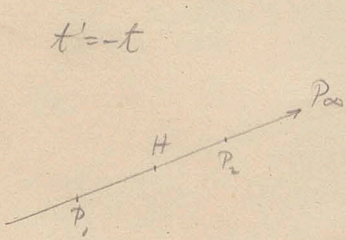
$$P_1=0 \qquad P_2=0$$

$P \equiv \alpha_1 P_1 - \alpha_2 P_2 = 0$ — t } harm. conj.

$P' \equiv \alpha_1 P_1 + \alpha_2 P_2 = 0$ — t'

$$t=\frac{\alpha_2 D_2}{\alpha_1 D_1} \qquad t'=-\frac{\alpha_2 D_2}{\alpha_1 D_1}$$

$$t'=-t$$

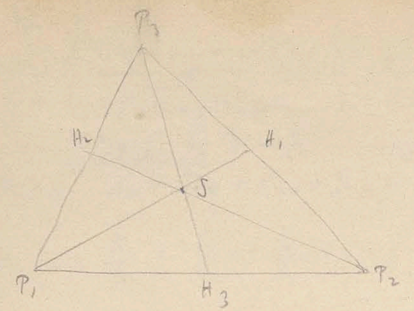


$P_1=0$, $P_2=0$ } N. F.

$P_1 - tP_2 = 0$ H — $t=-1$

$P_1 + P_2 = 0$ — H

$P_1 - P_2 = 0$ — P_∞



$P_1 = 0$

$P_2 = 0$

$P_3 = 0$

$H_1 \equiv P_2 + P_3 = 0$

$H_2 \equiv P_1 + P_3 = 0$ } linear!

$H_0 \equiv P_1 + P_2 = 0$

S —— $P_1 + \underbrace{(P_2 + P_3)}_{H_1} = 0$

etc.

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$P_1 = 0$ [illegible]

$P_2 = 0$

$P_1 - t P_2 = 0$ [illegible]

$P_1 + P_2 = 0$ [illegible] $P_1 - P_2 = \infty$ [illegible]

$P_1 \begin{cases} x_1 \\ y_1 \\ z_1 \end{cases}$ $P_2 \begin{cases} x_2 \\ y_2 \\ z_2 \end{cases}$

[illegible]

$P_1 = u x_1 + v y_1 + w z_1 + 1$

$P_2 = u x_2 + v y_2 + w z_2 + 1$

$P_1 - t P_2 = u(x_1 - t x_2) + v(y_1 - t y_2) + w(z_1 - t z_2) + 1 - t = 0$

$$u \frac{x_1 - t x_2}{1-t} + v \frac{y_1 - t y_2}{1-t} + w \frac{z_1 - t z_2}{1-t} + 1 = 0$$

$$\xi = \frac{x_1 - t x_2}{1-t}$$

$$\eta = \frac{y_1 - t y_2}{1-t}$$

$$\zeta = \frac{z_1 - t z_2}{1-t}$$

C [illegible]

$P_1 + P_2 + P_3$

$$\xi = \frac{x_1 + x_2 + x_3}{3}$$

$\eta =$

$\zeta =$

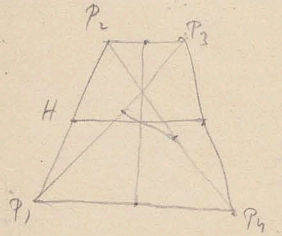
$P_1 = 0 \quad P_2 = 0 \quad P_3 = 0 \quad P_4 = 0$ [illegible]

$S: P_1 + P_2 + P_3 + P_4 = 0$

$\underbrace{P_1 + P_2} + P_3 + P_4 = 0$ [illegible] $P_1 + P_2 = 0$

$P_3 + P_4 = 0$

[illegible]



$$\left(\frac{P_1 + P_2}{2}\right) + \left(\frac{P_3 + P_4}{2}\right) = 0$$

[illegible]

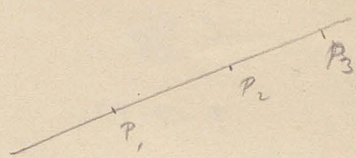
$S: (P_1 + P_2 + P_3) + P_4 = 0$

[illegible] P_4 [illegible] $\Delta\, P_1 P_2 P_3$

$P_i = A_i u + B_i v + C_i w + D_i$

$P_1 = 0$

$P_2 = 0$



[illegible]: $P_3 - \quad P_1 - \lambda P_2 = 0$ [illegible] $t = \frac{D_2}{D_1}\lambda$

$P_4 - \quad P_1 - \mu P_2 = 0 \qquad t' = \mu \frac{D_2}{D_1} \qquad \frac{t}{t'} = (P_1 P_2 P_3 P_4) = \frac{\lambda}{\mu}$

$P_1 = 0 \quad P_2 = 0 \qquad P_1 - \lambda P_2 = 0$

$P_1 - \mu P_2 = 0 \qquad (P_1 P_2 P_3 P_4) = \frac{\lambda}{\mu}$

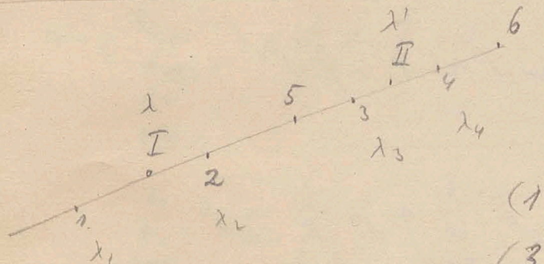
$$\left.\begin{array}{l} 1)\ P_1 - \lambda_1 P_2 = 0 \\ 2)\ P_1 - \lambda_2 P_2 = 0 \\ 3)\ P_1 - \lambda_3 P_2 = 0 \\ 4)\ P_1 - \lambda_4 P_2 = 0 \end{array}\right\} 4\ [illegible]$$

(!!!)

$$(1\,2\,3\,4) = \frac{\lambda_1 - \lambda_3}{\lambda_2 - \lambda_3} : \frac{\lambda_1 - \lambda_4}{\lambda_2 - \lambda_4}$$

[illegible] $(1234) = -1$ [illegible] harmon. [illegible]

$$\lambda_1 \lambda_2 - \tfrac{1}{2}(\lambda_1 + \lambda_2)(\lambda_3 + \lambda_4) + \lambda_3 \lambda_4 = 0$$



$$(1\ 2\ \mathrm{I}\ \mathrm{II}) = -1$$

$$(3\ 4\ \mathrm{I}\ \mathrm{II}) = -1$$

$$\left.\begin{array}{l} \lambda\lambda' - \tfrac{1}{2}(\lambda + \lambda')(\lambda_1 + \lambda_2) + \lambda_1 \lambda_2 = 0 \\ \lambda\lambda' - \tfrac{1}{2}(\lambda + \lambda')(\lambda_3 + \lambda_4) + \lambda_3 \lambda_4 = 0 \end{array}\right\} \begin{array}{l} \lambda' \\ \lambda = 0 \end{array}$$

$$\left.\begin{array}{l} \lambda\lambda' = \\ \lambda + \lambda' = \end{array}\right\} \lambda^2 + \alpha\lambda + \beta = 0$$

[illegible] 5 [illegible] 6 [illegible] ... [illegible] 123456
— Involu

[illegible]

$$\lambda\lambda' - \tfrac{1}{2}(\lambda + \lambda')(\lambda_5 + \lambda_6) + \lambda_5 \lambda_6 = 0$$

[illegible] $\lambda\ \lambda'$ elim. [illegible] Invol.

$P_1 = 0$	k_1	[illegible]
$P_2 = 0$	k_2	$k_1 P_1 + k_2 P_2 + k_3 P_3 \equiv 0$
$P_3 = 0$	k_3	[illegible] $P_1 P_2 P_3$ [illegible]

[illegible]

$P_1 = 0$

[illegible]

$P_2 = 0$

[illegible] $P_3 = 0$ [illegible] ; [illegible]

[illegible] $P_1 P_2$ [illegible] P_3 [illegible]

$P_1 = 0 \quad P_2 = 0$

$k_1 P_1 + k_2 P_2 = 0$ [illegible] $k_3 P_3 = 0$

$P_3 = 0$

[illegible] P_3 [illegible] $P_1 P_2$

[illegible] $P_3 \equiv -\frac{k_1}{k_3} P_1 - \frac{k_2}{k_3} P_2$

[illegible] $P_3 = 0$

[illegible] $-\frac{k_1}{k_3} P_1 - \frac{k_2}{k_3} P_2 = 0$

$P_1 - \lambda P_2 = 0$ [illegible] $P_1 - P_2$ [illegible]

$P_1 = 0$	k_1
$P_2 = 0$	k_2
$P_3 = 0$	k_3
$P_4 = 0$	k_4

[illegible]

[illegible]

$$k_1 P_1 + k_2 P_2 + k_3 P_3 + k_4 P_4 \equiv 0$$

[illegible]

P_1 — P_2 — P_3 [illegible] $u\,v\,w$

$$k_1 P_1 + k_2 P_2 + k_3 P_3 = 0$$

[illegible] $k_4 P_4 = 0$

$$P_4 = 0$$

$$P_4 \equiv \underbrace{-\frac{k_1}{k_4}}_{\mu_1} P_1 \underbrace{-\frac{k_2}{k_4}}_{\mu_2} P_2 \underbrace{-\frac{k_3}{k_4}}_{\mu_3} P_3 = 0$$

$$P_4 \equiv \mu_1 P_1 + \mu_2 P_2 + \mu_3 P_3 = 0$$ [illegible]

P_i — $x_i u + y_i v + z_i w + 1$

P_4 — $u(\mu_1 x_1 + \mu_2 x_2 + \mu_3 x_3) + v(\mu_1 y_1 + \mu_2 y_2 + \mu_3 y_3)$

$\underbrace{}_{x\,y\,z}$ $+ w(\mu_1 z_1 + \mu_2 z_2 + \mu_3 z_3) + \mu_1 + \mu_2 + \mu_3 = 0$

$$x = \frac{\mu_1 x_1 + \mu_2 x_2 + \mu_3 x_3}{\mu_1 + \mu_2 + \mu_3}$$

$$y = \frac{\mu_1 y_1 + \mu_2 y_2 + \mu_3 y_3}{\Sigma \mu}$$

$$z = \frac{\Sigma z}{\Sigma \mu}$$

$\frac{\mu_i}{\Sigma\mu} = \nu_i$ [illegible]

$$x = \nu_1 x_1 + \nu_2 x_2 + \nu_3 x_3$$
$$y = \nu_1 y_1 + \nu_2 y_2 + \nu_3 y_3$$
$$z = \nu_1 z_1 + \nu_2 z_2 + \nu_3 y_3$$

[illegible]: $\nu_1 + \nu_2 + \nu_3 = 1$ [illegible]

[illegible] homog. C.

$ux + vy + wz + 1 = 0$ [illegible]

x —	$\frac{x}{q}$	u —	$\frac{u}{r}$
y —	$\frac{y}{q}$	v	$\frac{v}{r}$
z —	$\frac{z}{q}$	w	$\frac{w}{r}$

$x\ y\ z\ q$ [illegible] homog. C
$u\ v\ w\ r$ [illegible] homog. C

$q = P =$

$ux + vy + wz + rq = 0$ [illegible] $x\ y\ z\ q$

[illegible] $u\ v\ w\ r$

$P_1 = 0$

$P_2 = 0$ [illegible] $\mu_1 P_1 + \mu_2 P_2 = 0$

$$\mu_1 (u x_1 + v y_1 + w z_1 + r q_1) +$$
$$+ \mu_2 (u x_2 + v y_2 + w z_2 + r q_2) = 0$$

$$u(\mu_1 x_1 + \mu_2 x_2) + v(\mu_1 y_1 + \mu_2 y_2) + w(\mu_1 z_1 + \mu_2 z_2) +$$
$$+ r(\mu_1 q_1 + \mu_2 q_2) = 0$$

C [illegible] $x\,y\,z\,w$ [illegible]:

$x = \mu_1 x_1 + \mu_2 x_2$

$y = \mu_1 y_1 + \mu_2 y_2$ $\qquad$ μ_1, μ_2 [illegible]

$z = \mu_1 z_1 + \mu_2 z_2$

$q = \mu_1 q_1 + \mu_2 q_2$

$P_1 \quad P_2 \quad P_3$

$$\left.\begin{aligned} x &= \mu_1 x_1 + \mu_2 x_2 + \mu_3 x_3 \\ y &= \mu_1 y_1 + \mu_2 y_2 + \mu_3 y_3 \\ z &= \mu_1 z_1 + \mu_2 z_2 + \mu_3 z_3 \\ q &= \mu_1 q_1 + \mu_2 q_2 + \mu_3 q_3 \end{aligned}\right\} = C \text{ [illegible]}$$

[illegible] μ_4 [illegible]

30/12 [illegible]

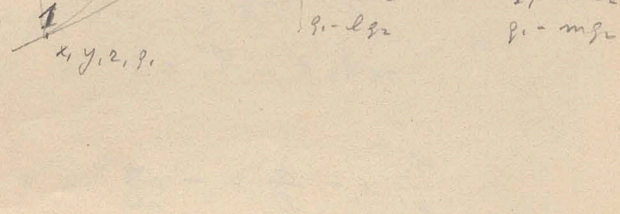
I $\varphi = 0$

II $\varphi' = 0$

III $\varphi - \lambda \varphi' = 0$

IV $\varphi + \mu \varphi' = 0$

$(\text{I II III IV}) = \dfrac{\lambda}{\mu}$



$(1\,2\,3\,4) = \dfrac{l}{m}$

$\varphi \equiv Ax + By + Cz + Dq$

$\varphi' \equiv A'x + B'y + C'z + D'q$

$x_i \; y_i \; z_i \quad q_i \qquad 6^{\circ} \quad x \; y \; z \; q$

$Ax_i + By_i + Cz_i + Dq_i \equiv \varphi_i$

cioè 1 . I V_i

$\varphi_1 = 0$

2 . II — $\varphi'_2 = 0$

3 . III $\quad Ax + By + Cz + Dq - \lambda(A'x + B'y + C'z + D'q) = 0$

$A(x_1 - lx_2) + B(y_1 - ly_2) + \text{---} \; D(q_1 - lq_2) -$

$-\lambda[A'(x_1 - lx_2) + B'(y_1 - ly_2) + \text{---} \; D'(q_1 - lq_2)] = 0$

$\varphi_1 - l\varphi_2 - \lambda[\varphi'_1 - l\varphi'_2] = 0$

$-l\varphi_2 - \lambda\varphi'_1 = 0$

$l\varphi_2 + \lambda\varphi'_1 = 0$

4 — IV $\quad 6^{\circ} \; \lambda \text{ -- } \mu$

$l \text{ -- } m$

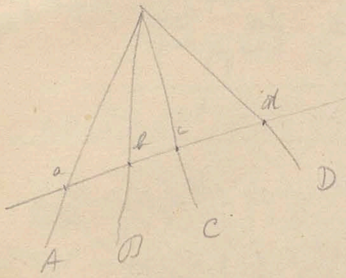
$m\varphi_2 + \mu\varphi'_1 = 0$

$$\frac{\varphi_2}{\varphi'_1} = -\frac{\lambda}{l} = -\frac{\mu}{m}$$

$-\frac{\lambda}{l} = -\frac{\mu}{m}$

$\frac{\lambda}{\mu} = \frac{l}{m}$ $\qquad (I\ II\ III\ IV) = (1\,2\,3\,4)$

[illegible]:



$(abcd) = (ABCD)$

[illegible]

Flächen zweiter Ordnung.

$$\left.\begin{array}{l} Ax + By + Cz + D = 0 \\ Ax + By + Cz + Dq = 0 \end{array}\right\} \text{Ebene}$$

[illegible]:

$$a_{11}x^2 + a_{22}y^2 + a_{33}z^2 + 2a_{12}xy + 2a_{13}xz + 2a_{23}yz +$$
$$+ 2a_1 x + 2a_2 y + 2a_3 z + a_0 = 0$$

$F(x\,y\,z) \equiv \uparrow$

homog. C.:

$$a_{11}x^2 + a_{22}y^2 + a_{33}z^2 + a_{44}q^2 + 2a_{12}xy + 2a_{13}xz + 2a_{23}yz$$
$$+ 2a_{14}xq + 2a_{24}yq + 2a_{34}zq = 0$$

[illegible] $q = 1$

$F(x\,y\,z\,q) \equiv \uparrow$

[illegible]

10 Coeff., [illegible]

[illegible]

$x_i\, y_i\, z_i\, q_i \qquad i = 1, 2, 3, \ldots\ 9$

$F(x_i\, y_i\, z_i\, q_i) = 0$ [illegible] $\dfrac{a_{11}}{a_{44}} \quad \dfrac{a_{12}}{a_{44}} \quad \ldots \quad \dfrac{a_{33}}{a_{44}}$

[illegible]

$1 \quad 2 \quad 3 \ \text{———}\ 8$ [illegible]

[illegible]

[illegible] Curve

$x_i\ y_i\ z_i\ q_i$

$F(x_i\, y_i\, z_i\, q_i) = 0$ [illegible]

[illegible] $= \lambda$

[illegible]
$a_i - \lambda b_i$ [illegible]

↓

$F(\quad)$

$\varphi(x\, y\, z\, q) - \lambda\, \psi(x, y, z, q)$

$\varphi - \lambda \psi = 0$

$\varphi = 0$ [illegible]
$\psi = 0$ " " " [illegible]

[illegible] $\varphi_q - \lambda \psi_0$

[illegible] $\lambda = \frac{\varphi_q}{\psi_q}$

[illegible] $\varphi_q = 0$

$\psi_q = 0$ [illegible] C. [illegible] φ [illegible] ψ.

