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The determination of Apex and Vertex from the Yale 50° — 55° Catalogue.

I. The catalogue of 8359 stars in the zone 50° — 55°, edited by Frank Schlesinger and Ida Barney¹⁾, gives the reobservation of this zone for the epoch 1925 and deduces the proper motions of these stars. As the data of some stars are incomplete, only 8276 stars could be taken into account in the present investigation. The position angles for all the stars were computed from the proper motion in right ascension and declination. The zone 50° — 55° was divided into 18 regions (in intervals of 1^h 20^m in right ascension), and the second method of Schwarzschild²⁾ of determining the positions of apex and vertex was applied. The Schwarzschild's method is well known. For each region the position angle ϑ_1 gives the direction to the apex and ϑ_0 the direction towards the vertex. The following table gives these angles calculated for all regions:

TABLE I.

Region	Intervals in right ascension	ϑ_1	ϑ_0
I	0 ^h 00 ^m — 1 ^h 20 ^m	310.7	191.2
II	1 20 — 2 40	314.8	173.0
III	2 40 — 4 00	344.2	163.4
IV	4 00 — 5 20	345.2	175.8
V	5 20 — 6 40	348.1	190.8
VI	6 40 — 8 00	349.5	189.1
VII	8 00 — 9 20	358.3	187.6
VIII	9 20 — 10 40	21.1	185.6
IX	10 40 — 12 00	24.6	191.9
X	12 00 — 13 20	72.5	163.2
XI	13 20 — 14 40	64.4	174.8
XII	14 40 — 16 00	34.4	175.1
XIII	16 00 — 17 20	4.4	179.4
XIV	17 20 — 18 40	316.2	183.1
XV	18 40 — 20 00	250.6	184.5
XVI	20 00 — 21 20	302.1	205.0
XVII	21 20 — 22 40	343.3	194.8
XVIII	22 40 — 24 00	307.1	195.8

¹⁾ Transactions of the Astronomical Observatory of Yale University. Vol. 4. New Haven. 1925.

²⁾ Nachrichten von d. Kgl. Gesellschaft der Wissenschaften zu Göttingen. Math.—physik. Klasse. 1908. Heft 2. Berlin. 1908.

Further, the quantities α, β, h were reckoned; they have following meaning: h is the velocity of the Sun in space projected on the region of the sky in question, α and β are the half major and minor axis of the velocity ellipse. The angles ϑ_1 and ϑ_0 and the ratios $\frac{\beta}{\alpha}$ being known, the following equations must be solved in order to determine the equatorial co-ordinates A and D of the apex for each region:

$$-x \sin \alpha + y \cos \alpha = \frac{h}{\beta} \sin \vartheta_1$$

$$-x \cos \alpha \sin \delta - y \sin \alpha \sin \delta + z \cos \delta = \frac{h}{\beta} \cos \vartheta_1$$

where α and δ are the co-ordinates of the centre of the region.

There are on the whole 36 equations. For the determination of the vertex similar equations were used:

$$-x \sin \alpha + y \cos \alpha = \sqrt{\frac{\alpha^2}{\beta^2} - 1} \sin \vartheta_0$$

$$-x \cos \alpha \sin \delta - y \sin \alpha \sin \delta + z \cos \delta = \sqrt{\frac{\alpha^2}{\beta^2} - 1} \cos \vartheta_0$$

The solution of the 36 equations by the method of the least squares gives:

the co-ordinates of the apex : $A = 276^{\circ}.6$ $D = + 63^{\circ}.6$
 and " " " " " vertex : $A' = 247.5$ $D' = - 81.8$

This calculation was verified by a geometrical method, proposed by prof. K. Jantzen. Consider a plane passing through the point G (the centre of the region), the centre of the celestial sphere and the apex. Let the equation of this plane be:

$$\lambda x + \mu y + \nu z = 0$$

where λ, μ, ν are the cosines of the angles between the pole of the resulting proper motion and the three axes of the equatorial system. We receive then:

$$\lambda = \sin \alpha \cos P - \cos \alpha \sin \delta \sin P$$

$$\mu = -\cos \alpha \cos P - \sin \alpha \sin \delta \sin P$$

$$\nu = \cos \delta \sin P$$

where α and δ are the co-ordinates of the point G and P is the position angle. The distance Δ of the point (x_0, y_0, z_0) from the plane $\lambda x + \mu y + \nu z = 0$ is $\Delta = \lambda x_0 + \mu y_0 + \nu z_0$. The number of planes defined above equals to that of our regions. For the apex, i. e. for the point, whose distances from all planes satisfy to the condition of minimum, we get:

$$\Sigma p \Delta^2 = \text{minimum, where } p \text{ corresponds to the weights.}$$

