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I. ASTRONOMIE
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I. ASTRONOMIE

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WILHELMINA IWANOWSKA.

Wyznaczenie funkcji rozkładu prędkości pewnej grupy gwiazd.

On the frequency-function of the peculiar velocities of a group of stars.

(Komunikat zgłoszony przez czł. Wł. Dziewulskiego na posiedzeniu w dniu 19.VI.31).

Introduction.

The matter of this paper is an attempt to find a frequency-function for the distribution of the peculiar motions of a group of stars with known parallaxes, proper motions and radial velocities. Although for a statistical investigation the abundance of the examined material is of great importance, I have confined myself to the study of the stars of late spectral types brighter than $3^m.0$ abs. in order to obtain a more homogeneous material, since these stars, as is evident from a former paper[1]), do not show appreciable differences in their motion relative to the sun and can be regarded as a connected group. Moreover, they represent the most numerous group of this kind, the stars of any other types or absolute magnitudes showing considerable relative motions. The number of collected stars is 910.

To the velocity-components of these stars was applied the ellipsoidal frequency-function:

$$F(x, y, z) = C\, e^{-(A_1x^2 + A_2y^2 + A_3z^2 + 2B_1yz + 2B_2zx + 2B_3xy)}$$

The coefficients of the function were determined according to a method given by Prof. W. Dziewulski[2]). The results obtained are as follows. The directions of three principal axes of the velocity-ellipsoid ex-

[1]) Bulletin de l'Observatoire Astronomique de Wilno, Nr. 9.

[2]) W. Dziewulski. On the determination of the vertex from the motus peculiares of stars. Bulletin de l'Académie des Sciences de Cracovie. 1916.

pressed in galactic coordinates (the galactic pole being $\alpha = 191{^\circ}\!,1$ $\delta = +26{^\circ}\!,8$ and $l = 0°$ in the ascending node):

a-axis:	$l = 358{^\circ}\!,3$	$b = +\ 6{^\circ}\!,2$
b-axis:	$l = 89,6$	$b = +12,2$
c-axis:	$l = 241,8$	$b = +76,3$

The ratios of the semiaxes are:

$$b/a = 0{,}70, \quad c/a = 0{,}53.$$

During the computations I noticed that the velocity-vectors are asymmetrically distributed around the centre. The amount of this asymmetry can be estimated from the graphical representation of the distributions of components of the velocity as given in figures 1, 2 and 3. The principal axes of the ellipsoid are taken as the axes of coordinates. The general character of the distribution shows that in the first approximation it may be considered as ellipsoidal (different dispersions on the three axes). But the obvious asymmetry, especially on the ξ-axis (the great axis of the ellipsoid) suggests a frequency-function composed of two ellipsoidal ones. To determine them a method of decomposition of a frequency-function of three variables into two ellipsoidal functions has been developed, being a generalization of the similar methods of C. V. L. Charlier and other writers.

Part I. (Theory)

Dissection of the frequency-function of three variates into two ellipsoidal distributions.

1. Let ξ, η, ζ be the coordinates in any rectangular system and let $F(\xi, \eta, \zeta)$ represent their frequency-function. The moment of *ijk* index about the point (ξ_0, η_0, ζ_0) is the integral:

$$N_{ijk} = \iiint_{-\infty}^{+\infty} (\xi - \xi_0)^i \, (\eta - \eta_0)^j \, (\zeta - \zeta_0)^k \, F(\xi, \eta, \zeta) \, d\xi \, d\eta \, d\zeta.$$

Arranging the moments according to the indices, we obtain following table:

The moments of the 0 — order: N_{000}.

„ „ „ „ 1 — st „ : N_{100}, N_{010}, N_{001}.

„ „ „ „ 2 — nd „ : N_{200}, N_{020}, N_{002}, N_{011}, N_{101}, N_{110}

„ „ „ „ 3 — rd „ : N_{300}, N_{030}, N_{003}, N_{210}, N_{201}, N_{120}, N_{021}, N_{102}, N_{012}, N_{111}.

„ „ „ „ 4 — th „ : N_{400}, N_{040}, N_{004}, N_{310}, N_{301}, N_{130}, N_{031}, N_{103}, N_{013}, N_{211}, N_{121}, N_{112}, N_{022}, N_{202}, N_{220}.

For every point $N_{000} = 1$. The moments of the first order about the point $(0, 0, 0)$ are the means of ξ, η, ζ, or, what is the same thing, the coordinates of the centre of distribution in the system considered; we denote them by a, b, c.

$$a = N_{100} = \iiint_{-\infty}^{+\infty} \xi \, F(\xi, \eta, \zeta) \, d\xi \, d\eta \, d\zeta$$

$$b = N_{010} = \iiint_{-\infty}^{+\infty} \eta \, F(\xi, \eta, \zeta) \, d\xi \, d\eta \, d\zeta$$

$$c = N_{001} = \iiint_{-\infty}^{+\infty} \zeta \, F(\xi, \eta, \zeta) \, d\xi \, d\eta \, d\zeta$$

The moments about the point (a, b, c) are called the central moments; let us denote them by M_{ijk}.

$$M_{ijk} = \iiint_{-\infty}^{+\infty} (\xi - a)^i \, (\eta - b)^j \, (\zeta - c)^k \, F(\xi, \eta, \zeta) \, d\xi \, d\eta \, d\zeta.$$

It follows from the definition that $M_{000} = 1$, $M_{100} = M_{010} = M_{001} = 0$.

If the moments N_{ijk} about a given point (ξ_0, η_0, ζ_0) are known, we can compute them for any other point (ξ_1, η_1, ζ_1). It can be effected by the substitution:

$$\begin{aligned} \xi - \xi_1 &= (\xi - \xi_0) + (\xi_0 - \xi_1) \\ \eta - \eta_1 &= (\eta - \eta_0) + (\eta_0 - \eta_1) \\ \zeta - \zeta_1 &= (\zeta - \zeta_0) + (\zeta_0 - \zeta_1) \end{aligned}$$

We obtain e. g.: $N'_{100} = \iiint_{-\infty}^{+\infty} (\xi - \xi_1) \, F(\xi, \eta, \zeta) \, d\xi \, d\eta \, d\zeta =$

$$= \iiint_{-\infty}^{+\infty} [(\xi - \xi_0) + (\xi_0 - \xi_1)] \, F(\xi, \eta, \zeta) \, d\xi \, d\eta \, d\zeta =$$

$$= \iiint_{-\infty}^{+\infty} (\xi - \xi_0) \, F \, d\xi \, d\eta \, d\zeta + (\xi_0 - \xi_1) \iiint_{-\infty}^{+\infty} F \, d\xi \, d\eta \, d\zeta =$$

$$= N_{100} + (\xi_0 - \xi_1).$$

Similarly:
$$\begin{aligned} N'_{010} &= N_{010} + (\eta_0 - \eta_1) \\ N'_{001} &= N_{001} + (\zeta_0 - \zeta_1) \\ N'_{200} &= N_{200} + 2\,(\xi_0 - \xi_1)\, N_{100} + (\xi_0 - \xi_1)^2. \text{ etc.} \end{aligned}$$

It is easily seen that the moments in a new system of coordinates x, y, z correlated with the old ones by the equations:

$$\begin{aligned} x &= \lambda_1 \xi + \mu_1 \eta + \nu_1 \zeta \\ y &= \lambda_2 \xi + \mu_2 \eta + \nu_2 \zeta \\ z &= \lambda_3 \xi + \mu_3 \eta + \nu_3 \zeta \end{aligned}$$

may be expressed by the moments in the old system ξ, η, ζ (N_{ij}) about the same point. From

$$N''_{ijk} = \iiint\limits_{-\infty}^{+\infty} x^i\, y^j\, z^k\, F\, dx\, dy\, dz =$$

$$= \iiint\limits_{-\infty}^{+\infty} (\lambda_1\xi + \mu_1\eta + \nu_1\zeta)^i\, (\lambda_2\xi + \mu_2\eta + \nu_2\zeta)^j\, (\lambda_3\xi + \mu_3\eta + \nu_3\zeta)^k\, F\, d\xi\, d\eta\, d\zeta$$

we find:

$$\begin{aligned}
N''_{100} &= \lambda_1 N_{100} + \mu_1 N_{010} + \nu_1 N_{001} \\
N''_{010} &= \lambda_2 N_{100} + \mu_2 N_{010} + \nu_2 N_{001} \\
N''_{001} &= \lambda_3 N_{100} + \mu_3 N_{010} + \nu_3 N_{001} \\
N''_{200} &= \lambda_1^2 N_{200} + \mu_1^2 N_{020} + \nu_1^2 N_{002} + 2\mu_1\nu_1 N_{011} + \\
&\quad + 2\nu_1\lambda_1 N_{101} + 2\lambda_1\mu_1 N_{110} \text{ etc.}
\end{aligned}$$

These results will be required later.

2. Returning to our problem, let us express the frequency-function $F(\xi, \eta, \zeta)$ as a sum of two components:

$$F(\xi, \eta, \zeta) = n_1 F_1(\xi, \eta, \zeta) + n_2 F_2(\xi, \eta, \zeta)$$

where n_1 and n_2 are the factors depending on the relative abundance of each distribution. Translating the origin of the system of coordinates to the centre of the whole distribution (a, b, c) and denoting the new coordinates by X, Y, Z, we hawe:

$$\begin{aligned}
X &= \xi - a \\
Y &= \eta - b \\
Z &= \zeta - c
\end{aligned}$$

According to the definition, the integral $M_{ijk} = \iiint\limits_{-\infty}^{+\infty} X^i\, Y^j\, Z^k\, F\, dX\, dY\, dZ$ is the central moment of the whole distribution, and the integrals

$$N'_{ijk} = \iiint\limits_{-\infty}^{+\infty} X^i\, Y^j\, Z^k\, F_1\, dX\, dY\, dZ$$

$$N''_{ijk} = \iiint\limits_{-\infty}^{+\infty} X^i\, Y^j\, Z^k\, F_2\, dX\, dY\, dZ$$

are the moments of the component distributions about the centre of the whole distribution (they are not central moments in relation to each of the component distributions). Thus we can write the relation:

$$M_{ijk} = n_1 N'_{ijk} + n_2 N''_{ijk} \qquad (1)$$

which is valid for all values of ijk.

Now, we shall represent the component-fnnctions F_1, F_2 in the form of the ellipsoidal frequency-functions:

$$F_1\,[(X,Y,Z)] = C_1\, e^{-\frac{1}{2}f_1}, \quad F_2\,[(X,Y,Z)] = C_2\, e^{-\frac{1}{2}f_2},$$

where $f_1 = A'_1\,(X-a_1)^2 + A'_2\,(Y-b_1)^2 + A'_3\,(Z-c_1)^2 +$
$+ 2B'_1\,(Y-b_1)(Z-c_1) + 2B'_2\,(Z-c_1)(X-a_1) + 2B'_3\,(X-a_1)(Y-b_1)$,

$$f_2 = A''_1\,(X-a_2)^2 + A''_2\,(Y-b_2)^2 + A''_3\,(Z-c_2)^2 +$$
$$+ 2B''_1\,(Y-b_2)(Z-c_2) + 2B''_2\,(Z-c_2)(X-a_2) + 2B''_3\,(X-a_2)(Y-b_2).$$

The problem consists in the determination of 20 parameters, viz.: n_1, n_2 — the total probabilities of each of the component distributions, a_1, b_1, c_1; a_2, b_2, c_2 — the coordinates of the centres of the component distributions and the coefficients A'_1 .... B'_3 A''_1 .... B''_3 or instead of them the semiaxes and the directions of the axes of the ellipsoids defined by the equations: $f_1 = const.$, $f_2 = const.$

The relation (1) gives necessary number of equations. It contains the moments N'_{ijk}, N''_{ijk} which can be expressed through the unkown parameters, the moments M_{ijk} being known for a given distribution. We will now derive the relations between the unknown parameters and the moments N'_{ijk}, N''_{ijk}. It is sufficient to do it for the first component-function only, the calculation for the second being quite analogous.