[illegible] 7 [illegible]

$F(x_i\ y_i\ z_i\ q_i) = 0 \qquad i = 1, 2, 3, \ldots\ldots 7$

[illegible] 2 [illegible]

$a_i + \lambda b_i + \mu c_i = 0$

$\downarrow$

$F(x y z q) = 0$

$\downarrow$

$\varphi + \lambda \psi + \mu \chi = 0$

[illegible]

BH. [illegible]

[illegible] C [illegible]

[illegible]

---

2 [illegible] C. 4 [illegible]

3 [illegible] 8 [illegible]

5/12

$\varphi + \lambda \psi = 0$

$\varphi = 0$
$\psi = 0$ [illegible]

[illegible] 9 [illegible]

[illegible] 8 [illegible] C 4 [illegible] ∞ [illegible]

[illegible] 7 [illegible]

[illegible] $\varphi\ \psi\ \chi$

$\varphi + \lambda \psi + \mu \chi = 0$

8 [illegible] 8 associierte P.

$e = 0$ [illegible]

$e_1 = 0$ "

$e e_1 = 0$ [illegible]

Ebenenpaar

$e_1 \; e_2 \qquad e_3 \; e_4$

$e_1 e_2 = 0 \qquad e_3 e_4 = 0$ [illegible]

$e_1 e_2 - \lambda e_3 e_4 = 0$ = [illegible]

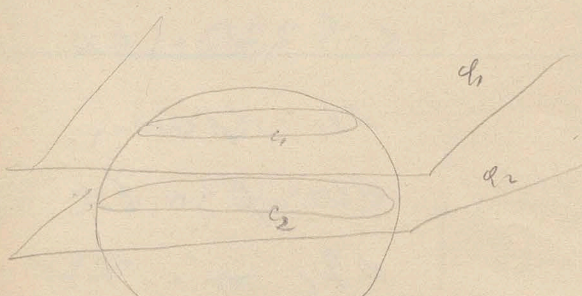
$F = 0$

$e_1 = 0 \quad e_2 = 0$

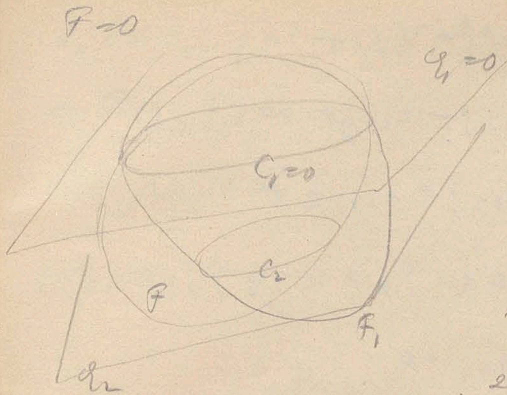
$e_1 e_2 = 0$

$F - \lambda e_1 e_2 = 0$ [illegible]

$e_1 e_2$ [illegible]



$(c_1 \; c_2)$ [illegible]



$F_1 \neq F - \lambda q_1 q_2 = 0$

[illegible] C_1 [illegible]

[illegible] Curve C_2

Parallele

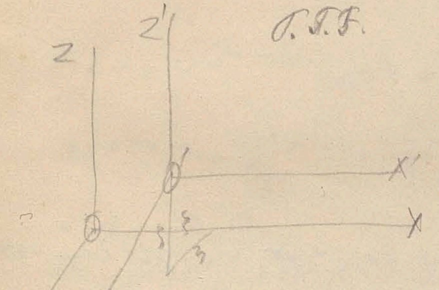
$x y z$

$F(xyz) \equiv$

$a_{11} x^2 + a_{22} y^2 + a_{33} z^2 + 2a_{12} xy + 2a_{13} xz + 2a_{23} yz +$

$+ 2a_{14} x + 2a_{24} y + 2a_{34} z + a_{44} = 0$

P.T.F.



$x = x' + \xi$

$y = y' + \eta$

$z = z' + \zeta$

$a_{11}(x'+\xi)^2 + a_{22}(y'+\eta)^2 + a_{33}(z'+\zeta)^2 +$

$+ 2a_{12}(x'+\xi)(y'+\eta) + 2a_{13}(x'+\xi)(z'+\zeta) + 2a_{23}(y'+\eta)(z'+\zeta)$

$+ 2a_{14}(x'+\xi) + 2a_{24}(y'+\eta) + 2a_{34}(z'+\zeta) + a_{44} = 0$

$x' \text{---} x$

$a_{11} x^2 + a_{22} y^2 + a_{33} z^2 + 2a_{12} xy + 2a_{13} xz + 2a_{23} yz +$

$+2x(a_{11}\xi + a_{12}\eta + a_{13}\zeta + a_{14}) +$

$+2(a_{12}\xi + a_{22}\eta + a_{23}\zeta + a_{24})y +$

$+2(a_{13}\xi + a_{23}\eta + a_{33}\zeta + a_{34})z +$

$a_{11}\xi^2 + a_{22}\eta^2 + a_{33}\zeta^2 + 2a_{12}\xi\eta + 2a_{13}\xi\zeta + 2a_{23}\eta\zeta$

$+2a \text{———} + a_{44} = 0$

[illegible]

[illegible]

$$\frac{\partial F}{\partial x} = 2(a_{11}x + a_{12}y + a_{13}z + a_{14})$$

$$\frac{\partial F}{\partial y} = 2(a_{12}x + a_{22}y + a_{23}z + a_{24})$$

$$\frac{\partial F}{\partial z} = 2(a_{13}x + a_{23}y + a_{33}z + a_{34})$$

$F(xyz) = 0 \quad --- \quad \xi\eta\zeta$

$$a_{11}x^2 + a_{22}y^2 + a_{33}z^2 + 2a_{12}xy + 2a_{23}yz + 2a_{13}xz +$$

$$+ \frac{\partial F}{\partial \xi}x + \frac{\partial F}{\partial \eta}y + \frac{\partial F}{\partial \zeta}z + F(\xi\eta\zeta) = 0$$

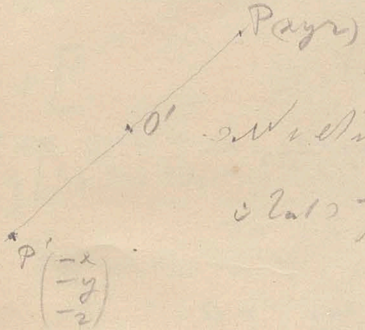
$$\left.\begin{aligned}\frac{\partial F}{\partial \xi} &= 0\\ \frac{\partial F}{\partial \eta} &= 0\\ \frac{\partial F}{\partial \zeta} &= 0\end{aligned}\right\} \quad \left.\begin{aligned} a_{11}\xi + a_{12}\eta + a_{13}\zeta + a_{14} &= 0\\ a_{12}\xi + a_{22}\eta + a_{23}\zeta + a_{24} &= 0\\ a_{13}\xi + a_{23}\eta + a_{33}\zeta + a_{34} &= 0\end{aligned}\right\}$$

$a_{11}x^2 + a_{22}y^2 + a_{33}z^2 + 2a_{12}xy + 2a_{1}$ — —

$+ F(\xi\eta\zeta) = 0$

[illegible]

[illegible] (xyz) [illegible] $(-x\ -y\ -z)$



[illegible]

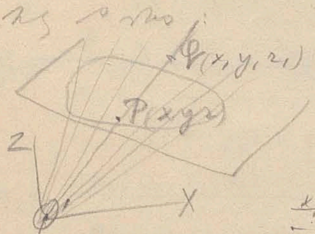
[illegible]

[illegible] $F(\xi\eta\zeta) = 0$ [illegible]

[illegible] homogi:

$a_{11}x^2 + a_{22}y^2 + a_{33}z^2 + 2a_{12}xy + 2a_{13}xz + 2a_{23}yz = 0$

= [illegible]



[illegible]

[illegible]

~~$x_1 : x = y_1 : y = z_1 : z$~~ $= \frac{1}{\lambda}$

$x = \lambda x_1$

$y = \lambda y_1$

$z = \lambda z_1$

$$\lambda_2 \left[a_{11} x_1^2 + a_{22} y_1^2 + a_{33} z_1^2 + \cdots \; 2a_{23} y_1 z_1 \right] = 0$$

$$a_{11} x_1^2 + \text{———} \; 2a_{23} y_1 z_1 = 0$$

[illegible]

[illegible]

12/12

$F(xyz) = 0$ [illegible]

$$a_{11} x^2 + a_{22} y^2 + a_{33} z^2 + 2a_{12} xy + 2a_{13} xz + 2a_{23} yz +$$

$$2a'_{14} x + 2a'_{24} y + 2a'_{34} z + a'_{44} = 0$$

$$\begin{cases} a'_{14} = a_{11}\xi + a_{12}\eta + a_{13}\zeta + a_{14} = 0 \\ a'_{24} = a_{12}\xi + a_{22}\eta + a_{23}\zeta + a_{24} = 0 \\ a'_{34} = a_{13}\xi + a_{23}\eta + a_{33}\zeta + a_{34} = 0 \end{cases}$$

$$a'_{44} = F(\xi\eta\zeta)$$

$$F(xyz) \equiv x(a_{11}x + a_{12}y + a_{13}z + a_{14}) + y(a_{12}x + a_{22}y + \cdots a_{24})$$

$$+ z(a_{13}x + a_{23}y + a_{33}z + a_{34}) + (a_{14}x + a_{24}y + a_{34}z + a_{44})$$

$$F(\xi\eta\zeta) = a_{14}\xi + a_{24}\eta + a_{34}\zeta + a_{44}$$

$\xi\eta\zeta$ [illegible]

$a_{21}\xi$ — a_{12}

$$a_{11}\xi + a_{12}\eta + a_{13}\zeta + a_{14} = 0 \quad a_{23}$$

$$a_{21}\xi + a_{22}\eta + a_{23}\zeta + a_{24} = 0 \quad a_{13} \quad a_{33}$$

$$a_{31}\xi + a_{32}\eta + a_{33}\zeta + a_{34} = 0 \quad a_{23}$$

$a_{ik} = a_{ki}$

ξ elim.:

$$\xi\,[a_{11}a_{23} - a_{21}a_{13}] + \eta\,[a_{12}a_{23} - a_{22}a_{13}] + a_{14}a_{23} - a_{24}a_{13} = 0$$

$$\xi\,[a_{21}a_{33} - a_{31}a_{23}] + \eta\,[a_{22}a_{33} - a_{32}a_{23}] + a_{24}a_{33} - a_{34}a_{23} = 0$$

η elim.

$$\xi\,[(a_{11}a_{23} - a_{21}a_{13})(\qquad) - (\qquad)(\qquad)]$$

$$+ [(\qquad)(\qquad) - (\qquad)(\qquad)] = 0$$

$$\xi\,[a_{11}a_{22}a_{33}a_{23} + a_{21}a_{13}a_{32}a_{23} + a_{21}\,\sout{a_{33}}\,a_{22}a_{13} +$$

$$a_{31}a_{23}a_{12}a_{23} - \times\;] + [\qquad] = 0$$

$$\xi\,a_{23}\,[a_{11}a_{22}a_{33} + a_{21}a_{13}a_{32} + a_{31}a_{12}a_{23} -$$

$$a_{11}a_{23}a_{32} - a_{22}a_{13}a_{31} - a_{33}a_{12}a_{21}\quad] +$$

$$+ a_{23}\,[a_{14}a_{22}a_{33} + a_{24}a_{13}a_{32} + a_{34}a_{12}a_{23} -$$

$$- a_{14}a_{23}a_{32} - a_{22}a_{13}a_{34} - a_{33}a_{12}a_{24}] = 0$$

$$\xi = -\frac{a_{14}a_{22}a_{33} + a_{24}a_{13}\ldots + \qquad +}{a_{11}a_{22}a_{33} + a_{21}a_{13}a_{32} + \qquad +}$$

$$\eta = -\frac{a_{11}a_{24}a_{33} + \text{———}}{a_{11}a_{22}a_{33} + \text{———}}$$

$$\zeta = -\frac{a_{11}a_{22}a_{34} + \text{— — —}}{a_{11}a_{22}a_{33} + \text{— — — —}}$$

$\omega\ \xi\eta\zeta$ [illegible]

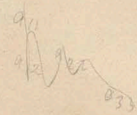
$\omega\ \xi\eta\zeta$ [illegible] [illegible]

$$a_{11}a_{22}a_{33} + 2a_{12}a_{23}a_{31} - a_{11}a_{23}^2 - a_{22}a_{13}^2 - a_{33}a_{12}^2 \gtrless 0.$$

[illegible]

$\omega < 20$ [illegible]

[illegible]



$F(xyz) = 0$

$a_{11}x^2 + \cdots\cdots \qquad + a_{44} = 0$

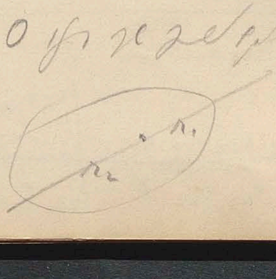
Polarc. [illegible]

$$\left.\begin{aligned} x &= r\cos\alpha \\ y &= r\cos\beta \\ z &= r\cos\gamma \end{aligned}\right\} \alpha\beta\gamma$$

$$r^2[a_{11}\cos^2\alpha + \cdots \qquad 2a_{13}\cos\alpha\cos\gamma] +$$

$$2r[a_{14}\cos\alpha + a_{24}\cos\beta + a_{34}\cos\gamma] + a_{44} = 0$$

$r_1,\ r_2$ [illegible] 0 [illegible]



[illegible] $= a_{44} = 0$

[illegible] O [illegible]

[illegible] $= 0$ [illegible] $a_1 = 0$

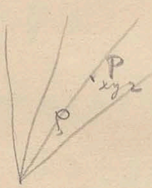
[illegible] $\left\{ \begin{array}{l} r_2 [\ \cos\alpha + \text{———} \ 2 a_{13} \cos\alpha \cos\gamma] + \\ + 2 [a'_{14} \cos\alpha + \text{———} \ a'_{34} \cos\gamma] = 0 \end{array} \right.$

[illegible] $= 0$ [illegible]

$a'_{14} \cos\alpha + a'_{24} \cos\beta + a'_{34} \cos\gamma = 0$

[illegible]

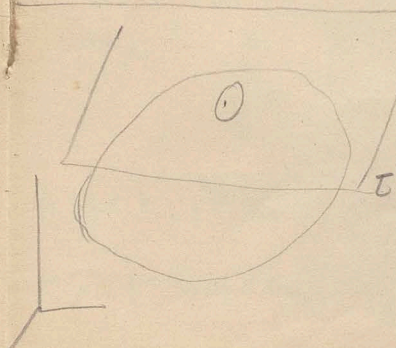
[illegible]



$a'_{14} x + a'_{24} y + a'_{34} z = 0$

[illegible]

[illegible]



O [illegible]

14/12 $F(xyz) = a_{11}x^2 + \text{——} \qquad a_{44} = 0$

CA $\left\{\begin{matrix}\xi\\ \eta\\ \zeta\end{matrix}\right.$ $= a_{11}x^2 + \qquad 2a_{23}yz +$

$2a'_{14}x + 2a'_{24}y + 2a'_{34}z + a'_{44} = 0$

$a'_{44} = F(\xi\eta\zeta)$

$a_{11}x^2 + \text{———} \qquad 2a'_{34}z = 0$

$a'_{14}x + a'_{24}y + a'_{34}z = 0$ = Tangentialeb.