Denoting approximate values of the co-ordinates of the apex by A_0, D_0 and putting: $A = A_0 + \Delta A, D = D_0 + \Delta D$, we receive the equations of the form:

$$\lambda \cos D_0 \Delta A + (\mu \sin D_0 + \nu \cos D_0) \Delta D - \mu \cos D_0 + \nu \sin D_0 = \Delta$$

The corrections ΔA and ΔD were found from the normal equations:

$$[p\lambda\lambda] \cos D_0 \Delta A + ([p\lambda\mu] \sin D_0 + [p\lambda\nu] \cos D_0) \Delta D + (-[p\lambda\mu] \cos D_0 + [p\lambda\nu] \sin D_0) = 0$$

$$([p\lambda\mu] \sin D_0 \cos D_0 + [p\lambda\nu] \cos^2 D_0) \Delta A +$$

$$([p\mu^2] \sin^2 D_0 + 2[p\mu\nu] \sin D_0 \cos D_0 + [p\nu^2] \cos^2 D_0) \Delta D +$$

$$([p\nu^2] - [p\mu^2]) \sin D_0 \cos D_0 - [p\mu\nu] \cos 2 D_0 = 0$$

After some trials (for A_0 we take $A_0 = 270^\circ$) the value $D_0 = + 60^\circ$ was chosen. The equations gave then for the co-ordinates of the apex the following values: $A = 278^\circ.0, D = + 62^\circ.5$.

Finally a third method was applied. Denoting by s the proper motion and by q the position angle we have the rectangular co-ordinates: $x = s \cos q, y = s \sin q$. The expressions for every region $X = \Sigma x, Y = \Sigma y$ represent the component of the resultant velocity towards the antiapex; the direction Q is given by $\text{tg } Q = Y/X$. The proper motions were here considered equal ($s = 1$). These calculations may be brought in accordance with those of Schwarzschild, who took into account the number of vectors of the proper motions only, but not their values. The directions calculated for all regions are compiled in table II:

T A B L E II.

Region	Direction Q	Region	Direction Q	Region	Direction Q
I	123.6 ⁰	VII	168.8 ⁰	XIII	182.4 ⁰
II	128.8	VIII	196.4	XIV	114.6
III	152.4	IX	204.4	XV	69.1
IV	162.2	X	242.6	XVI	119.3
V	155.8	XI	245.9	XVII	146.8
VI	158.0	XII	223.5	XVIII	116.5

This method gives the co-ordinates of the apex: $A = 259^\circ.3, D = + 63^\circ.7$.

II. The direction of apex and vertex calculated from the 8276 stars, lying between 50° — 55° , differ greatly from those generally accepted. Therefore, it seemed worth while to study the stars of different spectral types separately. Unfortunately 4541 stars (mostly faint) had to be eliminated their spectral types being unknown. The remaining 3735 stars were divided into 2 groups: the first involving 1708 stars of the types B, A, F— F_4 , the second — 2027 stars of the types F_5 — F_9 , G, K and M.

Schwarzschild's method of determining the apex and vertex was applied to each group. The number of stars being now smaller, the zone 50° — 55° was divided into 12 equal regions (in intervals of 2^h in right ascension). The white stars, constituting the first group, are distributed irregularly, hence the weights allowing for the number of stars in each region were introduced into the calculation. Table III gives the results of the investigation of the first group (white stars).

T A B L E III.