3. Let the semiaxes of the first ellipsoid be σ'_1, σ'_2, σ'_3, their direction-cosines in the XYZ – system λ'_1, μ'_1, ν'_1; λ'_2, μ'_2, ν'_2; λ'_3, μ'_3, ν'_3 and the coordinates of the system referred to the principal axes of the ellipsoid — x_1, y_1, z_1. We get the following formulae for the transformation of coordinates:

$$\begin{aligned} X &= a_1 + \lambda'_1 x_1 + \lambda'_2 y_1 + \lambda'_3 z_1 \\ Y &= b_1 + \mu'_1 x_1 + \mu'_2 y_1 + \mu'_3 z_1 \\ Z &= c_1 + \nu'_1 x_1 + \nu'_2 y_1 + \nu'_3 z_1 \end{aligned} \qquad (2)$$

where a_1, b_1, c_1 denote as before the coordinates of the centre of the first ellipsoid in the system X, Y, Z. We shall use the following moments of the function F_1:

$N'_{ijk} = \iiint_{-\infty}^{+\infty} X^i\, Y^j\, Z^k\, F_1\, dX\, dY\, dZ$ — the moments already used in the system X, Y, Z.

$M'_{ijk} = \iiint_{-\infty}^{+\infty} (X-a_1)^i\,(Y-b_1)^j\,(Z-c_1)^k\, F_1\, dX dY dZ$ — the central moments in the X, Y, Z — system,

$m'_{ijk} = \iiint_{-\infty}^{+\infty} x_1^{\,i}\, y_1^{\,j}\, z_1^{\,k}\, F_1\, dx_1\, dy_1\, dz_1$ — the central moments in the x_1, y_1, z_1 — system.

The moments of each kind can be easily transformed into another in the way previously shown.

We shall first prove that the moments N'_{ijk} of the higher order can be expressed by the moments of the same kind of the first and second order, and then deduce the relations between these moments and the elements of the ellipsoid.

Since the function F_1 agrees its simplest form in the x_1, y_1, z_1 — system, namely

$$F_1\,[(x_1, y_1, z_1)] = \frac{1}{(2\pi)^{3/2}\,\sigma'_1\,\sigma'_2\,\sigma'_3}\,e^{-\frac{1}{2}\left(\frac{x_1^2}{\sigma'^2_1}+\frac{y_1^2}{\sigma'^2_2}+\frac{z_1^2}{\sigma'^2_3}\right)},$$

we may compute the moments m'_{ijk} performing easy integrations of the type: $\int\limits_{-\infty}^{+\infty} t^n e^{-\frac{t^2}{2\sigma^2}}\,dt$. All the moments for which one of the indices i, j, k is odd vanish, the remaining ones up to the fourth order are easily calculated:

$$\begin{aligned}
m'_{000} &= 1\\
m'_{200} &= \sigma'^2_1\\
m'_{020} &= \sigma'^2_2\\
m'_{002} &= \sigma'^2_3\\
m'_{400} &= 3\,\sigma'^4_1\\
m'_{040} &= 3\,\sigma'^4_2\\
m'_{004} &= 3\,\sigma'^4_3\\
m'_{022} &= \sigma'^2_2\,\sigma'^2_3\\
m'_{202} &= \sigma'^2_1\,\sigma'^2_3\\
m'_{220} &= \sigma'^2_1\,\sigma'^2_2\,.
\end{aligned} \tag{3}$$

The moments N'_{ijk} can be expressed by the moments m'_{ijk} by means of the transformations given in § 1. We get

$$\begin{aligned}
N'_{000} &= 1\\
N'_{100} &= a_1\\
N'_{010} &= b_1\\
N'_{001} &= c_1\\
N'_{200} &= a_1^2 + \lambda'^2_1\sigma'^2_1 + \lambda'^2_2\sigma'^2_2 + \lambda'^2_3\sigma'^2_3\\
N'_{020} &= b_1^2 + \mu'^2_1\sigma'^2_1 + \mu'^2_2\sigma'^2_2 + \mu'^2_3\sigma'^2_3\\
N'_{002} &= c_1^2 + \nu'^2_1\sigma'^2_1 + \nu'^2_2\sigma'^2_2 + \nu'^2_3\sigma'^2_3\\
N'_{011} &= b_1\,c_1 + \mu'_1\,\nu'_1\,\sigma'^2_1 + \mu'_2\,\nu'_2\,\sigma'^2_2 + \mu'_3\,\nu'_3\,\sigma'^2_3\\
N'_{101} &= c_1\,a_1 + \nu'_1\,\lambda'_1\,\sigma'^2_1 + \nu'_2\,\lambda'_2\,\sigma'^2_2 + \nu'_3\,\lambda'_3\,\sigma'^2_3\\
N'_{110} &= a_1\,b_1 + \lambda'_1\,\mu'_1\,\sigma'^2_1 + \lambda'_2\,\mu'_2\,\sigma'^2_2 + \lambda'_3\,\mu'_3\,\sigma'^2_3\,.
\end{aligned} \tag{4}$$

Computing in a similar manner the moments of the third and fourth order and applying the formulae (4), we obtain:

$$
\begin{array}{lll}
N'_{300} = 3a_1 N'_{200} - 2a_1^3 & \text{similarly} & N'_{030},\ N'_{003} \\
N'_{210} = b_1 N'_{200} + 2a_1 N'_{110} - 2a_1^2 b_1 & \text{"} & N'_{201},\ N'_{120}, \\
 & & N'_{021},\ N'_{102},\ N'_{012} \\
N'_{111} = a_1 N'_{011} + b_1 N'_{101} + c_1 N'_{110} - 2a_1 b_1 c_1 & & \\
N'_{400} = 3N'^2_{200} - 2a_1^4 & \text{"} & N'_{040},\ N'_{004} \\
N'_{310} = 3N'_{200} N'_{110} - 2a_1^3 b_1 & \text{"} & N'_{301},\ N'_{130}, \\
 & & N'_{031},\ N'_{103},\ N'_{013} \\
N'_{022} = N'_{020} N'_{002} + 2N'^2_{011} - 2b_1^2 c_1^2 & \text{"} & N'_{202},\ N'_{220} \\
N'_{211} = N'_{200} N'_{011} + 2N'_{110} N'_{101} - 2a_1^2 b_1 c_1 & \text{"} & N'_{121},\ N'_{112}
\end{array}
\tag{5}
$$

It is therefore evident that, supposing an ellipsoidal distribution, all the moments N'_{ijk} can be reduced to those of the first and second order. On the other hand, the elements of the ellipsoid are defined by the moments N'_{ijk} of the first and second order: a theorem given by Charlier[1]) shows that the squares of the semiaxes of the ellipsoid σ'^2_1, σ'^2_2, σ'^2_3 are equal to the roots of the equation:

$$
G(t) = \begin{vmatrix} M'_{200} - t & M'_{110} & M'_{101} \\ M'_{110} & M'_{020} - t & M'_{011} \\ M'_{101} & M'_{011} & M'_{002} - t \end{vmatrix} = O, \tag{6}
$$

and the direction-cosines are defined by the relations:

$$
\begin{aligned}
\frac{\lambda_1'}{G_{11}(t_1)} = \frac{\mu_1'}{G_{12}(t_1)} = \frac{\nu_1'}{G_{13}(t_1)} &= \frac{1}{\sqrt{G^2_{11}(t_1) + G^2_{12}(t_1) + G^2_{13}(t_1)}} \\
\frac{\lambda_2'}{G_{11}(t_2)} = \frac{\mu_2'}{G_{12}(t_2)} = \frac{\nu_2'}{G_{13}(t_2)} &= \frac{1}{\sqrt{G^2_{11}(t_2) + G^2_{12}(t_2) + G^2_{13}(t_2)}} \\
\frac{\lambda_3'}{G_{11}(t_3)} = \frac{\mu_3'}{G_{12}(t_3)} = \frac{\nu_3'}{G_{13}(t_3)} &= \frac{1}{\sqrt{G^2_{11}(t_3) + G^2_{12}(t_3) + G^2_{13}(t_3)}}
\end{aligned}
\tag{7}
$$

where G_{11}, G_{12}, G_{13} are the subdeterminants of $G(t)$. The moments M'_{ijk} which appear in the equation (6) are the central moments which can be easily expressed by the moments N'_{ijk}, e. g.:

$$
M'_{200} = N'_{200} - a_1^2, \quad M'_{011} = N'_{011} - b_1 c_1 .
$$

The validity of (6) and (7) can be proved directly from (4).

Thus we have expressed the moments of higher orders N'_{ijk}, N''_{ijk} by means of similar moments of the first and second order;

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[1]) C. V. L. Charlier. „The Motion and the Distribution of Stars". Berkeley, California, 1926.

on the other hand the elements of the ellipsoids can be computed from the same moments. Since

n_1, a_1, b_1, c_1, σ_1', σ_2', σ_3' and three independent direction-cosines
n_2, a_2, b_2, c_2, σ_1'', σ_2'', σ_3'' „ „ „ „ „

can be replaced by

$$n_1,\ a_1,\ b_1,\ c_1,\ N'_{200},\ N'_{020},\ N'_{002},\ N'_{011},\ N'_{101},\ N'_{110}$$
$$n_2,\ a_2,\ b_2,\ c_2,\ N''_{200},\ N''_{020},\ N''_{002},\ N''_{011},\ N''_{101},\ N''_{110}.$$

4. The relation (1) applied to the various values of the indices i, j, k gives an arbitrary number of equations useful to find the unknown quantities. We will write these equations up to the fourth order inclusively:

0 order $1 = n_1 + n_2$

1-st „ $0 = n_1 a_1 + n_2 a_2$ and 2 similar equations

2-nd „
$$M_{200} = n_1 N'_{200} + n_2 N''_{200} \quad \text{„ „ „ „}$$
$$M_{011} = n_1 N'_{011} + n_2 N''_{011} \quad \text{„ „ „ „}$$

3-rd „
$$M_{300} = n_1(3a_1 N'_{200} - 2a_1^3) + n_2(3a_2 N''_{200} - 2a_2^3)$$
and 2 similar equations
$$M_{210} = n_1(b_1 N'_{200} + 2a_1 N'_{110} - 2a_1^2 b_1) + n_2(b_2 N''_{200} + 2a_2 N''_{110} - 2a_2^2 b_2)$$
and 5 similar equations
$$M_{111} = n_1(a_1 N'_{011} + b_1 N'_{101} + c_1 N'_{110} - 2a_1 b_1 c_1) + n_2(a_2 N''_{011} + b_2 N''_{101} + c_2 N''_{110} - 2a_2 b_2 c_2) \tag{8}$$

4-th „
$$M_{400} = n_1(3N'^2_{200} - 2a_1^4) + n_2(3N''^2_{200} - 2a_2^4)$$
and 2 similar equations
$$M_{310} = n_1(3N'_{200}N'_{110} - 2a_1^3 b_1) + n_2(3N''_{200}N''_{110} - 2a_2^3 b_2)$$
and 5 similar equations
$$M_{022} = n_1(N'_{020}N'_{002} + 2N'^2_{011} - 2b_1^2 c_1^2) + n_2(N''_{020}N''_{002} + 2N''^2_{011} - 2b_2^2 c_2^2)$$
and 2 similar equations
$$M_{211} = n_1(N'_{200}N'_{011} + 2N'_{110}N'_{101} - 2a_1^2 b_1 c_1) + n_2(N''_{200}N''_{011} + 2N''_{110}N''_{101} - 2a_2^2 b_2 c_2)$$
and 2 similar equations

Since there are 20 unknown quantities it would seem natural to solve 20 equations, preferentially those of the 0, 1-st, 2-nd and 3-rd order owing to their simple form. However, the 20 first equations are not independent; it can be easily shown that there are two relations between them. For this reason we shall include in our system of equations two independent equations of the higher order. The second difficulty arises from the fact that the real distribution deviates more or less from the ideal two-ellipsoidal distribution and the remaining relations are not satisfied by the roots of the chosen equations. Therefore the direct solution must be replaced by a least-squares solution of all equations having regard to their weights, as the equations are not equivalent; since the moments are the means of the products of the successive powers of the variables x, y, z, we may attribute a greater weight to the equations of the lower order. These are the general remarks concerning the solution of the equations.