[illegible]:

u_n [illegible] [– homog. [illegible]]

u_{n-1} – –

$u_n + u_{n-1} + u_{n-2} + \text{——} \quad u_3 + u_2 + u_1 + u_0 = 0$

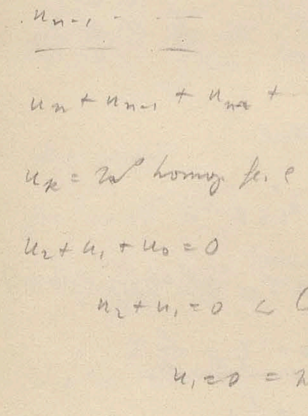
u_k = [illegible] homog. [illegible]

$u_2 + u_1 + u_0 = 0$

$u_2 + u_1 = 0$ [illegible]

$u_1 = 0$ = [illegible]

[illegible] P.



$a'_{14}x + a'_{24}y + a'_{34}z = 0$

$x \quad \text{——} \quad x - \xi$

$y \qquad y - \eta$

$z \qquad z - \zeta$

$a'_{14}(x-\xi) + a'_{24}(y-\eta) + a'_{34}(z-\zeta) = 0$

$a_{21} = a_{12}$

$(a_{11}\xi + a_{12}\eta + a_{13}\zeta + a_{14})(x-\xi) + (a_{21}\xi + a_{22}\eta + a_{23}\zeta + a_{24})(y-\eta)$

$+ (a_{31}\xi + a_{32}\eta + a_{33}\zeta + a_{34})(z-\zeta) = 0$

$(a_{11}\xi + \text{———} \quad)x + (a_{21}\xi + \text{———} \quad)y + (a_{31} \text{———} \quad)z =$

$= a_{11}\xi^2 + a_{22}\eta^2 + a_{33}\zeta^2 + 2a_{12}\xi\eta + 2a_{13}\xi\zeta + 2a_{23}\eta\zeta +$

$(a_{14}\xi + a_{24}\eta + a_{34}\zeta)$

ω < ερθ:

$a_{11}\xi^2 + a_{22}\eta^2 + \text{———}$ $\quad a_{14}\xi + a_{24}\eta + a_{34}\zeta = -(a_{14}\xi + \cdots) + a_{44}$

$(a_{11}\xi + \text{———} \quad)x + (a_{21}\xi + \text{———} \quad)y + (\qquad)z =$

$= -(a_{14}\xi + a_{24}\eta + a_{34}\zeta + a_{44})$

ης ε π. Ζ:

$(a_{11}\xi + a_{12}\eta + a_{13}\zeta + a_{14})x + (a_{21}\xi + a_{22}\eta + a_{23}\zeta + a_{24})y +$

$+ (a_{31}\xi + a_{32}\eta + a_{33}\zeta + a_{34})z + a_{41}\xi + a_{42}\eta + a_{43}\zeta + a_{44} = 0$

homog. C.

x	$\frac{x}{w}$	ξ	$\frac{\xi}{\omega}$
y	$\frac{y}{w}$	η	$\frac{\eta}{\omega}$
z	$\frac{z}{w}$	ζ	$\frac{\zeta}{\omega}$

$(a_{11}\xi + a_{12}\eta + a_{13}\zeta + a_{14}\omega)x + (a_{21}\xi + a_{22}\eta + a_{23}\zeta + a_{24}\omega)y +$

$+ (a_{31}\xi + a_{32}\eta + a_{33}\zeta + a_{34}\omega)z + (a_{41}\xi + a_{42}\eta + a_{43}\zeta + a_{44}\omega)w = 0$

[illegible] $\xi\ \eta\ \zeta$ [illegible].

$$\xi(a_{11}x + a_{12}y + a_{13}z + a_{14}w) + (a_{21}x + a_{22}y + a_{23}z + a_{24}w)\eta$$

$$+ (a_{31}x + a_{32}y + a_{33}z + a_{34}w)\zeta + (a_{41}x + a_{42}y + a_{43}z + a_{44}w)\omega = 0$$

[illegible] ζ -- --

$$F(x\,y\,z\,w) = a_{11}x^2 + a_{22}y^2 + a_{33}z^2 + a_{44}w^2 + 2a_{12}xy + 2a_{13}xz$$

$$+ 2a_{23}yz + 2a_{14}xw + 2a_{24}yw + 2a_{34}zw = 0$$

$$\frac{1}{2}\frac{\partial F}{\partial x} = a_{11}x + a_{12}y + a_{13}z + a_{14}w \qquad \frac{1}{2}\frac{\partial F}{\partial \xi} = \text{———}$$

$$\frac{1}{2}\frac{\partial F}{\partial y} = a_{21}x + a_{22}y + a_{23}z + a_{24}w \qquad \text{———}$$

$$\frac{1}{2}\frac{\partial F}{\partial z} = a_{31}x + \text{———} \quad a_{34}w \qquad \text{———}$$

$$\frac{1}{2}\frac{\partial F}{\partial w} = a_{41}x + \text{———} \quad a_{44}w \qquad \text{———}$$

$$x\frac{\partial F}{\partial \xi} + y\frac{\partial F}{\partial \eta} + z\frac{\partial F}{\partial \zeta} + w\frac{\partial F}{\partial \omega} = 0$$

[illegible]:

$$\xi\frac{\partial F}{\partial x} + \eta\frac{\partial F}{\partial y} + \zeta\frac{\partial F}{\partial z} + \omega\frac{\partial F}{\partial w} = 0$$

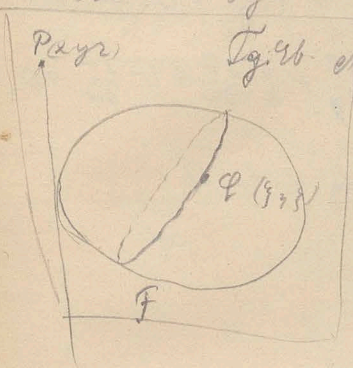
[illegible]:

$$x\frac{\partial F}{\partial \xi} + y\frac{\partial F}{\partial \eta} + z\frac{\partial F}{\partial \zeta} + w\frac{\partial F}{\partial \omega} \equiv \xi\frac{\partial F}{\partial x} + \eta\frac{\partial F}{\partial y} + \text{———}$$

$F(xyzw) = x(a_{11}x + \cdots\cdots) + y(a_{21}x + \cdots a_{24}w) +$

$+ z(a_{31}x + \cdots\cdots) + w(a_{41}x + \cdots\cdots a_{44}w)$

$$x\frac{\partial F}{\partial x} + y\frac{\partial F}{\partial y} + z\frac{\partial F}{\partial z} + w\frac{\partial F}{\partial w} \equiv 2F$$



$F(\xi\eta\zeta\omega) = 0$

$(a_{11}x + a_{12}y + \cdots\cdots)\xi + (\ )\eta + \cdots = 0$

[illegible] P [illegible] Π

[illegible]

[illegible] = Polare

P = [illegible] Π

[illegible] Π [illegible] Pol

$A\xi + B\eta + C\zeta + D\omega = 0$

$a_{11}x + a_{12}y + a_{13}z + a_{14}w = \lambda A$

$a_{21}x + a_{22}y + a_{23}z + a_{24}w = \lambda B$

$a_{31}x + a_{32}y + a_{33}z + a_{34}w = \lambda C$

$a_{41}x + \cdots\cdots = \lambda D$

x, y, z, w =

Polar-Reciprocität [illegible]

[illegible]

[illegible] $= $ [illegible] F [illegible] P [illegible]

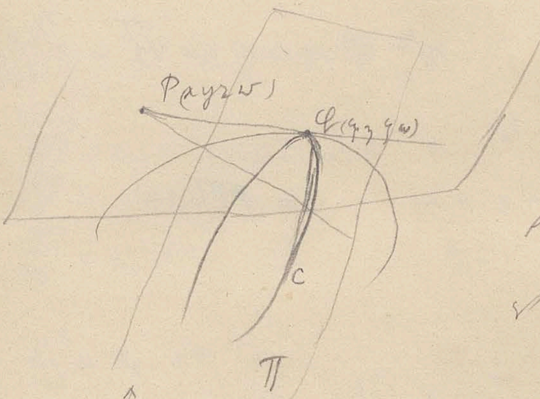
4/12 $\xi\ \eta\ \zeta\ \omega$

$x\ y\ z\ w$

$$\xi \frac{\partial F}{\partial x} + \eta \frac{\partial F}{\partial y} + \zeta \frac{\partial F}{\partial z} + \omega \frac{\partial F}{\partial w} = 0$$

$$\equiv x \frac{\partial F}{\partial \xi} + y \frac{\partial F}{\partial \eta} + z \frac{\partial F}{\partial \zeta} + w \frac{\partial F}{\partial \omega} = 0$$

[illegible] $xyzw$ [illegible] Q



[illegible] C. C [illegible]

[illegible] P C

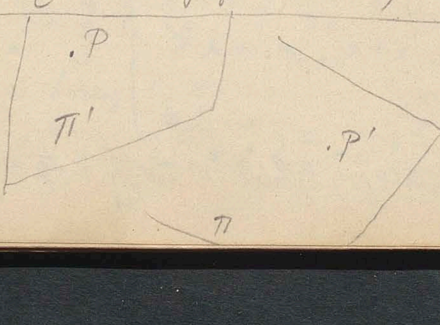
Q — Π

[illegible] Polar [illegible] P

P [illegible] " " " Q

[illegible]

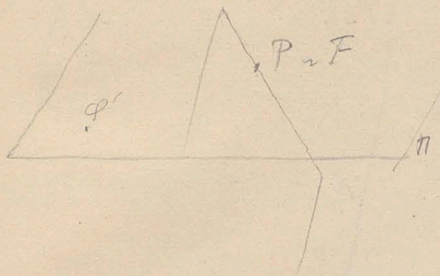
2 [illegible] : 2 conjugierte Pole [illegible]

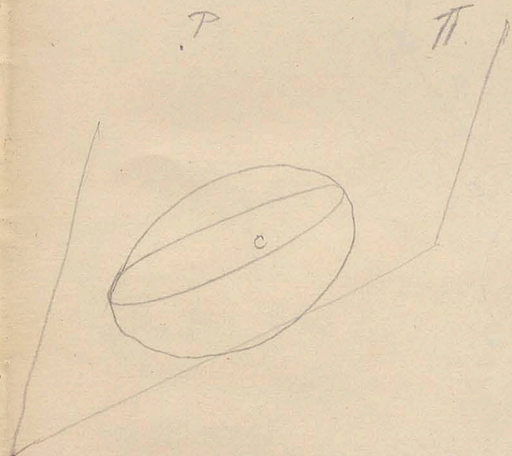


[illegible] P [illegible] Π [illegible]

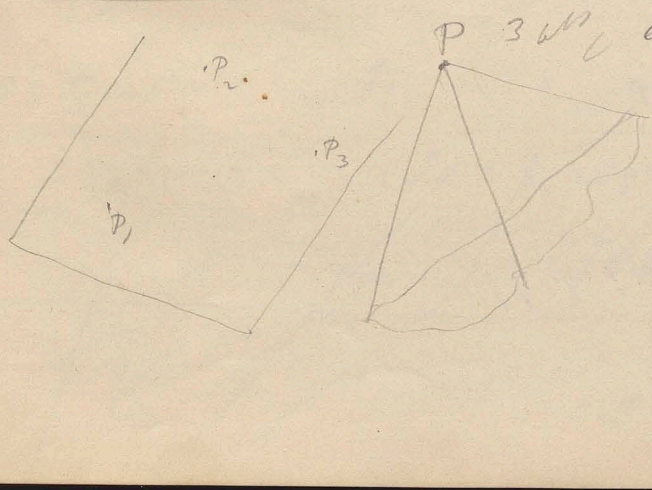
[illegible] P' [illegible]

$\Pi'_2\ P_2$





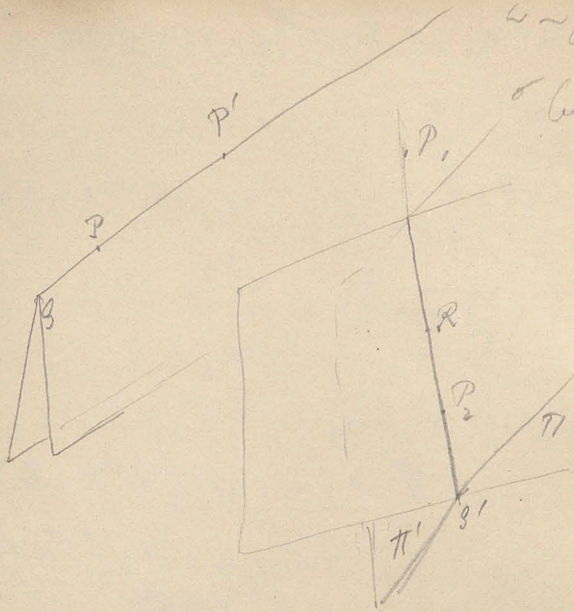
= P



$P_1 P_2 P_3$

[illegible]

[illegible]



[illegible]

$G = P_1, P_2$ [illegible]

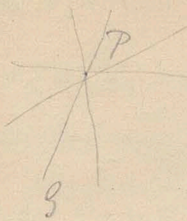
[illegible]

R — $P = G_1{}^2$

[illegible]

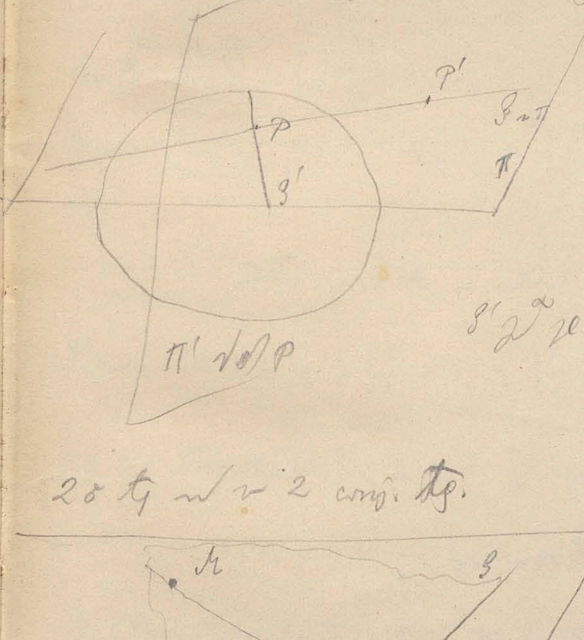
---

2 recipr. [illegible] ~~conj.~~



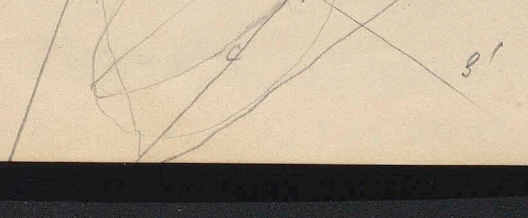
[illegible]

[illegible]



[illegible]

[illegible] conj. tg.



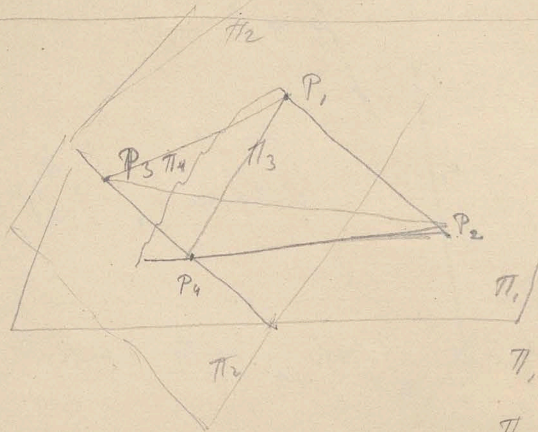
[illegible] conj. tg.

[illegible]

[illegible]

[illegible] P_2

– 2te [illegible] C



~ Tetraeder

[illegible] $P_1 P_2 P_3 P_4$

Π_1 = [illegible] $P_2 P_3 P_4$

Π_2 = $P_1 P_3 P_4$

Π_3 ———

1 2 [illegible] conj. [illegible] C

$P_1 P_2$, $P_3 P_4$ –

~ Quadrupel von conj. Polen = [illegible] conj. Tetr.

1 2 ~ [illegible] conj. C

1 2 conj. [illegible] [C [illegible] conj. C] [illegible] 1 2 [illegible] Tetr.