Region	Weights p	Number of stars	ϑ_1	ϑ_0	$\frac{\beta}{\alpha}$	$\frac{\alpha}{h}$	$\frac{\beta}{h}$
I	8.8	265	⁰ 300.5	⁰ 184.5	0.79	2.03	1.60
II	7.4	221	328.9	352.3	0.74	2.50	1.85
III	6.1	183	347.3	10.2	0.70	2.38	1.67
IV	3.9	116	358.0	14.1	0.36	1.85	0.67
V	2.6	77	22.0	20.0	0.38	4.05	1.52
VI	1.2	36	55.0	124.7	0.53	7.04	3.73
VII	1.0	30	102.5	86.8	0.36	4.21	1.51
VIII	1.5	46	100.0	86.9	0.49	5.96	2.94
IX	1.9	56	325.0	176.0	0.60	5.48	3.29
X	3.7	111	233.3	187.2	0.31	6.46	2.02
XI	7.6	228	276.2	207.0	0.57	3.50	2.00
XII	11.3	339	297.5	195.3	0.68	3.57	2.44

We receive for the direction of the apex:

with weights: $A = 271^{\circ}.3$ $D = + 44^{\circ}.6$

without weights: $A = 265^{\circ}.8$ $D = + 43^{\circ}.5$

and for the direction of the vertex:

with weights: $A' = 280^{\circ}.1$ $D' = - 19^{\circ}.6$

without weights: $A' = 279^{\circ}.4$ $D' = - 7^{\circ}.1$.

As a test of this computation the following table gives the values of ϑ_1 and $\frac{h}{\beta}$ observed (O) an calculated (C) and the differences (O — C).

TABLE IV.

Region	ϑ_1		O - C	$\frac{h}{\beta}$		O - C
	Observ.	Calc.		Observ.	Calc.	
I	⁰ 300.5	⁰ 309.1	- 8.6	0.6	0.6	0.0
II	328.9	327.8	+ 1.1	0.5	0.7	- 0.2
III	347.3	358.8	- 11.5	0.6	0.7	- 0.1
IV	358.0	1.0	- 3.0	1.5	0.7	+ 0.8
V	22.0	30.4	- 8.4	0.7	0.7	0.0
VI	55.0	49.3	+ 5.7	0.3	0.6	- 0.3
VII	102.5	66.9	+ 35.6	0.7	0.5	+ 0.2
VIII	100.0	85.6	+ 14.4	0.3	0.4	- 0.1
IX	325.0	119.3	+ 205.7	0.3	0.2	+ 0.1
X	233.3	234.9	- 1.6	0.5	0.1	+ 0.4
XI	276.2	272.5	+ 3.7	0.5	0.4	+ 0.1
XII	297.5	291.6	+ 5.9	0.4	0.5	- 0.1

This table shows that the difference O - C is very large for the region IX; we receive here rather the direction to the antiapex than that to the apex. As the number of stars in this region is very small (56 stars), we have discarded this region altogether and repeated the calculation for 11 regions. The results thus obtained are: $A = 271^{\circ}.9$, $D = +42^{\circ}.8$.

For the angles ν_0 determining the direction of the vertex and for the values $\sqrt{\frac{\alpha^2}{\beta^2} - 1}$ an analogous table has been compiled (Table V). It gives the observed and calculated values as well as their differences: O - C.

TABLE V.

Region	ϑ_0		O - C	$\sqrt{\frac{\alpha^2}{\beta^2} - 1}$		O - C
	Observ.	Calc.		Observ.	Calc.	
I	⁰ 184.5	⁰ 261.4	- 76.9	0.8	1.1	- 0.3
II	352.3	286.0	+ 66.3	0.9	1.0	- 0.1
III	10.2	319.6	+ 50.6	1.0	0.7	+ 0.3
IV	14.1	8.5	+ 5.6	2.6	0.7	+ 1.9
V	20.0	52.9	- 32.9	2.4	0.8	+ 1.6
VI	124.7	82.5	+ 42.2	1.6	1.0	+ 0.6
VII	86.8	106.0	- 19.2	2.6	1.2	+ 1.4
VIII	86.9	129.3	- 42.4	1.8	1.2	+ 0.6
IX	176.0	155.6	+ 20.4	1.3	1.2	+ 0.1
X	187.2	184.8	+ 2.4	3.1	1.1	+ 2.0
XI	207.0	213.4	- 6.4	1.4	1.2	+ 0.2
XII	195.3	238.5	- 43.2	1.1	1.2	- 0.1

We investigate now the second group of stars viz. the red stars of spectral types F_5 — F_9 , G, K, M. They are distributed in the zone $50^\circ - 55^\circ$ more regularly than the white ones. Therefore it is not necessary to give any weights to different regions. Table VI gives the results of our calculations.