5. The solution of the system of equations (8) is generally troublesome. The equations will be considerably simplified if we rotate the system of coordinates so as to bring the line joining the centres of the component distributions to coincidence with one of the axes, e. g. the X-axis. This can be done by turning the old XYZ — system twice: 1^0 — around the Z-axis by an angle Θ_1 in order to bring the line of centres into the ZX — plane. Denoting the auxiliary system by $\underline{X}$, $\underline{Y}$, $\underline{Z}$ and the moments by $\underline{M}_{ijk}$ we get:

$$\begin{aligned} \underline{X} &= X\cos\Theta_1 + Y\sin\Theta_1 \\ \underline{Y} &= -X\sin\Theta_1 + Y\cos\Theta_1 \\ \underline{Z} &= Z \end{aligned} \tag{9a}$$

2^0 — around the $\underline{Y}$-axis by an angle Θ_2 to obtain the coincidence of the line of centres with the $\underline{X}$-axis. If we denote the coordinates and moments in the new system by $\overline{X}$, $\overline{Y}$, $\overline{Z}$, $\overline{M}_{ijk}$ respectively, we find:

$$\begin{aligned} \overline{X} &= \underline{X}\cos\Theta_2 - \underline{Z}\sin\Theta_2 \\ \overline{Y} &= \underline{Y} \\ \overline{Z} &= \underline{X}\sin\Theta_2 + \underline{Z}\cos\Theta_2 \end{aligned} \tag{9b}$$

Finally, the following relations subsist between the primary coordinates X, Y, Z and the new ones $\overline{X}$, $\overline{Y}$, $\overline{Z}$:

$$\begin{aligned} \overline{X} &= X\cos\Theta_1\cos\Theta_2 + Y\sin\Theta_1\cos\Theta_2 - Z\sin\Theta_2 \\ \overline{Y} &= -X\sin\Theta_1 + Y\cos\Theta_1 \\ \overline{Z} &= X\cos\Theta_1\sin\Theta_2 + Y\sin\Theta_1\sin\Theta_2 + Z\cos\Theta_2 \end{aligned} \tag{9}$$

Using the formulae (9 a), (9 b), (9) we can express the moments relative to one system of axes by those of another.

Since the centres of the component distributions lie on the $\overline{X}$-axis, we obtain the following conditions:

from the first rotation $\underline{b}_1 = \underline{b}_2 = 0$,

from the second rotation $\overline{c}_1 = \overline{c}_2 = 0$ (evidently also $\overline{b}_1 = \overline{b}_2 = 0$).

Thus, as is evident from equations (8), several moments in the new system are equal to zero, viz.: $\overline{M}_{030} = \overline{M}_{003} = \overline{M}_{021} = \overline{M}_{012} = 0$ and the remaining equations are considerably simplified:

$$1 = n_1 + n_2$$
$$0 = n_1 \bar{a}_1 + n_2 \bar{a}_2$$
$$\overline{M}_{200} = n_1 \overline{N}'_{200} + n_2 \overline{N}''_{200}$$
and 5 similar equations for $\overline{M}_{020}$, $\overline{M}_{002}$, $\overline{M}_{011}$, $\overline{M}_{101}$, $\overline{M}_{110}$
$$\overline{M}_{300} = n_1 (3\bar{a}_1 \overline{N}'_{200} - 2\bar{a}_1^3) + n_2 (3\bar{a}_2 \overline{N}''_{200} - 2\bar{a}_2^3)$$
$$\overline{M}_{210} = n_1 2\bar{a}_1 \overline{N}'_{110} + n_2 2\bar{a}_2 \overline{N}''_{110}$$
and 1 similar equation for $\overline{M}_{201}$
$$\overline{M}_{120} = n_1 \bar{a}_1 \overline{N}'_{020} + n_2 \bar{a}_2 \overline{N}''_{020}$$
and 1 similar equation for $\overline{M}_{102}$
$$\overline{M}_{111} = n_1 \bar{a}_1 \overline{N}'_{011} + n_2 \bar{a}_2 \overline{N}''_{011} \tag{10}$$
$$\overline{M}_{400} = n_1 (3\overline{N}'^2_{200} - 2\bar{a}_1^4) + n_2 (3\overline{N}''^2_{200} - 2\bar{a}_2^4)$$
$$\overline{M}_{040} = n_1 3\overline{N}'^2_{020} + n_2 3\overline{N}''^2_{020}$$
and 1 similar equation for $\overline{M}_{004}$
$$\overline{M}_{310} = n_1 3\overline{N}'_{200} \overline{N}'_{110} + n_2 3\overline{N}''_{200} \overline{N}''_{110}$$
and 5 similar equations for $\overline{M}_{301}$, $\overline{M}_{130}$, $\overline{M}_{031}$, $\overline{M}_{103}$, $\overline{M}_{013}$
$$\overline{M}_{022} = n_1 (\overline{N}'_{020} \overline{N}'_{002} + 2\overline{N}'^2_{011}) + n_2 (\overline{N}''_{020} \overline{N}''_{002} + 2\overline{N}''^2_{011})$$
and 2 similar equations for $\overline{M}_{202}$, $\overline{M}_{220}$
$$\overline{M}_{211} = n_1 (\overline{N}'_{200} \overline{N}'_{011} + 2\overline{N}'_{110} \overline{N}'_{101}) + n_2 (\overline{N}''_{200} \overline{N}''_{011} + 2\overline{N}''_{110} \overline{N}''_{101})$$
and 2 similar equations for $\overline{M}_{121}$, $\overline{M}_{112}$.

The angles Θ_1, Θ_2 can be determined successively from the conditions $\overline{M}_{030} = 0$, $\overline{M}_{003} = 0$; using the formulae (9a) and (9b) we express the moment M_{003} by the primary moments M_{ijk} and the moment $\overline{M}_{003}$ by $\underline{M}_{ijk}$, we obtain the equations:

$$\begin{aligned} M_{300} tg^3\Theta_1 - 3M_{210} tg^2\Theta_1 + 3M_{120} tg\Theta_1 - M_{030} &= 0 \\ \underline{M}_{300} tg^3\Theta_2 + 3\underline{M}_{201} tg^2\Theta_2 + 3\underline{M}_{102} tg\Theta_2 + \underline{M}_{003} &= 0 \end{aligned} \tag{11}$$

from which Θ_1 and Θ_2 could be found.

It is however desirable to obtain a simultaneous determination of the angles from four conditions: $\overline{M}_{030} = \overline{M}_{003} = \overline{M}_{021} = \overline{M}_{012} = 0$.

In the system of equations (10) the number of unknown quantities is now reduced to 16:

$$n_1,\ \bar{a}_1,\ \overline{N}'_{200},\ \overline{N}'_{020},\ \overline{N}'_{002},\ \overline{N}'_{011},\ \overline{N}'_{101},\ \overline{N}'_{110}$$
$$n_2,\ \bar{a}_2,\ \overline{N}''_{200},\ \overline{N}''_{020},\ \overline{N}''_{002},\ \overline{N}''_{011},\ \overline{N}''_{101},\ \overline{N}''_{110}.$$

To find them it would suffice to take into account all the equations up to the third order and two equations of the fourth order. The solution is in this case very easy. For example, we can express all unknown parameters by means of a_1 and a_2 from the equations of the 0, 1-st, 2-nd and 3-rd order. We get:

$$n_1 = \frac{\bar{a}_2}{\bar{a}_2 - \bar{a}_1}, \quad N'_{200} = \bar{M}_{200} - \frac{\bar{M}_{300}}{3\bar{a}_2} + \tfrac{2}{3}\bar{a}_1(\bar{a}_1 + \bar{a}_2), \quad N'_{020} = \bar{M}_{020} - \frac{\bar{M}_{120}}{\bar{a}_2},$$

$$\bar{N}'_{002} = \bar{M}_{002} - \frac{\bar{M}_{102}}{\bar{a}_2}, \quad \bar{N}'_{011} = \bar{M}_{011} - \frac{\bar{M}_{111}}{\bar{a}_2},$$

$$\bar{N}'_{101} = \bar{M}_{101} - \frac{\bar{M}_{201}}{2\bar{a}_2}, \quad \bar{N}'_{110} = \bar{M}_{110} - \frac{\bar{M}_{210}}{2\bar{a}_2} \tag{12}$$

$$n_2 = \frac{\bar{a}_1}{\bar{a}_1 - \bar{a}_2}, \quad N''_{200} = \bar{M}_{200} - \frac{\bar{M}_{300}}{3\bar{a}_1} + \tfrac{2}{3}\bar{a}_2(\bar{a}_1 + \bar{a}_2), \quad N''_{020} = \bar{M}_{020} - \frac{\bar{M}_{120}}{\bar{a}_1},$$

$$\bar{N}''_{002} = \bar{M}_{002} - \frac{\bar{M}_{102}}{\bar{a}_1}, \quad \bar{N}''_{011} = \bar{M}_{011} - \frac{\bar{M}_{111}}{\bar{a}_1},$$

$$\bar{N}''_{101} = \bar{M}_{101} - \frac{\bar{M}_{201}}{2\bar{a}_1}, \quad \bar{N}''_{110} = \bar{M}_{110} - \frac{\bar{M}_{210}}{2\bar{a}_1}.$$

Then, substituting in two equations of the fourth order, i. e. those containing the moments $\bar{M}_{040}$ and $\bar{M}_{310}$, we obtain:

$$\bar{a}_1\bar{a}_2 = \frac{3\bar{M}^2_{120}}{3\bar{M}^2_{020} - \bar{M}_{040}},$$

$$\bar{a}_1 + \bar{a}_2 = \frac{1}{\bar{M}_{210}}\left(\bar{M}_{310} - 3\bar{M}_{200}\bar{M}_{110} + \frac{\bar{M}_{300}\cdot\bar{M}_{210}}{2\bar{a}_1\cdot\bar{a}_2}\right). \tag{13}$$

We can also use all the equations of the fourth order and take the means of the values $\bar{a}_1\bar{a}_2$ and $\bar{a}_1 + \bar{a}_2$.

The computation of $\bar{a}_1$ and $\bar{a}_2$ and then of all the remaining unknown quantities from the relations (11) is quite simple. It may be remarked that as $\bar{a}_1$ and $\bar{a}_2$ are determined from a quadratic equation, imaginary solutions can occur. In this case the resolution of the given distribution into two ellipsoidal functions would be impossible. A similar case may occur by the computation of the dispersions $\sigma'_1, \sigma'_2, \sigma'_3; \sigma''_1, \sigma''_2, \sigma''_3$ from the relations (7).