1 3 ~ [illegible] ~ Tripel von conj C

[illegible] : P_1 ; P_2 + Π_1 ; P_3 ; $P_3 P_4$

9/1

$$\left.\begin{matrix} x_1\, x_2\, x_3\, x_4 \text{ --- } C \\ u_1\, u_2\, u_3\, u_4 \text{ --- } \mathfrak{W} \end{matrix}\right\} C.$$

$$u_1 x_1 + u_2 x_2 + u_3 x_3 + u_4 x_4 = 0$$

$$F(x_1 x_2 x_3 x_4) \equiv F(x) = a_{11} x_1^2 + a_{22} x_2^2 + \ldots 2a_{12} x_1 x_2 + \ldots$$
$$+ 2a_{34} x_3 x_4$$

$$\equiv x_1 [a_{11} x_1 + a_{12} x_2 + a_{13} x_3 + a_{14} x_4] + x_2 [a_{21} x_1 + a_{22} x_2 +$$
$$+ a_{23} x_3 + a_{24} x_4] + x_3 [\quad] + x_4 [\quad] \qquad a_{ik} = a_{ki}$$

$$a_{11} x_1 + a_{12} x_2 + a_{13} x_3 + a_{14} x_4 = \frac{1}{2} \frac{\partial F(x)}{\partial x_1}$$
$$a_{21} x_1 + a_{22} x_2 + a_{23} x_3 + a_{24} x_4 = \frac{1}{2} \frac{\partial F(x)}{\partial x_2}$$

$$\text{------------} = \frac{1}{2} \frac{\partial F(x)}{\partial x_4}$$

$$F(x_1 x_2 x_3 x_4) \equiv x_1 \frac{1}{2} \frac{\partial F}{\partial x_1} + x_2 \frac{1}{2} \frac{\partial F}{\partial x_2} + x_3 \frac{1}{2} \frac{\partial F}{\partial x_3} + x_4 \frac{1}{2} \frac{\partial F}{\partial x_4}$$

C x ∂ f Polar z. u

x — $x_1 x_2 x_3 x_4$, u — $u_1 u_2 u_3 u_4$

$$\xi_1 (a_{11} x_1 + a_{12} x_2 + a_{13} x_3 + a_{14} x_4) + \xi_2 (a_{21} x_1 + \ldots a_{24} x_4) +$$
$$+ \xi_3 (a_{31} x_1 + \ldots a_{34} x_4) + \xi_4 (a_{41} x_1 + \ldots a_{44} x_4) \equiv$$
$$\equiv x_1 (a_{11} \xi_1 + \ldots a_{14} \xi_4) + x_2 (a_{21} \xi_1 + \ldots a_{24} \xi_4) + x_3 (a_{31} \xi_1 + \ldots)$$
$$+ x_4 (a_{41} \xi_1 + \ldots a_{44} \xi_4) \equiv \varphi(x\, \xi)$$

u — $\psi(x, y) = 0$ ψ ... 2 conj. ... $F = 0$ ...

$\xi_1 u_1 + \xi_2 u_2 + \xi_3 u_3 + \xi_4 u_4 = 0$

$$\text{I}\begin{cases} u_1 = a_{11} x_1 + a_{12} x_2 + a_{13} x_3 + a_{14} x_4 \\ u_2 = a_{21} x_1 + \cdots \\ u_3 = a_{31} x_1 + \cdots \\ u_4 = a_{41} x_1 + \cdots \end{cases}$$

$u_1\, u_2\, u_3\, u_4$ ...
$x_1\, x_2\, x_3\, x_4$ ...
$\xi_1\, \xi_2\, \xi_3\, \xi_4$ ...

... C.!

$u_1 x_1 + u_2 x_2 + u_3 x_3 + u_4 x_4 = F(x_1 x_2 x_3 x_4)$

$u_1 x_1 + \cdots + u_4 x_4 = 0$ ~ $F = 0$

$$A = \begin{vmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & \cdots & & \\ a_{31} & \cdots & & \\ a_{41} & a_{42} & a_{43} & a_{44} \end{vmatrix} \gtrless 0 \Big\} \ldots$$

A_{ik} = Unter Δ

$$\text{I} \times \begin{vmatrix} A_{11} & A_{12} \\ A_{21} & \\ A_{31} & \\ A_{41} & \end{vmatrix}$$

$$\text{II}\begin{cases} A x_1 = A_{11} u_1 + A_{21} u_2 + A_{31} u_3 + A_{41} u_4 \\ A x_2 = A_{12} u_1 + \cdots \\ A x_3 = A_{13} u_1 + \cdots \\ A x_4 = A_{14} u_1 + \cdots \end{cases} \Big\} \ldots$$

IV × | u_1
u_2
u_3
u_4

$$A\,(u_1x_1+u_2x_2+u_3x_3+u_4x_4) = A_{11}u_1^2 + A_{22}u_2^2 + A_{33}u_3^2 +$$

$$+ A_{44}u_4^2 + 2A_{12}u_1u_2 + 2A_{13}u_1u_3 + \sout{A_{13}\ldots} + \text{—}$$

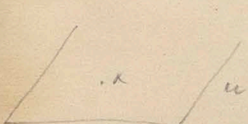
$$+ 2A_{34}u_3u_4 = \Phi(u_1\,u_2\,u_3\,u_4).$$

Φ j F. recipr.

$$u_1x_1+u_2x_2+u_3x_3+u_4x_4 = F(x_1\,x_2\,x_3\,x_4) = \frac{1}{A}\Phi(u_1\,u_2\,u_3\,u_4)$$

Wz. l:

x in u $\quad F(x_1x_2x_3x_4)=0 \quad$ z $\quad \Phi(u_1\,u_2\,u_3\,u_4)=0$



$\Phi = 0$ rów. [illegible] C.
rów. e F. z

$$a_{11}x_1 + a_{12}x_2 + \text{—}\ a_{14}x_4 + (-1)\,u_1 = 0$$
$$a_{21}x_1 + \text{—}\ a_{24}x_4 + (-1)\,u_2 = 0$$
$$a_{31}x_1 + \text{—}\ a_{34}x_4 + (-1)\,u_3 = 0$$
$$a_{41}x_1 + \text{—}\ a_{44}x_4 + (-1)\,u_4 = 0$$
$$u_1x_1 + u_2x_2 + \text{—}\ u_4x_4 + (-1)\,F(x_1x_2x_3x_4) = 0$$

$x_1\,x_2\,x_3\,x_4\,(-1)$ 5 homog. rów.

$$\begin{vmatrix} a_{11} & a_{12} & a_{13} & a_{14} & u_1 \\ a_{21} & a_{22} & a_{23} & a_{24} & u_2 \\ a_{31} & a_{32} & a_{33} & a_{34} & u_3 \\ a_{41} & a_{42} & a_{43} & a_{44} & u_4 \\ u_1 & u_2 & u_3 & u_4 & F \end{vmatrix} = 0$$

x z u $\quad F(x_1, x_2\, x_3\, x_4) = 0$ s $\Phi(u_1, u_2, u_3, u_4) = 0$

$\Phi = 0$ = r.ş e.f. r z C.

r.ş e Tg. r

$$a_{11} x_1 + a_{12} x_2 + \text{—} \; a_{14} x_4 + (-1) u_1 = 0$$

$$a_{21} x_1 + \text{———} \; a_{24} x_4 + (-1) u_2 = 0$$

$$a_{31} x_1 \qquad a_{34} x_4 + (-1) u_3 = 0$$

$$a_{41} x_1 \qquad a_{44} x_4 + (-1) u_4 = 0$$

$$u_1 x_1 + \; u_2 x_2 + \qquad u_4 x_4 + (-1) F(x_1 x_2 x_3 x_4) = 0$$

$x_1\, x_2\, x_3\, x_4\, (-1)$ — 5 homog. R.

$$\begin{vmatrix} a_{11} & a_{12} & a_{13} & a_{14} & u_1 \\ a_{21} & a_{22} & a_{23} & a_{24} & u_2 \\ a_{31} & a_{32} & a_{33} & a_{34} & u_3 \\ a_{41} & a_{42} & a_{43} & a_{44} & u_4 \\ u_1 & u_2 & u_3 & u_4 & F \end{vmatrix} = 0$$

$$\sin\left(x + \frac{\pi}{4}\right) = \sin x \cos\frac{\pi}{4} + \cos x \sin\frac{\pi}{4} = \sin\frac{\pi}{4}\left[\sin x + \cos x\right]$$

$$= \frac{d^{\frac{1}{2}} \sin x}{\sqrt{dx}}$$

$$d^{\frac{1}{2}} \sin x = \sqrt{\sin dx}\,\left[\sin x + \cos x\right] \sin\frac{\pi}{4}$$

Schram Sa: 8–9

IX Türkenst. 23

IX Berggasse

II. V. S. 1892/93

Dr. Emil Weyr

Analytische Geometrie des Raumes

M. Smoluchowski

11/1a

$F(x_1, x_2\, x_3\, x_4) = 0$ $\quad$ c C.

$\Phi(u_1, u_2\, u_3\, u_4) = 0$ $\quad$ z C.

$$\begin{vmatrix} a_{11} & a_{12} & a_{13} & a_{14} & u_1 \\ a_{21} & a_{22} & a_{23} & a_{24} & u_2 \\ a_{31} & a_{32} & a_{33} & a_{34} & u_3 \\ a_{41} & a_{42} & a_{43} & a_{44} & u_4 \\ u_1 & u_2 & u_3 & u_4 & F \end{vmatrix} = 0$$

$$F = \frac{1}{A}\Phi$$

$$\begin{vmatrix} a_{11} & a_{12} & a_{13} & a_{14} & 0+u_1 \\ a_{21} & a_{22} & a_{23} & a_{24} & 0+u_2 \\ a_{31} & a_{32} & a_{33} & a_{34} & 0+u_3 \\ a_{41} & a_{42} & a_{43} & a_{44} & 0+u_4 \\ u_1 & u_2 & u_3 & u_4 & \frac{1}{A}\Phi+0 \end{vmatrix} = 0$$

$$\begin{vmatrix} a_{11} & a_{12} & a_{13} & a_{14} & 0 \\ a_{21} & a_{22} & a_{23} & a_{24} & 0 \\ a_{31} & a_{32} & a_{33} & a_{34} & 0 \\ a_{41} & a_{42} & a_{43} & a_{44} & 0 \\ u_1 & u_2 & u_3 & u_4 & \frac{1}{A}\Phi \end{vmatrix} + \begin{vmatrix} a_{11} & a_{12} & a_{13} & a_{14} & u_1 \\ a_{21} & a_{22} & a_{23} & a_{24} & u_2 \\ a_{31} & a_{32} & a_{33} & a_{34} & u_3 \\ a_{41} & a_{42} & a_{43} & a_{44} & u_4 \\ u_1 & u_2 & u_3 & u_4 & 0 \end{vmatrix}$$

$$= A\ \frac{1}{A}\Phi(-1)^8 + \;\nearrow$$

$$\Phi = -\begin{vmatrix} a_{11} & a_{12} & a_{13} & a_{14} & u_1 \\ a_{21} & a_{22} & a_{23} & a_{24} & u_2 \\ a_{31} & a_{32} & a_{33} & a_{34} & u_3 \\ a_{41} & a_{42} & a_{43} & a_{44} & u_4 \\ u_1 & u_2 & u_3 & u_4 & 0 \end{vmatrix}$$

x — u
a — A $\Big\} = \Phi$

$= 0$ [illegible]

[illegible]

[illegible]:

Rel. [illegible] Δ [illegible] adj. [illegible]

$$\begin{vmatrix} A_{11} & A_{12} & A_{13} & A_{14} \\ A_{21} & \text{—} & & \\ A_{31} & \text{—} & & \\ A_{41} & \text{—} & & \end{vmatrix} = \underline{A}$$

$$\underline{A} \cdot A = \begin{vmatrix} A & 0 & 0 & 0 \\ 0 & A & 0 & 0 \\ 0 & 0 & A & 0 \\ 0 & 0 & 0 & A \end{vmatrix} = A^4$$

[illegible] $A \gtrless 0$

$$\underline{A} = A^3$$

$$a_{11} A_{11} + a_{12} A_{12} + \text{—} + a_{14} A_{14} = A \quad \Big| \; A_{11}$$

$$a_{11} A_{21} + a_{12} A_{22} + \quad + a_{14} A_{24} = 0 \quad \Big| \; A_{21}$$

$$a_{11} A_{31} + a_{12} A_{32} + \text{—} + a_{14} A_{34} = 0 \quad \Big| \; A_{31}$$

$$a_{11} A_{41} + a_{12} A_{42} + \quad + a_{14} A_{44} = 0 \quad \Big| \; A_{41}$$

$$a_{11} \underline{A} = A A_{11}$$

$$\underline{A} = A^3$$

$$a_{11} = \frac{A_{11}}{A^2_{(4)}} \qquad a_{ik} = \frac{A_{ik}}{A^2} = \frac{A_{ik}}{A^{\frac{2}{3}}}$$

[illegible]

$$\Phi(u_1 u_2 u_3 u_4) \equiv A_{11} u_1^2 + A_{22} u_2^2 + \ldots\ 2 A_{34} u_3 u_4 = 0$$

$$A = \begin{vmatrix} A_{11} & A_{12} & A_{13} & A_{14} \\ A_{21} & & & \\ A_{31} & & & \\ A_{41} & & & \end{vmatrix}$$

A_{ik} = [illegible] adj. [illegible] A_{ik} [illegible]

$$a = \frac{A_{ik}}{(A)^{\frac{2}{3}}}$$

$$F(x_1 x_2 x_3 x_4) = a_{11} x_1^2 + a_{22} x_2^2 + \ldots\ 2 a_{34} x_3 x_4 = 0$$

[illegible] homog. [illegible]

[illegible] $m_1\ m_2\ m_3\ m_4$

$\Phi(m_1\ m_2\ m_3\ m_4) = 0$ [illegible]

[illegible]

[illegible]

[illegible]

[illegible] Coeff. [illegible]: $a + b\lambda$ λ [illegible]

[illegible]

$$\Psi(u_1 u_2 u_3 u_4) + \lambda\, \Omega(u_1 u_2 u_3 u_4) = 0$$

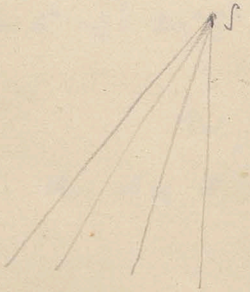
[illegible] ∞ [illegible]

[illegible] Ψ [illegible] Ω [illegible] $\Psi + \lambda\,\Omega$

[illegible] developables [illegible]

[illegible]

[illegible]



[illegible]

[illegible] 2 [illegible]

[illegible]

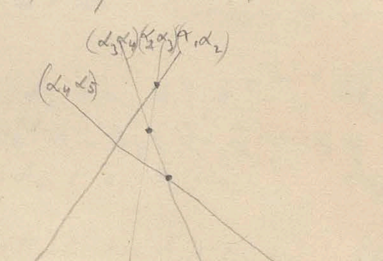
[illegible] = [illegible]

[illegible] = [illegible]

[illegible] dev. [illegible] [[illegible]

[illegible]]

1. 2 [illegible] ; 1 3 + [illegible]



[illegible]

[illegible]

[illegible]

= [illegible]

[illegible] = [illegible]

1/4

$a_{11}x_1^2 + \cdots \quad 2a_{14}x_3x_4 = 0$

$\sum a_{ik}x_ix_k = 0 \qquad a_{ik} = a_{ki}$

$x_4 = 1 \qquad x_3 = z \qquad x_2 = y \qquad x_1 = x$

$a_{11}x^2 + a_{22}y^2 + a_{33}z^2 + 2a_{12}xy + 2a_{13}xz + 2a_{23}yz +$

$+ 2a_{14}x + 2a_{24}y + 2a_{34}z + a_{44} = 0$

$x + \xi$

$y + \eta$

$z + \zeta$

$a_{11}\xi + a_{12}\eta + a_{13}\zeta + a_{14} = 0$

$a_{21}\xi + a_{22}\eta + a_{23}\zeta + a_{24} = 0$

$a_{31}\xi + a_{32}\eta + a_{33}\zeta + a_{34} = 0$

$a_{11}x^2 + a_{22}y^2 + a_{33}z^2 + 2a_{12}xy + 2a_{13}xz + 2a_{23}yz +$

$+ a'_{44} = 0$

$$A_{44} = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} \gtrless 0 \qquad \Delta = \begin{vmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \\ a_{41} & a_{42} & a_{43} & a_{44} \end{vmatrix}$$