TABLE VI.

Region	Number of stars	ϑ_1	ϑ_0	$\frac{\beta}{\alpha}$	$\frac{\alpha}{h}$	$\frac{\beta}{h}$
I	155	⁰ 317.5	⁰ 105.2	0.84	1.95	1.64
II	147	334.6	163.3	0.40	2.56	1.03
III	149	344.1	177.8	0.62	1.74	1.08
IV	181	354.2	193.2	0.64	1.77	1.13
V	196	0.6	175.5	0.60	2.04	1.22
VI	154	31.0	180.5	0.72	2.22	1.60
VII	164	76.4	348.0	0.62	2.16	1.34
VIII	207	50.0	352.2	0.41	9.72	3.97
IX	170	17.5	357.9	0.64	7.07	4.53
X	165	253.1	11.1	0.15	48.91	7.24
XI	152	275.0	21.9	0.32	9.23	2.93
XII	187	307.8	14.7	0.40	4.30	1.72

We find now for the direction of the apex: $A = 276^\circ.7$ $D = +56^\circ.0$
 and for that of the vertex: $A' = 273^\circ.1$ $D' = -35^\circ.7$

Tables VII and VIII contain the values of ϑ_1 resp. ϑ_0 and h/β resp. $\sqrt{\frac{\alpha^2}{\beta^2} - 1}$ observed (O) and calculated (C) as well as the differences: O—C.

TABLE VII.

Region	ϑ_1		O—C	$\frac{h}{\beta}$		O—C
	Observ.	Calc.		Observ.	Calc.	
I	⁰ 317.5	⁰ 315.8	+ 1.7	0.6	0.7	-- 0.1
II	334.6	330.6	+ 4.0	1.0	0.8	+ 0.2
III	344.1	347.3	- 3.2	0.9	0.8	+ 0.1
IV	354.2	4.8	- 10.6	0.9	0.8	+ 0.1
V	0.6	22.1	- 21.5	0.8	0.8	0.0
VI	31.0	37.8	- 6.8	0.6	0.7	- 0.1
VII	76.4	51.4	+ 25.0	0.7	0.6	+ 0.1
VIII	50.0	62.3	- 12.3	0.3	0.4	- 0.1
IX	17.5	65.6	- 48.1	0.2	0.2	0.0
X	253.1	309.7	- 56.6	0.1	0.1	0.0
XI	275.0	294.4	- 19.4	0.3	0.3	0.0
XII	307.8	302.7	+ 5.1	0.6	0.5	+ 0.1

T A B L E VIII.

Region	ϑ_0		O—C	$\sqrt{\frac{\alpha^2}{\beta^2} - 1}$		O—C
	Observ.	Calc.		Observ.	Calc.	
I	105.2 ⁰	74.3 ⁰	+ 30.9 ⁰	0.6	1.7	— 1.1
II	163.3	97.0	+ 66.3	2.3	1.3	+ 1.0
III	177.8	135.5	+ 42.3	1.3	0.8	+ 0.5
IV	193.2	211.7	— 18.5	1.2	0.7	+ 0.5
V	175.5	257.3	— 81.8	1.3	1.2	+ 0.1
VI	180.5	281.5	— 101.0	1.0	1.7	— 0.7
VII	348.0	301.6	+ 46.4	1.3	2.0	— 0.7
VIII	352.2	322.5	+ 29.7	2.2	2.1	+ 0.1
IX	357.9	345.5	+ 12.4	1.2	2.1	— 0.9
X	11.1	9.7	+ 1.4	6.6	2.1	+ 4.5
XI	21.9	33.1	— 11.2	3.0	2.1	+ 0.9
XII	14.7	54.3	— 39.6	2.3	2.0	+ 0.3

Finally a revue of all the determinations of the direction of apex and vertex is provided by table IX.

T A B L E IX.