6. Let us now consider a special case of the dissection, when component functions are circular, or Maxwellian functions:

$$F_1[(X,Y,Z)] = \frac{1}{(2\pi)^{3/2}\sigma'^3}\, e^{-\frac{1}{2\sigma'^2}[(X-a_1)^2 + (Y-b_1)^2 + (Z-c_1)^2]}$$

$$F_2[(X,Y,Z)] = \frac{1}{(2\pi)^{3/2}\sigma''}\, e^{-\frac{1}{2\sigma''^2}[(X-a_2)^2 + (Y-b_2)^2 + (Z-c_2)^2]}$$

Then the directions are undetermined and we can put $\lambda'_1 = \mu'_2 = \nu'_3 = 1$; $\lambda''_1 = \mu''_2 = \nu''_3 = 1$, the remaining direction-cosines being zero.

From (4) we get:

$$N'_{200} = a_1^2 + \sigma'^2,\ N'_{020} = b_1^2 + \sigma'^2,\ N'_{002} = c_1^2 + \sigma'^2,$$
$$N'_{011} = b_1 c_1,\ N'_{101} = a_1 c_1,\ N'_{110} = a_1 b_1$$
$$N''_{200} = a_2^2 + \sigma''^2,\ N''_{020} = b_2^2 + \sigma''^2,\ N''_{002} = c_2^2 + \sigma''^2,$$
$$N''_{011} = b_2 c_2,\ N''_{101} = a_2 c_2,\ N''_{110} = a_2 b_2$$

since

$$\sigma'_1 = \sigma'_2 = \sigma'_3 = \sigma';\quad \sigma''_1 = \sigma''_2 = \sigma''_3 = \sigma''.$$

The system of equations (8) up to the third order gets the following form [1]):

$$\begin{aligned}
1 &= n_1 + n_2 \\
0 &= n_1 a_1 + n_2 a_2 \\
0 &= n_1 b_1 + n_2 b_2 \\
0 &= n_1 c_1 + n_2 c_2 \\
M_{200} &= n_1 (\sigma'^2 + a_1^2) + n_2 (\sigma''^2 + a_2^2) \\
M_{020} &= n_1 (\sigma'^2 + b_1^2) + n_2 (\sigma''^2 + b_2^2) \\
M_{002} &= n_1 (\sigma'^2 + c_1^2) + n_2 (\sigma''^2 + c_2^2) \\
M_{011} &= n_1 b_1 c_1 + n_2 b_2 c_2 \\
M_{101} &= n_1 a_1 c_1 + n_2 a_2 c_2 \\
M_{110} &= n_1 a_1 b_1 + n_2 a_2 b_2 \\
M_{300} &= n_1 a_1 (3\sigma'^2 + a_1^2) + n_2 a_2 (3\sigma''^2 + a_2^2) \\
M_{030} &= n_1 b_1 (3\sigma'^2 + b_1^2) + n_2 b_2 (3\sigma''^2 + b_2^2) \\
M_{003} &= n_1 c_1 (3\sigma'^2 + c_1^2) + n_2 c_2 (3\sigma''^2 + c_2^2) \\
M_{210} &= n_1 b_1 (\sigma'^2 + a_1^2) + n_2 b_2 (\sigma''^2 + a_2^2) \\
M_{201} &= n_1 c_1 (\sigma'^2 + a_1^2) + n_2 c_2 (\sigma''^2 + a_2^2) \\
M_{120} &= n_1 a_1 (\sigma'^2 + b_1^2) + n_2 a_2 (\sigma''^2 + b_2^2) \\
M_{021} &= n_1 c_1 (\sigma'^2 + b_1^2) + n_2 c_2 (\sigma''^2 + b_2^2) \\
M_{102} &= n_1 a_1 (\sigma'^2 + c_1^2) + n_2 a_2 (\sigma''^2 + c_2^2) \\
M_{012} &= n_1 b_1 (\sigma'^2 + c_1^2) + n_2 b_2 (\sigma''^2 + c_2^2) \\
M_{111} &= n_1 a_1 b_1 c_1 + n_2 a_2 b_2 c_2
\end{aligned} \qquad (14)$$

These equations contain 10 unknown quantities:

$$n_1,\ a_1,\ b_1,\ c_1,\ \sigma';$$
$$n_2,\ a_2,\ b_2,\ c_2,\ \sigma''.$$

For resolving them the 10 equations up to the second order are not sufficient because there are two relations between them. We may add any two equations of the third order, or take all the equations of the third order and resolve the system by least-squares method. A direct solution of this case is given by Scigolev in his paper mentioned

[1]) cfr. a paper by Scigolev. Russian Astronomical Journal Vol. II. Pg. 1.

above. We will resolve the system applying a rotation of axes similar to that given above for the general case. From the conditions $\bar{b}_1 = \bar{b}_2 = 0$, $\bar{c}_1 = \bar{c}_2 = 0$ we get from the equations of the 2-nd order:

$$\bar{M}_{011} = \bar{M}_{101} = \bar{M}_{110} = 0, \; \bar{M}_{020} = \bar{M}_{002}; \tag{15}$$

from the equations of the 3-rd order:

$$\bar{M}_{030} = \bar{M}_{003} = \bar{M}_{210} = \bar{M}_{201} = \bar{M}_{021} = \bar{M}_{012} = \bar{M}_{111} = 0, \; \bar{M}_{120} = \bar{M}_{102}.$$

The remaining equations are:

$$\begin{aligned} 1 &= n_1 + n_2 \\ 0 &= n_1\bar{a}_1 + n_2\bar{a}_2 \\ \bar{M}_{200} &= n_1(\sigma'^2 + \bar{a}_1^2) + n_2(\sigma''^2 + \bar{a}_2^2) \\ \bar{M}_{020} &= n_1\sigma'^2 + n_2\sigma''^2 = \bar{M}_{002} \\ \bar{M}_{300} &= n_1\bar{a}_1(3\sigma'^2 + \bar{a}_1^2) + n_2\bar{a}_2(3\sigma''^2 + \bar{a}_2^2) \\ \bar{M}_{120} &= n_1\bar{a}_1\sigma'^2 + n_2\bar{a}_2\sigma''^2 \end{aligned} \tag{16}$$

From these equations n_1, $\bar{a}_1$, σ'; n_2, $\bar{a}_2$, σ'' can be computed like in the general case. The angles Θ_1, Θ_2 can be determined from the two conditions (15) of the second order, or from all of them by the least-squares method.

Part II.

The dissection of the distribution of the velocities of the *F, G, K, M*-type giants in two Maxwellian distributions.

The observational data for 910 stars mentioned in the introduction were collected from the following sources:

1) Catalogue of equatorial components of velocities of 1470 stars (Publications de l'Institut Astrophysique de Russie, Vol. III. Fasc. II. Moscou, 1926).

2) Catalogue of space velocities of 1488 stars (Bulletin de l'Institut Astronomique, N-o 11. Leningrad, 1925).

Besides this, for more than 100 stars the velocity components were computed using the proper motions from Boss' Catalogue, the spectroscopic parallaxes given by Adams, Joy, Strömberg (Astrophysical Journal XLVI, LIII, LXIV) and the radial velocities from the Publications of the Lick Observatory, Vol. XVI.

The principal axes of the ellipsoid derived by the method of Prof. W. Dziewulski were chosen as the axes of reference („first ellipsoid"), and the moments N'_{ijk} relative to the centre of the coordinates were calculated up to the third order. As a detailed description of these calculations is of little interest, I will mention only that space was divided in cubes of side of 8 km/sec and the velocity-vectors contained in the cubes counted; assuming for all vectors of each cube the coordinates of its centre, the moments, viz., the means of the products of successive powers of the components were reckoned. All stars having any velocity-component exceeding the limits

on the ξ-axis, or the a-axis of the ellipsoid:	from	-72 km/sec	to	$+72$ km/sec
„ „ η-axis, „ „ b-axis „ „ „ :	„	-56 „	„	$+56$ „
„ „ ζ-axis, „ „ c-axis „ „ „ :	„	-40 „	„	$+40$ „

were excluded. This reduced the number of stars used to 845.

The moments of the first order are very small, since the system of coordinates was referred to the centre of velocities of all 910 stars. Introducing a new system of axes XYZ by the translation indicated by the values of these moments, the new moments M_{ijk} were calculated.

$$\begin{aligned}
M_{000} &= 1 & M_{300} &= -5108{,}01\\
M_{100} &= M_{010} = M_{001} = 0 & M_{030} &= -2595{,}44\\
M_{200} &= +516{,}79 & M_{003} &= -346{,}61\\
M_{020} &= +300{,}23 & M_{210} &= -1587{,}81\\
M_{002} &= +180{,}49 & M_{201} &= -76{,}09\\
M_{011} &= -3{,}99 & M_{120} &= -447{,}65\\
M_{101} &= +4{,}35 & M_{021} &= -387{,}22\\
M_{110} &= -56{,}95 & M_{102} &= -802{,}07\\
& & M_{012} &= -398{,}48\\
& & M_{111} &= +133{,}26
\end{aligned}$$

It may be concluded from this table that this material is too scanty for being resolved into two ellipsoidal functions, since the fourth order moments are not quite reliable. It is clear that any accidental irregularities in the distribution affect the moments the more (especially those of higher order) the poorer the material used. Therefore I tried to resolve a less general problem, viz., to resolve the frequency-function into two Maxwellian functions. For this aim the moments of the 1-st, 2-nd and 3-rd order are sufficient.

At the outset a single velocity-ellipsoid was determined by the method of Charlier[1]), using the moments given above up to the second order. Resolving the equation

$$G(t) = \begin{vmatrix} M_{200}-t & M_{110} & M_{101}\\ M_{110} & M_{020}-t & M_{011}\\ M_{101} & M_{011} & M_{002}-t \end{vmatrix} = O,$$

[1]) C. V. L. Charlier. The Motion and the Distribution of Stars. Berkeley, California, 1926.

I obtained for the semiaxes of the ellipsoid:

$$a = 23{,}04 \text{ km/sec}, \quad b = 16{,}92 \text{ km/sec}, \quad c = 13{,}43 \text{ km/sec}$$

and from the formulae

$$\frac{\lambda}{G_{11}} = \frac{\mu}{G_{12}} = \frac{\nu}{G_{13}} = \frac{1}{\sqrt{G^2_{11} + G^2_{12} + G^2_{13}}}$$

their direction-cosines. The galactic coordinates of the axes of the ellipsoid and their angles with the axes of coordinates are:

	ξ	η	ζ
a	13°,9	103°,9	89°,3
b	76,2	13,9	91,5
c	90,4	88,1	2,0

a-axis: $l = 344°6$, $b = +\ 3°{,}8$
b-axis: $l = 75{,}4$, $b = +11{,}8$
c-axis: $l = 235{,}9$, $b = +77{,}8$

This ellipsoid will be called „the second ellipsoid". A difference in the directions of the axes of both ellipsoids is the consequence of the two methods of calculation being based on different suppositions. The first ellipsoid is adapted to the numbers of vectors in equal spacial angles and the second — to the numbers of vectors in a rectangular net of points.