A_{44} $a'_{44} = a_{11}\xi^2 + a_{22}\eta^2 + \ldots \quad a_{44}$

$= \xi(a_{11}\xi + a_{12}\eta + a_{13}\zeta + a_{14}) = 0$

$+ \eta(a_{21}\xi + a_{22}\eta + a_{23}\zeta + a_{24}) = 0$

$+ \zeta(a_{31}\xi + a_{32}\eta + a_{33}\zeta + a_{34}) = 0$

$+ a_{41}\xi + a_{42}\eta + a_{43}\zeta + a_{44}$

$a'_{44} = a_{41}\xi + a_{42}\eta + a_{43}\zeta + a_{44}$

$\xi = \dfrac{A_{41}}{A_{44}} \qquad \eta = \dfrac{A_{42}}{A_{44}} \qquad \zeta = \dfrac{A_{43}}{A_{44}}$

$a'_{44} = \dfrac{a_{41}A_{41} + a_{42}A_{42} + a_{43}A_{43} + a_{44}A_{44}}{A_{44}}$

$a'_{44} = \dfrac{\Delta}{A_{44}}$

$a_{11}x^2 + a_{22}y^2 + a_{33}z^2 + 2a_{12}xy + 2a_{13}xz + 2a_{23}yz$

$+ \dfrac{\Delta}{A_{44}} = 0$ a e sec = v e f

rz e C. :

$x = \alpha_1 x' + \alpha_2 y' + \alpha_3 z'$
$y = \beta_1 x' + \beta_2 y' + \beta_3 z'$ } ~ f
$z = \gamma_1 x' + \gamma_2 y' + \gamma_3 z'$

$a'_{11}x'^2 + a'_{22}y'^2 + \text{———}$ $2a'_{23}y'z' + \frac{\Delta}{A_{44}} = 0$

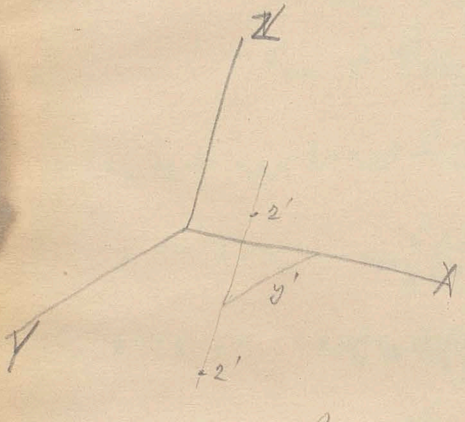
$a_{11}x^2 + \text{———}$ $2a_{23}yz = F$

$a'_{11}x'^2 + \text{———}$ $2a'_{23}y'z' = F'$

F [illegible] F'

$a'_{12} = 0$ $a'_{13} = 0$ $a'_{23} = 0$

$a'_{11}x'^2 + a'_{22}y'^2 + a'_{33}z'^2 + \frac{\Delta}{A_{44}} = 0$



[illegible]

$\frac{1}{2}$

$z' = \pm\sqrt{\dots\dots}$

[illegible]

[illegible]

[illegible]

Transf. [illegible] :

$a'_{12} = 0$ $a'_{23} = 0$ $a'_{13} = 0$

[illegible] XYZ ⊥

[illegible] X'Y'Z' [illegible] "

$x^2+y^2+z^2 = r^2 = x'^2+y'^2+z'^2$

~~[illegible]~~

1/5 F [illegible] Transf. [illegible] F' x

$F = a_{11}x^2 + \dots \qquad 2a_{23}yz = 0$

[illegible]

[illegible]

$$\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = 0$$

[illegible] F' [illegible]

$x^2+y^2+z^2 = x'^2+y'^2+z'^2$

$k(x^2+y^2+z^2) = k(x'^2+y'^2+z'^2)$

$F - F'$

$k(x^2+y^2+z^2) - F = k(x'^2+y'^2+z'^2) - F'$

$(k-a_{11})x^2 + (k-a_{22})y^2 + (k-a_{33})z^2 - 2a_{12}xy - 2a_{13}xz -$

$-2a_{23}yz = (k-a'_{11})x'^2 + (k-a'_{22})y'^2 + (k-a'_{33})z'^2 -$

$-2a'_{12}x'y' - 2a_{13}x'z' - 2a_{23}y'z'$

[illegible]

[illegible]:

$$\begin{vmatrix} k-a_{11} & -a_{12} & -a_{13} \\ -a_{12} & k-a_{22} & -a_{23} \\ -a_{13} & -a_{23} & k-a_{33} \end{vmatrix} = 0$$

[illegible] k [illegible] $\sim$ $\begin{matrix} k-a_{11} & -a_{12} & -a_{13} \\ & k-a_{22} & -a_{13} \end{matrix}$

$$\begin{vmatrix} k-a'_{11} & -a'_{12} & -a'_{13} \\ -a'_{12} & k-a'_{22} & -a'_{23} \\ -a'_{13} & -a'_{23} & k-a'_{33} \end{vmatrix} = 0$$

$$(k-a_{11})(k-a_{22})(k-a_{33}) - 2\,a_{12}\,a_{23}\,a_{13} - a_{13}^2(k-a_{22}) -$$

$$-a_{23}^2(k-a_{11}) - a_{12}^2(k-a_{33}) = 0$$

$$k^3 - k^2(a_{11}+a_{22}+a_{33}) + k(a_{11}a_{22} + a_{11}a_{33} + a_{22}a_{33} -$$

$$- a_{12}^2 - a_{23}^2 - a_{13}^2) - [a_{11}a_{22}a_{33} + 2a_{12}a_{23}a_{13} -$$

$$~~a_{12}^2 - a_{13}^2~~ \; - a_{12}^2 a_{33} - a_{23}^2 a_{11} - a_{13}^2 a_{22}] = 0$$

3 [illegible] k $\begin{matrix} k_1 \\ k_2 \\ k_3 \end{matrix}$ [illegible] k [illegible] ... [illegible]

$$k^3 - k^2(a'_{11}+a'_{22}+a'_{33}) + k(a'_{11}a'_{22} + a'_{11}a'_{33} + a'_{22}a'_{33} -$$

$$-a'^2_{12} - a'^2_{23} - a'^2_{13}) - [a'_{11}a'_{22}a'_{33} + 2a'_{12}a'_{23}a'_{13} - a'^2_{12}a'_{33} -$$

$a_{23}'^2 a_{11}' - a_{13}'^2 a_{22}'] = 0$

$k_1, \; k_2 \; k_3$ [illegible]

[illegible]: $a_{11}' + a_{22}' + a_{33}' = a_{11} + a_{22} + a_{33}$

$$\begin{cases} a_{11}' a_{22}' + a_{22}' a_{33}' + a_{33}' a_{11}' - a_{12}'^2 - a_{23}'^2 - a_{13}'^2 = \\ = a_{11} a_{22} + a_{22} a_{33} + a_{33} a_{11} - a_{12}^2 - a_{23}^2 - a_{13}^2 \end{cases}$$

$$\begin{cases} a_{11}' a_{22}' a_{33}' + 2 a_{12}' a_{23}' a_{13}' - a_{12}'^2 a_{33}' - a_{23}'^2 a_{11}' - \\ - a_{13}'^2 a_{22}' = a_{11} a_{22} a_{33} + 2 a_{12} a_{23} a_{31} - a_{12}^2 a_{33} - \\ - a_{23}^2 a_{11} - a_{13}^2 a_{22}. \end{cases}$$

[illegible]: $a_{13}' = 0 \quad a_{23}' = 0 \quad a_{12}' = 0$

[illegible]: $a_{11}' + a_{22}' + a_{33}' = a_{11} + a_{22} + a_{33}$

$$a_{11}' a_{22}' + a_{22}' a_{33}' + a_{33}' a_{11}' = a_{11} a_{22} + a_{22} a_{33} + a_{33} a_{11} - a_{12}^2 - a_{23}^2 - a_{13}^2$$

$$a_{11}' a_{22}' a_{33}' = a_{11} a_{22} a_{33} + 2 a_{12} a_{23} a_{13} - a_{12}^2 a_{33} - a_{23}^2 a_{11} - a_{13}^2 a_{22}$$

[illegible] $a_{11}' \; a_{22}' \; a_{33}'$:

$$u^3 - (a_{11} + a_{22} + a_{33}) u^2 + (a_{11} a_{22} + a_{22} a_{33} + a_{33} a_{11} - a_{12}^2 - a_{23}^2 - a_{13}^2) u - (a_{11} a_{22} a_{33} + 2 a_{12} a_{23} a_{13} - a_{12}^2 a_{33} - a_{23}^2 a_{11} - a_{13}^2 a_{22}) = 0$$

u_1, u_2, u_3

$u_1 = a'_{11}$

$u_2 = a'_{22}$

$u_3 = a'_{33}$

$$a'_{11} x'^2 + a'_{22} y'^2 + a'_{33} z'^2 + \frac{\Delta}{A_{44}} = 0$$

$$u_1 x^2 + u_2 y^2 + u_3 z^2 + \frac{\Delta}{A_{44}} = 0$$

Transf. ⊥

10/5 $$u_1 x^2 + u_2 y^2 + u_3 z^2 + \frac{\Delta}{A_{44}} = 0$$

$$u^3 - (a_{11} + a_{22} + a_{33}) u^2 + (a_{11} a_{22} + a_{22} a_{33} + a_{33} a_{11} -$$

$$- a_{12}^2 - a_{23}^2 - a_{13}^2) u - [a_{11} a_{22} a_{33} + 2 a_{12} a_{23} a_{13} -$$

$$- a_{12}^2 a_{33} - a_{13}^2 a_{22} - a_{23}^2 a_{11}] = 0$$

3

$$(u - a_{11})(u - a_{22})(u - a_{33}) - a_{12}^2 [u - a_{33}] - a_{23}^2 [u - a_{11}] -$$

$$- a_{13}^2 [u - a_{22}] - 2 a_{12} a_{23} a_{13} = 0$$

$$\equiv (u - a_{33}) \left[(u - a_{11})(u - a_{22}) - a_{12}^2 \right] - a_{23}^2 [u - a_{11}] - a_{13}^2 [u - a_{22}] -$$

$$- 2 a_{12} a_{23} a_{13} = 0$$

$$(u - a_{11})(u - a_{22}) - a_{12}^2 = 0$$

2

$$u^2 - u(a_{11} + a_{22}) + a_{11}a_{22} - a_{12}^2 = 0$$

$y \equiv$ [illegible]

$x \equiv u$

$$\left.\begin{matrix}\alpha\\ \beta\end{matrix}\right\} = \frac{a_{11}+a_{22}}{2} \pm \sqrt{\left(\frac{a_{11}+a_{22}}{2}\right)^2 + a_{12}^2 - a_{11}a_{22}}$$

$$= \frac{a_{11}+a_{22}}{2} \pm \sqrt{\frac{(a_{11}-a_{22})^2}{2} + a_{12}^2}$$

$= \omega$

[illegible]

$a_{11} a_{22}$ [illegible] α, β [illegible]

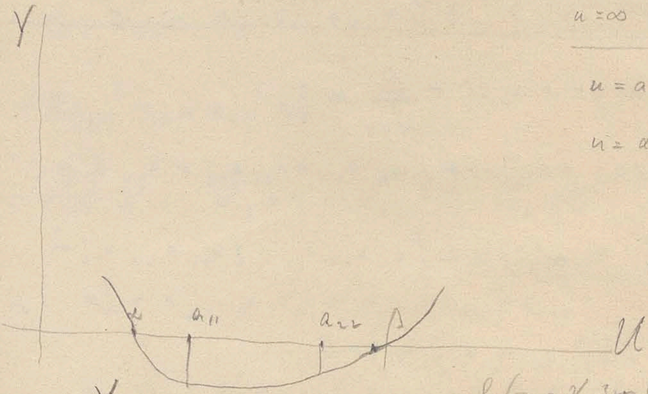
[illegible] $<$ [illegible] α [illegible] u [illegible]

$>$ [illegible]

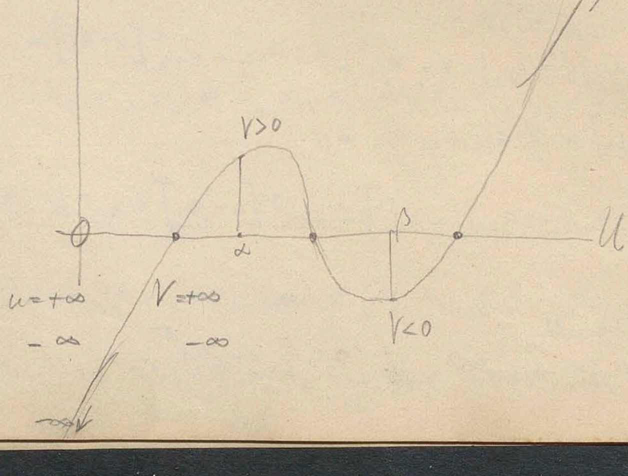
$u = \infty \quad y > 0$

$u = a_{11} \quad y = -a_{12}^2$

$u = a_{22} \quad y = -a_{12}^2$



[illegible]



[illegible] $= \alpha$

$$V = -a_{23}^2(u-a_{11}) - a_{13}^2(u-a_{22}) - 2a_{12}a_{23}a_{13}$$

[illegible] $\alpha =$ [illegible]

$$u = \alpha < a_{11} < a_{22}$$

$$V = a_{23}^2(a_{11}-u) + a_{13}^2(a_{22}-u) - 2a_{12}a_{23}a_{13}$$

$$(a_{11}-u)(a_{22}-u) = a_{12}^2$$

$$V = \left[a_{23}\sqrt{a_{11}-u} + a_{13}\sqrt{a_{22}-u}\right]^2 > 0$$

[illegible] $u = \beta$

$$V = -a_{23}^2\underbrace{(u-a_{11})}_{>0} - a_{13}^2\underbrace{(u-a_{22})}_{>0} - 2a_{12}a_{23}a_{13}$$

$$\beta > a_{11} > a_{22}$$

$$V = -\left[a_{23}^2(u-a_{11}) + a_{13}^2(u-a_{22}) + 2a_{12}a_{23}a_{13}\right]$$

$$= -\left[a_{23}\sqrt{u-a_{11}} + a_{13}\sqrt{u-a_{22}}\right]^2 < 0$$

[illegible] 3 [illegible] 3 [illegible]

[illegible]

$$u_1x^2 + u_2y^2 + u_3z^2 = -\frac{A}{A_{44}} = D$$

$D > 0$

I. $u_1, u_2, u_3 > 0$

II. $u_1, u_2 > 0 \quad u_3 < 0$

III $u_1 > 0, \quad u_2, u_3 < 0$

IV $u_1, u_2, u_3 < 0$

$u_1 x^2 + u_2 y^2 + u_3 z^2 = D$
> 0

IV. [illegible] ; [illegible] imag.

15/5 $u_1 x^2 + u_2 y^2 + u_3 z^2 = D$

$D > 0$

$$D = -\frac{\Delta}{A_{44}}$$

I. $u_1, u_2, u_3 > 0$

$$\frac{x^2}{\frac{D}{u_1}} + \frac{y^2}{\frac{D}{u_2}} + \frac{z^2}{\frac{D}{u_3}} = 1$$

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$$

Ellipsoid

II. $u_1, u_2 > 0 \quad u_3 < 0$

$$\frac{x^2}{\frac{D}{u_1}} + \frac{y^2}{\frac{D}{u_2}} + \frac{z^2}{\frac{D}{u_3}} = 1$$

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{z^2}{c^2} = 1$$ Hyperboloid I Art [v. r. p]

III $u_1 > 0, \quad u_2, u_3 < 0$

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} - \frac{z^2}{c^2} = 1$$ Hyperboloid II Art [illegible]

$\Delta = 0$

$D = 0$

$$u_1 x^2 + u_2 y^2 + u_3 z^2 = 0$$

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 0$$ [illegible] $x=0 \; y=0 \; z=0$

imagin. [illegible]

[illegible] xyz [illegible] $\partial x \; \partial y \; \partial z$

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{z^2}{c^2} = 0$$ [illegible]

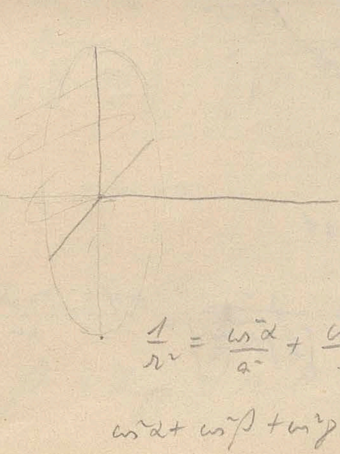
$$\frac{x^2}{a^2} - \frac{y^2}{b^2} - \frac{z^2}{c^2} = 0$$ [illegible] $x \lessgtr z$ [illegible]