Number of stars	Apex		Vertex		Method
	A	D	A'	D'	
8276	276.6 ⁰	+ 63.6 ⁰	247.5 ⁰	— 81.8 ⁰	Schwarzschild
"	278.0	62.5	—	—	Geometrical
"	259.3	63.7	—	—	"
1708	271.3	44.6	280.1	— 19.6	Schwarzschild
"	265.8	43.5	279.4	— 7.1	"
1652	271.9	42.8	—	—	"
2027	276.7	56.0	273.1	— 35.7	"

III. The results for the white stars and for the red ones differ considerably. As the number of dwarfs in the second group is probably much larger than in the first, absolute magnitudes of stars in both groups were computed by means of the formula $M = m + 5 + 5 \log \pi$, where m is the apparent magnitude and π the parallax of the star and use was made of the tables of Van Rhijn ¹⁾, giving the mean parallaxes for different spectral types as functions of the proper

¹⁾ Publications of the Kapteyn Astronomical Laboratory. № 34 Groningen 1923.

motions and the apparent magnitudes. The stars of the absolute magnitude 3.0 or more were reckoned among the dwarfs, those below the 3.0 among the giants. The distribution of the stars, for which the spectral types in the Yale Catalogue are given, among giants and dwarfs is as follows (The absolute magnitudes of B-stars were reckoned approximately):

Spectral type	Number of stars	Number of giants	Number of dwarfs
B	258	258	—
A	1201	1201	—
F — F ₄	249	218	31
F ₅ — F ₉	434	300	134
G	512	290	222
K	996	922	74
M	78	78	—
R — P — N	7	—	—

As in the first group only 31 (i. e. 1.8%) dwarf stars are present, hence it is obvious that they are of little influence on the results. In the second group (red stars) however out of 2027 stars 430 are dwarf stars, what amounts to 21.2%. The presence of these dwarfs can seriously affect the final results.

Our investigation has been already finished when a paper of John E. Merrill¹⁾, entitled: „Solar apex and galactic rotation from the Yale 50°—55° Catalogue“ appeared in the Astr. Journal. The coordinates of the apex $A = 285^{\circ}.9$, $D = +62^{\circ}.1$ calculated by this author by means of all the stars of the zone of Yale Catalogue are in agreement with the results given above.

This investigation was suggested by prof. Wł. Dziewulski, to whom I am greatly indebted for much advice and encouragement.

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¹⁾ The Astronomical Journal № 921. Vol XXXIX, № 11 Albany 1929.

WL. DZIEWULSKI.

On the motion of dwarf stars.

In his very interesting paper on: „Dynamics of the galactic system in the vicinity of the sun“. J. H. Oort¹⁾ has investigated the distribution of the velocity-vectors of the so called high velocity stars. The aim of the present investigation is precisely similar but it is confined to the motion of the dwarf stars, carried out on somewhat different lines.

Assuming for the direction of the solar motion: $\alpha = 270^\circ$, $\delta = + 30^\circ$ and for its speed 20 km per sec. Miss W. Iwanowska has calculated the space- or peculiar-velocities of these stars, for which the spectroscopic parallaxes, the radial velocities and the proper motions are known. I am very much indebted to Miss W. Iwanowska for these calculations, which she kindly put at my disposal.

The galactic plane was taken for the plane of reference; the co-ordinates of the pole of the galactic plane were assumed: $\alpha = 19^\circ$, $\delta = + 27^\circ$. The galactic co-ordinates and the galactic components of the peculiar velocities were reckoned.

Let us first consider the dwarf stars, whose peculiar velocities exceed 70 km per sec. There are 139 stars of this kind. It may be easily shown that this group of stars is moving as a whole in the direction: $l = 243^\circ.5$, $b = + 1^\circ.5$ with a velocity of 63 km per sec. This velocity depends obviously on the employed stars and has only a relative value. Subtracting this common motion, which represents the displacement of the group of dwarfs relatively to the system of giants, we find the movement of the dwarf stars relatively to the centre of gravity. We can now investigate the distribution of

¹⁾ Bulletin of the Astronomical Institutes of the Netherlands. Vol. IV. No 159. 1928.

the velocity - vectors in this system. As in our former investigations¹⁾ the three axis ellipsoidal distribution is considered. The sky was divided into regions and the stars moving towards each region were counted. The following zones and regions were chosen as follows:

- I zone from $- 15^{\circ}$ to $+ 15^{\circ}$ in lat. and every 45° in longit., on the whole 8 regions
- II zone from $+ 15$ to $+ 50$ in lat. and every 45 in longit., on the whole 8 regions
- III zone from $- 15$ to $- 50$ in lat. and every 45 in longit., on the whole 8 regions
- IV zone from $+ 50$ to $+ 90$ in lat. and every 90 in longit., on the whole 4 regions
- V zone from $- 50$ to $- 90$ in lat. and every 90 in longit., on the whole 4 regions

together 32 regions. In order to allow for the inequality of the areas of different regions the correcting factor 1.02 for the number of the vectors in the regions of the zone II and III should be introduced and 1.11 for the zone IV and V. As the number of vectors in these regions is small and the fractions of the vectors have been neglected, the numbers of vectors remain practically unmodified by this correction. Let: $Ax^2 + A_1y^2 + A_2z^2 + 2Byz + 2B_1zx + 2B_2xy + H = 0$ where x, y, z are the rectangular galactic co-ordinates, be the equation of the velocity ellipsoid. For the 32 regions we get 32 equations which we resolve by the method of least squares. When the constants are found, the axes (a, b, c) and their directions can be easily computed.

Table I contains the co-ordinates of each region and the observed number of stars therein. After determining the constants of the ellipsoid we calculate the number of stars in each region and build the differences: Observ.-Calcul. For the direction of the axes of the velocity-ellipsoid we get:

$$\begin{array}{ll} a \text{ -- axis} & : l = 326^{\circ}.7 \quad b = + 0^{\circ}.3 \\ b \text{ -- axis} & : l = 236.8 \quad b = - 5.2 \\ c \text{ -- axis} & : l = 233.4 \quad b = + 84.8 \end{array}$$

and for the ratios $\frac{b}{a}, \frac{c}{a}$: $\frac{b}{a} = 0.60, \frac{c}{a} = 0.38$.

Let us consider further the dwarf stars, whose peculiar velocities exceed not 70 but 50 km per sec. The number of velocity-

¹⁾ „On the determination of the velocities from the stars of F, G, K, M, types“. Bulletin de l'Observatoire astronomique de Wilno. № 8. 1926.

vectors is now larger: we find 214 stars. For this group we calculate again the common motion and find its direction: $l = 237^{\circ}.8$, $b = +2^{\circ}.2$ and velocity: 45 km per sec. As now the stars with smaller velocities are admitted, the resulting velocity gets smaller. (This velocity of the displacement of the group of dwarf stars has only, as mentioned, a relative value). After subtracting this common motion we receive the movements of the dwarf stars relatively to the centre of gravity. We take again into consideration the three axis ellipsoidal distribution of the velocity-vectors. The sky is divided again into 32 regions and the stars moving in the direction of each region are counted; 32 equations were formed and solved. For the directions of the axes of the velocity-ellipsoid we receive:

$$\begin{array}{lll} \text{a — axis} & : & l = 325^{\circ}.2 \quad b = -0^{\circ}.8 \\ \text{b — axis} & : & l = 235.3 \quad b = +1.3 \\ \text{c — axis} & : & \quad \quad \quad b = +90.0 \end{array}$$

and for the ratios $\frac{b}{a}$ and $\frac{c}{a}$: $\frac{b}{a} = 0.49$, $\frac{c}{a} = 0.34$.

Table II contains the observed and calculated numbers of stars in each region and their differences. It is to be noted that the results of the first (the peculiar velocities > 70 km per sec.) and the second calculation (the peculiar velocities > 50 km per sec.) are in good agreement, though the numbers of stars taken into consideration are rather small.

The investigated group of dwarf stars has a common motion relatively to the system of giant stars; corrected of this common motion the resulting motions show an ellipsoidal distribution. The ellipsoid of velocity distribution is very flattened; this shape shows distinctly two favoured (opposite) directions of the star movements parallel to the greatest axis. This greatest axis is directed to the point $l = 325^{\circ}$ (or 327°) and is practically lying in the galactic plane.

We consider now only those stars, whose motions are directed towards the regions, lying in the close proximity to the greatest axis. As limits for the latitude we choose: $+30^{\circ} - -30^{\circ}$ and for the longitudes: $115^{\circ} - 175^{\circ}$ and $295^{\circ} - 355^{\circ}$. Projecting the velocity-vectors on the galactic plane and on the direction of the great axis, the mean velocities of the observed velocities in the chosen regions were calculated: 75 km per sec. in the direction $l = 145^{\circ}$ and 100 km per sec. in the direction $l = 325^{\circ}$.