Afterwards I tried to find the two Maxwellian distributions. For this purpose the system of equations (14) was resolved by a method similar to that given by S c i g o l e v in his paper mentioned above, using all the equations of the third order and treating them by the least-squares method. The results (in the XYZ-system) are:

$$\begin{aligned} a_1 &= +7{,}1 \text{ km/sec} & a_2 &= -40{,}5 \text{ km/sec} \\ b_1 &= -2{,}1 \ \text{„} & b_2 &= +12{,}0 \ \text{„} \\ c_1 &= +0{,}1 \ \text{„} & c_2 &= -0{,}5 \ \text{„} \\ n_1 &= 0{,}85 & n_2 &= 0{,}15 \\ \sigma' &= 13{,}8 \text{ km/sec} & \sigma'' &= 21{,}1 \text{ km/sec} \end{aligned}$$

This solution is of a provisional character only, because some of the calculated quantities had the small moments M_{011}, M_{101}, M_{110} in the denominator. This is a consequence of the axes of coordinates being chosen so that the X-axis of the first ellipsoid is near to the line joining the centres of the component distributions. Therefore it seemed natural to resolve the problem by rotating axes of coordinates. The angles Θ_1, Θ_2 determined by a least-squares solution of the conditions $\overline{M}_{011} = \overline{M}_{101} = \overline{M}_{110} = 0$, $\overline{M}_{020} = \overline{M}_{002}$, are:

$$\Theta_1 = 13°{,}9, \quad \Theta_2 = 0°{,}8.$$

The rotation of the system by these angles gives the following values of the moments which, according to the theory, must be zero:

The moments of 2-nd order.	The moments of 3-rd order.
$\bar{M}_{011} = -2,81$	$\bar{M}_{030} = -3014,36$
$\bar{M}_{101} = -0,02$	$\bar{M}_{003} = -316,25$
$\bar{M}_{110} = +0,01$	$\bar{M}_{210} = -2373,11$
	$\bar{M}_{201} = -122,48$
	$\bar{M}_{021} = -296,55$
	$\bar{M}_{012} = -585,14$
	$\bar{M}_{111} = +217,20$

It can be seen that the moments of the second order satisfy the condition $\bar{M}_{ijk} = 0$ very well, for the angles Θ_1, Θ_2 were determined from the moments of the second order. Yet, the moments of the third order are rather large, but they have, as already mentioned, a small weight and are not convenient to be used for the determination of the angles Θ_1, Θ_2.

The final resolution in the XYZ-system (whose origine is situated in the centre of the whole distribution and the axes are parallel to those of the first ellipsoid) is:

$a_1 = +14,51$ km/sec $\qquad a_2 = -19,33$ km/sec
$b_1 = -3,59$ „ $\qquad b_2 = +4,78$ „
$c_1 = +0,22$ „ $\qquad c_2 = -0,29$ „
$n_1 = 0,57$ $\qquad n_2 = 0,43$
$\sigma' = 14,03$ km/sec $\qquad \sigma'' = 16,78$ km/sec.

Or, in the galactic rectangular system (x-axis in the ascending node of the galactic plane, y-axis at $l = 90^0$) the coordinates of the centres of the two distributions are:

$a'_1 = +14,37$ km/sec $\qquad a'_2 = -19,15$ km/sec
$b'_1 = -3,99$ „ $\qquad b'_2 = +5,32$ „
$c'_1 = +1,02$ „ $\qquad c'_2 = -1,36$ „

and the line joining the centres is directed toward the point whose galactic longitude and latitude are:

$$l = 344^{\circ}\!,5 \qquad b = +3^{\circ}\!,9$$

This line coincides very nearly with the great axis of the second ellipsoid.

Now, three frequency-functions are obtained:

1) the first ellipsoidal function:

$$F(\xi, \eta, \zeta) = 0{\cdot}00001213\ e^{-\frac{1}{2}\left[\frac{\xi^2}{(24{\cdot}23)^2} + \frac{\eta^2}{(16{\cdot}91)^2} + \frac{\zeta^2}{(12{\cdot}78)^2}\right]}$$

2) the second ellipsoidal function:

$$F(x, y, z) = 0{\cdot}00001213\ e^{-\frac{1}{2}\left[\frac{x^2}{(23{,}04)^2} + \frac{y^2}{(16{,}92)^2} + \frac{z^2}{(13{,}43)^2}\right]}$$

where
$$x = -0{,}78 + 0{,}971\,\xi + 0{,}239\,\eta$$
$$y = -2{,}06 - 0{,}239\,\xi + 0{,}971\,\eta$$
$$z = -0{,}49 + \zeta$$

3) two - Maxwellian function:

$$F(\xi, \eta, \zeta) = 0{,}57 \,.\, 0{,}00002299\ e^{-\frac{(\xi - 15{,}29)^2 + (\eta + 1{,}53)^2 + (\zeta - 0{,}71)^2}{2\,.\,(14{,}03)^2}} +$$
$$+ 0{,}43 \,.\, 0{,}00001344\ e^{-\frac{(\xi + 18{,}55)^2 + (\eta - 6{,}84)^2 + (\zeta - 0{,}20)^2}{2\,.\,(16{,}78)^2}}$$

In order to compare these functions with the observed distribution the numbers of vectors in the cubes were computed for every function and the differences „observed - calculated" were derived. Then the dispersions of O. - C. are:

1) 0,80
2) 0,79
3) 0,76

On the other hand a graphical representation of these functions for each coordinate was made. Integrating the frequency - function with respect to two coordinates we obtain respectively a Gaussian function or a sum of two such functions as the frequency - distribution of the remaining coordinate. These functions were represented graphically and compared with the observed distributions (Fig. 1, 2 and 3).

From the diagrams as well as from computed dispersions of O. - C. we can conclude that the double - Maxwellian - function is somewhat more convenient for the representing of actual distribution of the velocities of the considered stars.

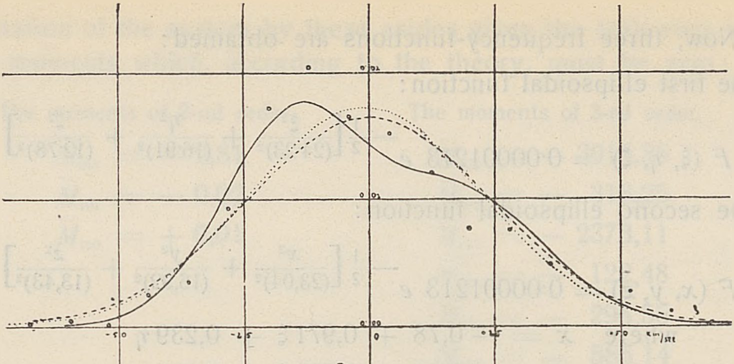

Fig. 1.

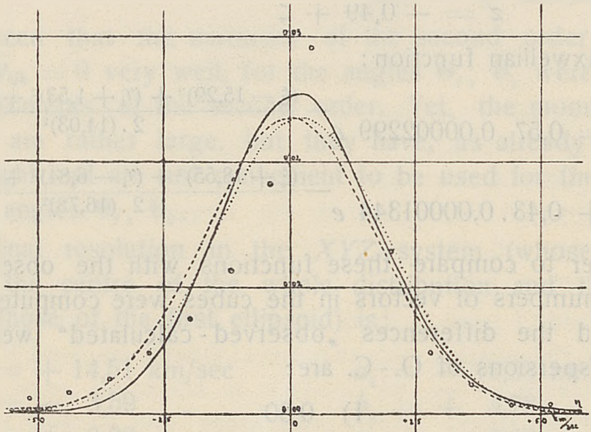

Fig. 2.

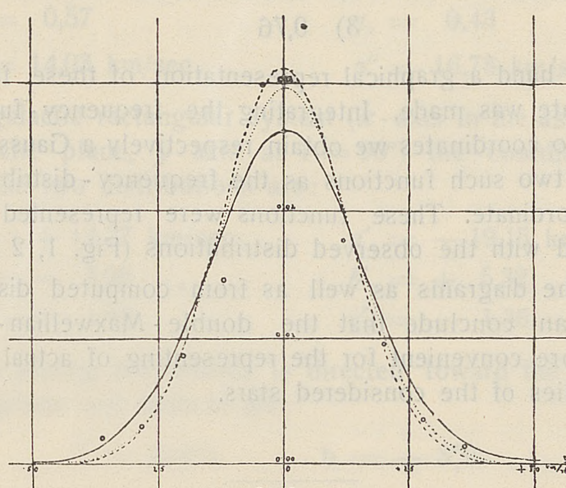

Fig. 3

The distribution of the ξ —, η —, ζ — velocity - components:
the circles represent the observed distribution,
the full line — the computed two-Maxwellian distribution,
the interrupted — the first ellipsoidal "
the dotted — the second ellipsoidal "

Streszczenie.

Zadaniem niniejszej pracy było zbadanie rozkładu prędkości swoistych gwiazd-olbrzymów o typach widmowych F, G, K, M. Gwiazd tych o znanych prędkościach radjalnych, ruchach własnych i paralaksach zebrano 910; po odrzuceniu najbardziej szybkich pozostało 845. Rozkłady rzutów prędkości tych gwiazd na 3 osie przedstawione są na rys. 1, 2, 3. Poza obserwowaną zwykle różnicą dyspersyj na 3-ch osiach, którąby odpowiadała funkcji elipsoidalnej, zauważono znaczną asymetrję krzywej częstości, zwłaszcza na pierwszej osi. Nasunęło to myśl zastosowania funkcji częstości, złożonej z sumy 2-ch funkcyj elipsoidalnych, mianowicie:

$$F\,(X, Y, Z) = n_1\, F_1\,(X, Y, Z) + n_2\, F_2\,(X, Y, Z),$$

przyczem $F_1(X, Y, Z) = C_1\, e^{-\frac{1}{2} f_1}$

gdzie $$f_1 = A_1'(X-a_1)^2 + A_2'(Y-b_1)^2 + A'_3(Z-c_1)^2 + 2B_1'(Y-b_1)(Z-c_1) + \\ + 2B_2'(X-a_1)(Z-c_1) + 2B_3'(X-a_1)(Y-b_1)],$$

$$F_2(X, Y, Z) = C_2\, e^{-\frac{1}{2} f_2}$$

gdzie $$f_2 = A_1''(X-a_2)^2 + A_2''(Y-b_2)^2 + A_3''(Z-c_2)^2 + 2B_1''(Y-b_2)(Z-c_2) + \\ + 2B_2''(X-a_2)(Z-c_2) + 2B_3''(X-a_2)(Y-b_2)].$$

W tym celu została opracowana metoda wyznaczenia parametrów tej funkcji dla uważanego rozkładu. Jest ona uogólnieniem prac Charlier'a i innych z tej dziedziny, polega na użyciu momentów rozkładu i zastosowaniu równania (1). Metoda ta, przedstawiona w części I tej pracy, jest wyprowadzona dla wypadku najbardziej ogólnego — 2-ch elipsoid dowolnie zorjentowanych. Zastosowanie zaś (część II) znalazł przypadek szczególny — 2-ch kul, t. zn. funkcjom składowym F_1, F_2 nadano następującą postać:

$$F_1\,(X, Y, Z) = \frac{1}{(2\pi)^{3/2}\,\sigma'^3}\, e^{-\frac{1}{2\sigma'^2}[(X-a_1)^2 + (Y-b_1)^2 + (Z-c_1)^2]},$$

$$F_2\,(X, Y, Z) = \frac{1}{(2\pi)^{3/2}\,\sigma''^3}\, e^{-\frac{1}{2\sigma''^2}[(X-a_2)^2 + (Y-b_2)^2 + (Z-c_2)^2]}.$$

Znalezione wartości parametrów a_1, b_1, c_1, σ'; a_2, b_2, c_2, σ'' podane są ostatecznie w układzie galaktycznym prostokątnym.

Wyznaczoną w ten sposób funkcję dwukulistą porównano (rys. 1, 2, 3) z rozkładem obserwowanym obok znalezionej jednocześnie funkcji elipsoidalnej, pojedyńczej. Ta ostatnia wyznaczona była 2-ma różnemi metodami. Jak widać z wykresów, funkcja dwukulista odpowiada nieco lepiej rozkładowi obserwowanemu.

WŁADYSŁAW DZIEWULSKI.

O ruchu gwiazd typu widmowego B.

On the motion of stars of the spectral type B.

(Komunikat zgłoszony na posiedzeniu w dniu 19.VI 1931 r.).