I Ell. [illegible] $\Sigma \dots^2 = 1$

$|x| \leqq a$

$|y| \leqq b$

$|z| \leqq c$



$x = r\cos\alpha$

$y = r\cos\beta$

$z = r\cos\gamma$

$$\frac{1}{r^2} = \frac{\cos^2\alpha}{a^2} + \frac{\cos^2\beta}{b^2} + \frac{\cos^2\gamma}{c^2}$$

$$\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$$

$$\frac{1}{r^2} = \frac{1}{a^2} + \cos^2\beta \underbrace{\left(\frac{1}{b^2} - \frac{1}{a^2}\right)}_{>0} + \cos^2\gamma \underbrace{\left(\frac{1}{c^2} - \frac{1}{a^2}\right)}_{>0}$$

$a > b > c$

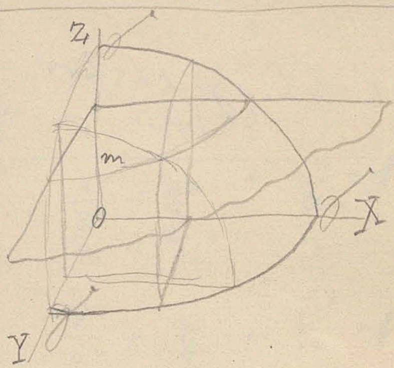
$\frac{1}{r^2} \geqq \frac{1}{a^2}$

$r \leqq a$

$$\frac{1}{r^2} = \frac{1}{c^2} - \cos^2\alpha \underbrace{\left(\frac{1}{c^2} - \frac{1}{a^2}\right)}_{>0} - \cos^2\beta \underbrace{\left(\frac{1}{c^2} - \frac{1}{b^2}\right)}_{>0}$$

$\frac{1}{r^2} \leqq \frac{1}{c^2}$

$r \geqq c$



$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ $z=0$

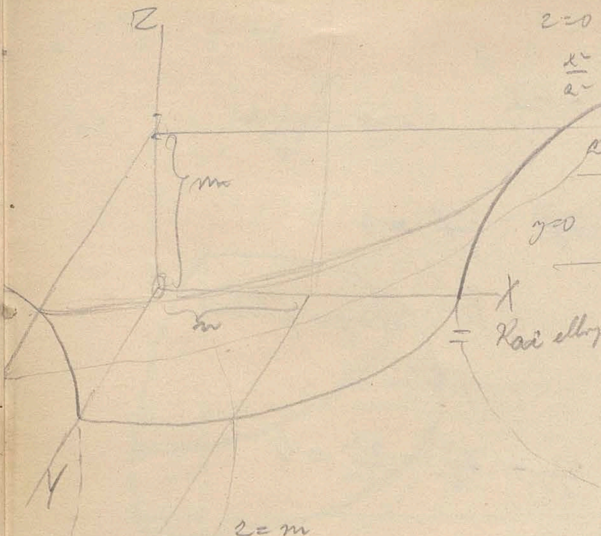
$\frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$ $x=0$

$\frac{x^2}{a^2} + \frac{z^2}{c^2} = 1$ $y=0$

$z = m$

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{m^2}{c^2} = 1$$

$$\frac{x^2}{\left[a\sqrt{1-\frac{m^2}{c^2}}\right]^2} + \frac{y^2}{\left[b\sqrt{1-\frac{m^2}{c^2}}\right]^2} = 1$$ un ell. si $|m| < c$



$z=0$

$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$

a, b

$y=0$ $\frac{x^2}{a^2}-\frac{z^2}{c^2}=1$

$x=0$ $\frac{y^2}{b^2}-\frac{z^2}{c^2}=1$

$z=m$

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}-\frac{m^2}{c^2}=1$$

$$\frac{x^2}{\left[a\sqrt{1+\frac{m^2}{c^2}}\right]^2}+\frac{y^2}{\left[b\sqrt{1+\frac{m^2}{c^2}}\right]^2}=1$$

$x=n$ $\frac{n^2}{a^2}+\frac{y^2}{b^2}-\frac{z^2}{c^2}=1$

$$\frac{y^2}{\left[b\sqrt{1-\frac{n^2}{a^2}}\right]^2}-\frac{z^2}{\left[c\sqrt{1-\frac{n^2}{a^2}}\right]^2}=1$$ Hyperbel

---

$n < a$ y ro = μ ro

$n > a$ z ro = μ ro

[illegible] : Hyp. [illegible]

Polar C.

$$\frac{1}{r^2} = \frac{\cos^2\alpha}{a^2} + \frac{\cos^2\beta}{b^2} - \frac{\cos^2\gamma}{c^2}$$

$r = \infty$ ω $\frac{\cos^2\alpha}{a^2} + \frac{\cos^2\beta}{b^2} - \frac{\cos^2\gamma}{c^2} = 0$

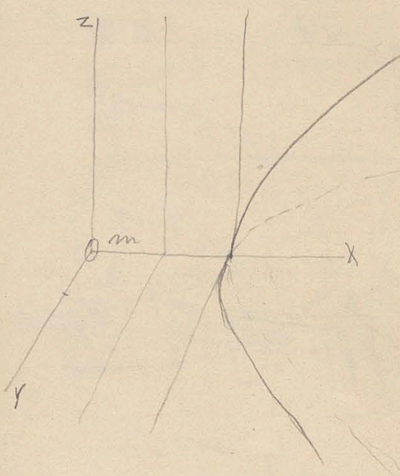
δ ... : Asymptoten

... = Asymptoten ... :

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{z^2}{c^2} = 0$$

17/5

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} - \frac{z^2}{c^2} = 1$$



$z = 0$

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

$y = 0$

$$\frac{x^2}{a^2} - \frac{z^2}{c^2} = 1$$

$x = 0$

$$-\frac{y^2}{b^2} - \frac{z^2}{c^2} = 1$$

$\frac{y^2}{b^2} + \frac{z^2}{c^2} = -1$ imag.

$x = m$

$$\frac{m^2}{a^2} - \frac{y^2}{b^2} - \frac{z^2}{c^2} = 1$$

$$\frac{y^2}{b^2} + \frac{z^2}{c^2} = \frac{m^2}{a^2} - 1$$

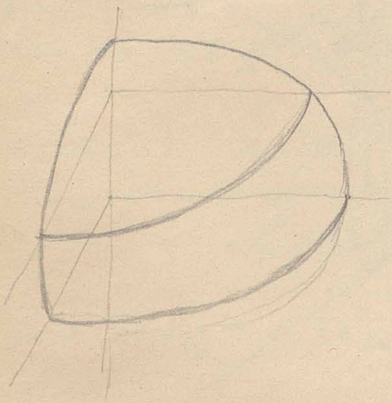
$$\frac{y^2}{\left(b\sqrt{\frac{m^2}{a^2}-1}\right)^2} + \frac{z^2}{\left(c\sqrt{\frac{m^2}{a^2}-1}\right)^2} = 1$$ Ellipse ... $\frac{m^2}{a^2} > 1$

$m > a$

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$$

$b=a$ $\qquad \frac{x^2+y^2}{a^2}+\frac{z^2}{c^2}=1$

Rotat. Ell.



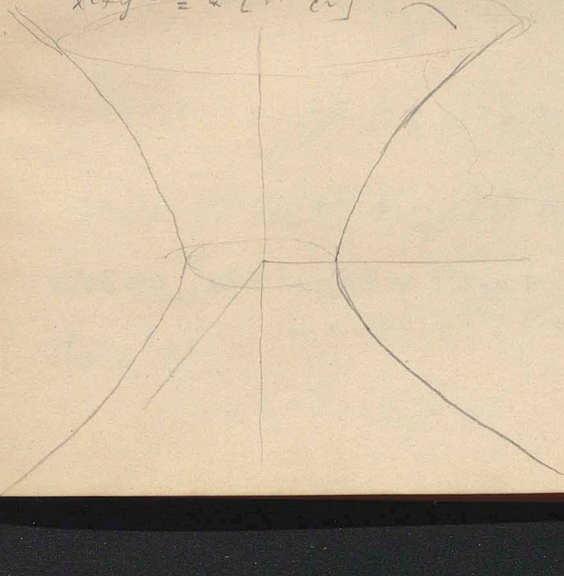
$a=b=c$ $\qquad x^2+y^2+z^2=a^2$

$\frac{x^2}{a^2}+\frac{y^2}{b^2}-\frac{z^2}{c^2}=1$ $\qquad a=b$

$\frac{x^2+y^2}{a^2}=1+\frac{z^2}{c^2}$ $\qquad z=m$

$x^2+y^2=a^2\left[1+\frac{m^2}{c^2}\right]$

Rotat. Hyperboloid



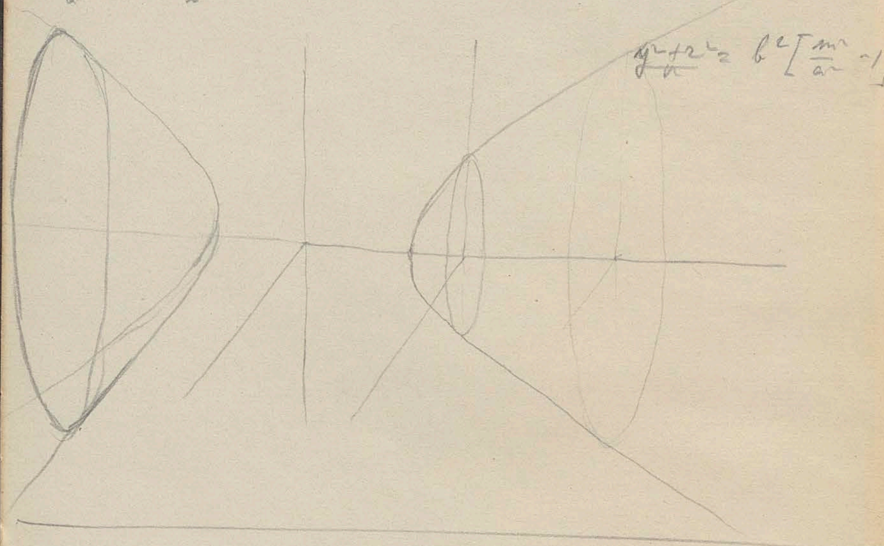
$$\frac{x^2}{a^2} - \frac{y^2}{b^2} - \frac{z^2}{c^2} = 1$$

$b = c$

$$\frac{x^2}{a^2} - \frac{y^2+z^2}{b^2} = 1$$

$x = m$

$$y^2+z^2 = b^2\left[\frac{m^2}{a^2} - 1\right]$$



c $A_{44} = 0$ [illegible] 2∞ [illegible]

$a_{11}x^2 + a_{22}y^2 + a_{33}z^2 + 2a_{12}xy + \ldots + a_{44} = 0$

RW C.S. [illegible] ‖ [illegible]

$F \quad + 2a_{14}x + 2a_{24}y + 2a_{34}z + a_{44} = 0$

$F' \quad + 2a'_{14}x' + 2a'_{24}y' + 2a'_{34}z' + a'_{44} = 0$

$u_1x'^2 + u_2y'^2 + \quad + u_3z'^2 + 2a'_{14}x' + 2a'_{24}y' + 2a'_{34}z' + a'_{44} = 0$

[illegible] u^3 ...

$$A_{44} = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = a_{11} a_{22} a_{33} + 2 a_{12} a_{23} a_{33} -$$

$\Delta_{pq} = 0$ ez [illegible]

$u_3 = 0$ $\qquad u_2, u_1$

[illegible] e^2 [illegible]

$$u_1 x'^2 + u_2 y'^2 + 2a'_{14} x' + 2a'_{24} y' + 2a'_{34} z' +$$

$$+ a'_{44} = 0$$

$\parallel$ [illegible] $\qquad x' = x'' + \alpha$

$\qquad y' = y'' + \beta$

$$u_1 x''^2 + u_2 y''^2 + 2a''_{14} x'' + 2a''_{24} y' + 2a'_{34} z' +$$

$$+ a''_{44} = 0$$

α s β [illegible]

$a''_{14} = 0 \qquad u_1 x''^2 + u_2 y''^2 + 2a'_{34} z' + a''_{44} = 0$

$a''_{24} = 0$

$\parallel$ [illegible]: $z' = z'' + \gamma \qquad u_1 x''^2 + u_2 y''^2 + 2a'_{34} z'' = 0$

$$a''_{44} + 2a'_{34}\gamma = 0$$

$u_1 x''^2 + y_2 y''^2 = -2a'_{34} z''$ [illegible]

> 0

$u_1 x^2 + u y^2 = 2 c z$

Discrim. e [illegible] =

29/5 Discr. e transf. [illegible] = Discr. e [illegible]

$a_{11} x_1^2 + a_{22} x_2^2 + a_{33} x_3^2 + a_{44} x_4^2 + \cdots$

$+ 2 a_{34} x_3 x_4 = 0$

$x_1 = m_{11} x'_1 + m_{12} x'_2 + m_{13} x'_3 + m_{14} x'_4$

$x_2 = m_{21} x'_1 + m_{22} x'_2 + m_{23} x'_3 + m_{24} x'_4$

$x_3 = m_{31} x'_1 + m_{32} x'_2 + m_{33} x'_3 + m_{34} x'_4$

$x_4 = m_{41} x'_1 + m_{42} x'_2 + m_{43} x'_3 + m_{44} x'_4$

$$\Delta = \begin{vmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & - & a_{34} \\ a_{41} & a_{42} & - & a_{44} \end{vmatrix}$$

$a'_{11} x'^2_1 + \cdots 2a'_{34} x'_3 x'_4 = 0$

$$\Delta' = \begin{vmatrix} a'_{11} & a'_{12} & a'_{13} & a'_{14} \\ a' & & & \\ & & & \\ a'_{41} & - & & a'_{44} \end{vmatrix}$$

$$\delta = \begin{vmatrix} m_{11} & \cdots & m_{14} \\ & & \\ m_{41} & \cdots & m_{44} \end{vmatrix}$$

$$\Delta' = \delta^2 \Delta$$

$$a'_{11} = a_{11} m_{11}^2 + a_{22} m_{21}^2 + a_{33} m_{31}^2 + a_{44} m_{41}^2$$
$$+ 2a_{12} m_{11} m_{21} + 2a_{13} m_{11} m_{31} + 2a_{14} m_{11} m_{41}$$
$$+ 2a_{23} m_{21} m_{31} + 2a_{24} m_{21} m_{41} + 2a_{34} m_{31} m_{41}$$

$$a'_{11} = m_{11}\left[a_{11} m_{11} + a_{12} m_{21} + a_{13} m_{31} + a_{14} m_{41}\right]$$
$$+ m_{21}\left[a_{12} m_{11} + a_{22} m_{21} + a_{32} m_{31} + a_{42} m_{41}\right]$$
$$+ m_{31}\left[a_{13} m_{11} + a_{23} m_{21} + a_{33} m_{31} + a_{43} m_{41}\right]$$
$$+ m_{41}\left[a_{14} m_{11} + a_{24} m_{21} + a_{34} m_{31} + a_{44} m_{41}\right]$$

$\delta a'_{ik}$

$$\Delta' = \delta \begin{vmatrix} a_{11} m_{11} + a_{12} \cdots & , a_{12} m_{11} + \cdots \\ \end{vmatrix}$$

$\Delta' = \delta . \delta \left| \begin{matrix} a_{11} & a_{12} & \text{—} \end{matrix} \right.$

$= \delta^2 \Delta$

---

x_1 — x $\qquad$ x'_1 — x'

x_2 — y $\qquad$ x'_2 — y'

x_3 — z $\qquad$ x'_3 — z'

x_4 — 1 $\qquad$ x'_4 — 1

$m_{14} = 0$ $\qquad$ $m_{21} = \cos\beta_1$ $\qquad$ $m_{31} = \cos\gamma_1$ | $m_{32} = \cos\gamma_2$

$m_{24} = 0$

$m_{34} = 0$ $\qquad$ $m_{41} = 0$ $\quad$ $m_{42} = 0$

$m_{44} = 1$

$m_{44} = 1$

$$\delta = \begin{vmatrix} \cos\alpha_1 & \cos\alpha_2 & \cos\alpha_3 & 0 \\ \cos\beta_1 & \cos\beta_2 & \cos\beta_3 & 0 \\ \cos\gamma_1 & \cos\gamma_2 & \cos\gamma_3 & 0 \\ 0 & 0 & 0 & 1 \end{vmatrix} = \begin{vmatrix} \cos\alpha_1 & \cos\alpha_2 & \cos\alpha_3 \\ \cos\beta_1 & \cos\beta_2 & \cos\beta_3 \\ \cos\gamma_1 & \cos\gamma_2 & \cos\gamma_3 \end{vmatrix} =$$

$$= \cos\alpha_1 \underbrace{(\cos\beta_2 \cos\gamma_3 - \cos\beta_3 \cos\gamma_2)}_{\cos\alpha_1} + \cos\beta_1 \underbrace{(\cos\gamma_2 \cos\alpha_3 - \cos\gamma_3 \cos\alpha_2)}_{\cos\beta_1} + \cos\gamma_1 \underbrace{(\cos\alpha_2 \cos\beta_3 - \cos\alpha_3 \cos\beta_2)}_{\cos\gamma_1}$$