Having regard to the common displacement of the dwarf group we get as resultant directions of the velocities of these two groups in longitude: $l = 174^{\circ}$ and $l = 301^{\circ}$. These results are in good

agreement with those of J. H. Oort, who found two maxima of his curve at 162° and 310° galactic longitudes.

Unfortunately our data are too meager to be applied to the study of the rotation in the galaxy.

TABLE I.

Zone	Region	Co-ordinates		Number of stars		O—C
		α	δ	Observ.	Calc.	
I	1	16.0°	$+ 1.4^{\circ}$	11	6	+ 5
	2	73.4	+ 6.3	1	4	- 3
	3	120.0	+ 1.0	20	11	+ 9
	4	150.3	- 1.5	15	17	- 2
	5	210.2	+ 4.2	4	4	0
	6	239.4	+ 9.0	2	3	- 1
	7	304.0	- 1.4	7	12	- 5
	8	345.1	- 5.5	13	12	+ 1
II	9	15.2	+ 22.5	8	4	+ 4
	10	67.5	+ 32.5	0	3	- 3
	11	116.2	+ 21.8	9	6	+ 3
	12	152.2	+ 30.7	3	4	- 1
	13	186.7	+ 18.7	3	5	- 2
	14	225.4	+ 38.2	1	2	- 1
	15	294.3	+ 27.7	5	5	0
	16	350.0	+ 24.4	2	6	- 4
III	17	11.8	- 26.2	6	3	+ 3
	18	67.5	- 32.5	0	2	- 2
	19	108.3	- 28.0	5	3	+ 2
	20	153.5	- 30.6	4	4	0
	21	205.3	- 17.4	1	4	- 3
	22	251.2	- 16.9	3	3	0
	23	305.9	- 31.4	3	4	- 1
	24	343.1	- 30.2	6	4	+ 2
IV	25	53.8	+ 61.7	2	1	+ 1
	26	154.8	+ 65.6	1	1	0
	27	225.0	+ 70.0	0	1	- 1
	28	315.0	+ 70.0	0	1	- 1
V	29	45.0	- 70.0	0	1	- 1
	30	174.8	- 63.5	1	1	0
	31	243.4	- 74.4	1	1	0
	32	347.2	- 67.8	2	1	+ 1

T A B L E II.

Zone	Region	Co-ordinates		Number of stars		O—C
		α	δ	Observ.	Calc.	
I	1	15.0 ⁰	+ 1.0 ⁰	15	16	+ 9
"	2	64.6	+ 6.2	6	4	+ 2
"	3	117.7	+ 1.6	28	17	+ 11
"	4	156.5	— 1.2	24	29	— 5
"	5	212.4	— 4.7	5	5	0
"	6	244.0	+ 9.7	4	4	0
"	7	298.3	— 1.7	9	17	— 8
"	8	343.1	— 5.0	21	24	— 3
II	9	22.4	+ 24.8	4	4	0
"	10	72.4	+ 22.9	1	4	— 3
"	11	113.9	+ 25.5	13	7	+ 6
"	12	146.8	+ 25.6	10	10	0
"	13	198.9	+ 24.5	4	5	— 1
"	14	228.5	+ 32.6	1	3	— 2
"	15	289.7	+ 23.2	4	7	— 3
"	16	342.2	+ 29.4	9	6	+ 3
III	17	17.5	— 38.3	5	3	+ 2
"	18	67.5	— 32.5	0	3	— 3
"	19	117.5	— 23.9	10	8	+ 2
"	20	155.6	— 31.2	9	6	+ 3
"	21	200.0	— 24.4	2	4	— 2
"	22	250.3	— 24.7	3	4	— 1
"	23	302.4	— 21.0	7	10	— 3
"	24	342.8	— 27.6	11	8	+ 3
IV	25	32.6	+ 73.2	2	2	0
"	26	90.0	+ 58.5	1	2	— 1
"	27	185.6	+ 61.0	2	2	0
"	28	315.0	+ 70.0	0	2	— 2
V	29	45.0	— 70.0	0	2	— 2
"	30	135.0	— 70.0	0	2	— 2
"	31	247.2	— 55.0	2	2	0
"	32	305.5	— 61.4	2	2	0