For the study of the distribution of star velocities, 406 stars of the spectral type B with known spectroscopic parallaxes, radial velocities nad proper motions have been collected by Miss W. Iwanowska and their peculiar velocities calculated on the assumption that the speed of the solar motion is 20 km per second and its direction: $\alpha = 270^{0}$, $\delta = +30^{0}$.

Considering exclusively this material, in fact rather scarce, the movement of the B-stars, as a whole, will be studied. It should be mentioned that the correction of radial velocities, resulting from the K—term, was left out of consideration.

Assuming the coordinates of the galactic pole: $\alpha = 191^{0}$, $\delta = +27^{0}$, the galactic rectangular coordinates of stars were reckoned and the plane of symmetry for the B-stars was determined, Building nine groups of stars the conditional equations were resolved and the inclination *(i)* of this plane to the galactic plane: $i = 12^{\circ}\!.2$ and the longitude of the descending node (☋) of this plane of symmetry: ☋ $= 37^{\circ}\!.5$ were calculated. The circle of the galactic latitude, passing through the point of the galactic longitude $l = 0^{0}$, intersects the plane of symetry in a point whose galactic latitude: $b = +7^{\circ}5$.

Les us now consider the distribution of the peculiar velocities of the investigated stars. As in our previous investigations the three axis ellipsoidal distribution is considered. The sky was divided into regions and the stars moving towards each region were counted. The following zones and regions were chosen as follows:

I	zone from	— 15°	to + 15°	in Decl. and every	30°	in R. A., on the whole	12 regions
II	„ „	+ 15	„ + 45	„ „ „ „	„	„ „	12 „
III	„ „	— 15	„ — 45	„ „ „ „	„	„ „	12 „
IV	„ „	+ 45	„ + 75	„ „ „ „	60	„ „	6 „
V	„ „	— 45	„ — 75	„ „ „ „	„	„ „	6 „
VI	„ „	+ 75	„ + 90	„ „ — —	—	— „ „	1 „
VII	„ „	— 75	„ — 90	„ „ — —	—	— „ „	1 „

together 50 regions. Taking the regions of the zone I as unity and allowing for the inequality of the areas of different regions, the correcting factors for the number of the vectors in other zones were introduced viz. 1.16 for the zone II and III, and 1.26 for the zone VI and VII. Accordingly the number of stars moving in the directions of these regions were multiplied by these factors.

Let: $Ax^2 + A_1y^2 + A_2z^2 + 2Byz + 2B_1zx + 2B_2xy + H = o$

where x, y, z are the rectangular aequatorial coordinates, be the equation of the velocity ellipsoid. For the 50 regions we get 50 equations, which we resolve by the method of least squares. When the constants are found, the axes (a, b, c) and their directions can be easily determined.

The following table contains the coordinates of each region and the observed number of stars therein. After determining the constants of the ellipsoid we calculate the number of stars in each region and build the differences: Observ. — Calcul. For the direction of the axes of the velocity ellipsoid we get in the galactic coordinates:

$$a \text{ — axis}: l = 359^\circ\!.6 \qquad b = +\ 7^\circ\!.4$$
$$b \text{ — axis}: l = 264.3 \qquad b = +\ 3.3$$
$$c \text{ — axis}: l = 105.1 \qquad b = +\ 77.6$$

and for the ratios $\frac{b}{a}, \frac{c}{a}$: $\frac{b}{a} = 0.81$, $\frac{c}{a} = 0.68$.

The direction of the greatest axis shows the favoured directions of the star movements. The calculated direction ($l = 0°$, $b = + 7°$) is situated nearly in the plane of symmetry.

Table.

Zone	Region	Coordinates		Number of stars		O.—C.
		α	δ	Observ.	Calc.	
I	1	9.0°	+ 5.1°	3	6	— 3
„	2	46.6	+ 1.5	4	8	— 4
„	3	73.9	+ 2.3	4	12	— 8
„	4	101.4	+ 3.6	13	16	— 3
„	5	136.3	+ 3.1	10	9	+ 1
„	6	168.0	— 3.8	8	6	+ 2
„	7	197.6	— 4.6	4	6	— 2
„	8	226.2	— 1.2	6	8	— 2
„	9	253.2	+ 1.1	19	12	+ 7
„	10	285.3	0.0	11	15	— 4
„	11	318.3	+ 1.0	6	9	— 3
„	12	341.0	— 5.0	3	6	— 3
II	13	17.5	+ 22.2	6	7	— 1
„	14	42.2	+ 28.1	12	9	+ 3
„	15	77.6	+ 34.0	17	13	+ 4
„	16	103.2	+ 32.7	10	12	— 2
„	17	132.6	+ 29.7	17	8	+ 9
„	18	168.3	+ 31.5	8	5	+ 3
„	19	196.5	+ 30.4	13	5	+ 8
„	20	224.0	+ 23.7	6	7	— 1
„	21	256.3	+ 25.6	22	11	+ 11
„	22	287.6	+ 29.7	20	13	+ 7
„	23	315.5	+ 30.5	14	11	+ 3
„	24	347.1	+ 32.2	10	8	+ 2
III	25	18.3	— 32.7	2	5	— 3
„	26	46.2	— 27.5	5	7	— 2
„	27	78.0	— 33.6	5	10	— 5
„	28	109.3	— 31.5	8	12	— 4
„	29	133.0	— 30.9	15	11	+ 4
„	30	166.5	— 29.1	14	8	+ 6
„	31	198.8	— 31.2	6	8	— 2
„	32	227.4	— 29.0	14	10	+ 4
„	33	259.9	— 30.0	19	13	+ 6
„	34	288.7	— 27.6	6	12	— 6
„	35	313.2	— 31.8	2	8	— 6
„	36	344.8	— 24.0	2	6	— 4
IV	37	33.6	+ 58.6	10	9	+ 1
„	38	88.5	+ 60.1	7	9	— 2
„	39	149.3	+ 59.8	7	6	+ 1
„	40	214.1	+ 58.2	6	6	0
„	41	274.0	+ 52.7	9	9	0
„	42	333.6	+ 55.8	7	9	— 2
V	43	29.0	— 60.5	1	6	— 5
„	44	91.5	— 68.3	1	8	— 7
„	45	148.8	— 56.5	9	9	0
„	46	210.6	— 53.6	11	9	+ 2
„	47	269.6	— 56.5	16	9	+ 7
„	48	333.4	— 63.5	1	9	— 8
VI	49	43.4	+ 85.9	8	8	0
VII	50	78.3	— 84.8	8	7	+ 1

Taking now this symmetrical plane as the $\xi\eta$ plane, let us direct the ξ-axis to the point with galactic coordinates: $l = 0^{\circ}$, $b = + 7^{\circ}$, the η-axis perpendicular to it, in the direction of the increasing galactic longitudes, and the ζ-axis perpendicular to $\xi\eta$ plane. The coordinates of all stars and the components of their velocities (u, v, w) were calculated with reference to this system. The coordinates are expressed in parsecs. Four stars, whose peculiar velocities exceed 80 km/sec, were excluded. We have thus on the whole 402 stars.

Let us investigate the distribution of the velocities in the $\xi\eta$ plane. Considering first the movement of the stars relatively to the ξ-axis, which corresponds, as we have seen, to the favoured direction of star movements, we take three groups of stars according to the values of their coordinates, namely:

$$+40 < \xi \qquad -60 < \xi \leq +40 \qquad \xi \leq -60$$

and consider for these groups the velocities of stars in two directions, where the u — component are positive and negative. We receive the following results:

	$+40 < \xi$		$-60 < \xi \leq +40$		$\xi \leq -60$	
	Number of stars	Veloc. km/sec	Number of stars	Veloc. km/sec	Number of stars	Veloc. km/sec
u — component positive	101	17.7	69	17.3	44	16.3
negative	39	14.8	54	16.1	95	17.6

We take now only the u — components of the velocities of stars and receive:

	$+40 < \xi$		$-60 < \xi \leq +40$		$\xi \leq -60$	
	Number of stars	Veloc. km/sec	Number of stars	Veloc. km/sec	Number of stars	Veloc. km/sec
u — component positive	101	+ 11.3	69	+ 10.8	44	+ 10.5
negative	39	— 6.8	54	— 6.8	95	— 10.7

The character of these velocities is analogous.

We divide now the stars into groups with reference to the η — axis. Including to the first group all stars, whose η — coordinates are less than + 40 parsecs and to the second the remaining ones, we receive the following results:

	$\eta < +40$		$\eta > +40$	
u — component of velocities	Number of stars	Veloc. km/sec	Number of stars	Veloc. km/sec
positive	84	15.9	130	18.1
negative	118	17.0	70	16.0

Considering the distribution of the velocity-vectors along the ξ-axis, we see that the average velocities increase or decrease, the u — component being positive or negative.

The collected material of the peculiar velocities of the stars of B-type is unfortunately too scarce to permit of any reliable conclusions.

Streszczenie.

Materjał obserwacyjny obejmuje 406 gwiazd typu widmowego B, dla których P. Wilhelmina Iwanowska wyliczyła ruchy swoiste, przytem w prędkościach radjalnych nie uwzględniono poprawki, związanej z wyrazem K. Mając spółrzędne gwiazd w układzie drogi mlecznej, szukano płaszczyzny symetrji układu gwiazd typu B. Płaszczyzna ta jest nachyloną do płaszczyzny drogi mlecznej pod kątem $12^0.2$, a długość węzła zstępującego wynosi $37^0.5$. Na powierzchni sklepienia niebios uwzględniono 50 obszarów i zbadano rozkład wektorów prędkościowych, skierowanych do tych obszarów. Zastosowano rozkład elipsoidalny, wyznaczono kierunki osi elipsoidy oraz stosunki długości tych osi, wreszcie obliczono rozkład teoretyczny. Wyniki zawiera podana tablica.

Kierunek wielkiej osi elipsoidy leży praktycznie w płaszczyźnie symetrji.

W płaszczyźnie symetrji skierowano oś ξ do punktu, którego spółrzędne galaktyczne wynoszą: $l = 0^0$, $b = +7^0$, oś η w tejże płaszczyźnie o 90^0 w kierunku wzrastających długości, oś ζ prostopadle do płaszczyzny symetrji. Przeliczono spółrzędne gwiazd i ich składowe prędkości w nowym układzie osi i badano rozkład prędkości gwiazd z jednej strony w różnych grupach, utworzonych wzdłuż osi ξ, z drugiej strony w odniesieniu do osi η. W rozkładzie wzdłuż osi ξ daje się zauważyć pewien bieg. Niestety, materjał obserwacyjny jest zbyt szczupły.

WŁODZIMIERZ ZONN.

Obserwacje fotograficzne gwiazdy zmiennej RZ Cassiopeiae.

Photographic observations of the variable RZ Cassiopeiae.

(Komunikat zgłoszony przez czł. Wł. Dziewulskiego na posiedzeniu w dniu 19.VI 1931 r.).

This star was announced by Müller[1]) in 1906 as a variable of the Algol type. Its elements[2]) are following:

$$\text{Min.} = 2417355.4200 + 1.1952525\ E$$

The observations were made in 1930: on February 27th and on August 30th. They consist of extrafocal observations, made with a 150 mm Zeiss-triplet with a wire screen; the grating was made of parallel wires: the diameter of the wire and the spacing between the wires were nearly 0.8 mm. The time of exposure was 10—15 minutes; the plates Lumière Opta were developed with Rodinal (1 : 20) during 10 minutes.

During these two nights 40 exposures were made on 10 plates.

The blackness of the images was measured with a Hartmann microphotometer. Each plate was measured twice. The blackness of the area adjacent to the image of each star was also determined in order to account for the lack of homogeneity of the plate. The results of measurements were reduced by the well known method of Schwarzschild, modified by Hertzsprung[3]). The value of the constant of the grating was determined empirically to $0^m.90$, the theoretical value being $0^m.98$.