$= 1$

$\Delta' = \Delta$

~~$u_1 x'^2 + u$~~

$u_1 x^2 + u_2 y^2 = 2cz$

$u_1 x^2 + u_2 y^2 - 2cz = 0$

$$\Delta' = \begin{vmatrix} u_1 & 0 & 0 & 0 \\ 0 & u_2 & 0 & 0 \\ 0 & 0 & 0 & -c \\ 0 & 0 & -c & 0 \end{vmatrix} = -u_1 u_2 c^2 = \Delta$$

$$c = \sqrt{-\frac{\Delta}{u_1 u_2}} \qquad \Delta = \begin{vmatrix} a_{11} & a_{12} & - \\ & & \end{vmatrix}$$

$u_1 x^2 + u_2 y^2 = 2cz$

$u_1 > 0$ $u_1 > 0$ $u_1 < 0$

~~$u_2 > 0$~~ $u_2 < 0$ $u_2 < 0$ law + 2 roi if $\sqrt{-2}$

7/6 $A_{44} = 0$

$u_1 x^2 + u_2 y^2 = 2cz$

I. $u_1, u_2 > 0$

$u_1 = \frac{1}{a^2}$ $u_2 = \frac{1}{b^2}$ $c = \frac{1}{m}$

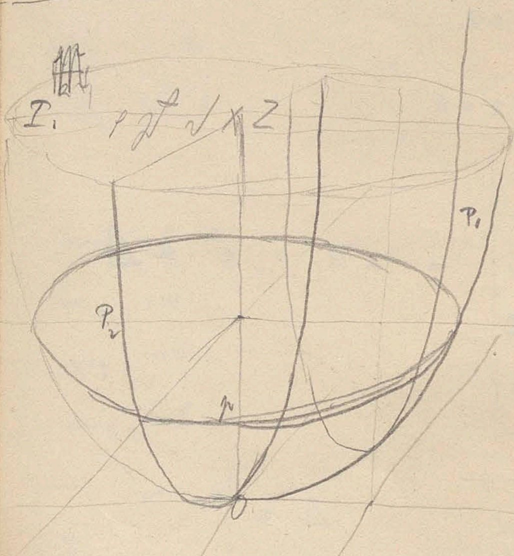
$\frac{x^2}{a^2} + \frac{y^2}{b^2} = \frac{2z}{m}$

$u_1 > 0,\ u_2 < 0$

II. $\frac{x^2}{a^2} - \frac{y^2}{b^2} = \frac{2z}{m}$

Elliptisches Paraboloid

I.



$y = 0$

$\frac{x^2}{a^2} = \frac{2z}{m}$

$x^2 = \frac{2a^2}{m} z$

$\frac{y^2}{b^2} = \frac{2z}{m}$

$y^2 = \frac{2b^2}{m} z$

X $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 0$

$x = 0, y = 0$

$\frac{x^2}{a^2} + \frac{y^2}{b^2} = \frac{2z}{m}$

~~$\frac{x^2}{(\sqrt{\frac{a}{2}})^2} + \frac{y^2}{(\sqrt{\ldots})^2} =$~~

$p > 0$ [illegible]

$p < 0$ imag.

$\mathfrak{f} \parallel ZY$ Ebene

$x = \rho$

$$\frac{\rho^2}{a^2} + \frac{y^2}{b^2} = \frac{2z}{m}$$

$$y^2 = \frac{2b^2}{m} z - \frac{b^2}{a^2}\rho^2$$ [illegible]

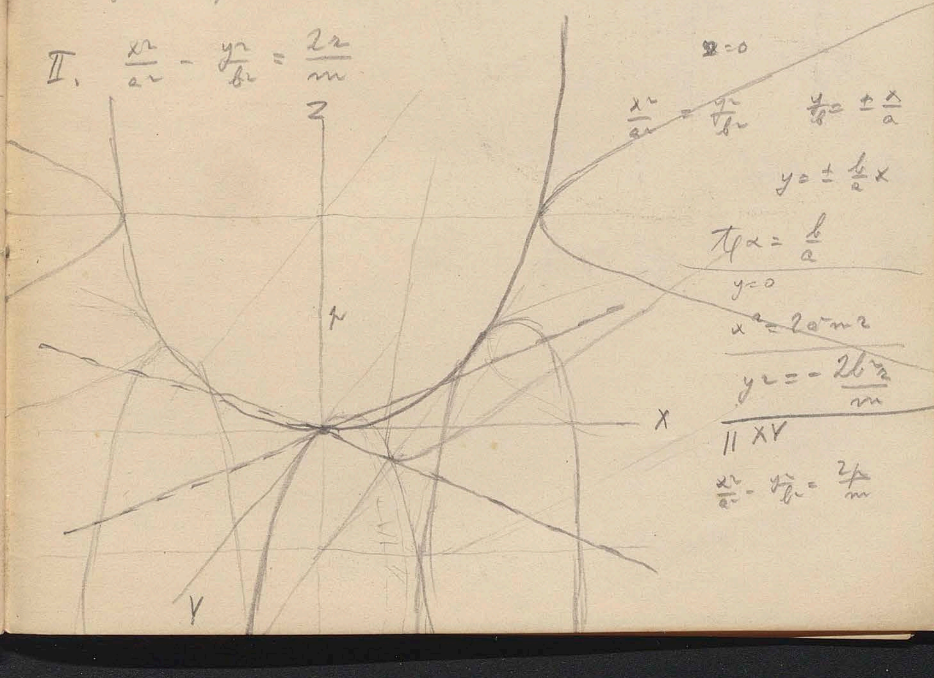
[illegible] Parabel [illegible] $\equiv$

[illegible] YZ Parabel

[illegible] Parabel $P_2 \parallel$ [illegible]

[illegible] Par. P_1 [illegible]

II. $$\frac{x^2}{a^2} - \frac{y^2}{b^2} = \frac{2z}{m}$$



$z = 0$

$$\frac{x^2}{a^2} = \frac{y^2}{b^2} \qquad \frac{y}{b} = \pm\frac{x}{a}$$

$$y = \pm\frac{b}{a}x$$

$$\operatorname{tg}\alpha = \frac{b}{a}$$

$y = 0$

$x^2 = 2a^2 m z$

$$y^2 = -\frac{2b^2 z}{m}$$

$\parallel XY$

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = \frac{2p}{m}$$

1 YZ ⊥ $x = \rho$

$$\frac{\rho^2}{a^2} - \frac{y^2}{b^2} = \frac{2z}{m}$$

$$y^2 = -\frac{b^2 z}{m} + \frac{b^2 \rho^2}{a^2}$$ Parabel

[illegible]

[illegible] YZ Parabel [illegible] XZ Parabel

[illegible]

windschiefes Paraboloid [illegible]
hyperbolisches

Ellipsoid $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$

Hyperbol. I. $\frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{z^2}{c^2} = 1$

" II. $\frac{x^2}{a^2} - \frac{y^2}{b^2} - \frac{z^2}{c^2} = 1$

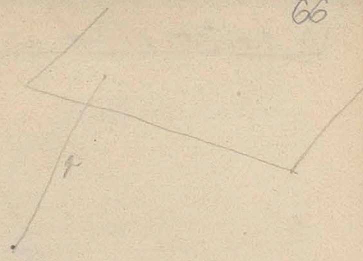
Kegel $\frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{z^2}{c^2} = 0$

} centrale [illegible]

El. Paraboloid $\frac{x^2}{a^2} + \frac{y^2}{b^2} = \frac{2z}{m}$

Hyp. " $\frac{x^2}{a^2} - \frac{y^2}{b^2} = \frac{2z}{m}$

6 $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$ · ad p



$x'y'z'$ Polar Ebene

$\frac{xx'}{a^2} + \frac{yy'}{b^2} + \frac{zz'}{c^2} = 1$

$\frac{xx'p}{a^2} + \frac{yy'p}{b^2} + \frac{zz'p}{c^2} = p$

$x\cos\alpha + y\cos\beta + z\cos\gamma = p$

$\cos\alpha = \frac{px'}{a^2} \quad \cos\beta = \frac{py'}{b^2} \quad \cos\gamma = \frac{pz'}{c^2}$

Polar z

Tg. E :

$a\cos\alpha = \frac{px'}{a} \quad b\cos\beta = \frac{py'}{b} \quad c\cos\gamma = \frac{pz'}{c}$

$a^2\cos^2\alpha + b^2\cos^2\beta + c^2\cos^2\gamma = p^2\underbrace{\left[\frac{x'^2}{a^2} + \frac{y'^2}{b^2} + \frac{z'^2}{c^2}\right]}_{=1}$

$p = \sqrt{\quad}$

---

3 Tg. E a. J ⊥

$p'^2 = a^2\cos^2\alpha' + b^2\cos^2\beta' + c^2\cos^2\gamma'$

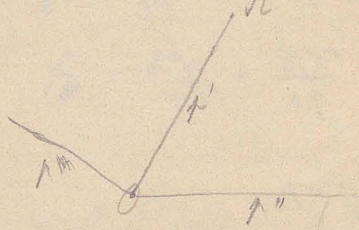
$p''^2 = a^2\cos^2\alpha'' + b^2\cos^2\beta'' + c^2\cos^2\gamma''$

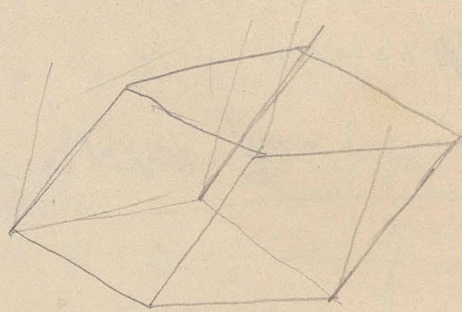
$p^2 + p'^2 + p''^2 = a^2[\cos^2\alpha + \cos^2\alpha' + \ldots] + b^2[\quad] + c^2[\quad]$

$\alpha\ \alpha'\ \alpha''$ ∡ e X ∨ $p\ p'\ p''$ b ⊥

$\Sigma\cos^2 = 1$

$p^2+p'^2+p''^2 = a^2+b^2+c^2$





$OM^2 = p^2+p'^2+p''^2 = a^2+b^2+c^2$

∴ [illegible] M [illegible] = const

∴ [illegible] M [illegible] R = $\sqrt{}$

σ: $x\ y\ z$ $\quad r\begin{cases}\alpha\\ \beta\\ \gamma\end{cases}$

$x = r\cos\alpha$

$r^2\frac{\cos^2\alpha}{a^2} + \frac{r^2\cos^2\beta}{b^2} + \frac{r^2\cos^2\gamma}{c^2} = 1$

$\frac{1}{r^2} = \frac{\cos^2\alpha}{a^2} + \frac{\cos^2\beta}{b^2} + \frac{\cos^2\gamma}{c^2}$

$$\frac{1}{r'^2} = \frac{\cos^2\alpha'}{a^2} + \frac{\cos^2\beta'}{b^2} + \frac{\cos^2\gamma'}{c^2} \qquad r \perp r' \perp r''$$

$$\frac{1}{r''^2} = \frac{\cos^2\alpha''}{a^2} + \frac{\cos^2\beta''}{b^2} + \frac{\cos^2\gamma''}{c^2}$$

$$\frac{1}{r^2} + \frac{1}{r'^2} + \frac{1}{r''^2} = \frac{1}{a^2} + \frac{1}{b^2} + \frac{1}{c^2}$$

[illegible]

[illegible]

$$Ax + By + Cz = D$$

$$\frac{xx'}{a^2} + \frac{yy'}{b^2} + \frac{zz'}{c^2} = 1$$

$$\frac{xx'D}{a^2} + \frac{yy'D}{b^2} + \frac{zz'D}{c^2} = D$$

$$A = \frac{x'D}{a^2} \qquad B = \frac{y'D}{b^2} \qquad C = \frac{z'D}{c^2}$$

$$Aa = \frac{x'D}{a} \qquad Bb = \frac{y'D}{b} \qquad Cc = \frac{z'D}{c}$$

$$A^2a^2 + B^2b^2 + C^2c^2 = D^2$$

[illegible] Coord.

$$ux + vy + wz + 1 = 0$$

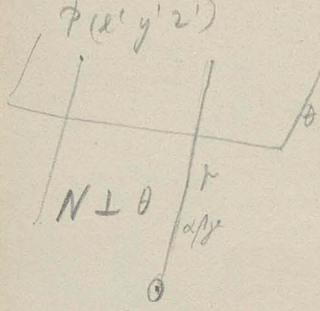
$$A \sim u \qquad B \sim v \qquad C \sim w \qquad D \sim -1$$

$$A^2u^2 + B^2v^2 + C^2w^2 = 1$$ [illegible]

[illegible]

[illegible] Coord.

zs e Norm. e.



$$\frac{x-x'}{\cos\alpha} = \frac{y-y'}{\cos\beta} = \frac{z-z'}{\cos\gamma}$$

$$\frac{a^2(x-x')}{x'} = \frac{b^2(y-y')}{y'} = \frac{c^2(z-z')}{z'}$$

pb e N. a f r pl

14/6 $$\frac{a^2(x-x')}{x'} = \frac{b^2(y-y')}{y'} = \frac{c^2(z-z')}{z'} = s$$

$$x-x' = \frac{s x'}{a^2}$$

$$y-y' = \frac{s y'}{b^2}$$

$$z-z' = \frac{s z'}{c^2}$$

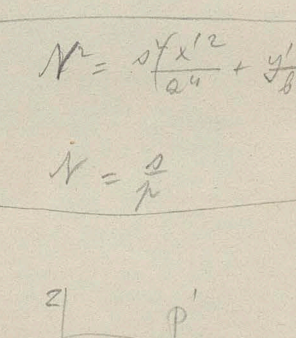
$$N^2 = s^2\left(\frac{x'^2}{a^4} + \frac{y'^2}{b^4} + \frac{z'^2}{c^4}\right) = \frac{s^2}{p^2}$$

$$N = \frac{s}{p}$$

$n \mid z=0$

$$s = -c^2$$

$$N_z = -\frac{c^2}{p} \quad \text{e} \; \perp xy$$

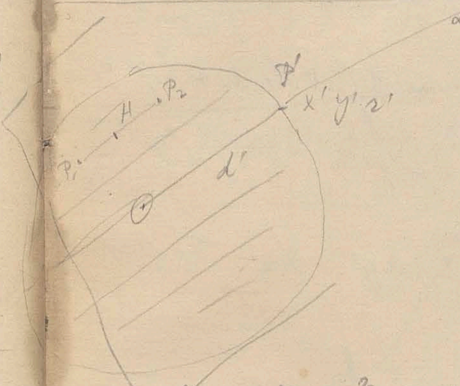


$$N_x = -\frac{a^2}{p}$$

$$N_y = -\frac{b^2}{p}$$

$N_x : N_y : N_z = a^2 : b^2 : c^2$

[illegible]



[illegible]

$$\frac{x_1^2}{a^2} + \frac{y_1^2}{b^2} + \frac{z_1^2}{c^2} = 1$$

$$\frac{x_2^2}{a^2} + \frac{y_2^2}{b^2} + \frac{z_2^2}{c^2} = 1$$

$$\frac{(x_1 - x_2)(x_1 + x_2)}{a^2} + \frac{(y_1 - y_2)(y_1 + y_2)}{b^2} + \frac{(z_1 - z_2)(z_1 + z_2)}{c^2} = 0$$

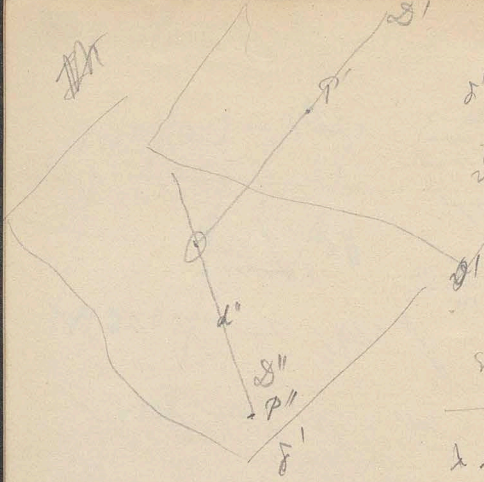
$$x = \frac{x_1 + x_2}{2} \quad y = \frac{y_1 + y_2}{2} \quad z = \frac{z_1 + z_2}{2}$$

$$x_1 - x_2 : y_1 - y_2 : z_1 - z_2 = \cos\alpha' : \cos\beta' : \cos\gamma'$$

$$\frac{x \cos\alpha'}{a^2} + \frac{y \cos\beta'}{b^2} + \frac{z \cos\gamma'}{c^2} = 0 \,\Big|\, d'$$

$$\frac{x x'}{a^2} + \frac{y y'}{b^2} + \frac{z z'}{c^2} = 0$$ [illegible]

= [illegible] d' conj. [illegible]



$\delta' \quad \frac{xx'}{a^2} + \frac{yy'}{b^2} + \frac{zz'}{c^2} = 0$

$D' \quad \frac{xx'}{a^2} + \frac{yy'}{b^2} + \frac{zz'}{c^2} = 1$

∵ ∥ [illegible]

[illegible] D'

$\lambda x' \quad \lambda y' \quad \lambda z'$ [illegible] C.