Table I gives the comparison stars. The photographic magnitudes were taken from the Henry Draper Catalogue. For each photograph a brightness curve was drawn, the magnitudes of the compari-

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[1]) Astronomische Nachrichten. Bd 171, p. 357. 1906.

[2]) Kat. und Eph. der veränd. Sterne. 1930.

[3]) Astronomische Nachrichten. Bd 186, p. 177. 1911.

son stars and of RZ Cassiopeiae were derived. Then from all the plates the magnitudes of the comparison stars, received with the Zeiss-triplet, were calculated. These values are given in table I.

Table I.

Comparison stars.

B. D.	H. D.	m H. D.	m Wilno
+ 70° 182	15472	7.8	7.93
66 219	15648	8.1	8.26
67 215	15784	7.11	7.10
66 223	15849	7.71	7.76
67 217	16066	8.0	8.12
69 171	16393	8.0	7.51
67 224	16769	5.90	6.19
68 208	17929	8.7	8.06
68 209	18056	8.1	8.19
68 212	18267	8.0	8.24
69 203	19906	7.7	7.73
69 205	20273	6.68	6.72

It is seen that the magnitudes of the stars: B. D. + 69°171, 68°208, 67°224 greatly differ from those of H. D. Catalogue. The magnitudes of two first stars probably are determined inaccurately in H. D. Catalogue, the last one (B. D. + 67°224) was too bright on our plates and therefore its measurement was rather uncertain.

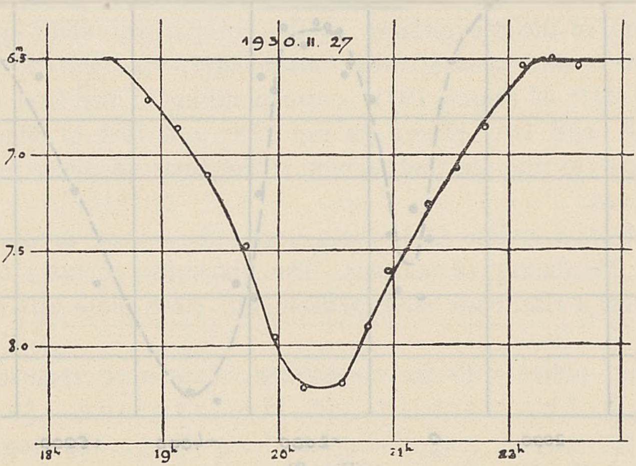
The magnitudes of RZ Cassiopeiae were derived from the system of magnitudes, determined at Wilno. Table II gives the mean moments of each exposure, calculated from the minimum, and the calculated magnitudes of RZ Cassiopeiae.

Table II.

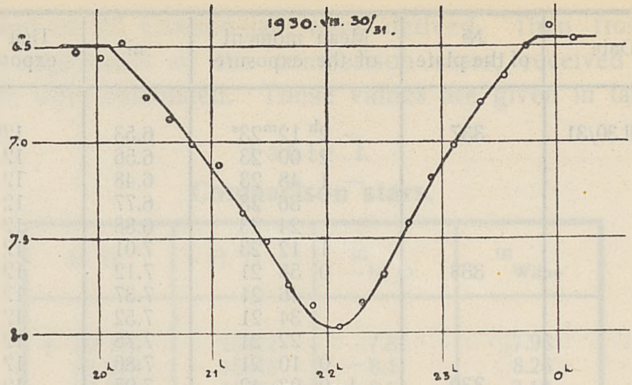
Date	№ of the plate	Mean moment of the exposure	m	Time of exposure
1930 II.27	241	— $1^h\,30^m00^s$	6.71	15^m
		15 00	6.86	15
		— 1 00 00	7.11	15
	242	— 0 40 00	7.48	15
		25 00	7.96	15
		— 0 10 00	8.23	15
	243	+ 0 10 00	8.20	15
		22 30	7.90	10
		35 00	7.60	15
	244	+ 0 55 00	7.26	15
		+ 1 10 00	7.07	15
		25 00	6.86	15
	245	+ 1 45 00	6.53	15
		+ 2 00 00	6.50	15
		15 00	6.53	15

Date	№ of the plate	Mean moment of the exposure	m	Time of exposure
VIII.30/31	337	$-2^{h}\,12^{m}23^{s}$	6.53	12^{m}
		$-2\;00\;23$	6.56	12
		$-1\;48\;23$	6.48	12
		$36\;23$	6.77	12
		$24\;23$	6.88	12
		$-1\;12\;23$	7.01	12
	338	$-0\;58\;21$	7.12	12
		$46\;21$	7.37	12
		$34\;21$	7.52	12
		$22\;21$	7.75	12
		$-0\;10\;21$	7.86	12
	339	$+0\;03\;42$	7.97	12
		$15\;42$	7.84	12
		$27\;42$	7.69	12
		$39\;42$	7.43	12
		$+0\;51\;42$	7.20	12
		$+1\;03\;42$	7.02	12
	340	$17\;45$	6.80	12
		$29\;45$	6.66	12
		$41\;45$	6.50	12
		$+1\;53\;45$	6.40	12
		$+2\;05\;45$	6.45	12

Fig. 1 gives the light curve for two days of observations (Civil M. T. Greenwich). The light varies during $3^{\mathrm{h}}\,30^{\mathrm{m}}$. The curve is symmetrical and has a sharp minimum. The highest magnitude of RZ Cassiopeiae is $6^{\mathrm{m}}.5$, whereas the minimum is $7^{\mathrm{m}}98$ and $8^{\mathrm{m}}.22$ according to my observations. The mean error of each observation amounts from $0^{\mathrm{m}}.04$ to $0^{\mathrm{m}}.10$. In some cases a large mean error was probably due to the fact that the comparison stars were very faint on the plates and their measurement rather difficult.



(Fig. 1).

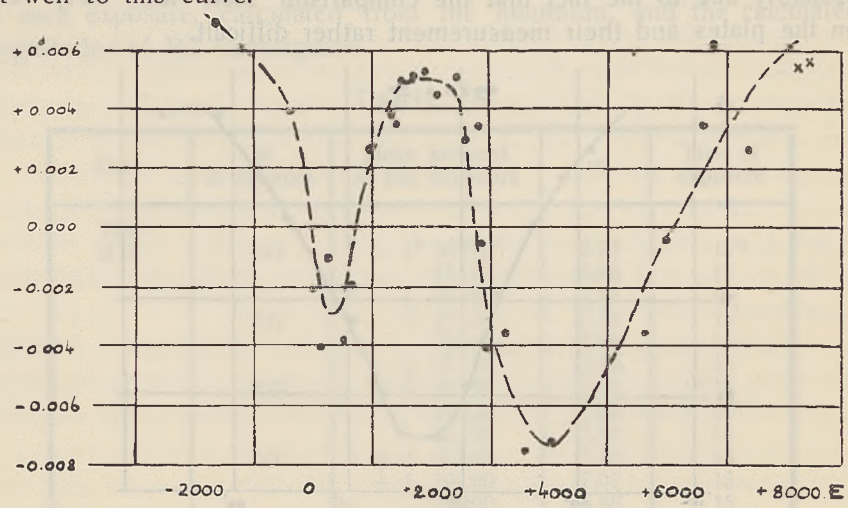


(Fig. 1).

The moments of the two heliocentric minima (Astr. M. T. Greenwich) are given in the table below: observed and calculated with the elements given above:

Epoch	J. D. Obs.	J. D. Calc.	Obs. — Calc.
7262	2426035.349	2426035.3437	+ 0.0053
7416	2426219.418	2426219.4125	+ 0.0055

The observations give a correction to the calculated minima. On the other hand it follows from the investigation of Hellerich[1]), that the period of variability of RZ Cassiopeiae is not constant but subjected to oscillations. Fig 2 shows the corrections to be applied to the observations of different authors. All the corrections were calculated with the elements given above. It is obvious that the corrections, following from the Wilno observations (indicated by crosses), fit well to this curve.



(Fig. 2).

[1]) Astronomische Nachrichten. Bd 233, p. 134. 1928.

Streszczenie.

Zapomocą astrokamery (o średnicy objektywu 150 mm.) dokonano dwóch seryj zdjęć gwiazdy zmiennej RZ Cassiopeiae w tym czasie, gdy gwiazda osiągała minimum swego blasku, a mianowicie 27 II 1930 i 30 VIII 1930. W czasie zdjęć przed objektywem umieszczano siatkę z drutów równoległych (grubość drutów i odstępy między niemi wynosiły około 0.8 mm.). Stała siatki wynosiła $0^m.90$. Zdjęcia trwały od 10 do 15 minut. Uzyskano w sumie 40 zdjęć na 10 kliszach.

Zaczernienia obrazów na kliszach mierzono zapomocą mikrofotometru Hartmann'a; materjał obserwacyjny opracowano zapomocą metody Schwarzschild'a i Hertzsprung'a.

Tablica I daje listę gwiazd, użytych do porównania, wraz z wielkościami gwiazd, wyprowadzonemi w Wilnie, a opartemi na wielkościach katalogu harvardzkiego. Tablica II podaje materjał obserwacyjny i wyprowadzone wielkości gwiazdy zmiennej z każdego zdjęcia.

Rysunek 1 daje krzywe zmian jasności w ciągu 2 wspomnianych nocy. Jasność RZ Cassiopeiae wynosi w maximum $6^m.5$, a w minimum otrzymano raz $7^m.98$, za drugim razem — $8^m.22$. Na podstawie krzywych zmian jasności wyprowadzono momenty, kiedy nastąpiło minimum.

Z badań Hellerich'a wynika, że okres zmienności ulega wahaniom. Rys. 2 daje przebieg obserwowanych zmian długości okresu. Obserwacje wileńskie, oznaczone krzyżykami, odpowiadają dobrze zaobserwowanemu zjawisku.

WŁODZIMIERZ ZONN.

Obserwacje fotograficzne zmian jasności planetoidy Eros.

Photographic observations of the variability of light of the minor planet Eros.

(Komunikat zgłoszony przez czł. Wł. Dziewulskiego na posiedzeniu w dniu 19.VI.1931 r.)

During the last opposition of Eros the weather at Wilno was very bad and only two somewhat longer sets of observations succeeded: viz. on 1931 January 23^{d} and February 10th. At that time the altitude of Eros was unfortunately not very great, the planet going then from the northern to the southern hemisphere; therefore the observations could not persist for a longer time.

The observations were made with a 150 mm Zeiss - triplet. They consisted in extra - focal photographs of Eros, the exposures being 10 minutes. As Eros has a rapid motion, the images of stars were not sharp, therefore from time to time the photographs of stars were made, and one of the stars lying near to Eros was chosen as a guiding star. The whole material of observations consists of 29 exposures of Eros and 18 exposures of stars on 9 plates. For the determination of the magnitudes of the comparison stars special exposures with a wire grating were made later, viz. two exposures on the 3^{d} and 20th of March 1931, corresponding to the first series of observations, and two others on the 17th and 19th of March 1931, corresponding to the second series. The time of these exposures was two hours.

The plates Lumière Opta were used; they were developed with Rodinal (1 : 20) during 10 minutes. The blackness of the images was measured with a Hartmann microphotometer. Each plate was measured twice.