$\frac{\lambda xx'}{a^2} + \lambda \frac{yy'}{b^2} + \frac{\lambda zz'}{c^2} = 1$ ∵ ∥

D'' / − [illegible] δ'

[illegible] conj. [illegible] D' [illegible]

= conj. [illegible]

$\frac{xx''}{a^2} + \frac{yy''}{b^2} + \frac{zz''}{c^2} = 0$

$\frac{x''x'}{a^2} + \frac{y''y'}{b^2} + \frac{z''z'}{c^2} = 0$ [illegible] P'' [illegible] δ'

[illegible] P' [illegible] D' conj. [illegible]

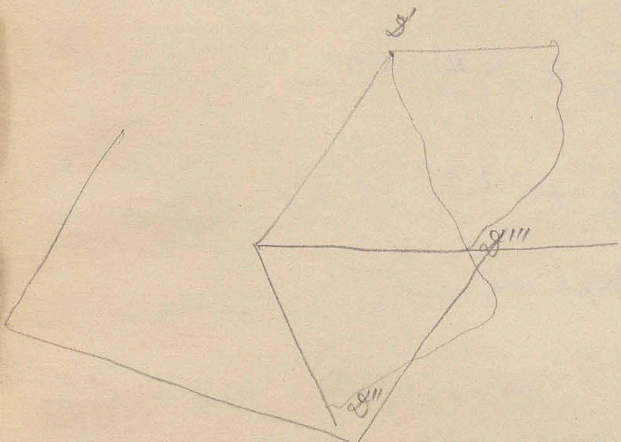
[illegible]

[illegible]

$d'' = OP''$ $\quad \sphericalangle\, \alpha''\ \beta''\ \gamma''$

$$\frac{\cos\alpha'\cos\alpha''}{a^2} + \frac{\cos\beta'\cos\beta''}{b^2} + \frac{\cos\gamma'\cos\gamma''}{c^2} = 0$$ } [illegible]

[illegible] Tripels conj. [illegible]



3 conj. [illegible]

[illegible]

$$Ax^2 + By^2 + Cz^2 = D$$

$$\frac{x^2}{\frac{D}{A}} + \frac{y^2}{\frac{D}{B}} + \frac{z^2}{\frac{D}{C}} = 1$$

$y = 0$
$z = 0$ $\quad x = d' \quad \frac{D}{A} = d'^2$ etc

$$\frac{x^2}{d'^2} + \frac{y^2}{d''^2} + \frac{z^2}{d'''^2} = 1$$

$$\frac{x'x''}{a^2} + \frac{y'y''}{b^2} + \frac{z'z''}{c^2} = 0 \quad \Big| \quad \frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$$

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = m^2$$

ff concentr. : $\frac{x^2}{(ma)^2} + \text{———}$

---

2 conj. Ab . ff

$$\frac{x'x''}{(ma)^2} + \frac{y'y''}{(mb)^2} + \frac{z'z''}{(mc)^2} = 0$$

$$\frac{x'x''}{a^2} + \frac{y'y''}{b^2} + \frac{z'z''}{c^2} = 0 =$$

6 . 12 Tripel conj. Ab

$m = 0$

$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 0$ = Asympt. ff

---

17/6 $\frac{x'x''}{a^2} + \frac{y'y''}{b^2} + \frac{z'z''}{c^2} = 0$

$$\frac{\cos\alpha'\cos\alpha''}{a^2} + \frac{\cos\beta'\cos\beta''}{b^2} + \frac{\cos\gamma'\cos\gamma''}{c^2} = 0$$

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$$

$\left.\begin{array}{l} \frac{x}{a} = \cos\lambda \\ \frac{y}{b} = \cos\mu \\ \frac{z}{c} = \cos\nu \end{array}\right\}$

$x = a\cos\lambda$ $\quad x' = a\cos\lambda'$ $\quad x'' = a\cos\lambda''$

$y = b\cos\mu$ $\quad y' = b\cos\mu'$ $\quad y'' = b\cos\mu''$

$z = c\cos\nu$ $\quad z' = c\cos\nu'$ $\quad z'' = c\cos\nu''$

~~[illegible]~~

$\cos\lambda'\cos\lambda'' + \cos\mu'\cos\mu'' + \cos\nu'\cos\nu'' = 0$

[illegible] $\perp$ [illegible] conj. [illegible]

$\sum \rho^{2t}$ [illegible] conj. $\frac{1}{2}$ [illegible] = const

$d' \mid d'' \mid d'''$

$x'y'z' \mid x''y''z'' \mid x'''y'''z'''$

$d'^2 = x'^2 + y'^2 + z'^2 = a^2\cos^2\lambda' + b^2\cos^2\mu' + c^2\cos^2\nu'$

$d''^2 = a^2\cos^2\lambda'' + b^2\cos^2\mu'' + c^2\cos^2\nu''$

$d'''^2 = a^2\cos^2\lambda''' + b^2\cos^2\mu''' + c^2\cos^2\nu'''$

$$d'^2 + d''^2 + d'''^2 = a^2\underbrace{(\cos^2\lambda' + \cos^2\lambda'' + \cos^2\lambda''')}_{=1} + b^2(\quad) + c^2(\quad)$$

[illegible] $\perp$ [illegible]

$d'^2 + d''^2 + d'''^2 = a^2 + b^2 + c^2$

[illegible] Parall. [illegible] 3 conj. $\frac{1}{2}$ [illegible] = const.

$$= \begin{vmatrix} x' & y' & z' \\ x'' & y'' & z'' \\ x''' & y''' & z''' \end{vmatrix}$$

$$= \begin{vmatrix} a\cos\lambda' & b\cos\mu' & c\cos\nu' \\ a\cos\lambda'' & b\cos\mu'' & c\cos\nu'' \\ a\cos\lambda''' & b\cos\mu''' & c\cos\nu''' \end{vmatrix}$$

$$= abc \begin{vmatrix} \cos\lambda' & \cos\mu' & \cos\nu' \\ \cos\lambda'' & \cos\mu'' & \cos\nu'' \\ \text{———} & & \end{vmatrix}$$

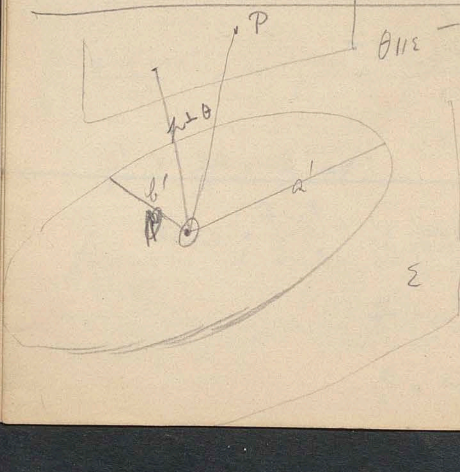
$$\Delta = \overbrace{\cos\lambda'\left[\cos\mu''\cos\nu''' - \cos\mu'''\cos\nu''\right]}^{\cos\lambda' \quad \overset{\shortparallel}{1}} + \cos\mu'\left[\cos\nu''\cos\lambda'' -\right.$$

$$\left. - \cos\nu'''\cos\lambda''\right] + \cos\nu'\underbrace{\left[\cos\lambda''\cos\nu''' - \cos\lambda'''\cos\nu''\right]}_{\cos\nu'}$$

[illegible]

$$= \cos^2\lambda' + \cos^2\mu' + \cos^2\nu' = 1$$

$$\begin{vmatrix} x' & y' & z' \\ x'' & y'' & z'' \\ x''' & y''' & z''' \end{vmatrix} = abc \qquad = \frac{1}{6}\,\text{Vol.}\left[O\ P'_x\ P''_x\ P'''\right]$$



θ ∥ Σ — [illegible]

a' b' [illegible] ½ [illegible]

[illegible]

[illegible] 1 [illegible] θ ∥ Σ [illegible]

[illegible] OP = 3το [illegible]

e ☐ $a'b'\gamma$ = $\underline{a'b'p = abc}$

$\sum$ ρ 2te ρ Proj. α 3 axes ½ 20 γ − β γl = const

d'

$$\text{Proj. } d' = x'\cos\alpha + y'\cos\beta + z'\cos\gamma$$

$$d'' = x''\cos\alpha + y''\cos\beta + z''\cos\gamma$$

$$d''' = x'''\cos\alpha + y'''\cos\beta + z'''\cos\gamma$$

$$\text{proj } d' = a\cos\lambda'\cos\alpha + b\cos\mu'\cos\beta + c\cos\nu'\cos\gamma$$

$$\text{"} \quad d'' = a\cos\lambda''\cos\alpha + \text{— — —}$$

$$\text{"} \quad d''' = a\cos\lambda'''\cos\alpha + \text{— — —}$$

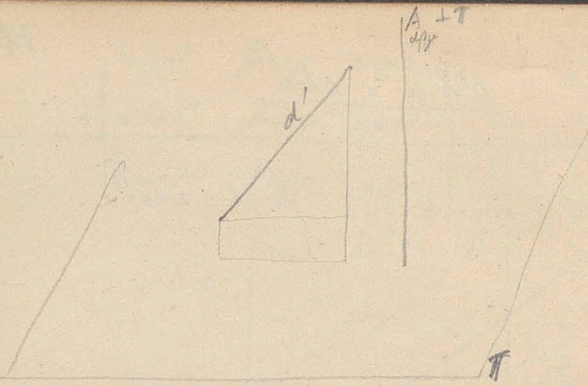
$$(\text{proj } d')^2 + (\quad)^2 + (\quad)^2 = a^2\cos^2\alpha + b^2\cos^2\beta + c^2\cos^2\gamma$$

$$= \text{const.}$$

ρ ρ: a 3 axes ½ 20 $d'd''d'''$ γ − β z proj. 6 + ρ $\sum$

ρ 2te ρ Proj. const.

$$d'^2 + d''^2 + d'''^2 = \text{const}$$



Σ e — = const

Σ e — = const } !!!

[illegible] central [illegible]

$x \cos\alpha + y \cos\beta + z \cos\gamma = 0$

[illegible]

19/6

$x \cos\alpha + y \cos\beta + z \cos\gamma = 0$

[illegible]

[illegible]

$\frac{1}{a'^2} + \frac{1}{b'^2} + \frac{1}{c'^2} = \frac{1}{a^2} + \frac{1}{b^2} + \frac{1}{c^2}$

$\frac{1}{a'^2} + \frac{1}{b'^2} = \frac{1}{a^2} + \frac{1}{b^2} + \frac{1}{c^2} - \frac{1}{c'^2}$

c' { [illegible]

$\xi = c' \cos\alpha$ $\qquad \frac{1}{a'^2}$

$\eta = c' \cos\beta$

$\zeta = c' \cos\gamma$

$$\frac{\xi^2}{a^2} + \frac{\eta^2}{b^2} + \frac{\zeta^2}{c^2} = 1$$

$$c'^2\left[\frac{\cos^2\alpha}{a^2} + \frac{\cos^2\beta}{b^2} + \frac{\cos^2\gamma}{c^2}\right] = 1$$

$$\frac{1}{c'^2} = \frac{\cos^2\alpha}{a^2} + \frac{\cos^2\beta}{b^2} + \frac{\cos^2\gamma}{c^2}$$

$$\frac{1}{a'^2} + \frac{1}{b'^2} = \frac{\sin^2\alpha}{a^2} + \frac{\sin^2\beta}{b^2} + \frac{\sin^2\gamma}{c^2}$$

$p \mid$ $\quad a'b'p = abc$

$$\frac{1}{a'b'} = \frac{p}{abc}$$

$$\frac{1}{a'^2b'^2} = \frac{p^2}{a^2b^2c^2} \qquad p^2 = a^2\cos^2\alpha + b^2\cos^2\beta + c^2\cos^2\gamma$$

$$\frac{1}{a'^2} \cdot \frac{1}{b'^2} = \frac{\cos^2\alpha}{b^2c^2} + \frac{\cos^2\beta}{c^2a^2} + \frac{\cos^2\gamma}{a^2b^2}$$

$$\frac{1}{u^4} - \left[\frac{\sin^2\alpha}{a^2} + \frac{\sin^2\beta}{b^2} + \frac{\sin^2\gamma}{c^2}\right]\frac{1}{u^2} + \frac{\cos^2\alpha}{b^2c^2} + \frac{\cos^2\beta}{a^2c^2} + \frac{\cos^2\gamma}{a^2b^2} = 0$$

$$u = \begin{cases} a' \\ b' \end{cases}$$

$u\, u > 0$ Ell.

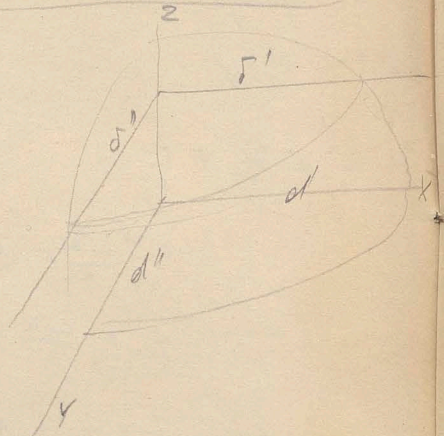
— Hyp.

$u < 0$ imaginär

[illegible]:

$$\frac{a^2\cos^2\alpha}{a^2-u^2}+\frac{b^2\cos^2\beta}{b^2-u^2}+\frac{c^2\cos^2\gamma}{c^2-u^2}=0$$

[illegible]



[illegible]

$$\frac{x^2}{d'^2}+\frac{y^2}{d''^2}+\frac{z^2}{d'''^2}=1$$

$z=0$

$$\frac{x^2}{d'^2}+\frac{y^2}{d''^2}=1$$

$z=m$

$$\frac{x'^2}{d'^2}+\frac{y'^2}{d''^2}=1-\frac{m^2}{d'''^2}$$

$$\frac{x'^2}{\underbrace{\left[d'\sqrt{1-\frac{m^2}{d'''^2}}\right]}_{\delta'}{}^2}+\frac{y'^2}{\underbrace{\left[d''\sqrt{1-\frac{m^2}{d'''^2}}\right]}_{\delta''}{}^2}=1$$

$$d':d''=\delta':\delta''$$

[illegible]

$a'^2 = b'^2$ [illegible]

2 ame. [illegible]

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$$

$$\frac{x^2}{a_1^2} + \frac{y^2}{b_1^2} + \frac{z^2}{c_1^2} = 1$$

[illegible]

$f = 0$

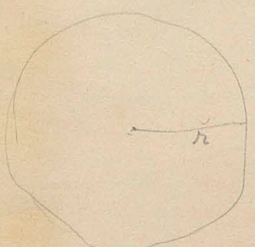
$f' = 0$

$f - \lambda f' = 0$ [illegible]

$$x^2\left[\frac{1}{a^2} - \frac{1}{a_1^2}\right] + y^2\left[\frac{1}{b^2} - \frac{1}{b_1^2}\right] + z^2[\ldots] = 0$$

[illegible]

[illegible]



[illegible]

$$\frac{x^2}{r^2} + \frac{y^2}{r^2} + \frac{z^2}{r^2} = 1$$

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$$

$$x^2\left[\frac{1}{a^2} - \frac{1}{r^2}\right] + y^2\left[\frac{1}{b^2} - \frac{1}{r^2}\right] + z^2\left[\frac{1}{c^2} - \frac{1}{r^2}\right] = 0$$

[illegible]

[illegible]

[illegible] $r = b$ $\quad x^2\left[\frac{1}{a^2} - \frac{1}{b^2}\right] + z^2\left[\frac{1}{c^2} - \frac{1}{b^2}\right] = 0$

1/5 15/4 28/2 15/2