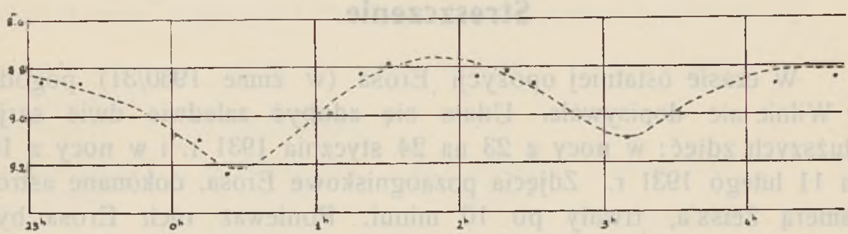
The four plates with the comparison stars were reduced by the method of Schwarzschild and Hertzsprung. As starting point the photographic magnitudes of Henry Draper Catalogue were taken. The following magnitudes were received.

B. D.	H. D.	$m_{H.D.}$	m_{Wilno}
$+5^0 2333$	90293	8.9	8.37
4 2333	90572	8.22	8.19
4 2337	90651	7.7	7.62
3 2372	90825	8.6	8.92
4 2343	90863	9.7	9.81
3 2373	90933	8.8	8.98
4 2344	90983	9.4	9.29
5 2344	91193	8.7	8.44
3 2387	91482	9.0	9.44
3 2388	91500	9.3	9.42
4 2351	91535	10.02	9.98
4 2353	91547	10.1	9.98

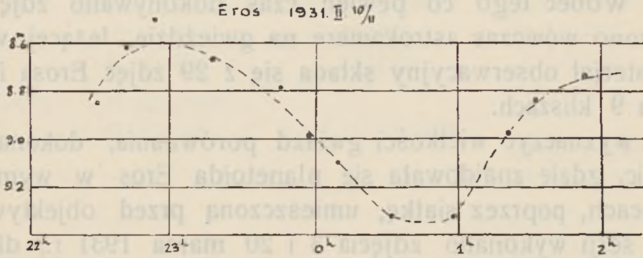
B. D.	H. D.	$m_{H.D.}$	m_{Wilno}
$-14^0 3002$	86913	7.61	7.82
14 3005	87036	7.40	7.60
14 3010	87097	9.4	8.93
15 2966	87098	8.4	8.10
14 3029	87827	8.14	8.20
14 3032	87890	9.2	9.17
14 3039	88084	8.24	7.93
12 3098	88108	8.69	9.07
14 3044	88285	8.97	9.18
14 3046	88346	8.18	8.56
12 3108	88479	9.2	9.20
14 3056	88613	9.4	9.22
14 3059	88683	8.01	8.06
14 3062	88766	8.1	7.92

The photographic magnitudes of the comparison stars being determined, the relation between the blackness of the images and the magnitudes of stars was represented graphically. As the exposure of Eros took place between those of comparison stars, its magnitude was determined from the two successive curves expressing this relation, an interpolation being used, if necessary, for the moment of the exposition of Eros. The results are given in the following figures (in mean civil Greenwich time):

Eros 1931. I 23/24



Eros 1931. II 10/11



From the curves the moments (Astr. M. Greenwich time) of minima were found:

J. D. 2426365.519
365.631
J. D. 2426383.531

The elements of Jacchia and of Müller being:

Müller: $m_1 = 2426266.4235 + 0.219711\ E$
$m_2 = 2426266.6108 + 0.219555\ E$
Jacchia: $m_1 = 2426266.488 + 0.21959\ E$
$m_2 = 2426266.607 + 0.21959\ E$

the following table gives the comparison with the elements of Müller and Jacchia.

Epoch	Observ.	Calc.	O. — C.	
451	2426365.519	2426365.513	+ 0.006	m_1 Müller
		.523	— 0.004	m_1 Jacchia
451	365.631	365.630	+ 0.001	m_2 Müller
		.642	— 0.011	m_2 Jacchia
533	383.531	383.529	+ 0.002	m_1 Müller
		.529	+ 0.002	m_1 Jacchia

Streszczenie.

W czasie ostatniej opozycji Erosa (w zimie 1930/31) pogoda w Wilnie nie dopisywała. Udało się zdobyć zaledwie dwie serje dłuższych zdjęć: w nocy z 23 na 24 stycznia 1931 r. i w nocy z 10 na 11 lutego 1931 r. Zdjęcia pozaogniskowe Erosa, dokonane astrokamerą Zeiss'a, trwały po 10 minut. Ponieważ ruch Erosa był znaczny, przeto w tym czasie obrazy gwiazd już cokolwiek przesunęły się. Wobec tego co pewien czas dokonywano zdjęć gwiazd i prowadzono wówczas astrokamerę na gwieździe, leżącej w pobliżu Erosa. Materjał obserwacyjny składa się z 29 zdjęć Erosa i 18 zdjęć gwiazd na 9 kliszach.

Aby wyznaczyć wielkości gwiazd porównania, dokonano zdjęć tych okolic, gdzie znajdowała się planetoida Eros w wymienionych dwóch nocach, poprzez siatkę, umieszczoną przed objektywem; dla pierwszej serji wykonano zdjęcia 3 i 20 marca 1931 r., dla drugiej serji — 17 i 19 marca 1931 r. Zdjęcia te trwały po dwie godziny. Po opracowaniu tych zdjęć, za skalę odniesienia przyjęto wielkości z katalogu harvardzkiego i wyznaczono wielkości, otrzymane astrokamerą wileńską. Dwie tabliczki zawierają wielkości gwiazd porównania.

Na podstawie wyznaczonych wielkości gwiazd porównania szukano graficznie związku pomiędzy zaczernieniem gwiazd na kliszach, na których były zdjęcia Erosa, i wielkościami gwiazd. Ponieważ były zawsze zdjęcia gwiazd zarówno poprzedzające zdjęcia Erosa, jak i następujące po nich, stosowano interpolację, by znaleźć wielkości Erosa. Na rysunkach punkty przedstawiają otrzymane wielkości fotograficzne Erosa. Poprzez te punkty przeprowadzono krzywe, które pozwalają odczytać momenty, gdy następowało minimum jasności Erosa.

WŁADYSŁAW DZIEWULSKI, WILHELMINA IWANOWSKA,
WŁODZIMIERZ ZONN.

Obserwacje zmian jasności planetoidy Eros.

Observations of the variability of light of the minor planet Eros.

(Komunikat zgłoszony na posiedzeniu w dniu 19.VI 1931 r.).

During the winter 1930/31 the regular observations at Wilno were hardly possible as the weather was very bad. Eros could be observed for the first time on January 22^{d} 1931, then on January 23^{d}, 29^{th}, February 3^{d} and 9^{th}, later the observations had to be interrupted, as the minor planet was passing rapidly to the southern hemisphere. The observations were made with the 150 mm short focus refractor (the magnifying power: 20), the brightness of Eros being compared with that of comparison stars. The set of observations of every observer was reduced separately and represented graphically. For each light curve the moments of maxima and minima were determined.

The following moments in J. D. (M. T. Greenwich) were found:

1931	Maximum	Minimum	Maximum	Minimum
22 I				2426364.542 Iw
			2426364.479 Dz	364.538 Dz
23 I	2426365.469 Iw	2426365.514 Iw	2426365.575 Iw	2426365.642 Iw
		365.507 Z	365.574 Z	365.637 Z
23 I	2426365.698 Iw			
29 I			2426371.492 Dz	2426371.564 Dz
3 II	2426376.461 Iw	2426376.497 Iw	2426376.550 Iw	2426376.607 Iw
	376.447 Dz	376.510 Dz	376.574 Dz	376.616 Dz
	376.447 Z	376.503 Z	376.554 Z	376.611 Z
9 II			2426382.483 Iw	2426382.556 Iw
		2426382.422 Dz	382.475 Dz	382.550 Dz
9 II	2426382.597 Iw			
	382.608 Dz			

Iw = W. Iwanowska
Dz = W. Dziewulski
Z = W. Zonn.

Streszczenie.

W czasie zimy 1930/31 pogoda w Wilnie była bardzo niepomyślną, dlatego też dokonano stosunkowo niewiele obserwacyj, dotyczących zmian jasności planetoidy Eros. Dłuższą serję obserwacyj uzyskaliśmy w czasie nocy 22, 23, 29 stycznia, 3 i 9 lutego 1931 r. Obserwowaliśmy lunetą krótkoogniskową o średnicy objektywu 150mm. Obserwacje, polegające na porównaniu jasności Erosa i wybranych gwiazd porównania, opracowaliśmy dla każdego obserwatora niezależnie. Otrzymane krzywe przebiegu zmian jasności pozwoliły ustalić momenty maximum i minimum, które zestawiono w załączonej powyżej tablicy.

WŁADYSŁAW DZIEWULSKI.

Obserwacje gwiazdy zmiennej ε Aurigae w czasie minimum w okresie 1928 — 1930.

Observations of the variable star ε Aurigae during the minimum 1928 — 1930.

(Komunikat zgłoszony na posiedzeniu w dniu 19.VI. 1931 r.)

55 observations were made by means of a Zeiss binocular with 6-fold magnification since April 26th 1928 until Mai 3d 1930. As some observations were made in small altitudes, a correction for the extinction was applied by means of the Potsdam tables.

Table I gives the comparison stars used in the observations; it should be noted that the star γ Persei served only once as comparison star, the brightness of the stars were taken from the Draper Catalogue:

Table I.

Desing.	Star	Magn.	Steps
		m	
a	γ Persei	3.08	(23.1)
b	η Aurigae	3.28	19.1
c	ζ Aurigae	3.94	10.7
d	ν Aurigae	4.18	2.7
e	μ Aurigae	4.28	0.0

Table II gives the observations, viz. the moments of observations, expressed in J. D. (M. T. Greenwich), the comparison stars and the brightness, calculated in steps and then converted into magnitudes.

Table II.

J. D. (M. T. Greenwich)	Comparison stars	Magn.
		m
2425363.31	b, c	3.45
368.39	a, b	3.36
561.27	c, d	3.84
562.25	c, d	3.84
568.22	c, d	3.89
589.30	c, d	3.94
599.18	c, d	3.92
618.24	c, d	3.94
619.20	c, d, e	3.94
620.17	c, d, e	3.94
624.17	c, d, e	3.94
635.19	c, d	3.94
642.19	c, d, e	3.99
643.19	c, d	3.99
644.20	c, d	3.99
648.21	c, d	3.99
654.21	c, d	4.04
682.27	c, d	4.04
714.29	c, d, e	4.05
715.30	c, d, e	4.05
734.31	c, d, e	4.09
744.37	c, d	4.06
870 37	c, d	3.92
871.27	c, d	3.92
872.35	c, d	3.92
874.29	c, d	3.90
892.28	c, d	3.84
897.24	b, c	3.73
903.22	b, c	3.68
909.26	b, c	3.68
969.27	b, c	3.52
982.15	b, c	3.50
985.25	b, c	3.50
996.20	b, c	3.55
999.21	b, c	3.52
2426000.26	b, c	3.50
015.25	b, c	3.50
030.28	b, c	3.50
035.36	b, c	3.50
052.28	b, c	3.45
057.31	b, c	3.45
058.38	b, c	3.45
059.29	b, c	3.50
066.29	b, c	3.45
067.27	b, c	3.50
074.30	b, c	3.47
080.34	b, c	3.45
082.36	b, c	3.46
085.35	b, c	3.46
088.32	b, c	3.51
092.36	b, c	3.47
094.32	b, c	3.48
097.35	b, c	3.48
100.35	b, c	3.48

As the observations in 1929 were interrupted on Mai 12$^{\underline{th}}$ (near the minimum), it is difficult to calculate exactly the moment of minimum. It seems to be near to the J. D. 2425734.

Streszczenie.

Obserwacje gwiazdy zmiennej ε Aurigae, wykonane lornetką Zeiss'a o sześciokrotnem powiększeniu, obejmują okres czasu od 28 IV 1928 do 3 V 1930. Tablica I zawiera gwiazdy porównawcze, tablica II — materjał obserwacyjny.